Layoffs, Lemons and Temps*

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Abstract

We develop a dynamic equilibrium model of labor demand with adverse selection. Firms learn the quality of newly hired workers after a period of employment. Adverse selection makes it costly to hire new workers and to release productive workers. As a result, firms hoard labor and under-react to labor demand shocks. The adverse selection problem also creates a market for temporary workers. In equilibrium, firms hire a buffer stock of permanent workers and respond to changing business conditions by varying their temp workers. A hiring subsidy or tax can improve welfare by discouraging firms from hoarding too many productive workers. We extend the model to analyze dynamic variation in adverse selection in response to aggregate shocks. In the model with aggregate dynamics, the adverse selection problem causes labor demand to under-react to shocks relative to a frictionless benchmark. While the aggregate shocks in the model are temporary, the equilibrium change in temp employment is permanent.

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1 Introduction

Labor is often modeled as a variable factor of production which firms are free to vary as they see fit. In reality, firms do not continually adjust the size of their workforce suggesting that there are frictions that impede and discourage firms from adjusting too often. One potential friction which can give rise to inertia in employment is adverse selection. Adverse selection makes it costly to hire productive new workers and discourages firms from releasing productive workers already on the payroll. Unlike other costs of adjustment, adverse selection costs are endogenous. This endogeneity is important for two reasons. First, the degree of the adverse selection, and therefore, the degree of the adjustment cost, may respond to economic policy. Second, the adverse election cost may vary systematically over the business cycle and thus may dynamically influence aggregate employment over time.

We develop a model for analyzing labor demand in a market subject to adverse selection. In the model, there are both good and bad workers in the unemployment pool. Firms cannot *ex ante* identify which workers are good and which are bad. The lack of information regarding the worker's productivity is the source of the adverse selection problem. In this environment, hiring a worker is an investment decision of sorts. Because they must commit to an initial evaluation period, firms do not hire unless their current employment level is sufficiently lower than their desired employment level. Similarly, because replacing productive workers is costly, firms do not fire workers unless their current employment level is sufficiently higher than their desired level. Indeed, in the model, many firms make no adjustment to their workforce at all.

Changes in the degree of adverse selection have two separate effects on labor demand. First, increasing the average quality of available workers reduces the cost of hiring and gives firms an incentive to expand employment. We refer to this as the *hiring effect*. Second, an increase in the average quality of available workers reduces the incentive to hoard labor and thus gives firms an incentive to lay off workers when demand is low. We refer to this as the *hoarding effect*. Under certain conditions, the hoarding effect can be strong enough to generate multiple equilibria. If firms perceive adverse selection to be mild then few firms hoard labor. In this case, there are many good workers in the unemployment pool, rationalizing the firms' perception. If adverse selection is severe, firms hoard labor causing the pool to have few good workers, rationalizing the high level of adverse selection.

While the adverse selection problem causes firms to under-react to labor demand shocks, it creates a market for temporary workers. In our model, temporary workers are supplied by temp agencies that identify productive workers and then lease them out to firms for a fee. The temporary workers are more costly to the firms on a flow basis but they do not require a costly screening process. By providing a form of insurance for firms against temporary changes in labor demand, temp agencies reduce the severity of the adverse selection problem. If the agency fee for the temps reflects only the cost of screening productive workers, then the market for temp workers is so attractive that firms hire only temps and never hire permanent workers. If the temp fee exceeds the screening costs (say because temp workers sometime remain idle or because their skills are not a perfect match for the client firms or because they need to adjust to a new work environment, etc.), then firms hire a buffer stock of permanent workers and use temp workers to accommodate swings in demand.

We then use the model to analyze the effects of hiring-taxes or hiring-subsidies. The policy intervention we consider is a subsidy or tax for hiring new workers and thus is similar in spirit to the Small Business Jobs and Wages Tax Cut proposed by the Obama administration in early 2010 and to the New Jobs Tax Credit passed in 1977. The optimal hiring subsidy is determined by whether firms hire more workers or hoard fewer workers in response to an improvement in the quality of the labor pool – i.e., the optimal policy depends on whether the hiring effect or the hoarding effect dominates. In the model, the optimal policy calls for reducing overall employment to improve the quality of the pool and encourage firms to adjust their labor force.

Finally, we use a quantitative version of the model to analyze the out-of-steady-state dynamics of adverse selection and employment in response to aggregate shocks. Adverse selection always causes the labor market to under-react to labor demand shocks. This under-reaction (or excess stability) arises because the quality in the pool of available workers is counter-cyclical. After a positive innovation to labor demand, firms have relatively many productive workers on staff and thus, firms that want to expand further face a pool of available workers that is less favorable. There are fewer productive workers available in an expansion and this discourages firms from hiring more workers. In contrast, the "silver lining" of a recession is that it is relatively easy to hire productive workers for firms that want to do so. The model with aggregate dynamics also shows that there are long-lived effects on the employment of temps. In response to a positive labor demand shock, temp agencies recruit more workers to meet a rising demand for temps. Because recruiting temp workers is effectively a sunk cost to temp agencies, temps are in permanently greater supply even though the shock itself is transitory.

The remainder of the paper is set out as follows: Section II analyzes the adverse selection model when temp workers are not available. In Section III we analyze the model with temp workers. Section IV considers the welfare effects of hiring taxes and hiring subsidies. Section V analyzes aggregate shocks. Section VI considers extensions and the related literature. Section VII concludes.

2 The Model without Temps

In this section we present the basic adverse selection model without temporary workers. There is a continuum of firms of measure one. Each firm's production function is $zF(n)$ where $F(\cdot)$ is a concave production function with the usual properties.[1] The firm discounts future payoffs according to the constant discount factor $\beta = (1+i)^{-1}$ where i is a net interest rate. Firms differ in terms of their idiosyncratic cost or demand shocks z, which are i.i.d. over time and distributed according to the distribution function $G(z)$ with density $g(z)$. The shock z could reflect changes in the firm's productivity or changes in the price of the good the firm sells. These shocks are bounded on the interval $[0, Z]$ where $Z < \infty$. We assume the probability of drawing $z = 0$ is zero. Labor is the only input into production.

There are two types of workers: productive workers and unproductive workers. Unproductive workers have zero productivity so no firm would knowingly employ them. A worker's type is private information. Firms do not know whether a given applicant is productive or unproductive. Instead, a worker's type is revealed only after working at the firm for one period. At the end of each period, the firms release the unproductive workers and begin the next period with only good workers on their payroll.

Our assumptions on the nature of interactions in the labor market deserve some additional discussion. First, we assume that firms have no memory of workers they release – that is, we rule out the possibility of recalling workers when demand picks up. While somewhat stylized, this assumption qualitatively captures the problem confronting firms in the real world. In reality, firms may try to recall laid-off workers at a later date. However, these firms inevitably have to find new productive workers since many of the laid-off workers will have found new jobs in the meantime. Second, we rule out contracts which could mitigate the adverse selection problem. For instance, a firm could offer payment only in the event that the worker is productive. Such contracts could in principle be used to reduce the level of

[1] I.e., $F(0) = 0$, $F' > 0$, $F'' < 0$, $\lim_{n\to 0} F'(n) = \infty$, $\lim_{n\to\infty} F'(n) = 0$.

adverse selection though a firm would always have the incentive to declare that the workers were unproductive to avoid paying them. This would create a moral hazard problem on the part of the firm.[2]

To keep the analysis simple, aggregate labor is supplied perfectly elastically at the exogenous wage w up to a full-employment level L (the labor supply curve looks like a backwards "L"). The wage can be interpreted as a constant disutility of working for each worker or as the workers' outside opportunity cost. The overall fraction of productive workers in the labor force is $\phi \in (0, 1)$. Let μ be the fraction of good workers available for hire in the unemployment pool. Because firms keep only good workers on their payrolls, μ must be less than ϕ. Importantly, μ is an endogenous variable. In the model, firms take μ as given when they make their employment decisions. After discussing firm behavior, we go on to analyze the endogenous determination of μ itself.

There is no uncertainty for the firm as to how many productive workers it obtains for a given number of hires. If a firm wishes to raise the number of productive workers on its staff from n to $n' > n$, it must hire $\frac{1}{\mu}(n' - n)$ workers.[3] On the other hand, since the firm has only good workers employed at the beginning of a period, if it wishes to reduce the number of productive workers from n to $n' < n$, it simply dismisses $n - n'$ workers. Let $e(n' - n)$ be the additional number of workers a firm must hire to change its employment of good workers from n to n'. Given our assumptions above,

$$e(n' - n) = \begin{cases} \frac{1-\mu}{\mu}(n' - n) & \text{for } n' > n, \\ 0 & \text{for } n' \leq n. \end{cases} \quad (1)$$

That is, $e(n' - n)$ is the "bycatch" of unproductive workers hired to get $n' - n$ new productive workers.

2.1 Optimal Decisions

We analyze the firm's optimization problem recursively. At the beginning of a period, the state of a firm is summarized by its idiosyncratic shock z and the number of productive workers

[2]Fairburn and Malcomson (2001) analyze a model in which firms use promotions to provide incentives for workers to exert effort. In their model, the worker's type is unknown to both the firm and the worker but performance provides the firm with a noisy signal of the worker's type.

[3]This 'Law of Large Numbers' assumption facilitates the analysis greatly but is clearly unrealistic for smaller firms.

n on its payroll. The firm takes the wage w and the average quality in the unemployment pool μ as given. Let $v(n, z)$ be the value of a firm with n productive workers and current idiosyncratic shock z.[4] The firm's value function $v(n, z)$ satisfies the Bellman equation

$$v(n, z) = \max_{n' \geq 0} \{zF(n') - wn' - w \cdot e(n' - n) + \beta V(n')\}, \tag{2}$$

where $V(n')$ is the expected continuation value for a firm that enters next period with n' productive workers.

Since we are analyzing a stationary equilibrium with constant w and μ, the only uncertainty the firm faces arises from the idiosyncratic shocks. Thus, $V(n') = \int_{z'} v(n', z') dG(z')$. Using standard dynamic programming arguments, it is straightforward to show that there is a unique value function v which satisfies (2). Both the value function $v(n, z)$, and the expected continuation value $V(n)$, are continuous, differentiable and concave in n.

To characterize the optimal choice of employment, we examine the first order condition for n'. This is problematic since the function $e(\cdot)$ in equation (2) is not differentiable at $n = n'$. If the firm chooses $n' \neq n$, however, the term is differentiable. In this case, it hires or fires to the point at which the marginal benefit of an additional productive worker is offset by the marginal cost of acquiring, or retaining, this worker. Given each z, we define two critical employment levels, $n_L(z)$ and $n_H(z)$, as follows:

$$zF_n(n_L(z)) + \beta V'(n_L(z)) = \frac{w}{\mu}, \tag{3}$$

and

$$zF_n(n_H(z)) + \beta V'(n_H(z)) = w. \tag{4}$$

The term $zF_n(n) + \beta V'(n)$ is positive and strictly decreasing in n so $n_L(z)$ and $n_H(z)$ are unique. Clearly, $n_L(z) < n_H(z)$. These two values define a range of employment for a given z. If a firm finds itself with employment outside of this range, it makes an adjustment. If the firm has too few workers then it increases its effective employment to $n' = n_L(z)$. If the firm has too many productive workers then it reduces its effective employment to $n' = n_H(z)$. If the firm has a current staff of workers inside the range (i.e., if $n_L(z) < n < n_H(z)$) then the

[4] The value function and the associated policy functions are themselves functions of the endogenous variable μ. When there is no loss of clarity, we suppress the dependence on μ and simply write $v(n, z)$ rather than $v(n, z; \mu)$. When it is needed, we include μ as an argument explicitly.

firm simply continues to operate with its current workforce.[5] We summarize these results in Proposition 1. All proofs are in Appendix I.

Proposition 1: *Given μ, there is a unique value function $v(n,z)$ which solves the dynamic programming problem in (2). The value function $v(n,z)$ is continuous, differentiable and concave in n for any z. Moreover, $V(n)$ is continuous, differentiable and concave in n. The optimal policy function $n'(n,z)$ for the firm is*

$$n'(n,z) = \begin{cases} n_L(z) & \text{if } n < n_L(z), \\ n & \text{if } n \in [n_L(z), n_H(z)], \\ n_H(z) & \text{if } n > n_H(z), \end{cases} \tag{5}$$

where $n_L(z)$ and $n_H(z)$ are given by (3) and (4).

To characterize the expected shadow value of an additional worker $V'(n)$ we define two critical values of z for any given n. The first critical value, $z_H(n)$, is the solution to

$$z_H(n) F_n(n) + \beta V'(n) = \frac{w}{\mu}. \tag{6}$$

Any firm currently with n workers will increase employment whenever it gets a shock $z > z_H(n)$. The second critical value $z_L(n)$ is implicitly defined as the solution to

$$z_L(n) F_n(n) + \beta V'(n) = w. \tag{7}$$

Any firm currently with n workers will decrease employment whenever it gets a shock $z < z_L(n)$. Firms that get shocks between $z_L(n)$ and $z_H(n)$ make no change to their employment.

Now consider the term $V'(n)$. This is the expected marginal benefit of an additional worker to a firm with n employees. If the firm draws a shock greater than $z_H(n)$, it increases its employment of productive workers. Hiring good workers is costly because the firm also pays the costs of inadvertently hiring unproductive workers. The additional hiring cost is the difference between the effective cost of hiring a new productive worker (w/μ) and the cost of paying a productive worker already on staff (w). Thus, in this case, the marginal benefit of an extra productive worker is $w(1-\mu)/\mu$. If the firm draws a shock less than $z_L(n)$, it reduces employment and so, the marginal benefit of an extra worker is zero. If z is between $z_L(n)$ and

[5] This form of the firm's optimal policy – in which many firms make no adjustment – is often referred to as an (s,S) policy. The range of inaction is often referred to as the (s,S) band.

$z_H(n)$, the firm makes no change to its employment. In this case, the marginal value of an additional worker is $zF_n(n) + \beta V'(n) - w > 0$ where the inequality follows from $z > z_L(n)$. We can now write the expected marginal shadow value $V'(n)$ as

$$V'(n) = \int_{z_L(n)}^{z_H(n)} (zF_n(n) + \beta V'(n) - w) g(z) dz + \int_{z_H(n)}^{\infty} \frac{w(1-\mu)}{\mu} g(z) dz. \tag{8}$$

The expected marginal shadow value $V'(n)$ is an important object in the model. This object captures the benefits to the firm of retaining productive workers. One can show that $V'(n)$ is decreasing in the average quality μ of the pool of available workers. When μ increases, the adverse selection problem is less severe and the benefit of hoarding labor decreases. Intuitively, as the pool of unemployed workers improves (i.e., as μ increases), the cost of obtaining another productive worker falls. As a result, the marginal benefit of having an additional productive worker on staff drops as μ increases.

If there were no adverse selection (i.e., if μ were 1 and thus $V'(n) = 0$), optimal employment $n^*(z)$ would be independent of the firm's initial employment n and would depend only on the idiosyncratic shock z. This frictionless employment level is the solution to

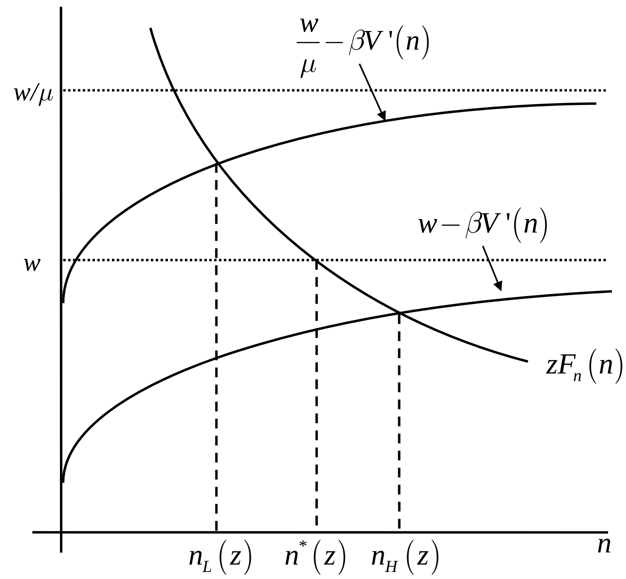
$$zF_n(n^*(z)) = w. \tag{9}$$

With adverse selection $V'(n) > 0$ and thus the frictionless employment level $n^*(z)$ is between $n_L(z)$ and $n_H(z)$ (it is in the (s, S) band). Because the shadow value $V'(n)$ is decreasing in μ, as the quality in the unemployment pool increases, $n_L(z)$ increases while $n_H(z)$ falls and the (s, S) band, $[n_L(z), n_H(z)]$, collapses around the frictionless level $n^*(z)$. These observations are summarized in Lemma 1.

