Adverse Selection and the Financial Accelerator

Christopher L. House [a, *]

University of Michigan

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Abstract

Many economists believe that credit market distortions create a financial accelerator which destabilizes the economy. This paper shows that when credit market distortions arise from adverse selection they sometimes stabilize the economy rather than destabilize it. The stabilizing forces are closely related to forces that cause overinvestment in static models. When investment projects are equity financed, or when contracts are written optimally, the distortions always stabilize the economy. Thus, stabilizing equilibria are a robust feature of the model. The empirical distinction between accelerator and stabilizer equilibria is subtle. Many empirical tests are unable to distinguish between accelerator and stabilizer equilibria.

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[a] Corresponding author. *Christopher L. House, Department of Economics, University of Michigan, 238 Lorch Hall, Ann Arbor, MI 48109-1220, USA.* Tel. + 1 734 764 2364; fax: + 1 734 764 2769. *E-mail address:* chouse@umich.edu

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1. Introduction

The financial accelerator hypothesis says that credit market distortions magnify economic shocks. Disturbances that would be small if markets were efficient are exaggerated and prolonged due to imperfections in credit and loan markets. In other words, credit market distortions destabilize the economy. The financial accelerator hypothesis is important because standard business cycle models require large, persistent disturbances to mimic the business cycles observed in the data. Because the financial accelerator amplifies and propagates shocks, it can potentially explain why business cycles are so significant even though the observed shocks are not.

This article uses an adverse selection model to reexamine the relationship between credit market imperfections and economic instability. The adverse selection problem distorts loan markets in a dynamic equilibrium model of business cycle fluctuations. The model has three central messages. First, while in some cases the distortions destabilize the economy, in others they cause the economy to be excessively stable. Stabilizing outcomes in the dynamic model are closely related to overinvestment outcomes in static models of credit market failure. Second, if investments are equity-financed or if borrowers and lenders write optimal contracts, the only equilibria that emerge are "stabilizer equilibria". Third, in the adverse selection model, the empirical distinction between accelerators and stabilizers is surprisingly subtle. Many statistics seen as evidence in support of a financial accelerator are consistent with stabilizer equilibria.

An important contribution of the paper is that it decomposes the total amplification caused by credit market distortions into separate channels. Because each channel, or effect, has economic meaning, the decomposition clarifies the way that credit market distortions amplify shocks. Specifically, in the adverse selection model, a shock that increases internal funds has three separate effects on investment. First, the increase in internal funds causes the premium on borrowed funds to fall. With a lower premium, investors have a greater incentive to invest. This is the "agency cost" channel emphasized by much of the existing literature.

Second, since borrowers internalize more of the costs and benefits of their projects when their net worth is higher, the level of investment is closer to the efficient level. This causes investment to increase in some settings; in others, however, investment may fall. This second effect is the dynamic analog of over- or underinvestment in static adverse selection models. If there is underinvestment in the static environment, investment will rise when internal funds increase. In the dynamic model, this causes shocks to be amplified. If there is overinvestment in the static environment, investment falls when internal funds increase. Consequently, the financial market imperfections mitigate shocks in the associated dynamic model. This duality between stabilization in dynamic models and overinvestment in static models has not been pointed out by the existing literature.

Finally, the allocation of investment becomes more efficient when internal funds rise. Investment increases for projects with high expected returns and falls for projects with low expected returns. Thus, even if the total volume of investment is unchanged, shocks are amplified because investment is allocated more appropriately.

The total effect on investment is the sum of these three effects. It some instances, the second effect is negative and dominates the other two effects. In such cases, the adverse selection problem inefficiently stabilizes the economy.

While the basic model can exhibit either accelerator or stabilizer equilibria, the accelerator equilibria are not robust to other forms of financing. Specifically, when firms use either equity financing or optimal contracting, the only equilibria that emerge are stabilizer equilibria. Furthermore, the stabilizer equilibria do not have overtly counterfactual implications (such as a procyclical interest rate spread). Many well-known empirical findings that have been cited as evidence for a financial accelerator are consistent with the stabilizer equilibria in this model. This suggests that stabilizing outcomes are not mere curiosities.

To understand the dynamic adverse selection model, it is easiest to first examine two static adverse selection environments. These static models are the building blocks for the dynamic model analyzed later.