Lemma 1 *For any n, $dV'(n)/d\mu \leq 0$ and $\lim_{\mu \to 1} V'(n) = 0$. For any z, (i) $n_L(z) \leq n^*(z) \leq n_H(z)$; (ii) $dn_L(z)/d\mu \geq 0$, $dn_H(z)/d\mu \leq 0$ and $\lim_{\mu \to 1} n_H(z) = \lim_{\mu \to 1} n_L(z) = n^*(z)$.*

Figure 1 shows the determination of $n_L(z)$, $n_H(z)$ and $n^*(z)$ for a given μ. The number of productive workers n is on the horizontal axis. The downward sloping line is the marginal product curve $zF_n(n)$. The upward sloping curves give the effective net cost of hiring additional productive workers $(\frac{w}{\mu} - \beta V'(n))$ and the effective net benefit of releasing productive workers $(w - \beta V'(n))$. These lines slope up and approach w/μ and w since $V''(n) < 0$ and $\lim_{n \to \infty} V'(n) = 0$. The hiring cutoff $n_L(z)$ is given by the intersection of the marginal product curve and the effective net benefit curve. The firing cutoff $n_H(z)$ is given by the intersection of the marginal product with the effective net cost curve. Finally, $n^*(z)$ is given by the intersection of the marginal product with the wage (between $n_L(z)$ and $n_H(z)$).

Figure 1: Optimal Policy Without Temporary Workers



2.2 Equilibrium

In the previous section, we considered labor demand for firms taking μ as given. Here, we extend our analysis to include the endogenous determination of μ. We focus on stationary equilibria with constant μ (we consider dynamic variations in μ in Section 5). In a stationary equilibrium, given μ, firms make employment decisions to maximize profits. The employment decisions imply the average quality of workers in the unemployment pool μ. The equilibrium can be analyzed as the fixed point of a mapping from perceived μ's to implied μ's. Given a perceived μ, the optimal policies are given by Proposition 1. The optimal policies should imply a distribution of employment across firms and a total level of employment of productive workers. Given the total number of employed productive workers, we can calculate the implied μ.

In principle, the mapping outlined above should be relatively straight-forward to construct. However, if the degree of adverse selection is too severe, the distribution of employment across firms may be indeterminate. To see this, recall that the firm's optimal policy entails an upper and lower adjustment trigger for any z. These cutoffs $n_L(z)$ and $n_H(z)$ define a range of inaction for any level of productivity. Lemma 1 says that the range of inaction $[n_L(z), n_H(z)]$ is a decreasing function of μ. That is, when adverse selection is severe, the range of inaction is wide. If μ is low enough, the range of inaction (for each z) can be so wide that firms never respond to shocks. This occurs when the upper cutoff for the lowest productivity shock $n_H(0)$ exceeds the lower cutoff for the highest productivity shock $n_L(Z)$. In this case, firms with $n \in [n_L(Z), n_H(0)]$ never adjust their employment and therefore the stationary distribution of employment would be indeterminate in this range. To resolve this indeterminacy, we use a modified version of an otherwise standard stationary equilibrium.

Our solution concept requires that the equilibrium be robust to a vanishingly small perturbation of the policy function. The perturbation we consider calls for firms to follow the optimal policy function $n'(n, z)$ with probability $1-\varepsilon$ but to follow an alternate policy function, $\hat{n}(n, z)$, with probability ε. The alternative policy we consider is $\hat{n}(n, z) = 0$, which could be interpreted as a ε probability that firms "die" or go bankrupt and release all of their workers. Under this perturbation, there is a unique stationary equilibrium for any μ as $\varepsilon \to 0$. We call this modified equilibrium a stationary ε-equilibrium.

Definition: A *Stationary ε-Equilibrium Without Temporary Workers* consists of a value function $v(n, z)$, a policy function $n'(n, z)$, a fraction μ, and a distribution of employment across firms $H(n)$ such that

1. Given μ, $v(n, z)$ and $n'(n, z)$ are given by Proposition 1.

2. The distribution satisfies $H(n) = \lim_{\varepsilon \to 0} H_\varepsilon(n)$ for all n where $H_\varepsilon(n)$ is a solution to

$$H_\varepsilon(n) = \int \left\{ \int [(1-\varepsilon)\mathbb{I}(n'(s, z) \leq n) + \varepsilon] \, dH_\varepsilon(s) \right\} dG(z) \tag{10}$$

   and where the indicator function $\mathbb{I}(x \leq n) = 1$ iff $x \leq n$.

3. The fraction of productive workers in the unemployment pool is

$$\mu = \frac{\phi L - N}{L - N} \tag{11}$$

   where total employment N is given by $N = \int n dH(n)$.

Equation (10) says that the measure of firms with employment less than n consists of two groups. First, there is the group of firms that follow the optimal policy and choose employment

n' less than n. Second, there is the group of firms that follow the alternate policy of setting employment to zero. The total number of firms with employment less than n is the sum of these two groups. Equation (11) gives the quality in the unemployment pool implied by the optimal behavior. Clearly, $0 < \mu < 1$. Moreover, it is easy to show that $\mu = \phi - (1 - \phi) \frac{N}{L-N}$ and so $\mu \leq \phi$.

To find a stationary distribution, we follow Elsby and Michaels (2013) and equate inflows and outflows for each interval $[0, n]$. The following Lemma establishes the uniqueness of, and characterizes, the limiting distribution of employment given μ. To reflect this dependence, we write the distribution as $H(n; \mu)$. This Lemma, and its proof, is only a slight modification of Proposition 5 in Elsby and Michaels (2013).

Lemma 2 *Given μ and the perturbed policy function $\hat{n}(n, z) = 0$, if $n_H(0; \mu) \geq n_L(Z; \mu)$, then*

$$H(n; \mu) = \begin{cases} 0 & \text{for } n < n_L(Z; \mu) \\ 1 & \text{for } n \geq n_L(Z; \mu) \end{cases}. \tag{12}$$

If $n_H(0; \mu) < n_L(Z; \mu)$, then

$$H(n; \mu) = \frac{G(z_L(n; \mu))}{1 - [G(z_H(n; \mu)) - G(z_L(n; \mu))]}. \tag{13}$$

If $n_L(Z) \leq n_H(0)$ then the distribution is given by a point mass at $n_L(Z)$. This result is due in part to our modified equilibrium concept. Without the modification, the stationary distribution of employment would be indeterminate in the range $[n_L(Z), n_H(0)]$. The modified equilibrium concept breaks this indeterminacy. With a vanishingly small probability firms set $n' = 0$. Firms that have $n = 0$ gradually increase employment over time. For each productivity draw z the firm adjusts to $n_L(z)$. Since $n_H(0) > n_L(Z) > n_L(z)$ firms never adjust down. Employment rises whenever the firms receive higher z's and, in the limit, this "ratcheting effect" leads all firms to have employment $n_L(Z)$.

If $n_H(0) < n_L(Z)$ then the density of firms is determinate and is nonzero only for $n \in [n_H(0), n_L(Z)]$. Firms outside this interval move inside and firms inside this interval never get shocks that move them outside.

We refer to equilibria with a distribution that is a point mass as *complete hoarding* equilibria and equilibria with a distribution over more than one point as *incomplete* (or *partial*) *hoarding* equilibria. The following Lemma establishes that there is a critical $\bar{\mu}$ such that there is complete hoarding if and only if μ is less than or equal to $\bar{\mu}$.

Lemma 3 *There exists a unique $\bar{\mu} \in (0,1)$ such that for $\mu \leq \bar{\mu}$ the stationary distribution is given by (12) (complete hoarding) and for $\mu > \bar{\mu}$ the stationary distribution is given by (13) (incomplete hoarding).*

Intuitively, if the adverse selection problem is severe (so that the average quality of unemployed workers μ is low), firms adopt an extreme form of labor hoarding in which they do not respond to shocks to the profit function and instead maintain a constant level of employment. If the adverse selection problem is mild (so that μ is relatively high) then firms respond to the shocks and there is a non-trivial distribution of employment levels across firms. As adverse selection falls ($\mu \to 1$), the shadow value of retaining workers $V'(n)$ falls. In the extreme case where $\mu = 1$, the value of retaining additional workers is zero. In this case, the firms simply release workers if z turns out to be low and hire workers at the frictionless competitive wage w if z turns out to be high.

To prove the existence of an equilibrium we define a mapping $T : [0,1] \to [0,1]$ as follows. Let $\mu \in [0,1]$ be given. Let $v(n,z)$ and $n'(n,z)$ be the solution to the dynamic programming problem in (2) as given in Proposition 1. Let $H(n;\mu)$ be the unique distribution implied from Lemma 2 and let $N(\mu) = \int n dH(n;\mu)$ be aggregate employment. Then, define $T(\mu)$ as

$$T(\mu) = \begin{cases} \frac{\phi L - N(\mu)}{L - N(\mu)} & \text{if } \phi L > N(\mu), \\ 0 & \text{otherwise.} \end{cases} \tag{14}$$

Proposition 2 *The mapping $T : [0,1] \to [0,1]$ defined in (14) has at least one fixed point. The fixed points μ^* of this mapping and the associated value functions $v(n,z)$, policy functions $n'(n,z)$ and distributions $H(n)$ are the stationary ε-equilibria.*

2.3 Discussion

The Hiring Effect and the Hoarding Effect: In the model, adverse selection exerts two opposing forces on overall employment. First, adverse selection increases the effective cost of hiring good workers. This effect puts downward pressure on employment. We call this first effect the *Hiring Effect*. Second, adverse selection increases the benefit of retaining workers for future production. This effect puts upward pressure on employment. We call this second effect the *Hoarding Effect*. The balance of these two forces determines whether overall employment is higher or lower than in a frictionless equilibrium.

To find an expression for the change in employment with respect to μ, begin by writing total employment as

$$N = \int_0^Z \left[(1 - H(n_H(z)))\, n_H(z) + H(n_L(z))\, n_L(z) + \int_{n_L(z)}^{n_H(z)} dH(n) \right] g(z)\, dz. \tag{15}$$

where we have suppressed the dependence of the variables on μ for clarity. The measure of employment for firms with shock z is made up of three terms. First, firms that began with $n > n_H(z)$ reduce their employment to $n' = n_H(z)$. There are $1 - H(n_H(z))$ such firms. Second, firms that began with $n < n_L(z)$ increase their employment to $n' = n_L(z)$. There are $H(n_L(z))$ such firms. Finally, firms that began with $n_L(z) \leq n \leq n_H(z)$ make no adjustment to their workforce. The last term in the bracket captures total employment for these firms.

The change in employment with respect to a change in adverse selection is

$$\begin{aligned} \frac{\partial N}{\partial \mu} = & \int_0^Z \Bigg\{ \underbrace{\frac{H(n_L(z))}{\Lambda(z, n_L(z))} \frac{w}{\mu^2}}_{\text{The Hiring Effect}} \\ & + \beta \times \underbrace{\left(\frac{1 - H(n_H(z))}{\Lambda(z, n_H(z))} \frac{\partial V'(n_H(z))}{\partial \mu} + \frac{H(n_L(z))}{\Lambda(z, n_L(z))} \frac{\partial V'(n_L(z))}{\partial \mu} \right)}_{\text{The Hoarding Effect}} \\ & - \int_{n_L(z)}^{n_H(z)} \frac{\partial H(n)}{\partial \mu} dn \Bigg\} g(z)\, dz \end{aligned} \tag{16}$$

where $\Lambda(z, n) = -[zF_{nn}(n) + \beta V''(n)] > 0$.

This expression is easiest to understand in the context of Figure 1. When μ changes there are two separate effects on firms' employment decisions. Most directly, when μ increases, the effective hiring wage w/μ falls. The hiring effect corresponds to the decrease in w/μ holding $V'(n)$ fixed. Thus, in Figure 1, the hiring effect would be a downward shift in the curve $w/\mu - \beta V'(n)$ and would naturally lead to an increase in $n_L(z)$. The strength of the hiring effect depends on the number of firms with shock z that are hiring (i.e., $H(n_L(z))$). An increase in μ also has an indirect effect on labor demand. Specifically, when μ increases, the marginal shadow value $V'(n)$ falls. The hoarding effect corresponds to the decrease in $V'(n)$ holding w/μ fixed. In the figure, the hoarding effect is an upward shift in both $w/\mu - \beta V'(n)$ and $w - \beta V'(n)$ and thus decreases both $n_L(z)$ and $n_H(z)$. In (16), the hoarding effect is the sum of the two terms involving $\partial V'/\partial \mu$. The last term ($\int [\partial H(n)/\partial \mu]\, dn$) captures additional changes in employment caused by changes in the overall distribution of employment H. For details on the derivation of (16) see Appendix II.

The hiring effect and the hoarding effect determine whether $T(\mu)$ is increasing or decreasing in μ. As μ increases (as adverse selection falls), the hiring effect encourages firms to hire more workers and puts upward pressure on $N(\mu)$. Since $T(\cdot)$ is a decreasing function of N, the hiring effect tends to make T a decreasing function of μ. The hoarding effect reduces the incentive to retain workers and thus tends to make T an increasing function of μ. If T is decreasing (so that $T_\mu < 0$) we say that the equilibrium has a dominant hiring effect. If T is increasing then we say that there is a dominant hoarding effect. In complete hoarding equilibria, the hiring effect always dominates so $T_\mu < 0$.

The incentive to hoard labor is present in essentially any model with hiring or firing costs. Firms hoard labor to avoid paying the cost too often. The hoarding incentive in our model comes from the adverse selection problem. Unlike exogenous hiring and firing costs, the adverse selection costs are endogenous and will vary depending on economic conditions and on the behavior of other firms.

Multiple Equilibria: If the hoarding effect is sufficiently strong, the model can have multiple equilibria. If firms anticipate that adverse selection is severe (i.e., that μ is low), they will be reluctant to release good workers and instead hoard labor. As a result, if this hoarding incentive is strong enough, the pool of potential workers is relatively low quality in equilibrium. In contrast, if firms anticipate that adverse selection is mild (i.e., that μ is high), they will be willing to release good workers when productivity is low. As a result, the pool of potential workers has many good workers in it and the average quality is high in equilibrium. The following example demonstrates the possibility of multiple equilibria.[6]

Example. Consider a case with two possible shocks $z_l < z_h$. The probability of getting z_h is p. The marginal product function is piecewise linear with F_n given by

$$F_n(n) = \begin{cases} a_1 - b_1 n & \text{for } n \leq \hat{n} \\ a_2 - b_2 n & \text{for } n > \hat{n} \end{cases}$$

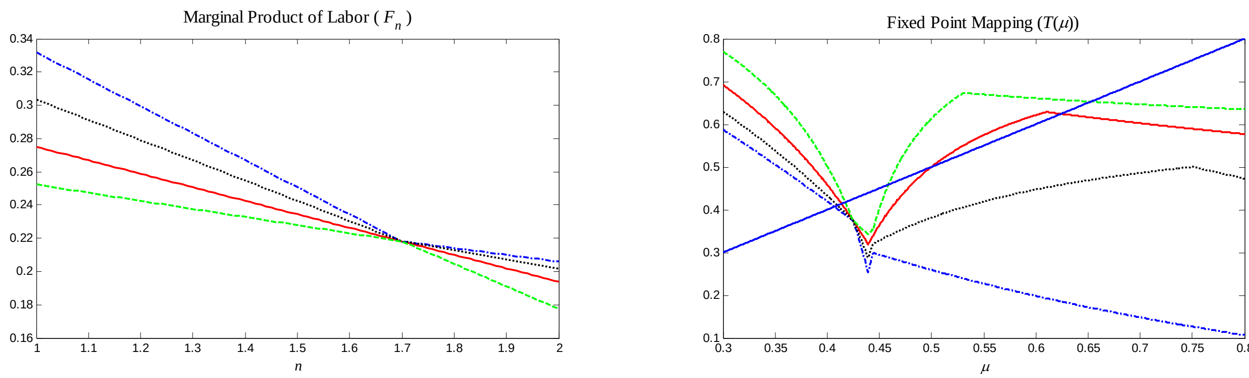
with $a_1 - b_1\hat{n} = a_2 - b_2\hat{n}$. If $b_1 = b_2$, F_n is linear. If $b_1 > (<) b_2$ the marginal product convex (concave) in n. The left panel of Figure 2 shows different examples of F_n. In an incomplete

[6] The potential for multiple equilibria in adverse selection models is well-known. See e.g. Wilson (1980).

hoarding equilibrium, firms switch between two employment levels $n_L(z_h)$ and $n_H(z_l)$ and aggregate employment is $N(\mu) = pn_L(z_h;\mu) + (1-p)\,n_H(z_l;\mu)$. We choose parameters to ensure that $n_H(z_l) < \hat{n} < n_L(z_h)$ so $F_{nn}(n_H(z_l)) = b_1$ and $F_{nn}(n_L(z_h)) = b_2$. For this example, equation (16) is

$$\frac{\partial N(\mu)}{\partial \mu} = \underbrace{\frac{w}{\mu^2}\left[\frac{p}{z_h b_2}\right]}_{\text{The Hiring Effect}} - \beta \underbrace{\left(\frac{w}{\mu^2}\frac{1-p}{z_l b_1} + \frac{w}{\mu^2}\frac{p}{z_h b_2}\right)}_{\text{The Hoarding Effect}}$$

Figure 2: Optimal Policy Without Temporary Workers



Notes: The left panel plots hypothetical examples of the marginal product function $F_n(n)$ for the piecewise linear case described in the text. The right panel plots the mappings $T(\mu)$ corresponding to the marginal

Not surprisingly, the slopes (b_1 and b_2) of the marginal product function govern the intensity of the hiring and hoarding effects. If b_2 is relatively low, then as μ increases, firms can hire more workers without sharply reducing the marginal product of labor and the hiring effect is strong. (The hiring effect matters only for the high z. Firms with low z are releasing workers which does not entail a screening cost.) If b_1 and b_2 are both low, then as μ increases, the firm can release many workers without sharply increasing the marginal product of labor and thus the hoarding effect is strong. Since firms want to release workers when they draw low z's, and hire workers when they draw the high z's, if the marginal product F_n is steeper for the high z's and flatter for the low z's, the hoarding effect will tend to dominate. (In the example, the hoarding effect dominates whenever $b_2 > b_1\left(\frac{1-\beta}{\beta}\frac{p}{1-p}\frac{z_l}{z_h}\right)$.) The right panel of Figure 2 shows the mappings $T(\mu)$ corresponding to the marginal product functions in the left panel. The more concave marginal product functions have weaker hiring effects and stronger hoarding effects and thus feature multiple steady state equilibria. The convex marginal product functions have stronger hiring effects and weaker hoarding effects and have unique equilibria.

Unique Equilibria: It is possible to place restrictions on the model to rule out multiple equilibria. Multiple equilibria require two conditions. First, changes in the quality of the pool must have a dominant hoarding effect on labor demand. That is, when the pool improves (i.e., higher μ), firms' willingness to release good workers when productivity is low overcomes their willingness to hire good workers when productivity is high. Second, there must be many firms on the hiring-firing margin so that small changes in demand cause substantial changes in the equilibrium μ. Thus, one way to ensure that there is a unique stationary equilibrium is to limit the number of potential marginal firms.

The following assumption bounds the number of marginal firms. Recall that Z is the maximum possible productivity. Let $n^*(Z)$ be the efficient number of workers associated with Z, (i.e., $ZF_n(n^*(Z)) = w$) and let $\hat{\mu} = [\phi L - n^*(Z)] / [L - n^*(Z)]$. That is, $\hat{\mu}$ is the quality in the unemployment pool that would arise if all firms continuously maintained an employment level of $n^*(Z)$.

Assumption 1 *Assume the shock distribution has a density $g(z)$ and that $g(z) < \bar{g}$ where*

$$\bar{g} = \frac{1-\beta}{\beta}\hat{\mu}\frac{\phi L - n^*(Z)}{Zn^*(Z)}.$$

The following proposition states that if the density is sufficiently low (so that g is everywhere less than $\bar{g}$), then the stationary equilibrium must be unique.

Proposition 3 *Let $G(z)$ satisfy Assumption 1. Then there is a unique stationary equilibrium μ in the model without temps.*

The hiring effect and the hoarding effect cause firms at the margin to change their employment decisions in response to changes in μ. Assumption 1 limits the extent to which this change in behavior influences the equilibrium μ essentially by limiting the number of marginal firms. If Assumption 1 is satisfied, there are never very many firms at the margin and so variations in μ have only small effects on N and T.

3 The Model with Temps

This section extends the model to allow for temp agencies. The temp agencies play an insurance role by providing costly workers to firms with temporarily high productivity. Like the other firms, the temp agencies confront the adverse selection problem when they hire workers. Section 3.1 analyzes the supply of temp workers. Section 3.2 analyzes the demand for both permanent and temporary workers. Section 3.3 studies the equilibrium.

3.1 The Supply of Temp Workers

Temp workers are supplied by competitive temp agencies. The temp agencies hire workers at the market wage w and lease productive workers to firms at a temp wage w^m. Like the other firms, temp agencies retain only good workers. Thus, after the evaluation period (the first period of employment), the temp agency gets the temp wage w^m for each temp worker and pays each temp worker w. In this respect, temp agencies are just like the other firms. They have no informational advantage when they hire new workers and they must incur the same evaluation cost to determine which workers are productive and which are not.

Temp agencies differ in two important dimensions from the other firms however. First, the temp firms can lease workers to other firms without losing contact with the workers. In the case without temps, if a firm dismissed a worker, it could not re-establish contact with him later on. (Unlike regular firms, temp agencies have a "memory" of their workers.) Second, we allow the temp firms to write contracts which we do not allow for the other firms. Specifically, the temp agencies get paid by the firms only if the workers supplied are good. We do not allow firms to do this on their own directly with the workers. If we did not allow the temp agencies to make this contract, the temp agencies would offer no relief from the adverse selection problem. The ability to keep track of workers and to write contingent contracts requires that the temp agencies pay a flow cost $x \geq 0$ (x may also include other costs of operating the temp agency – see below). The model in Section 2, without temp agencies, can be thought of as a case in which the cost of operating a temp agency x is prohibitively high.

The temp industry is competitive and thus (expected) profits at temp agencies are zero. The cost to the temp agency of acquiring each productive worker is the evaluation cost $w\,(1-\mu)\,/\mu$. The benefit to the temp agency is the present discounted value of markups $w^m - w$. If the evaluation cost were the only cost paid by the temp agency, in equilibrium it should be that $w(1-\mu) = \mu\,(w^m - w)\,/(1-\beta)$. We refer to this wage as the *efficient temp wage* and denote it by $\bar{w}^m$. That is,

$$\bar{w}^m = \frac{w}{\mu}\left[1 - \beta\,(1-\mu)\right]. \tag{17}$$

It is straight-forward to show that $w < \bar{w}^m < w/\mu$. Notice that if $\beta \to 1$ (i.e., if the temp agencies are very patient) then $\bar{w}^m \to w$.

As we mentioned above, we allow for the possibility that there could be additional flow costs of providing temp workers. One interpretation of the flow cost x is a cost of maintaining accurate records and contact information, processing payments, necessary time between jobs, etc. Another interpretation of x is as a measure of fit or match quality between a temp worker and his or her employer. Temp workers likely do not have the exact skill set or familiarity with the work environment that a permanent worker does. As a result, temp workers are effectively more costly than permanent workers because they are on average worse matches for the firm. If there are additional costs, then the temp wage will exceed $\bar{w}^m$. To allow for this possibility, we write the temp wage as the efficient temp wage plus an additional flow cost x so that $w^m = \bar{w}^m + x$. We refer to case with $x > 0$ as the *inefficient temp case* though our terminology is not meant to imply that there are any additional distortions in the labor market. If the additional overhead costs cause the temp wage to be greater than w/μ then the model collapses to the case without temp workers.

It is worth pointing out that there is nothing which precludes the possibility that a firm could function both as a unit of production and as a temp agency simultaneously. If the firm is willing to incur the per worker flow cost x then it can maintain a stock of temp workers in addition to its permanent staff. The firm could then employ some of its own temp workers directly or it could lease them out to other firms as temp workers.

Formally, the representative temp agency's value function is

$$v^T(m) = \max_{m' \geq 0} \left\{ m' w^m - m'(w + x) - w \cdot e(m' - m) + \beta v^T(m') \right\}$$

where m is the current stock of temp workers on staff, and the function $e(\cdot)$ is again given by (1) (the superscript T indicates the value function is for the temp agency). Like the individual firms' demand for labor, the temp agency faces a kinked adjustment cost when it changes its labor force. If the temp agency optimally chooses to hire workers ($m' > m$) then

$$w^m - (w + x) - w\left(\frac{1-\mu}{\mu}\right) + \beta \frac{dv^T(m')}{dm'} = 0.$$

If $m' < m$ then

$$w^m - (w + x) + \beta \frac{dv^T(m')}{dm} \leq 0$$

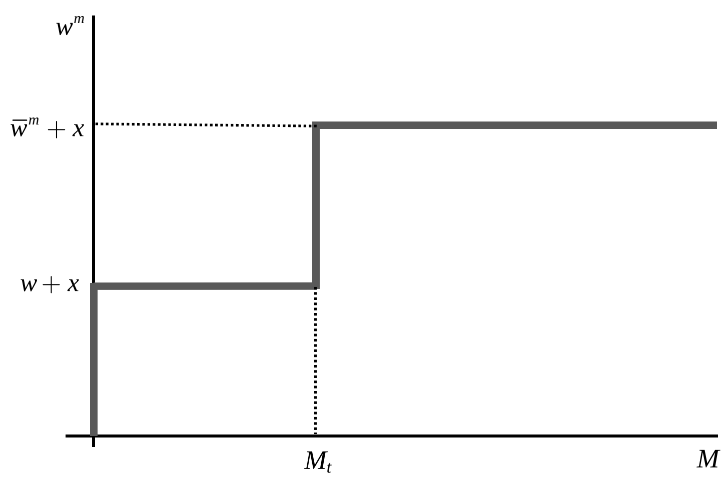
with equality if $m' > 0$. The shadow value of good temp workers to the temp agency is

$$\frac{dv^T(m)}{dm} = \begin{cases} 0 & m' < m \\ w^m - (w+x) + \beta \frac{dv^T(m')}{dm'} & m' = m \\ w\left(\frac{1-\mu}{\mu}\right) & m' > m \end{cases}.$$

In a steady state, $m' = m$ and thus $\frac{dv^T(m)}{dm} = \frac{w^m - (w+x)}{1-\beta}$. If the temp agencies were hiring as they approached the steady state then the temp wage would be $w^m = \bar{w}^m + x$. (In the analysis below, the temp wage is $\bar{w}^m + x$ which implicitly assumes that the temp agency was hiring as the economy entered the steady state.) On the other hand, if the temp agencies were firing as they approached the steady state (if, for some reason, the temp agency begins with more workers than necessary) then, $w^m = w + x$. Intuitively, if the temp agency is hiring, the discounted sum of temp wage payments must be enough to cover both the discounted flow costs plus the initial evaluation cost $w\left(\frac{1-\mu}{\mu}\right)$.

Figure 3 shows the steady state supply function for temp agencies that have a total of M_t temp workers currently on staff. If $w^m = \bar{w}^m + x$, the temp agency is willing to supply $M \geq M_t$ since w^m would cover both the search cost and the flow cost of the worker. If $w^m = w + x$, the temp agency is willing to supply $0 \leq M \leq M_t$ since the temp wage would cover only the flow cost of the worker. If $w + x \leq w^m \leq \bar{w}^m + x$ the temp agency supplies exactly M_t workers.

Figure 3: The Supply of Temporary Workers



3.2 Labor Demand

With temps, firms can respond to changing conditions either by hiring workers directly or by hiring workers from the temp agencies. The Bellman equation governing the firm's optimization problem is

$$v(n,z) = \max_{n',m} \{zF(n'+m) - wn' - we(n'-n) - w^m m + \beta V(n')\}, \tag{18}$$

where m and n' are temporary workers and productive permanent workers respectively. We use the term "permanent worker" to distinguish these workers from temporary workers. Note that the firm never begins a period with temp workers on hand, thus there is no continuation value associated with hiring temps. Because the permanent workers have continuation values, the decision to hire additional permanent staff is similar to an investment decision.

If a firm wants to increase its employment it has two options. It can hire a permanent worker at an effective cost $w/\mu - \beta V'(n)$, or it can hire a temporary worker at an effective cost w^m. As before, if the firm hires permanent workers, it hires to the point at which the marginal product of labor is equal to the effective hiring cost. Thus, if the firm hires permanent workers, $zF_n(n'+m) + \beta V'(n') = w/\mu$. Similarly, if the firm releases permanent workers then $zF_n(n'+m) + \beta V'(n') = w$.

Unlike the earlier model, the firm can hire temporary workers in addition to its permanent workforce. Intuitively, because retaining a permanent worker currently on staff is less costly than hiring a temporary worker, if a firm chooses to hire temp workers, it will not fire any existing permanent workers. Moreover, if the firm hires temporary workers, it will choose m so that the marginal product of labor will be equal to the temp wage, $zF_n(n'+m) = w^m$. If the firm hires no temporary workers, then $zF_n(n') \leq w^m$. Define $n_M(z)$ as the solution to

$$zF_n(n_M(z)) = w^m. \tag{19}$$

Notice that $n_M(z)$ is a purely static variable. No forward- or backward-looking elements enter the determination of this variable. This employment level is the one the firm would choose if it freely interacted with a spot labor market for productive workers at a spot price w^m. If the firm chooses to hire any temporary workers, then total employment at the firm (including both permanent and temporary workers) must be $n_M(z)$.