2. Static Models of Adverse Selection in Credit Markets

In general, credit market imperfections can result in either overinvestment or underinvestment. In the adverse selection model, over- or underinvestment is determined by the distribution of investment opportunities. The basic intuition can be seen by comparing two polar cases: the Stiglitz and Weiss (1981) model (hereafter 'SW') and the De Meza and Webb (1987) model (hereafter 'DW'). This section contains a brief analysis of these simple static models since they provide important insights into the dynamic model analyzed later.

2.1. Basic Setup

Several features of the model are common to both SW and DW. The model is a two-period world in which entrepreneurs interact with savers. In the first period, entrepreneurs that choose to do so, borrow and invest. In the second period, investment outcomes are realized, entrepreneurs pay back their debts (if they can) and consume whatever is left over. Normalize the number of entrepreneurs to be 1 and assume that savers supply $S > 1$ inelastically.

The savers have a safe outside option that yields a gross rate of return $\bar{\rho} > 0$. Competition ensures that the rate of return in the credit market will also be $\bar{\rho}$. Let R be the rate of interest charged to the entrepreneurs. The only difference between $\bar{\rho}$ and R is the default rate Δ. Thus, Δ is the interest rate spread and $\bar{\rho} = R[1 - \Delta]$.

Entrepreneurs are risk neutral and care only about consumption in the second period. Each entrepreneur has a project which either succeeds or fails. There are three numbers associated with each project: the probability of success p, the payoff in the event of success x, and the expected payoff $r = px$. The distribution of projects can be described by a joint distribution f over any two of these numbers since the third is redundant. Importantly, entrepreneurs know the characteristics of their projects while savers do not. This is the source of asymmetric information in the model. In addition, borrowers have limited liability. If a project fails, the lenders cannot extract further payments from the entrepreneur.

Activating a project requires an investment of one unit in the first period. Entrepreneurs have personal, "internal" funds $w < 1$ so they cannot completely self-finance. If an entrepreneur wants to activate his project, he must borrow $(1 - w)$ "external" funds in the credit market. For the time being, all credit market interactions are assumed to be described by standard debt contracts. Equity financing and optimal contracting are discussed later.

An increase in the risky interest rate R discourages some entrepreneurs from investing. Consequently, when R changes, the pool of borrowers changes. It could be the case that relatively more safe borrowers leave the loan pool so that Δ increases as R increases. Alternatively, if more risky borrowers leave, then Δ falls as R rises. Clearly the equilibrium depends critically on the distribution of projects. It is along this dimension that SW and DW differ.

2.2. Stiglitz and Weiss Go to the Bank

In Stiglitz and Weiss (1981), all of the projects have the same expected return r but differ in their success probability p. The expected return to an entrepreneur from activating his project is $r - pR(1-w)$. This is decreasing in p: payoffs are higher if the agent has a riskier project. Thus, there is a cutoff probability $\hat{p}$ such that all agents with $p < \hat{p}$ apply for credit. If there are any entrepreneurs in the loan pool, they are from the "risky tail" of the distribution as in Figure 1.A. Since the entrepreneurs have the option to save w at the safe rate $\bar{\rho}$, the cutoff probability $\hat{p}$ solves $w\bar{\rho} = r - \hat{p}R(1-w)$.

Typically, there is underinvestment in the SW model.[1] The reason for the underinvestment is simple. In equilibrium,

$$\bar{\rho} = R(1-\Delta) = \frac{R\int_0^{\hat{p}} pf(p)dp}{F(\hat{p})}.$$

Consider the entrepreneur at the cutoff $\hat{p}$. This is the safest project in the loan pool. If lenders could identify this entrepreneur, they would offer him a low interest rate since it is relatively unlikely that he will default. Unfortunately, this project is indistinguishable from the other riskier projects and thus gets the high interest rate that is appropriate for the average riskiness of the loan pool. As a result, the marginal entrepreneur's incentive to invest is too low.

Notice that investment rises as the level of internal funds increases. Because $\hat{p}R > \bar{\rho}$, the cutoff probability $\hat{p}$ is increasing in w ($\frac{\partial \hat{p}}{\partial w} = \frac{\hat{p}R - \bar{\rho}}{R(1-w)} > 0$). Since total investment consists of all entrepreneurs with $p < \hat{p}$, investment increases with w.

To summarize, *in the SW model there is underinvestment in equilibrium. Only the riskiest projects are undertaken. Furthermore, increases in internal funds increase investment.*

2.3. De Meza and Webb Go to the Bank

At the other extreme is the distribution considered by De Meza and Webb (1987). Here, all of the projects have the same actual outcome x if they succeed. As in the SW model, projects differ in their probability of success p. The expected

[1] If $r < \bar{\rho}$ then zero investment is optimal and this is the case (if $\hat{p} > 0$ then $\bar{\rho} \leq \hat{p}R$ implies $\bar{\rho}(1-w) \leq r - w\bar{\rho} \Rightarrow \bar{\rho} \leq r$, which contradicts $\bar{\rho} > r$). If $r > \bar{\rho}$ then let $\mu_p = \int_0^1 pf(p)dp$ and $r^* = \bar{\rho} + (1-w)\bar{\rho}\frac{1-\mu_p}{\mu_p}$. If $r > r^*$ then $\hat{p} = 1$ and all projects are activated (which is optimal since $r > \bar{\rho}$). If $\bar{\rho} < r < r^*$ then $0 < \hat{p} < 1$ and there is underinvestment.

payoff to an entrepreneur who activates his project is $p\left[x - R(1-w)\right]$, which is increasing in p. Again there is a cutoff $\hat{p}$ but now the projects that are activated are those with $p \geq \hat{p}$ – i.e., the "safe tail." See Figure 1.B. The cutoff satisfies $\hat{p}\left[x - R(1-w)\right] = w\bar{\rho}$.

In equilibrium,

$$\bar{\rho} = R\frac{\int_{\hat{p}}^{1} pf(p)dp}{1 - F(\hat{p})}.$$

It is easy to show that $\bar{\rho} \geq \hat{p}x$ so the expected return on the marginal project is below its social opportunity cost. Thus, in DW there is overinvestment. The intuition is similar: the marginal project is the riskiest project and should get a high interest rate. This entrepreneur cannot be distinguished from the safer projects and thus gets an inefficiently low interest rate. Consequently, the incentive to invest for the projects at the margin is too high.

Unlike the SW model, increases in internal funds reduce investment. Again, $\hat{p}$ is increasing in w but since investment consists of all entrepreneurs with $p > \hat{p}$, investment falls as w rises.

To summarize, *in the DW model there is overinvestment in equilibrium. Selection is toward the safer projects and increases in internal funds reduce investment.*

2.4. General Distributions

Not surprisingly, general distributions have mixed results. Nevertheless, it is useful to distinguish underinvestment equilibria from overinvestment equilibria.

Consider any joint density function $f(r,p)$. For any p, define $\hat{r}(p)$ as

$$\hat{r}(p) = \bar{\rho} + (1-w)\left[pR - \bar{\rho}\right]. \tag{1}$$

This is the cutoff expected return for projects that succeed with probability p. Any entrepreneur with a project (r,p) for which $r \geq \hat{r}(p)$ will find it profitable to invest. In equilibrium the diversified rate of return in the credit market must equal the safe rate of return.[2]

$$\bar{\rho} = R\left[1 - \Delta(R, \bar{\rho}, w)\right],$$

[2] There is no credit rationing in this model. The safe saving option pins down the rate of return in the credit market and effectively rules out rationing. For a formal proof of this see House (2004). As in Mankiw (1986), it is possible for the credit market to shut down completely. Throughout the paper, attention is restricted to equilibria with active credit markets.

where $\Delta(R, \bar{\rho}, w)$, the average default rate, is given by

$$\Delta(R, \bar{\rho}, w) = \frac{\int_0^1 \left\{ \int_{\hat{r}(p)}^{\infty} (1-p) f(r,p) dr \right\} dp}{\int_0^1 \left\{ \int_{\hat{r}(p)}^{\infty} f(r,p) dr \right\} dp}.$$

$\Delta(\cdot)$ depends on $R, \bar{\rho}$, and w because the cutoffs $\hat{r}(p)$ depend on those variables. One can view $\Delta(\cdot)$ as a function that captures the important features of the adverse selection problem.

Consider the change in output caused by increasing investment by some small amount $dI > 0$. The opportunity cost of activating a project is $\bar{\rho}$. At the margin, the expected payoff of a project is $\hat{r}(p)$. The net change in output is

$$\left[\int_0^1 \hat{r}(p) f(\hat{r}, p) dp - \bar{\rho} \int_0^1 f(\hat{r}, p) dp \right] dI.$$