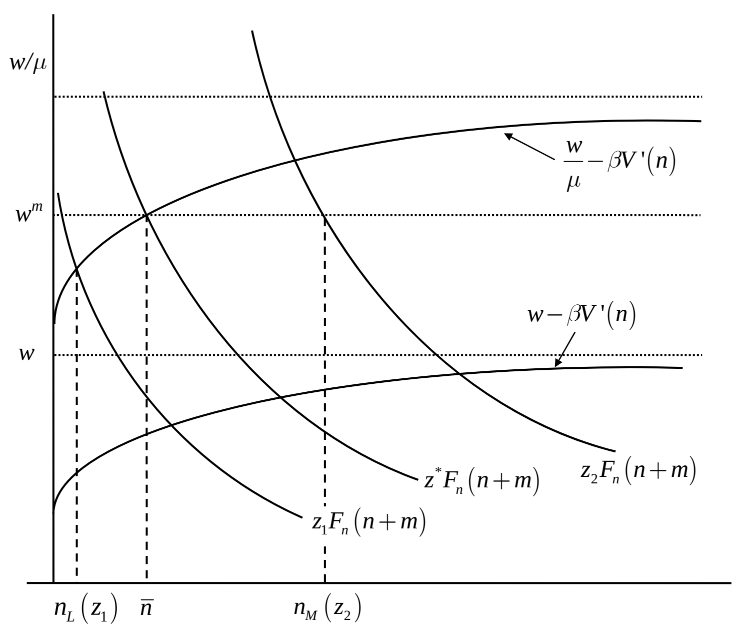
If the temp workers are not too costly, firms will likely hire a mix of permanent and temporary workers. Define $\bar{n}$ as the critical employment level where the effective cost of hiring a permanent worker and the cost of hiring a temporary worker are equal.

$$w^m = \frac{w}{\mu} - \beta V'(\bar{n}). \tag{20}$$

This critical employment level will always exist provided that $w^m < w/\mu$. Since $V'(n)$ is decreasing in n, $\bar{n}$ is unique. For $n < \bar{n}$, hiring an additional permanent worker is effectively cheaper than hiring a temp. For $n > \bar{n}$, the continuation value $\beta V'(n)$ is sufficiently low that hiring a temp is effectively cheaper than hiring a permanent worker.

Figure 4 depicts the optimal choice of permanent and temporary workers for three productivity levels $z_1 < z^* < z_2$. Each productivity level implies a different marginal product of labor $zF_n(n+m)$ (the downward sloping lines in the figure). The two upward sloping curves correspond to the effective net cost of hiring permanent workers $(w/\mu - \beta V'(n))$ and the effective net benefit of firing permanent workers $(w - \beta V'(n))$. The employment level $\bar{n}$, at which it is just as costly to hire a permanent worker as a temporary worker, is defined by the intersection of the effective hiring cost line $w/\mu - \beta V'(n)$ with the temp wage w^m. The figure is drawn under the assumption that $w/\mu - \beta V'(0) < w^m$ and so firms will want to hire at least some permanent staff.

Figure 4: Optimal Policy With Temporary Workers



Consider the hiring decisions for a firm that begins the period with $n = 0$ workers. If the firm draws the low productivity level (z_1) the firm uses only permanent workers and the firm's decision is the same as the case without temps. The firm hires to the point at which the effective cost of permanent workers is equal to the benefit. This point is $n_L(z_1)$. At $n_L(z_1)$, the effective cost of permanent workers is less than w^m and thus there is no reason to hire any temps. The medium productivity level z^* is the highest z for which the firm will not use temps. This critical productivity level z^* is implicitly defined by

$$z^* F_n(\bar{n}) = w^m. \tag{21}$$

If a firm were to draw z^*, it would hire $\bar{n}$ permanent workers and zero temps. At this level of employment $(\bar{n})$ the effective cost of hiring permanent workers is the same as the effective cost of hiring temporary workers. If the firm draws the high productivity level (z_2), it hires both permanent and temporary workers. Total employment $n_M(z_2)$ is defined by the intersection of the marginal product curve and the temp wage line. At this employment level the firm hires $\bar{n}$ permanent workers and $n_M(z_2) - \bar{n}$ temporary workers.

The optimal hiring decisions of firms that begin with $n > 0$ can also be inferred from the figure. A firm that begins with $\bar{n}$ workers, will not release any permanent workers if it draws the low shock (i.e., z_1). A firm that begins with n between $\bar{n}$ and $n_M(z_2)$ and draws the z_2 shock, retains all of its permanent workers and hires temporary workers so that total employment is $n_M(z_2)$. The following Proposition summarizes the optimal policy function for a firm in the model with temps.

Proposition 4 *In the model with temporary workers, given μ and a wage for temporary workers $\bar{w}^m \leq w^m < \frac{w}{\mu}$, the optimal policy is given by an employment policy for permanent workers $n'(n,z)$ and an employment policy for temporary workers $m(n,z)$. If $z \leq z^*$ then*

$$n'(n,z) = \begin{cases} n_L(z) & \text{for } n \leq n_L(z) \\ n & \text{for } n_L(z) < n \leq n_H(z) \\ n_H(z) & \text{for } n_H(z) < n \end{cases},$$

and $m(n,z) = 0$. If $z > z^*$ then

$$n'(n,z) = \begin{cases} \bar{n} & \text{for } n \leq \bar{n} \\ n & \text{for } \bar{n} < n \leq n_H(z) \\ n_H(z) & \text{for } n_H(z) < n \end{cases},$$

and $m(n,z) = \max\{0, n_M(z) - n'(n,z)\}$. *Here* $n_M(z)$, $\bar{n}$ *and* z^* *are defined in (19), (20) and (21) and* $n_L(z)$ *and* $n_H(z)$ *are defined in (3) and (4).*

One can show that, given μ, the continuation value $V'(n)$ is identical to the continuation value in the no temp model for any $n \leq \bar{n}$. Thus, for $z \leq z^*$ firms employ $n_L(z) \leq \bar{n}$ permanent workers. This is exactly the same hiring policy as in the no temp model. Put differently, if the firm is not actively hiring temp workers in a given period, its hiring policy for permanent workers is unaffected by the availability of temps. Also, for $z > z^*$, firms employ more *total* workers than in the model without temps. Specifically, firms employ $\bar{n} + m(n,z) = n_M(z) \geq n_L(z)$ total workers. The composition of these workers differs however. Firms that draw $z > z^*$ continue to operate with their buffer-stock $\bar{n}$ but respond to the high shock by hiring temp workers. Taken together, these observations imply that, for a given μ, total firm-level employment will be more volatile if temp workers are available. Not surprisingly, the increased employment volatility is due to variation in temp employment. Variation in permanent employment drops if temps are available.

3.3 Equilibrium

The definition of an equilibrium is similar to the case without temps. As before, we appeal to an ε perturbation to ensure uniqueness of the distribution of employment across firms.

Definition: A *Stationary ε-Equilibrium for the Model with Temporary Workers* consists of a fraction μ, a temp wage $w^m \geq \bar{w}^m(\mu)$, a value function $v(n,z)$, an employment policy function for permanent workers $n'(n,z)$, an employment policy function for temporary workers $m(n,z)$, and a distribution of permanent workers across firms $H(n)$ such that

1. Given μ and w^m, $n'(n,z)$ and $m(n,z)$ solve the firm's problem in (18) and imply $v(n,z)$.

2. $H(n)$ satisfies $H(n) = \lim_{\varepsilon \to 0} H_\varepsilon(n)$ for all n where $H_\varepsilon(n)$ is a solution to

$$H_\varepsilon(n) = \int \int [(1-\varepsilon)\,\mathbb{I}(n'(n,z) \leq n) + \varepsilon]\, dH_\varepsilon(n)\, dG(z)$$

   and where the indicator function $\mathbb{I}(x \leq n) = 1$ iff $x \leq n$.

3. The fraction of productive workers in the unemployment pool is $\mu = (\phi L - N - M)/(L - N - M)$ where $N = \int n dH(n)$ is the total employment of permanent workers and $M = \int \int m(n,z)\, dH(n)\, dG(z)$ is the total employment of temporary workers.

There are two broad types of equilibria in the model with temp workers. There is the equilibrium that arrises when the temp wage is efficient (when $w^m = \bar{w}^m$) and the equilibria when the temp wage is above the efficient level (when $w^m = \bar{w}^m + x$). We refer to this second case as the "costly temps" case. We discuss these two cases separately. We begin by characterizing the equilibrium with a perfectly efficient market for temporary workers.

3.3.1 Efficient Temps

If $w^m = \bar{w}^m$ the temp wage is "efficient." There are no additional overhead costs or markups beyond the cost of recovering the initial evaluation for the temp agency. Here we show that firms never hire permanent workers in the efficient temp case – all employees are temporary workers.

To demonstrate that only temp workers will be employed, we show that, for a firm with no employees, the effective cost of hiring the "first" permanent worker is equal to the efficient temp wage.[7] A firm with zero current employees will increase employment next period with probability one. The discounted shadow value of a permanent worker for such a firm is $\beta V'(0) = w(1-\mu)/\mu$. Using the definition of the efficient temp wage (equation 17), we conclude that $w/\mu - \beta V'(0) = \bar{w}^m$ and the effective cost of hiring the first permanent worker is the same as the cost of hiring a temp. Because the shadow value of permanent workers is decreasing in n, the effective cost of hiring the second or third permanent worker is strictly greater than $\bar{w}^m$. As a result, if $w^m = \bar{w}^m$, the buffer-stock $\bar{n}$ of permanent workers is zero and the firms never hire permanent staff.

That firms hire only temps is not surprising. The shocks to labor demand are idiosyncratic and the temp agency offers to provide productive workers at a wage which covers the workers' opportunity costs plus the minimal evaluation costs to the temp agency. In a sense, the temp agencies are providing an insurance service to the production firms. Given this option, the firms "perfectly insure" against variations in labor demand caused by idiosyncratic shocks and the temp workers are the only ones ever hired.

Since firms hire only temps, we can calculate total employment given μ using the static condition (19). Employment for a firm with shock z is $n_M(z;\mu)$. Aggregate employment of permanent workers is $N(\mu) = 0$ and aggregate employment of temporary workers is $M(\mu) = \int_0^Z n_M(z;\mu)\, g(z)\, dz$. As before, the equilibrium is a fixed point in μ. Given total employment

[7]This argument goes through even if the actual temp wage exceeds the efficient temp wage. That is, even if $w^m > \bar{w}^m(\mu)$, we still have $w/\mu - \beta V'(0) = \bar{w}^m$.

$M(\mu)$, the implied degree of adverse selection in the unemployment pool is

$$\mu' = T^{\bar{m}}(\mu) = 1 - (1-\phi)\frac{L}{L - M(\mu)}$$

where the notation $T^{\bar{m}}$ distinguishes this mapping from the mapping in the model without temps. The fixed point mapping $T^{\bar{m}}(\mu)$ is strictly decreasing in μ and thus, there is a unique equilibrium μ^* for the efficient temp case. Moreover, since $\bar{w}^m > w$, $n_M(z;\mu) < n^*(z)$ for any z, and aggregate employment is strictly less than the efficient level of employment. The following proposition summarizes this discussion.

Proposition 5 *If* $w^m = \bar{w}^m$ *then, (i) the only workers employed in equilibrium are temp workers and* $\bar{n} = 0$; *(ii) the equilibrium is unique; (iii) aggregate employment is less than the efficient level of employment.*

3.3.2 Costly Temps

The case in which the provision of temp workers entails costs above and beyond the search cost (i.e., in which $x > 0$) is somewhat more complicated. Given μ, optimal behavior for each firm is given by Proposition 4. Unlike the efficient temp case, if $w^m > \bar{w}^m$, firms hire a strictly positive buffer-stock of permanent workers and use temporary workers to accommodate transitory labor demand shocks. In a stationary equilibrium, firms employ at most $\bar{n}$ permanent workers in any period where $\bar{n}$ is defined by (20). We now proceed to characterize the equilibrium.

To prove existence, we again construct a fixed point mapping in μ. Given μ, let $v(n,z;\mu)$ be the value of a firm with current shock z and current permanent staff n that behaves optimally. Let $m(z,n;\mu)$ and $n'(z,n;\mu)$ be the associated policy functions characterized by the cutoff functions $n_L(z;\mu)$, $n_H(z;\mu)$, $\bar{n}(\mu)$ and $n_M(z;\mu)$.

Complete Hoarding: If there is complete hoarding (so that $n_H(0;\mu) \geq \min\{\bar{n}(\mu), n_L(Z;\mu)\}$) then the distribution of permanent workers, $H(n;\mu)$, is given by

$$H(n;\mu) = \begin{cases} 0 & \text{for } n < \min\{\bar{n}(\mu), n_L(Z;\mu)\} \\ 1 & \text{for } n \geq \min\{\bar{n}(\mu), n_L(Z;\mu)\} \end{cases}. \tag{22}$$

Firms may or may not use temps however. If $\bar{n}(\mu) \geq n_L(Z;\mu)$ then no temps are hired and aggregate employment is $N(\mu) = n_L(Z;\mu)$. If $\bar{n}(\mu) \leq n_L(Z;\mu)$ then firms hire both temps and permanent workers though they never adjust their permanent staff. The permanent staff at each firm is $\bar{n}(\mu)$ while total employment is $\max\{n_M(z;\mu), \bar{n}(\mu)\}$ and employment of temps is $m(z,n;\mu) = \max\{n_M(z;\mu) - \bar{n}(\mu), 0\}$. In this case, total employment of good workers in equilibrium is $N(\mu) = \bar{n}(\mu) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} (n_M(z;\mu) - \bar{n}(\mu))\, dG(z)$.

Incomplete Hoarding: If there is incomplete hoarding ($n_H(0;\mu) < \min\{\bar{n}(\mu), n_L(Z;\mu)\}$), then the distribution of permanent workers, $H(n;\mu)$, is

$$H(n;\mu) = \begin{cases} 0 & \text{for } n < n_H(0;\mu) \\ \frac{G(z_L(n;\mu))}{1-[G(z_H(n;\mu))-G(z_L(n;\mu))]} & \text{for } n_H(0;\mu) \leq n \leq \min\{\bar{n}(\mu), n_L(Z;\mu)\} \\ 1 & \text{for } \min\{\bar{n}(\mu), n_L(Z;\mu)\} < n \end{cases} \tag{23}$$

Again, firms may or may not use temps. If $\bar{n}(\mu) \geq n_L(Z;\mu)$ then no temps are hired and aggregate employment is $N(\mu) = \int_0^\infty n dH(n;\mu)$. If $\bar{n}(\mu) \leq n_L(Z;\mu)$ then firms hire both temps and permanent workers. Firms with $z < z_M(\bar{n}(\mu);\mu)$ hire no temporary workers and instead operate only using their permanent staff. Firms with $z > z_M(\bar{n}(\mu);\mu)$ choose total employment $n_M(z;\mu)$ and thus hire temporary workers $m(z,n;\mu) = n_M(z;\mu) - \bar{n}(\mu)$. Total employment of good workers is $N(\mu) = \int_0^\infty n dH(n;\mu) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} (n_M(z;\mu) - \bar{n}(\mu))\, dG(z)$.

In the above cases, the level of employment $N(\mu)$ implies $T(\mu) = \frac{\phi L - N(\mu)}{L - N(\mu)}$. The mapping $T : [0,1] \to [0,1]$ is continuous and thus existence of at least one equilibrium is guaranteed. This discussion is summarized in the following proposition:

Proposition 6. *In the model with inefficient temps, the mapping T has at least one fixed point. The fixed points μ^* of this mapping and associated value function $v(n,z;\mu^*)$, policy functions $m(z,n;\mu^*)$ and $n'(z,n;\mu^*)$ and distribution $H(n;\mu^*)$ are the stationary ε-equilibrium.*

In equilibrium, firms' employment choices exhibits a buffer-stock adjustment pattern. Specifically, there is a natural tendency for firms to "bunch up" at $\bar{n}$. Thus, $\bar{n}$ might be considered the normal employment level for a typical firm in this industry. For these firms, any increases in employment will be accomplished by temporary employment. Only if the firm gets a very low shock will it reduce its permanent staff. For modest changes in z, the firm will either make no adjustment to permanent employment or it will hire temp workers. Notice, based on (22) or (23), in a stationary equilibrium, no firm ever has $n > \bar{n}$.

4 Welfare and Policy

In this section we briefly consider the welfare in the labor market with adverse selection. After calculating welfare, we then ask whether hiring subsidies or taxes improve aggregate welfare relative to the case without government policy. Typically policy interventions entail a dynamic transition from the initial steady state to a new steady state consistent with the new policy instrument. For our purposes, we compare the steady state flow of welfare for different policy choices. This comparison essentially adopts a "long-horizon" view which would be technically appropriate only for very patient firms. Thus, in our policy calculations, we consider welfare comparisons under the assumption that β is close to 1 and we ignore the welfare consequences of the dynamic transition paths from one stationary equilibrium to another.

Calculating aggregate welfare for the model is straightforward. In either the model with temps or the model without temps, we can characterize both the equilibrium degree of adverse selection and the equilibrium distribution of employees across firms. We calculate welfare for each firm that draws a particular shock z and then aggregate over all of the firms. While the definition of welfare is natural, the calculations themselves are rather tedious. Exact expressions for aggregate welfare are included in Appendix I.

Hiring Subsidies and Hiring Taxes. We consider the effects of a hiring subsidy (or tax). If the labor market were frictionless, subsidizing firms to hire more workers would cause deadweight losses. However, because of the adverse selection problem, subsidies or taxes to hiring could work to undo some of the existing distortions in the market. A hiring subsidy is a natural candidate for a policy in this environment since adverse selection effectively increases the costs of changing the level of employment at a firm.

If τ is the hiring subsidy, the effective after-tax cost of hiring a worker is $\frac{w(1-\mu)(1-\tau)}{\mu}$. From the firms' perspective, this is equivalent to an environment without a subsidy but with a different μ, say $\tilde{\mu}$, with $\frac{1-\tilde{\mu}}{\tilde{\mu}} = \frac{(1-\mu)(1-\tau)}{\mu}$. To avoid tracking the additional parameter τ, we imagine that the government simply chooses μ directly. This choice will not be consistent with a stationary equilibrium if the government chooses a μ for which $T(\mu) \neq \mu$. Thus, while the firms behavior is governed according to μ, the true hiring costs to society are governed by $T(\mu)$. The difference between the actual amount of adverse selection in the subsidized equilibrium $T(\mu)$ and the effective costs to the firm μ is accounted for by the government hiring subsidy. The implicit subsidy in the stationary equilibrium is $\tau = \frac{\mu - T(\mu)}{\mu(1-T(\mu))}$. If $\mu > T(\mu)$ then the implicit subsidy is positive (i.e., the actual hiring costs are greater than the perceived hiring costs). If $\mu < T(\mu)$ then the implicit subsidy is negative (the government taxes hiring).

We do not consider the revenue consequences of the tax or subsidy. This approach implicitly ignores additional welfare effects caused by either raising additional tax revenue or by lowering other distortionary tax rates. Instead, we assume the government either has access to the necessary funds or can tax or transfer funds to agents lump sum. As mentioned above, we judge policies based on how they change the average flow of welfare in the steady state and so our analysis applies only if firms are sufficiently patient (i.e., only if β is sufficiently close to 1).

The inefficiency in the model arises because firms do not internalize the effects of their own labor hoarding on the quality in the pool of available workers. Firms hoard labor to avoid the costs of finding productive workers when demand is high. The cost of this behavior to the firm is the value of wages paid during periods of average or below average demand. Firms internalize both of these costs and benefits when they make their hiring and firing decisions. Holding productive workers off the market however, also imposes costs on other firms. By hoarding labor, firms restrict the supply of productive workers available for hire and thus indirectly increase the cost of hiring to other employers. Firms do not internalize this cost and thus there is room for a corrective tax or subsidy.

Not surprisingly, whether it is optimal to tax or to subsidize hiring depends on the relative strength of the hiring and hoarding effects. If the hiring effect is dominant, then a tax on hiring improves welfare. If the hoarding effect is dominant, then a hiring subsidy improves welfare. The following results summarize our main result in this section:

Proposition 7. *For β near 1, (i) in complete hoarding equilibria (with or without temps), or in the model with efficient temps, the marginal social benefit of a hiring subsidy (or hiring tax) is zero; (ii) in incomplete hoarding equilibria (with or without temps), a hiring subsidy will increase the steady state flow of welfare if there is a dominant hoarding effect in equilibrium (i.e., if $T_{\mu}(\mu) > 0$); a hiring tax will increase the steady state flow of welfare if there is a dominant hiring effect in equilibrium (i.e., if $T_{\mu}(\mu) < 0$).*

The intuition underlying Proposition 7 is somewhat subtle. When a firm hires workers, it imposes negative externalities on other firms by lowering the quality of the pool of available workers. Firms do not internalize this effect and as a result, locally, welfare would increase if firms reduced employment overall. Whether the government uses a tax or a subsidy depends on the strength of the hiring effect and the hoarding effect. If the hiring effect is dominant then a hiring tax will reduce employment and improve the quality of the pool of available workers. If the hoarding effect is dominant, a hiring subsidy causes firms to reduce employment because it lowers their incentive to retain workers. In either case, the optimal policy improves the pool of available workers by discouraging firms from hoarding.

5 Aggregate Dynamics

So far, we have been focused on properties of the adverse selection model and the effects of policy in steady states. In this section we consider the dynamic reaction of the model to aggregate shocks to labor demand (e.g., business cycle variation). To do this, we extend the model to allow for time variation in the endogenous variables μ_t, N_t and, in the model with temp workers, w_t^m and M_t. For simplicity, we maintain the assumption that the base wage w is constant over time and confine our attention to how aggregate shocks influence adverse selection and the demand for labor.

The precise dynamic setting we consider is as follows. We assume the economy begins in the stationary equilibrium described by Propositions 2 and 6. We then consider the equilibrium that arises following an unanticipated aggregate shock. The aggregate shock temporarily shifts aggregate labor demand causing changes in the endogenous variables μ_t, N_t, etc. To model this shock, we make a modest change to the firms' production functions. Specifically, production for firm i at time t is $A_t z_{it} F(n_{it})$ where z_{it} is the idiosyncratic shock for firm i and A_t is the aggregate shock common to all firms. While the initial change in A_t is unanticipated, subsequent time path of A_t is perfectly forecastable. In the initial period, productivity rises by 10 percent relative to its steady state value, $A_0 = 1.10$. For periods $t > 0$, A_t returns gradually to its steady state value $\bar{A} = 1$ according to $A_t = (1-\rho) + \rho A_{t-1}$ with $\rho < 1$. We then consider a perfect foresight equilibrium. Firms anticipate a time path of μ_t (and w_t^m in the model with temps) for $t = 0...\infty$. Based on the anticipated time paths for A_t and μ_t, firms optimally choose how many workers to hire and fire each period. In equilibrium, their decisions must be consistent with the anticipated time paths.

Because the dynamic model is substantially more complex than the stationary model, we use numerical techniques to analyze and quantify the equilibrium. The production function F is assumed to be quadratic: $F(n_{it}) = Bn_{it} - \frac{n_{it}^2}{2}$. Where B is a scaling parameter chosen to ensure that labor demand does not exceed our employment grid space. The idiosyncratic shock z is uniformly distributed on a grid with mean 1 and standard deviation 0.15. The time periods are assumed to be years. The firm's annual discount factor β is 0.95. The autoregressive root ρ for the aggregate shock is set to 0.70 annually. For the model with temp workers, we assume that the additional flow cost of temp workers is ten percent of the base wage (i.e., $x = 0.10$). We assume that the aggregate quality share ϕ is 0.95 and we choose total labor supply L to ensure that $\mu = 0.75$ in the steady state. The baseline parameters are summarized in Table 1. The remaining details of the numerical solution are in Appendix III.

Table 1: Model Parameters

Parameter	Value
Wage (w)	1.00
Quality in unemployment pool (μ)	0.75
Persistence of aggregate shock (ρ, annual)	0.70
Temp flow cost (x)	0.10
Discount factor (β, annual)	0.95
Average idiosyncratic productivity ($\bar{z}$)	1.00
Std dev. of idiosyncratic productivity (σ_z)	0.15

We consider four models: (i) the model without temps, (ii) the model with efficient temps ($x = 0$), (iii) the model with costly temps ($x > 0$) and (iv) a frictionless model with no adverse selection problem. Table 2 reports the steady state values of aggregate employment across the models. Comparing the frictionless case (column 1) with the adverse selection model without the option of temp workers (column 2) shows that the hoarding effect is strong enough to induce firms to employ more workers than they would in the frictionless model. Allowing firms to use temp workers reduces the incentive to hoard labor and causes aggregate employment to fall. In the costly temp case (column 3) employment again exceeds the efficient level. In the efficient temp case (column 4), total employment is below the efficient level given the greater cost of the temp workers.

Table 2: Steady State Values

Variable	Frictionless Model	Model without Temps	Costly Temps	Efficient Temps
Temp wage (w^m)	–	–	1.117	1.017
Aggregate employment ($N + M$)	0.668	0.701	0.689	0.651
Permanent workers (N)	0.668	0.701	0.601	0.000
Temp workers (M)	–	–	0.088	0.651

Note: We choose the total labor supply (L) and the fraction of productive workers (ϕ) to guarantee $\mu = 0.75$ across each simulation.

Figure 5 shows the impulse response functions for the aggregate shock. The figures depict reactions of each variable as a ratio of their steady state level. The upper left panel shows the shock. The top middle panel reports total employment for each of the four models. Not surprisingly, the model with the frictionless labor market responds most to the shock. Firms in this model quickly expand employment following the shock and then release the workers as the shock fades. In the models with adverse selection, the response is substantially more muted but also more persistent.

The endogeneity of the hiring cost in our model is clear in the figure. The upper right panel shows the quality (μ) in the unemployment pool following the shock. Because firms expand employment, they draw more productive workers out of the unemployment pool lowering the average quality of available workers and effectively making hiring more costly. This is in sharp contrast to a model with hiring or firing costs which are exogenous features of the environment.

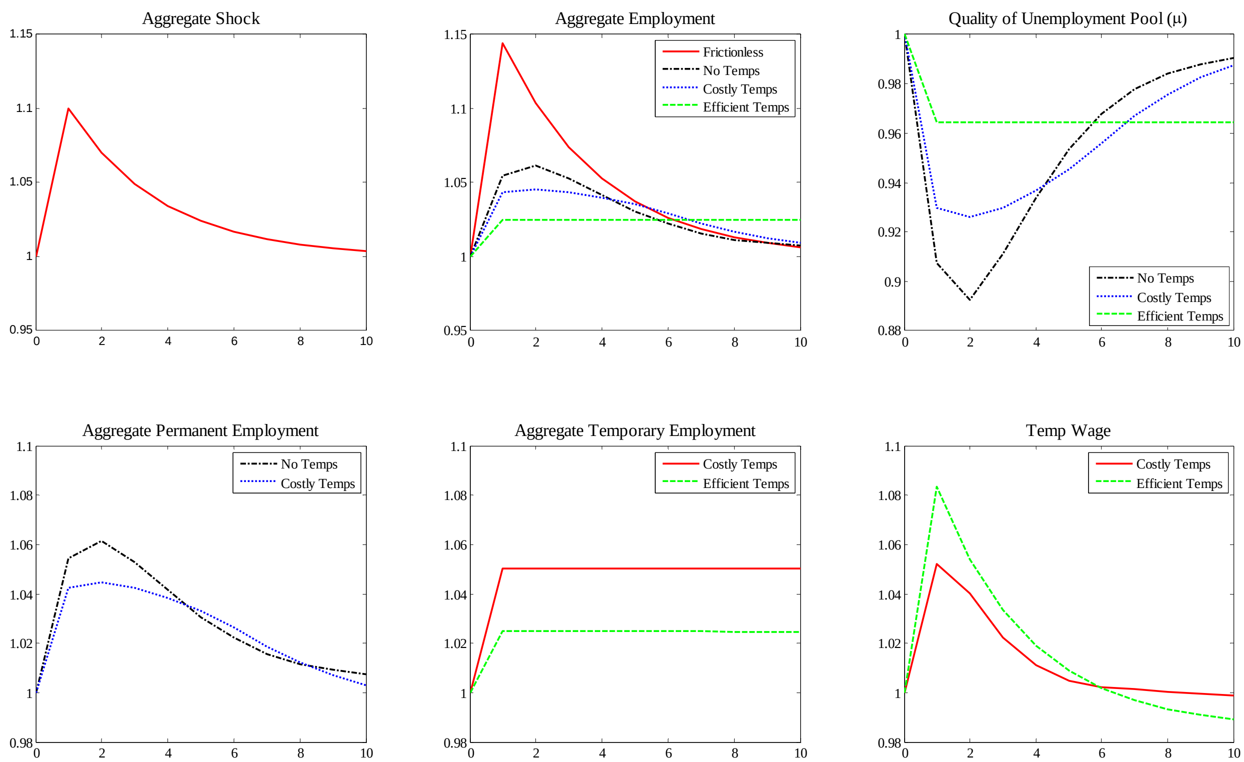
While adverse selection causes inefficiency and misallocation, the inefficiencies do not result in a more volatile market. Instead, adverse selection causes firms to under-react to the shock and thus the distorted market is inefficiently stable (insufficiently responsive to shocks). In part, this excessively stable labor market arises because the degree of adverse selection varies over the cycle. In particular, the adverse selection costs rise with labor demand. This phenomenon is undoubtedly a feature of real-world labor markets. In good times, firms hire and retain many good workers leaving few productive workers available overall. In bad times, firms release productive workers creating an opportunity for other firms that might want to expand employment.[8]

In the lower panels, we decompose the employment response into permanent workers and temp workers. Total employment of permanent workers rises more when temp workers are not available (in the model with efficient temps, firms never hire permanent workers in accordance with Proposition 5). The lower middle panel compares the model with efficient temps to the model with costly temps. Notice that, as a percent of steady state, temp employment rises more in the costly temp case. This is because there are fewer temps employed on average and because most firms expand employment by hiring temps rather than by expanding their stock of permanent workers.

Notice that the change in temp workers is permanent. Because of the search costs of finding productive workers, temp agencies hire only if the temp wage is high enough to cover these costs. If the temp wage is not sufficiently high however, the temp agency continues to

[8]See House (2006) for a similar "stabilizing" effect in the context of financial market distortions.

Figure 5: Response To An Aggregate Shock



Notes: The figure reports the responses of the variables as a ratio of their steady state values to an unanticipated shock at date 1. Time, in years, is on the horizontal axis.

supply the current stock of temp workers. The supply of temp workers would decline only if w^m fell below $w + x$ (see Figure 3). Thus, the aggregate adjustment of temp workers in our model is highly persistent as shown in the lower panels of Figure 5. An additional consequence of this persistence is that the temp wage remains lower than the initial equilibrium even after the shock has faded. Temps hiring thus permanently increases in the long run and firms can reduce their employment of permanent staff because of the greater availability of temp workers going forward.[9]

6 Discussion and Related Literature

Interpretation of the Temp Wage. In the model, the temp workers get wages that exceed the wages of the permanent workers. This may strike the reader as unrealistic since many temp workers are paid less than permanent staff. Two points are worth mentioning. First, the temp wage w^m includes both the wage payment to the temp worker and the additional fee paid to the temp agency. From the standpoint of the worker, the temp workers are paid no more than the permanent workers in the model.

Second, in our model there are no fundamental differences between temp workers and permanent workers. In reality, workers select into career tracks which determine whether they are a temp worker or a permanent worker. It would seem most likely that the most productive workers seek out permanent work while lower productivity workers provide work on a temporary basis. Indeed, one of the interpretations of the additional flow cost x in the model with costly temps is as costs arising because the temp workers are inadequately matched with the needs of the firm.