Since $\hat{r}(p) - \bar{\rho} = (1-w)[pR - \bar{\rho}]$, the change in output can be expressed as:

$$\left\{ (1-w) \int_0^1 [pR - \bar{\rho}] f(\hat{r}, p) dp \right\} dI.$$

The derivative of $\Delta(\cdot)$ with respect to $\bar{\rho}$ is:

$$\frac{\partial \Delta}{\partial \bar{\rho}} = \frac{w}{IR} \left[\int_0^1 (pR - \bar{\rho}) f(\hat{r}(p), p) dp \right],$$

where total investment is $I \equiv \int_0^1 \left\{ \int_{\hat{r}(p)}^{\infty} f(r,p) dr \right\} dp > 0$. Consequently, the sign of $\frac{\partial \Delta}{\partial \bar{\rho}}$ distinguishes overinvestment models from underinvestment ones.[3] To understand the intuition for this result, first note that an increase in $\bar{\rho}$ always discourages investment ($\frac{\partial \hat{r}(p)}{\partial \bar{\rho}} = w \; \forall p$ so every cutoff increases). If $\frac{\partial \Delta}{\partial \bar{\rho}} > 0$, then, on average, the projects at the margin are safer than the average project in the loan pool. In this case, the average marginal entrepreneur is charged an interest rate that is too high and investment should be increased. If $\frac{\partial \Delta}{\partial \bar{\rho}} < 0$, then the projects at the margin are, as a group, riskier than the average project in the loan pool and are therefore getting inefficiently low interest rates. In this case, investment should be restrained. These effects are clear in the SW and DW models. In SW the marginal investors have the safest projects so an increase in $\bar{\rho}$ raises the default

[3] Mankiw (1986) derives a similar efficiency condition.

rate by chasing away relatively safe projects. In DW the marginal projects are the risky ones so increasing $\bar{\rho}$ lowers the default rate. Thus, in SW investment should be increased (there is underinvestment) while in DW investment should be reduced (there is overinvestment).

The derivative of $\Delta(\cdot)$ with respect to internal funds w is

$$\frac{\partial \Delta}{\partial w} = \frac{-1}{IR} \int_0^1 (Rp - \bar{\rho})^2 f(\hat{r}(p), p) dp < 0.$$

Thus, $\frac{\partial \Delta}{\partial w} < 0$ for any distribution f. This observation has two implications. First, because it reduces the default rate, an increase in internal funds reduces the interest rate spread. This is important since it is well known that interest rate spreads are strongly countercyclical. Second, it can be shown that, in general, a change in investment improves efficiency only if the loan pool becomes safer. Because increases in internal funds always make the loan pool safer, they necessarily improve efficiency. The increase in efficiency arises because the absolute difference between the equilibrium cutoffs $\hat{r}(p)$ and the efficient cutoff $\bar{\rho}$ is decreasing in the level of internal funds w. As w increases, $\hat{r}(p) \to \bar{\rho}$ for every p and the allocation of funds becomes more and more efficient. If internal funds increase to the point where entrepreneurs can completely self-finance, the equilibrium cutoffs would be $\hat{r}(p) = \bar{\rho}$ and the market would be efficient.

3. Adverse Selection and Credit Market Dynamics

This section presents a dynamic model of credit market failure. The dynamic model has several features designed to make the analysis transparent and to allow for a direct comparison with the static models presented above. An advantage of the model is that it permits a decomposition of the dynamic effects of the adverse selection problem into components that have economic meaning. Specifically, an increase in internal funds has three separate effects on dynamics: it reduces the interest premium, which stimulates investment; it causes more efficient use of investment; and it causes the level of investment to move towards the efficient level.

3.1. Setup

The economy consists of overlapping generations of two-period-lived agents. Each generation has savers and entrepreneurs. As before, entrepreneurs are risk neutral and only value consumption in the second period of life. The number of entrepreneurs is normalized to 1. Entrepreneurs may invest in projects that, if successful, yield productive capital in the following period. Capital fully depreciates after use, so the payoff to having one unit of capital is just its marginal product. Each project requires an initial investment of one unit of the consumption good. Entrepreneurs inelastically supply one unit of labor in youth and receive wages w_t.[4]

All agents have access to a safe investment technology that yields $\bar{\rho}$ goods in period $t+1$ for each unit saved at time t. The safe savings technology does not produce capital. Rather it simply yields units of consumable output the period after the saving took place. Thus the entire capital stock comes from the market with the adverse selection problem. In a model with additional capital markets that are free of distortions, one would expect a significant amount of substitution between the two markets which would partially offset the dynamic effects of the credit market frictions. Entrepreneurs can either save their income w_t to get $w_t\bar{\rho}$ or they can borrow $(1-w_t)$ and finance their project. Attention is restricted to equilibria in which w is strictly less than 1 so that entrepreneurs cannot completely self finance.