Hiring Subsidies in Practice. The Federal Government only rarely uses subsidies to new hiring as a policy instrument and when it does, the rationale is typically as a counter-cyclical measure. For instance, in January 2010, President Barack H. Obama proposed the *Small Business Jobs and Wages Tax Cut.* The purpose of the proposal was to "accelerate the pace of job growth by providing businesses – particularly America's small businesses – with a tax cut for putting more Americans back to work." While there were other provisions in the proposal, the center piece was to be a $5,000 tax credit for every net new employee that a business adds in 2010. Ultimately, the actual 2010 legislation did not include the proposed hiring subsidy but instead

[9]In the model, total employment of temps and permanent workers is bounded by the total supply of productive workers ϕL. In the numerical example, total employment is always below this limit.

temporarily reduced payroll taxes for all employees. (Thus, the final bill had features of an employment subsidy but not a *hiring* subsidy.)[10]

In 1977, the Federal government passed the *New Jobs Tax Credit* which provided for a marginal wage credit for new hires over and above the previous year's employment level. The tax credit lasted from 1977 to 1978 and was targeted more towards low income and part time workers rather than permanent staff. (See Perloff and Wachter (1979) for more details.)

Related Literature. Our paper connects a wide variety of research. First and foremost, the paper contributes to the interrelated study of adverse selection in labor markets and the role of temporary workers in the labor force. The study of adverse selection dates back to Akerlof (1970) and Wilson (1977, 1980).[11] Adverse selection in labor markets has been studied by, among others, Greenwald (1986), Gibbons and Katz (1991), Weiss (1991), Montgomery (1999), and Nakamura (2008). Kugler and Saint-Paul (2004) present a model studying worker flows in an adverse selection environment where firms face additional costs of hiring (beyond the adverse selection cost). In their model, firms can hire either unemployed workers or workers that are employed elsewhere. Their evidence indicates that as hiring costs rise, firms are more likely to hire currently employed workers. This suggests that adverse selection does indeed play a role in firms hiring decisions. Research on temporary workers has become more active recently though most of the work is empirical in nature. Prominent examples include Abraham and Taylor (1996), Autor (2001), Houseman (2001) and Segal and Sullivan (1997).[12] Ono and Sullivan (2006) present a model of labor demand for permanent and temporary workers which is sufficiently tractable to take to the data. While many of these papers mention adverse selection and screening costs as an important cost of hiring permanent workers and an important incentive for firms to use temps, none of these papers presents a model of temps arising because of the adverse selection problem. We should note that Autor (2009) explicitly points out that one of the functions of labor market intermediaries like temp agencies is to alleviate adverse selection problems. Altonji and Pierret (2001) and Arcidiacono et al. (2008) provide empirical evidence suggesting that asymmetric information about worker productivity

[10]For details on the Small Business Jobs and Wages Tax Cut see the Office of the Press Secretary 2010 and Congressional Budget Office 2010. Details of the 2010 stimulus act are in S.A. 4753: The Reid-McConnell Tax Relief, Unemployment Insurance Reauthorization and Job Creation Act of 2010 Amendment to H.R. 4853.

[11]See Stiglitz and Weiss (1981), Mankiw (1986), de Meza and Webb (1987), and Eisfeldt (2004) for equilibrium models of adverse selection in financial markets. House and Leahy (2004) present an analysis of adverse selection in (s, S) models. Hendel and Lizzeri (1999) analyze adverse selection in durable goods markets.

[12]See also Autor and Houseman (2006, 2010), Böheim and Cardoso (2009), Foote and Folta (2002), Heinrich et al. (2009) and Kvasnicka (2009).

is present in actual labor markets. Our paper adds to this literature by presenting, to our knowledge, the first equilibrium model of temp-employment in an adverse selection setting.

Our paper also relates to the literatures on labor hoarding (see Clark 1973, Fay and Medoff 1985, and Fair 1985) and wage cyclicality. Solon et al. (1994) show that because the composition of the labor force varies systematically over the business cycle (as firms release the lowest productivity workers first), measured average real wages appear to be less pro-cyclical. Our model implies a similar effect occurs in unobserved productivity. Similarly, Burnside et al. (1993) show that labor hoarding causes measured variations in total factor productivity to be substantially overstated. Wen (2005) analyzes the interaction between labor hoarding and firms' inventory management behavior.

Finally, our work contributes to the growing literature on (s, S) adjustment in labor markets (Bentolila and Bertola 1990). Recent contributions to this literature include King and Thomas (2006) and Elsby and Michaels (2013).[13] The paper by Elsby and Michaels deserves special mention. They analyze an equilibrium model of labor demand in which firms face kinked adjustment cost functions and display (s, S) behavior as in our model. In their model, the adjustment cost arises from search frictions. In addition to making a number of substantive contributions to the literature on costly labor adjustment and kinked-adjustment costs more broadly, Elsby and Michaels provide a number of technical contributions to the analysis of such models.

7 Conclusion

We have presented an equilibrium model of labor demand in the presence of adverse selection. Adverse selection raises the cost of hiring new productive workers and makes firms reluctant to vary their permanent workforce in response to transitory labor demand shocks. Unlike many other types of hiring costs, the adverse selection cost is endogenous. If the adverse selection problem is severe, then it is optimal to hoard labor. The labor hoarding in turn rationalizes the severity of the adverse selection problem. If the adverse selection problem is mild, then few firms hoard labor, rationalizing the low degree of adverse selection.

We then embed a market for temporary workers in the adverse selection model. Just

[13]The (s, S) model was initially developed by Arrow et al. (1951) to study inventory management and was later extended to the study of consumer durables by Grossman and Laroque (1990). See, for example, Bertola and Caballero (1990), Bar-Ilan and Blinder (1992), Caballero (1993), Eberly (1994), Carroll and Dunn (1997), Adda and Cooper (2000), Caplin and Leahy (1999), Leahy and Zeira (2005), and House and Leahy (2004).

as adverse selection in the pool of unemployed workers makes firms reluctant to vary their permanent workforce, it simultaneously creates a demand for temporary workers. The optimal hiring policy for the firms is to maintain a buffer stock of permanent workers and to use temps to accommodate transitory shocks to labor demand. The model predicts that, at the firm level, variation in temp workers should exceed the variation in permanent staff.

Not surprisingly, the equilibria are inefficient. The inefficiency arises because firms do not internalize the costs they impose on other firms by hoarding good workers in periods of low demand. The model suggests that allocations would be improved by a policy which discourages firms from employing too many workers. This policy could take the form of a hiring tax but could also take the form of a hiring subsidy if the incentive to hoard workers is sufficiently strong.

Finally, using a dynamic version of the model, we analyze how the market reacts to aggregate shocks to labor demand. Adverse selection causes the labor market to under-react to labor demand shocks. Shocks which cause firms to hire more workers lead to a reduction in the quality of productive workers in the pool of available labor. The reduction in quality reduces the incentive for other firms to hire. Similarly, during a recession, because many workers are laid off, there is a disproportionate increase in the supply of productive workers available and thus some firms find it attractive to hire. In the dynamic model, aggregate shocks also lead to variations in temp employment. Because temp agencies pay an upfront search cost to identify productive workers, aggregate shocks cause permanent changes in the number of temp workers supplied in equilibrium.

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Appendix I: Proofs of the Propositions

This appendix provides a sketch of the proofs of the propositions. Detailed proofs, as well as the proofs of the lemmas, are available in an online appendix or from the authors upon request.

Proof of Proposition 1: Since z is bounded, the payoff function $zF(n') - wn' - we(n' - n)$ is bounded. It is straightforward to show that the Bellman equation in (2) satisfies assumptions 9.4–9.7, 9.10 and 9.11 in Stokey, Lucas and Prescott [1989]. Theorems 9.6 and 9.8 imply that there exists a unique $v(n, z)$ which is continuous and concave. Thus, $V(n)$ is also continuous and concave in n.

To show differentiability, consider the set of functions $\mathcal{B}$ such that $b \in \mathcal{B}$ implies that $B(n) = \int b(n, z)\, dG(z)$ is differentiable and concave in n. Consider the mapping T defined by

$$r(n, z) = (Tb)(n, z) = \max_{n'} \left\{ zF(n') - wn' - we(n' - n) + \beta B(n') \right\}.$$

Since B is differentiable, maximizing the expression inside the brackets gives the optimal choice of $n'(n, z)$ as (5). The image of the function b under T is then,

$$r(n, z) = \begin{cases} zF(n_L(z)) - w\frac{1}{\mu}n_L(z) + wn\left(\frac{1}{\mu} - 1\right) + \beta B(n_L(z)) & \text{if } n < n_L(z) \\ zF(n) - wn + \beta B(n) & \text{if } n \in [n_L(z), n_H(z)] \\ zF(n_H(z)) - wn_H(z) + \beta B(n_H(z)) & \text{if } n > n_H(z) \end{cases}.$$

This function is clearly differentiable away from $n_L(z)$ and $n_H(z)$. It is also differentiable at these points, which can be verified by direct computation. Thus, $r(n, z)$ is differentiable in n for all z. Thus, $r \in \mathcal{B}$. Since the set $\mathcal{B}$ maps into itself, the unique solution is in $\mathcal{B}$. If it is in $\mathcal{B}$, $V(n) = \int v(n, z)\, dG(z)$ is differentiable and thus each small v is also differentiable as demonstrated above. Accordingly, the optimal policy function is given by (5), where $n_L(z)$ and $n_H(z)$ are given by (3) and (4). $\square$

Proof of Proposition 2: To prove the existence of equilibria, we first prove the continuity of the mapping T on $[0, 1]$. Clearly, T is continuous on $[0, \bar{\mu})$ and $(\bar{\mu}, 1]$, where $\bar{\mu}$, together with $\bar{n}$, is given in the proof to Lemma 3. To show that T is continuous at $\bar{\mu}$, it is sufficient to show that employment $N(\mu)$ is continuous at $\bar{\mu}$. For any $\mu > \bar{\mu}$, total employment is bounded by $n_H(0; \mu) \leq N(\mu) \leq n_L(Z; \mu)$. As μ approaches $\bar{\mu}$ from above, both $n_H(0; \mu)$ and $n_L(Z; \mu)$ converges to $\bar{n}$. This implies that $N(\mu)$ approaches $\bar{n}$. For any $\mu < \bar{\mu}$, each firm employs the same number of workers $n_L(Z; \mu)$. Thus, total employment is $N(\mu) = n_L(Z; \mu)$. As μ approaches $\bar{\mu}$ from below, $n_L(Z; \mu)$ converges to $\bar{n}$, which implies that $N(\mu)$ approaches $\bar{n}$. Finally, by definition, $N(\bar{\mu}) = \bar{n}$ and $T(\bar{\mu}) = (\phi L - \bar{n})/(L - \bar{n})$. Therefore, $N(\mu)$ and $T(\mu)$ are continuous at $\bar{\mu}$.

We next consider two case: $T(\bar{\mu}) > \bar{\mu}$ and $T(\bar{\mu}) \leq \bar{\mu}$. We first demonstrate the existence of the stationary equilibrium when $T(\bar{\mu}) \leq \bar{\mu}$. In this case there is one and only one stationary complete-hoading equilibrium in which all firms have the same employment level independent of productivity. When $\mu \leq \bar{\mu}$, $N(\mu) = n_L(Z; \mu)$, which monotonically and continuously increases with μ. Thus, $T(\mu)$ monotonically and continuously decreases with μ for any $\mu \leq \bar{\mu}$. Consider the continuous and decreasing function $s(\mu) = T(\mu) - \mu$ on $(0, \bar{\mu}]$. By assumption $s(\bar{\mu}) = T(\bar{\mu}) - \bar{\mu} \leq 0$. Since T is decreasing in μ in this range and since by assumption $T(\bar{\mu}) \leq \bar{\mu}$ it must be the case that $T(T(\bar{\mu})) \geq T(\bar{\mu})$ thus, $s(T(\bar{\mu})) = T(T(\bar{\mu})) - T(\bar{\mu}) \geq 0$. By the intermediate value theorem there is a point $\mu^* \in [T(\bar{\mu}), \bar{\mu}]$ such that $s(\mu^*) = 0$, which implies $T(\mu^*) = \mu^*$. Since $s(\mu)$ is strictly decreasing in μ this μ^* is unique. Thus, we have one and only one complete-hoarding equilibrium with μ^* between $\bar{\mu}$ and $T(\bar{\mu})$.

We next consider the case where $T(\bar{\mu}) > \bar{\mu}$. Notice that $T(\mu)$ monotonically decreases with μ for $\mu < \bar{\mu}$. Thus, for $\mu < \bar{\mu}$, $T(\mu) > T(\bar{\mu}) > \bar{\mu} > \mu$, which implies that there is no stationary complete-hoarding equilibrium. From the definition of the mapping $T(\mu)$, $\mu' = T(\mu)$ is bounded by ϕ on interval $(0, 1)$. Consider the interval $(\bar{\mu}, 1)$, we have $s(\bar{\mu}) = T(\bar{\mu}) - \bar{\mu} > 0$ and $s(\mu) = T(\mu) - \mu \leq \phi - 1 < 0$ as μ approaches 1. By the intermediate value theorem, there exists at least one solution μ^* such that $s(\mu^*) = 0$ since $s(\mu)$ is continuous on $(\bar{\mu}, 1)$. That is, we have at least one stationary equilibrium with $\mu^* \in [\bar{\mu}, 1)$. $\square$

Proof of Proposition 3: One sufficient condition for the uniqueness of the fixed point is that $T'(\mu) < 1$. Differentiation gives $T'(\mu) = -\frac{1-T(\mu)}{L-N(\mu)} N'(\mu)$. When $\mu < \bar{\mu}$, it is easy to show that $T'(\mu) < 0$. When $\mu \geq \bar{\mu}$, $N(\mu)$ can be written as

$$N(\mu) = \int_{n_H(0;\mu)}^{n_L(Z;\mu)} [1 - H(n;\mu)]\, dn = n_L(Z;\mu) - n_H(0;\mu) - \int_{n_H(0;\mu)}^{n_L(Z;\mu)} H(n;\mu)\, dn.$$

Differentiating with respect to μ gives $N'(\mu) = -\frac{\partial n_H(0;\mu)}{\partial \mu} - \int_{n_H(0;\mu)}^{n_L(Z;\mu)} \frac{\partial H(n;\mu)}{\partial \mu} dn$, where,

$$\frac{\partial H(n;\mu)}{\partial \mu} = \frac{g(z_L(n;\mu))\left(1 - H(n;\mu)\right) \frac{\partial z_L(n;\mu)}{\partial \mu} + g(z_H(n;\mu)) H(n;\mu) \frac{\partial z_H(n;\mu)}{\partial \mu}}{1 - [G(z_H(n;\mu)) - G(z_L(n;\mu))]}.$$

The proof of Lemma 1 provides expressions for $\frac{\partial z_H(n;\mu)}{\partial \mu}$ and $\frac{\partial z_L(n;\mu)}{\partial \mu}$. Using these together with (13) gives

$$\frac{\partial H(n;\mu)}{\partial \mu} < \frac{w}{\mu^2} \frac{\beta}{F_n(n)} \frac{g(z_L(n;\mu))\left(1 - H(n;\mu)\right)^2}{1 - \beta\left[G\left(z_H(n;\mu)\right) - G\left(z_L(n;\mu)\right)\right]} < \frac{w}{\mu^2} \frac{1}{F_n(n^*(Z))} \frac{\beta}{1-\beta} \bar{g}.$$

Plugging the above inequality into the expression for $N'(\mu)$ and using $\frac{\partial n_H(Z_l;\mu)}{\partial \mu} < 0$, gives

$$T'(\mu) < \frac{1 - T(\mu)}{L - N(\mu)} \frac{w}{\mu^2} \frac{n^*(Z)}{F_n(n^*(Z))} \frac{\beta}{1-\beta} \bar{g} < \frac{1}{L - n^*(Z)} \frac{w}{\hat{\mu}^2} \frac{n^*(Z)}{F_n(n^*(Z))} \frac{\beta}{1-\beta} \bar{g},$$

since μ is at least $\hat{\mu} = [\phi L - n^*(Z)] / [L - n^*(Z)]$. Then, $T'(\mu) < 1$ is guaranteed by Assumption 1.$\square$

Proof of Proposition 4: We start by illustrating that $\bar{n}(\mu)$ is well defined. For any μ, define $b(n) = \frac{w}{\mu} - \beta V'(n;\mu) - w^m$. $b(n)$ is strictly increasing with n for any $n < n_{\max}$, where $n_{\max}$ is defined as the largest support of the stationary distribution. Moreover, $b(0) = \frac{w}{\mu} - \beta V'(0;\mu) - w^m = \bar{w}^m - w^m \leq 0$, and $b(n)$ approaches $w/\mu - w^m > 0$ as n approaches $n_{\max}$. Thus, there exists one unique $\bar{n}(\mu)$ such that $b(\bar{n}(\mu)) = 0$, and in particular $\bar{n}(\mu) = 0$ when $w^m = \bar{w}^m$. The effective cost of hiring one additional permanent worker, $\frac{w}{\mu} - \beta V'(n;\mu)$, is at least as small as the cost of hiring a temporary worker, w^m, if and only if $n \leq \bar{n}(\mu)$.

Lemma 1 shows that $n_L(z;\mu)$ monotonically increases with z. Consider firms with z sufficiently small such that $n_L(z;\mu) \leq \bar{n}(\mu)$. By the definition of $n_L(z;\mu)$ and $\bar{n}(\mu)$, we have $F_n(n_L(z;\mu), z) + \beta V'(n_L(z;\mu);\mu) = \frac{w}{\mu}$ and $w^m + \beta V'(\bar{n}(\mu);\mu) = \frac{w}{\mu}$. Taking the difference of these two equations on both sides gives

$$F_n(n_L(z;\mu), z) - w^m = \beta V'(\bar{n}(\mu);\mu) - \beta V'(n_L(z;\mu);\mu) \leq 0.$$

This implies that $F_n(n_L(z;\mu), z) < w^m$ and $n_M(z;\mu) < n_L(z;\mu)$. Any firm with $n < n_L(z;\mu)$ hires permanent workers $n_L(z;\mu)$ and no temporary workers; the effective cost of hiring temps exceeds the effective cost of hiring permanent workers. Firms with $n_L(z;\mu) \leq n \leq n_H(z;\mu)$ maintain the employment of permanent workers at n; these firms also have no incentive to hire temps since $n > n_M(z;\mu)$. Firms with $n > n_H(z;\mu)$ reduce employment of permanent workers to $n_H(z;\mu)$, and they will not simultaneously hire a temp. This is because the cost of retaining a good worker w is less than the cost of hiring a temp w^m. Thus, the optimal policy is the same as the case without temps.

Consider firms with z such that $n_L(z;\mu) \geq \bar{n}(\mu)$. A firm with current employment $n < \bar{n}(\mu)$ increases permanent workers to $\bar{n}(\mu)$ and hires $n_M(z;\mu) - \bar{n}(\mu)$ temps. If the firm does not hire temps, then it hires permanent workers to $n_L(z;\mu)$. At this level of employment, however, $F_n(n_L(z;\mu), z) > w^m$ so the firm will deviate and hire temps. Since the firm must hire temps, it must be the case that $n'(n, z;\mu) = \bar{n}(\mu)$ and $m(n, z) = n_M(z) - \bar{n}(\mu)$. Using similar reasoning, firms with $\bar{n}(\mu) \leq n \leq n_M(z)$ also must hire temps.

However, in this range the effective cost of additional permanent workers $\frac{w}{\mu} - \beta V'(n;\mu)$ exceeds the marginal product w^m so the firm doesn't increase its permanent workforce (they also don't fire permanent workers). Thus, for such firms, $n'(n,z;\mu) = n$ and $m(n,z;\mu) = n_M(z;\mu) - n \geq 0$. Firms with $n_M(z;\mu) \leq n \leq n_H(z;\mu)$ maintain $n'(n,z;\mu) = n$ but hire no temps $m(n,z;\mu) = 0$. Firms with $n > n_H(z;\mu)$ reduce employment of permanent workers until $n'(n,z;\mu) = n_H(z;\mu)$ and do not simultaneously hire temps $m(n,z;\mu) = 0$. $\square$

Proof of Proposition 5:
1. According to Proposition 4, $\bar{n}(\mu)$ is defined as the solution to $\bar{w}^m = \frac{w}{\mu} - \beta V'(\bar{n}(\mu);\mu)$. Given the efficient temp wage $\bar{w}^m = (w - w\beta(1-\mu))/\mu$, we thus have $V'(\bar{n}(\mu)) = w(1-\mu)/\mu$. Since V' is monotonically decreasing on $[0, n_{\max}]$ and bounded above by $w(1-\mu)/\mu$. Thus, $\bar{n}(\mu) = 0$ under the efficient temp wage. That is, firms never hire permanent workers regardless of their shocks. In the equilibrium, the only workers employed are temporary workers.
2. Under the efficient temp wage, the total employment $N(\mu)$ is given by $N(\mu) = \int_0^Z n_M(z;\mu)\, dG(z)$, where $n_M(z;\mu)$ is the solution to $zF_n(n_M(z;\mu)) = \bar{w}^m = w\left(\frac{1-\beta}{\mu} + \beta\right)$ for any positive z, and $n_M(0;\mu) = 0$. Taking derivatives of $n_M(z;\mu)$ with respect to μ gives

$$\frac{\partial n_M(z;\mu)}{\partial \mu} = -\frac{1}{zF_{nn}(n_M(z;\mu))} \frac{w(1-\beta)}{\mu^2} > 0.$$

Thus, aggregate employment increases with μ: $N_\mu(\mu) = \int_0^Z \frac{\partial n_M(z;\mu)}{\partial \mu} dG(z) > 0$ which implies $T_\mu(\mu) < 0$ and thus the equilibrium is unique.
3. When μ approaches 1, we have the efficient level of employment. Given $N_\mu > 0$, it must be the case that aggregate employment is strictly less than the efficient level of employment for any $\mu < 1$. $\square$

Proof of Proposition 6: To show existence, we prove the continuity of $T(\mu)$ on $[0,1]$. The proof is similar to the proof of Proposition 2 (though somewhat more complicated) and is thus omitted. By definition, given the fixed point μ^*, $v(n,z;\mu^*)$, $m(n,z;\mu^*)$ and $n'(n,z;\mu^*)$ solve the firms' dynamic programming problems. By construction, the distribution $H(n;\mu^*)$ is implied by $m(\cdot)$ and $n'(\cdot)$. $\square$

Proof of Proposition 7:
i: Complete Hoarding. (a) In complete hoarding equilibria without temps, all firms have permanent workers $n_L(Z;\mu)$. Thus, steady-state flow welfare is $W(\mu) = (Ez)F(n_L(Z;\mu)) - wn_L(Z;\mu)$ and

$$\frac{dW(\mu)}{d\mu} = \left[(Ez)F_n(n_L(Z;\mu)) - w\right] \frac{\partial n_L(Z;\mu)}{\partial \mu}.$$

Lemma 1 shows that $\frac{\partial n_L(Z;\mu)}{\partial \mu} > 0$. The marginal shadow value of additional workers at $n_L(Z;\mu)$ satisfies $V'(n_L(Z;\mu))(1-\beta) = (Ez)F_n(n_L(Z;\mu)) - w$. Since $V' > 0$, $(Ez)F_n(n_L(Z;\mu)) - w > 0$. Thus, for $\beta < 1$, $\frac{dW(\mu)}{d\mu} > 0$. As $\beta \to 1$, $\frac{dW(\mu)}{d\mu} \to 0$.

(b) Now consider complete hoarding equilibria with temps. There are two cases: $\bar{n}(\mu) \geq n_L(Z;\mu)$ and $\bar{n}(\mu) \leq n_L(Z;\mu)$. When $\bar{n}(\mu) \geq n_L(Z;\mu)$, firms maintain permanent workers $n_L(Z;\mu)$ and hire no temporary workers. This is the same as the complete hoarding equilibria without temps.

When $\bar{n}(\mu) \leq n_L(Z;\mu)$, firms hire both temps and permanent workers though they never adjust their permanent staff. Permanent staff at each firm is $\bar{n}(\mu)$, total employment is $\max\{n_M(z;\mu), \bar{n}(\mu)\}$ and employment of temps is $m(z,n;\mu) = \max\{n_M(z;\mu) - \bar{n}(\mu), 0\}$. The total flow welfare is

$$W(\mu) = \int_0^{z_M(\bar{n}(\mu);\mu)} \left[zF(\bar{n}(\mu)) - w\bar{n}(\mu)\right] dG(z) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} \left[zF(n_M(z;\mu)) - (w+x)n_M(z;\mu) + x\bar{n}(\mu)\right] dG(z).$$

Differentiating with respect to μ, using the definition of $\bar{w}^m(\mu)$, $n_M(z_M(\bar{n};\mu);\mu) = \bar{n}$ and the fact that when

$n_M > 0$, $zF_n(n_M(z;\mu)) = \bar{w}^m(\mu) + x$ gives

$$\frac{dW(\mu)}{d\mu} = \int_0^{z_M(\bar{n}(\mu);\mu)} [zF_n(\bar{n}(\mu)) - w] \frac{d\bar{n}(\mu)}{d\mu} dG(z) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} \left[(1-\beta) w \left(\frac{1-\mu}{\mu} \right) \frac{\partial n_M(z;\mu)}{\partial \mu} + x \frac{d\bar{n}(\mu)}{d\mu} \right] dG(z).$$

Aggregate employment is $N(\mu) = \bar{n}(\mu) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} (n_M(z;\mu) - \bar{n}(\mu)) dG(z)$ and thus,

$$N_\mu(\mu) = \frac{d\bar{n}(\mu)}{d\mu} + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} \left(\frac{\partial n_M(z;\mu)}{\partial \mu} - \frac{d\bar{n}(\mu)}{d\mu} \right) dG(z).$$

The marginal shadow value $V'(\bar{n}(\mu);\mu)$ satisfies

$$V'(\bar{n}(\mu);\mu)(1-\beta) = \int_0^{z_M(\bar{n}(\mu);\mu)} [zF_n(\bar{n}(\mu)) - w] dG(z) + \int_{z_M(\bar{n}(\mu);\mu)}^{Z} \left[(1-\beta) w \left(\frac{1-\mu}{\mu} \right) + x \right] dG(z).$$

Combining these three equations gives

$$\frac{dW(\mu)}{d\mu} = (1-\beta) \left[w \left(\frac{1-\mu}{\mu} \right) N_\mu(\mu) + \frac{d\bar{n}(\mu)}{d\mu} \left(V'(\bar{n}(\mu);\mu) - w \left(\frac{1-\mu}{\mu} \right) \right) \right].$$

As $\beta \to 1$, $\frac{dW(\mu)}{d\mu} \to 0$.

(c) For efficient temp equilibria, welfare is $W(\mu) = \int_0^Z [zF(n_M(z;\mu)) - wn_M(z;\mu)] dG(z)$ and

$$\frac{dW(\mu)}{d\mu} = \int_0^Z [zF_n(n_M(z;\mu)) - w] \frac{\partial n_M(z;\mu)}{\partial \mu} dG(z) > 0$$

where the inequality holds because $\frac{\partial n_M(z;\mu)}{\partial \mu} > 0$ and $zF_n(n_M(z;\mu)) - w > 0$. When $\beta = 1$, $\bar{w}^m(\mu) = w$ and $zF_n(n_M(z;\mu)) - w = 0$ so $\frac{dW(\mu)}{d\mu} = 0$.

<u>ii: Incomplete Hoarding</u>. (a) With incomplete hoarding, $n_H(0;\mu) < \min\{\bar{n}(\mu), n_L(Z;\mu)\}$ and $H(n;\mu)$ is given by (23). Again there are two cases to consider.

Case 1: $\bar{n}(\mu) \geq n_L(Z;\mu)$. In this case no temporary workers are hired. This case is the same as the model without temps. Total welfare is $W(\mu) = \int_0^{n^{\max}(\mu)} w(n;\mu) dH(n;\mu)$, where

$$\begin{aligned} w(n;\mu) &= \int_0^{z_L(n;\mu)} [zF(n_H(z;\mu)) - wn_H(z;\mu)] dG(z) + \int_{z_L(n;\mu)}^{z_H(n;\mu)} [zF(n) - wn] dG(z) \\ &\quad + \int_{z_H(n;\mu)}^{\infty} [zF(n_L(z;\mu)) - wn_L(z;\mu)] dG(z) - w \frac{1-\mu}{\mu} \int_{z_H(n;\mu)}^{\infty} (n_L(z;\mu) - n) dG(z) \end{aligned}$$

or, using integration by parts, $W(\mu) = w(n^{\max}(\mu);\mu) - \int_0^{n^{\max}(\mu)} \frac{\partial w(n;\mu)}{\partial n} H(n;\mu) dn$. Differentiating gives

$$W_\mu(\mu) = \frac{\partial w(n^{\max};\mu)}{\partial \mu} - \int_0^{n^{\max}(\mu)} \frac{\partial^2 w(n;\mu)}{\partial n \partial \mu} H(n;\mu) dn - \int_0^{n^{\max}(\mu)} \frac{\partial w(n;\mu)}{\partial n} \frac{\partial H(n;\mu)}{\partial \mu} dn. \tag{24}$$

Consider each term in the expression. For the first term, differentiate $w(n;\mu)$ with respect to μ and evaluate at $n = n^{\max}$ to get

$$\frac{\partial w(n^{\max};\mu)}{\partial \mu} = \int_0^{z_L(n^{\max};\mu)} [zF_n(n_H(z;\mu)) - w] \frac{\partial n_H(z;\mu)}{\partial \mu} dG(z).$$

After a change of integration, (from z to n) we get

$$\frac{\partial w(n^{\max};\mu)}{\partial \mu} = \int_0^{n^{\max}(\mu)} [z_L(n;\mu) F_n(n) - w] \frac{\partial n_H(z_L(n;\mu);\mu)}{\partial \mu} \frac{\partial z_L(n;\mu)}{\partial n} g(z_L(n;\mu)) dn.$$

Using expressions for $\frac{dn_H(z;\mu)}{d\mu}$, $\frac{dz_L(n;\mu)}{dn}$ and $1-H(n;\mu)$ and letting $\beta \to 1$ we obtain

$$\frac{\partial w\left(n^{\max};\mu\right)}{\partial \mu} = -\frac{w}{\mu^2}\int_0^{n^{\max}(\mu)} \frac{z_L\left(n;\mu\right)F_n\left(n\right)-w}{F_n(n)}\left(1-H\left(n;\mu\right)\right)g\left(z_L\left(n;\mu\right)\right)dn.$$

Now consider the second term in (24). Differentiating $w\left(n;\mu\right)$ with respect to n gives

$$\frac{\partial w\left(n;\mu\right)}{\partial n} = \int_{z_L(n;\mu)}^{z_H(n;\mu)} \left[zF_n\left(n\right)-w\right]dG\left(z\right) + w\frac{1-T\left(\mu\right)}{T\left(\mu\right)}\left[1-G\left(z_H\left(n;\mu\right)\right)\right]. \tag{25}$$

Further differentiating with respect to μ gives

$$\begin{aligned}\frac{\partial^2 w\left(n;\mu\right)}{\partial n\partial\mu} &= \frac{\partial z_H\left(n;\mu\right)}{\partial\mu}\left[z_H\left(n;\mu\right)F_n\left(n\right)-\frac{w}{T\left(\mu\right)}\right]g\left(z_H\left(n;\mu\right)\right)-\frac{\partial z_L\left(n;\mu\right)}{\partial\mu}\left[z_L\left(n;\mu\right)F_n\left(n\right)-w\right]g\left(z_L\left(n;\mu\right)\right)\\&\quad-\frac{w}{\left(T\left(\mu\right)\right)^2}T_\mu\left(\mu\right)\left[1-G\left(z_H\left(n;\mu\right)\right)\right].\end{aligned}$$