The project distribution is $f(p,k)$, where k is the expected capital payoff and p is the probability of success. Consider a group of projects with the same probability of success p, i.e., a cross-section of the joint density f. The cutoff project $\hat{k}\left(p\right)$, for this group satisfies

$$\hat{k}_t(p) = \frac{1}{r_{t+1}}\left[\bar{\rho} + (1-w_t)\left\{pR_t - \bar{\rho}\right\}\right], \tag{2}$$

where r_{t+1} is the expected marginal product of capital in the next period. This is the dynamic version of equation (1) in the general static model. As before, there are different cutoffs for each $p \in [0,1]$. All entrepreneurs with projects (k,p) with $k > \hat{k}(p)$ demand funding. The efficient cutoffs are $\hat{k} = \frac{\bar{\rho}}{r}$ for all p, but these are

[4] Most of these features can be relaxed without changing the main results. Risk neutrality and two-period-lived agents allow me to focus on credit market distortions as distinct from risk aversion and learning. Risk aversion changes the steady state critical values but does not undo the basic logic of the results. Longer-lived agents would gradually reveal their type over time. In this case, younger firms would have severe adverse selection problems while established firms would not. While this is an interesting possibility, it is very complicated because the distribution of successful entrepreneurs would be an endogenous state variable. Full depreciation of capital makes credit market distortions quantitatively important. Durable capital would damp the effects of credit market distortions because a given change in investment would have a smaller effect on the capital stock.

not the equilibrium cutoffs. The critical values differ from the efficient cutoffs by an amount proportional to the amount of external financing. As internal funds increase, the cutoffs approach the efficient cutoffs. Projects that pull the average payoff up (i.e., those for which $pR_t > \bar{\rho}$) set a cutoff that is too high. Projects with expected returns less than $\bar{\rho}$ have cutoffs that are too low. Only projects for which $p = 1 - \Delta$ have $\hat{k} = \frac{\bar{\rho}}{r}$.

Consider again a cross-section of the distribution f for a given p. All projects for which $k > \hat{k}(p)$ are activated. By the law of large numbers, the contribution to the capital stock next period from this slice of the distribution is $\int_{\hat{k}(p)}^{\infty} kf(k,p)dk$. To get the total capital stock next period, simply integrate over p to get

$$K_{t+1} = \int_0^1 \int_{\hat{k}_t(p)}^{\infty} kf(k,p)dkdp. \tag{3}$$

Competitive firms produce output according to a Cobb-Douglas technology

$$Y_t = z_t K_t^{\alpha}, \tag{4}$$

where z_t is productivity at date t. Factor demands satisfy

$$w_t = (1-\alpha) z_t K_t^{\alpha}, \tag{5}$$

$$r_t = \alpha z_t K_t^{\alpha - 1}. \tag{6}$$

Finally, the no-arbitrage condition is

$$\bar{\rho} = R_t \left[1 - \Delta\left(w_t, r_{t+1}, \bar{\rho}, R_t\right)\right], \tag{7}$$

where

$$\Delta\left(w_t, r_{t+1}, \bar{\rho}, R_t\right) = \frac{\int_0^1 \int_{\hat{k}_t(p)}^{\infty} (1-p) f(k,p)dkdp}{\int_0^1 \int_{\hat{k}_t(p)}^{\infty} f(k,p)dkdp}. \tag{8}$$

Again, Δ depends on $w_t, r_{t+1}, \bar{\rho}$ and R_t because the cutoffs $\hat{k}_t(p)$ depend on these variables.

3.2. Dynamics

Productivity shocks generate dynamics in the model. Assume that z_t follows the AR(1) process

$$z_t = (1-q)z + qz_{t-1} + \eta_t, \tag{9}$$

where q is the autoregressive root and η_t is i.i.d.

Equations (3), (5), (6), (7), and (9) govern the behavior of the model. An approximate solution can be computed with log-linear approximations to these equations in the neighborhood of a (stable) steady state characterized by constant K, w, r, and Δ.[5] Linearizing (3) yields

$$\tilde{K}_{t+1} = \frac{-1}{K} \int_0^1 \hat{k}(p) f(\hat{k}(p), p) \left[R \frac{\partial \hat{k}}{\partial R_t} \tilde{R}_t + r \frac{\partial \hat{k}}{\partial r_{t+1}} \tilde{r}_{t+1} + w \frac{\partial \hat{k}}{\partial w_t} \tilde{w}_t \right] dp, \tag{10}$$