Using the definition of $z_H\left(n;\mu\right)$, and $z_L\left(n;\mu\right)$, evaluating $\frac{\partial^2 w(n;\mu)}{\partial n\partial\mu}$ at $T\left(\mu\right)=\mu$ and letting $\beta\to 1$, gives

$$\begin{aligned}\frac{\partial^2 w\left(n\right)}{\partial n\partial\mu} &= -\frac{\partial z_H\left(n\right)}{\partial\mu}\left[\int_{z_L(n)}^{z_H(n)}(zF_n(n)-w)dG(z)+\frac{w(1-T\left(\mu\right))}{T\left(\mu\right)}\left[1-G\left(z_H\left(n\right)\right)\right]\right]\frac{\left(1-H\left(n\right)\right)g\left(z_H\left(n\right)\right)}{1-G\left(z_H\left(n\right)\right)}\\&\quad+\frac{\partial z_L\left(n\right)}{\partial\mu}\left[\int_{z_L(n)}^{z_H(n)}(zF_n(n)-w)dG(z)+\frac{w(1-T\left(\mu\right))}{T\left(\mu\right)}\left[1-G\left(z_H\left(n\right)\right)\right]\right]\frac{H\left(n\right)}{G\left(z_L\left(n\right)\right)}g\left(z_L\left(n\right)\right)\\&\quad-\frac{w}{\left(T\left(\mu\right)\right)^2}T_\mu\left(\mu\right)\left[1-G\left(z_H\left(n\right)\right)\right].\end{aligned}$$

Finally, consider the third term in (24). Note that

$$\frac{\partial H\left(n\right)}{\partial\mu} = \left(\frac{\partial z_L\left(n\right)}{\partial\mu}\frac{g(z_L(n))}{G(z_L(n))}+\frac{\partial z_H\left(n\right)}{\partial\mu}\frac{g(z_H(n))}{1-G(z_H(n))}\right)H\left(n\right)\left(1-H\left(n\right)\right).$$

Using (25) we have

$$\begin{aligned}&\int_0^{n^{\max}}\frac{\partial w\left(n\right)}{\partial n}\frac{\partial H\left(n\right)}{\partial\mu}dn\\=&\int_0^{n^{\max}}\frac{\partial z_L\left(n\right)}{\partial\mu}\left[\int_{z_L(n)}^{z_H(n)}\left(zF_n\left(n\right)-w\right)dG(z)+\frac{w\left(1-T\left(\mu\right)\right)\left(1-G\left(z_H\left(n\right)\right)\right)}{T\left(\mu\right)}\right]\frac{H\left(n\right)\left(1-H\left(n\right)\right)g(z_L(n;\mu))}{G(z_L(n;\mu))}dn\\&+\int_0^{n^{\max}}\frac{\partial z_H\left(n\right)}{\partial\mu}\left[\int_{z_L(n)}^{z_H(n)}\left(zF_n\left(n\right)-w\right)dG\left(z\right)+\frac{w\left(1-T\left(\mu\right)\right)\left(1-G\left(z_H\left(n\right)\right)\right)}{T\left(\mu\right)}\right]\frac{H\left(n\right)\left(1-H\left(n\right)\right)g(z_H(n))}{1-G(z_H(n))}dn.\end{aligned}$$

Combining all three terms and letting $\beta\to 1$ gives (after some algebra),

$$W_\mu\left(\mu\right)=\frac{wT_\mu\left(\mu\right)}{\left(T\left(\mu\right)\right)^2}\int_0^{n^{\max}}\left[1-G\left(z_H\left(n\right)\right)\right]H\left(n\right)dn.$$

Case 2: $\bar{n}(\mu) < n_L(Z;\mu)$. In this case both permenant and temporary workers are hired. For $n < \bar{n}(\mu)$,

$$\begin{aligned} w(n;\mu) &= \int_0^{z_L(n;\mu)} [zF(n_H(z;\mu)) - wn_H(z;\mu)]\, dG(z) + \int_{z_L(n;\mu)}^{z_H(n;\mu)} [zF(n) - wn]\, dG(z) \\ &+ \int_{z_H(n;\mu)}^{z_M(\bar{n}(\mu);\mu)} [zF(n_L(z;\mu)) - wn_L(z;\mu)]\, dG(z) - w\frac{1-\mu}{\mu} \int_{z_H(n;\mu)}^{z_M(\bar{n}(\mu);\mu)} (n_L(z;\mu) - n)\, dG(z) \\ &+ \int_{z_M(\bar{n}(\mu);\mu)}^{\infty} [zF(n_M(z;\mu)) - w\bar{n}(\mu) - (w+x)\{n_M(z;\mu) - \bar{n}(\mu)\}]\, dG(z) \\ &- w\frac{1-\mu}{\mu} \int_{z_M(\bar{n}(\mu);\mu)}^{\infty} (\bar{n}(\mu) - n)\, dG(z). \end{aligned}$$

For $n = \bar{n}(\mu)$,

$$\begin{aligned} w(\bar{n}(\mu);\mu) &= \int_0^{z_L(\bar{n}(\mu);\mu)} [zF(n_H(z;\mu)) - wn_H(z;\mu)]\, dG(z) + \int_{z_L(\bar{n}(\mu);\mu)}^{z_M(\bar{n}(\mu);\mu)} [zF(\bar{n}(\mu)) - w\bar{n}(\mu)]\, dG(z) \\ &+ \int_{z_M(\bar{n}(\mu);\mu)}^{\infty} [zF(n_M(z;\mu)) - w\bar{n}(\mu) - (w+x)\{n_M(z;\mu) - \bar{n}(\mu)\}]\, dG(z). \end{aligned}$$

Total welfare is $W(\mu) = w(\bar{n}(\mu);\mu) - \int_0^{\bar{n}(\mu)} \frac{\partial w(n;\mu)}{\partial n} H(n;\mu)\, dn$. Differentiating gives

$$W_\mu(\mu) = \frac{\partial w(\bar{n}(\mu);\mu)}{\partial \mu} - \int_0^{\bar{n}(\mu)} \frac{\partial^2 w(n;\mu)}{\partial n \partial \mu} H(n;\mu)\, dn - \int_0^{\bar{n}(\mu)} \frac{\partial w(n;\mu)}{\partial n} \frac{\partial H(n;\mu)}{\partial \mu} dn.$$

Again there are three terms to consider. Following an approach similar to that used in Case 1, gives

$$W_\mu(\mu) = \frac{wT_\mu(\mu)}{(T(\mu))^2} \int_0^{n^{\max}} [1 - G(z_H(n))]\, H(n)\, dn.$$

This completes the proof. □

Appendix II: Deriving Equation (14)

Total employment can be written as

$$N(\mu) = \int_0^Z \eta(z;\mu)\, g(z)\, dz \tag{26}$$

where $\eta(z;\mu)$ is total steady state employment for all firms with productivity draw z.

$$\begin{aligned}
\eta(z;\mu) &= n_H(z;\mu) \lim_{s \searrow n_H(z;\mu)} (1 - H(n_H(z;\mu);\mu)) + n_L(z;\mu) \lim_{s \nearrow n_L(z;\mu)} H(s;\mu) + \int_{n_L(z;\mu)}^{n_H(z;\mu)} n dH(n;\mu) \\
&= n_H(z;\mu)(1 - H(n_H(z;\mu);\mu)) + n_L(z;\mu) \lim_{s \nearrow n_L(z;\mu)} H(s;\mu) + \int_{n_L(z;\mu)}^{n_H(z;\mu)} nh(n;\mu)\, d\mu.
\end{aligned}$$

Differentiating (26) with respect to μ gives:

$$\frac{\partial N(\mu)}{\partial \mu} = \int_0^Z \frac{\partial \eta(z;\mu)}{\partial \mu} g(z)\, dz$$

where,

$$\begin{aligned}
\frac{\partial \eta(z;\mu)}{\partial \mu} &= \frac{\partial n_H(z;\mu)}{\partial \mu}(1 - H(n_H(z;\mu);\mu)) + H(n_L(z;\mu);\mu)\frac{\partial n_L(z;\mu)}{\partial \mu} \\
&\quad + \frac{\partial H(n_L(z;\mu);\mu)}{\partial \mu} n_L(z;\mu) - \frac{\partial H(n_H(z;\mu);\mu)}{\partial \mu} n_H(z;\mu) + \int_{n_L(z;\mu)}^{n_H(z;\mu)} n \frac{\partial h(n;\mu)}{\partial \mu} dn
\end{aligned}$$

which, using integration by parts, is

$$\frac{\partial \eta(z;\mu)}{\partial \mu} = \frac{\partial n_H(z;\mu)}{\partial \mu}(1 - H(n_H(z;\mu);\mu)) + H(n_L(z;\mu);\mu)\frac{\partial n_L(z;\mu)}{\partial \mu} - \int_{n_L(z;\mu)}^{n_H(z;\mu)} \frac{\partial H(n;\mu)}{\partial \mu} dn$$

The first two terms in this equation show how employment for firms with shock z changes due to changes in the firms' optimal behavior holding the distribution H constant. That is, these terms quantify the changes in employment caused by changes in $n_H(z;\mu)$ and $n_L(z;\mu)$. The last term shows how employment (for firms with shock z) changes due to changes in the distribution H holding their policy fixed (i.e., holding the cutoff-triggers $n_L(z;\mu)$ and $n_H(z;\mu)$ constant). Thus, the first two terms are a "direct effect" while the last term represents a "feedback" effect. Recall that the cutoffs $n_L(z;\mu)$ and $n_H(z;\mu)$ are given by

$$F_n(n_L(z;\mu), z) + \beta V'(n_L(z;\mu);\mu) = \frac{w}{\mu},$$

$$F_n(n_H(z;\mu), z) + \beta V'(n_H(z;\mu);\mu) = w.$$

Differentiating with respect to μ gives

$$\begin{aligned}
\frac{\partial n_L(z;\mu)}{\partial \mu} &= -\left[zF_{nn}(n_L(z;\mu)) + \beta V''(n_L(z;\mu);\mu)\right]^{-1}\left[\frac{w}{\mu^2} + \beta \frac{\partial V'(n_L(z;\mu);\mu)}{\partial \mu}\right] \\
&= \frac{1}{\Lambda(z, n_L(z;\mu);\mu)}\left[\frac{w}{\mu^2} + \beta \frac{\partial V'(n_L(z;\mu);\mu)}{\partial \mu}\right]
\end{aligned}$$

$$\begin{aligned}
\frac{\partial n_H(z;\mu)}{\partial \mu} &= -\left[zF_{nn}(n_H(z;\mu)) + \beta V''(n_H(z;\mu);\mu)\right]^{-1} \beta \frac{\partial V'(n_H(z;\mu);\mu)}{\partial \mu} \\
&= \frac{1}{\Lambda(z, n_L(z;\mu);\mu)} \beta \frac{\partial V'(n_H(z;\mu);\mu)}{\partial \mu}
\end{aligned}$$

where $\Lambda(z, n; \mu)$ is

$$\Lambda(z, n; \mu) = -\left[zF_{nn}(n) + \beta V''(n; \mu)\right] > 0.$$

Combining these expressions in $\frac{\partial \eta(z;\mu)}{\partial \mu}$ we have

$$\frac{\partial \eta(z; \mu)}{\partial \mu} = \frac{(1 - H(n_H(z; \mu); \mu))}{\Lambda(z, n_L(z; \mu); \mu)} \beta \frac{\partial V'(n_H(z; \mu); \mu)}{\partial \mu} + \frac{H(n_L(z; \mu); \mu)}{\Lambda(z, n_L(z; \mu); \mu)} \left[\frac{w}{\mu^2} + \beta \frac{\partial V'(n_L(z; \mu); \mu)}{\partial \mu}\right] - \int_{n_L(z)}^{n_H(z)} \frac{\partial H(n)}{\partial \mu} dn$$

which is rearranged to give equation (16) in the text.

Appendix III: Computational Algorithm

In this appendix, we describe the numerical solution method used for calculating the non-stationary equilibria for the model with aggregate uncertainty described in Section 5.

First, consider the model without temporary workers. We assume the economy starts at date 0 with the invariant distribution H given by Proposition 2 and that the economy returns to the steady state in $T = 30$ periods. We then solve for the transition path associated with the aggregate shock sequence. The solution algorithm begins with an initial guess of $\{\mu_t^{(0)}\}_{t=0}^T$. Given any guess $\{\mu_t^{(j)}\}_{t=0}^T$ (here j denotes the j^{th} guess), we solve for the time-dependent value functions and associated policy functions for firms using standard value function iteration techniques with discrete grid spaces for employment n and the idiosyncratic shock z. The policy functions, together with the initial distribution H_0, generate subsequent employment distributions $\{H_t^{(j)}\}_{t=1}^T$. For each period, we sum across firms to compute aggregate employment $\{N_t^{(j)}\}_{t=0}^T$. The sequence of aggregate employment implies a new sequence $\{\tilde{\mu}_t^{(j)}\}_{t=0}^T$. The updated guess is

$$\mu_t^{(j+1)} = \zeta \mu_t^{(j)} + (1 - \zeta)\, \tilde{\mu}_t^{(j)}, \text{ for } t = 1...T. \tag{27}$$

Here $\zeta \in (0, 1)$ is an attenuation parameter. We repeat this procedure to convergence. Our convergence criterion is $\max\{|\mu_t^{(j)} - \tilde{\mu}_t^{(j)}|\}_{t=0}^T \leq \mu \times 10^{-4}$.

Now consider the model with temporary workers. We again assume that the economy starts at date 0 with the invariant distribution H given by Proposition 6 and that the economy returns to the steady state after $T = 30$ periods. The algorithm uses two primary nested loops. In the outer loop, we iterate over the sequence $\{\mu_t\}_{t=0}^T$. In the inner loop, we iterate over the sequence of temp wages $\{w_{mt}\}_{t=0}^T$. The calculation for the inner loop proceeds as follows. Given any guess $\{\mu_t^{(k)}\}_{t=0}^T$ and a guess $\{w_{mt}^{(j)}\}_{t=0}^T$, we solve the firms' dynamic programming problems using value function iteration. The time-dependent policy functions, together with the initial distribution H_0, generate employment distributions $\{H_t^{(j)}\}_{t=1}^T$. For each period, we sum across firms to compute total demand for permanent workers $\{N_t^{(j)}\}_{t=0}^T$ and total demand for temp workers $\{M_t^{D,(j)}\}_{t=0}^T$. Given $\{\mu_t^{(k)}\}_{t=0}^T$ and $\{w_{mt}^{(j)}\}_{t=0}^T$, we solve the temp agency's dynamic programming problem using value function iteration. The policy functions for the temp agency implies a supply of temp workers $\{M_t^{S,(j)}\}_{t=0}^T$. We update the guess of the temp wage according to

$$w_{mt}^{(j+1)} = w_{mt}^{(j)} + \zeta^m \left(M_t^{D,(j)} - M_t^{S,(j)} \right), \text{ for } t = 1...T, \tag{28}$$

where $\zeta^m > 0$ is an adjustment parameter. We repeat this procedure to convergence. Our convergence criterion is $\max\{|M_t^{D,(j)} - M_t^{S,(j)}|\}_{t=0}^T \leq M \times 10^{-4}$. Once we have the sequence $\{w_{mt}\}_{t=0}^T$ associated with the sequence $\{\mu_t^{(k)}\}_{t=0}^T$, we use the associated paths for temps and permanent workers $\{M_t, N_t\}_{t=0}^T$ to calculate a new sequence $\{\tilde{\mu}_t^{(k)}\}_{t=0}^T$. We then construct a new guess $\{\mu_t^{(k+1)}\}_{t=0}^T$ according to (27). We repeat this procedure to convergence with the same convergence criteria as above.