where $\tilde{v}$ denotes the percent deviation of v from its steady state value. Note that because $\bar{\rho}$ is constant, the linear version of (7) is

$$\tilde{R}_t = \frac{w \frac{\partial \Delta}{\partial w} \tilde{w}_t + r \frac{\partial \Delta}{\partial r} \tilde{r}_{t+1}}{\rho'}, \tag{11}$$

where $\rho' \equiv 1 - \Delta - \frac{\partial \Delta}{\partial R} R$ is the derivative of $R[1 - \Delta(\cdot)]$ with respect to R. In equilibrium ρ' must be positive, otherwise lenders could reduce R and increase their return per dollar without reducing demand. Finally,

$$\tilde{w}_t = \tilde{z}_t + \alpha \tilde{K}_t, \tag{12}$$

$$\tilde{r}_t = \tilde{z}_t + (\alpha - 1) \tilde{K}_t, \tag{13}$$

$$\tilde{z}_t = q \tilde{z}_{t-1} + \eta_t. \tag{14}$$

Equations (10), (11), (12), (13), and (14) characterize the local dynamics of the system.

Without the adverse selection problem, investment would respond only to anticipated changes in the future marginal product of capital r_{t+1}. Apart from being an indicator of future productivity, investment would not react to variations in internal funds. Let $\left.\frac{\partial I}{\partial r_{t+1}}\right|_{\text{PI}}$ denote the change in investment that would occur under perfect information. One can show that, in the presence of adverse selection,

[5] Existence of a steady state is not guaranteed. w, r, and $\Delta(.)$ are continuous in K. However, the equilibrium R is the minimum R such that $\bar{\rho} = R[1 - \Delta(\cdot)]$. In general, this R is not continuous in K. In addition, the mapping $K \to K'$ may have downward jumps; thus existence arguments using continuity or monotonicity do not work. The existence of a steady state is taken as an assumption for the purposes of this analysis,

the change in the capital stock satisfies the following equation:

$$
\begin{aligned}
\tilde{K}_{t+1} \quad = \quad & \underbrace{\frac{\bar{\rho}}{K} \cdot \left.\frac{\partial I}{\partial r_{t+1}}\right|_{\mathrm{PI}}}_{\text{“Perfect Information Dynamics”}} \cdot \tilde{r}_{t+1} \quad + \quad \underbrace{\frac{I}{K}\frac{\bar{\rho}}{r}\frac{w}{\rho'}\frac{\partial \Delta}{\partial w}\varepsilon_{IR}}_{\text{“Agency Cost Effect”}} \cdot \tilde{w}_t \qquad (15) \\
& + \underbrace{\frac{I}{K}\frac{\bar{\rho}}{r}R\frac{\partial \Delta}{\partial \bar{\rho}}}_{\text{“Investment Effect”}} \cdot \tilde{w}_t \quad - \underbrace{(1-w)\frac{I}{K}\frac{\bar{\rho}}{r}\frac{w}{\rho'}\frac{\partial \Delta}{\partial w}}_{\text{“Efficiency Gain”}} \cdot \tilde{w}_t \\
& \underbrace{+\frac{\bar{\rho} I}{K}\left[\frac{1}{\rho'}\frac{\partial \Delta}{\partial r}\left(\varepsilon_{IR} - (1-w)\right) + \frac{(1-w)}{w} R \frac{\partial \Delta}{\partial \bar{\rho}}\right]}_{\text{“Payoff Effect”}} \cdot \tilde{r}_{t+1}.
\end{aligned}
$$

Here, $I_t = \int_0^1 \int_{\hat{k}_t(p)}^{\infty} f(k,p)dkdp$ is total investment and $\varepsilon_{IR} < 0$ is the interest elasticity of investment demand. In the static models the function $\Delta(\cdot)$ captured the adverse selection effects. Not surprisingly, the dynamic system is also governed by the first order properties of this function.

Equation (15) shows that the total effect of a productivity shock can be decomposed into five components. The first term is the normal change in investment that occurs under perfect information. The other terms represent deviations from the full information path caused by adverse selection.

The second term in (15) is the *Agency Cost Effect*. Recall that, for any distribution, $\frac{\partial \Delta}{\partial w}$ is negative. As cash flow increases, the loan pool becomes safer and the premium on external finance falls. Because the interest elasticity of investment is negative, the sign of the coefficient is positive. For positive shocks, lower interest rates further stimulate investment. For negative shocks, interest rates rise causing investment to contract further. This channel, which always serves to amplify disturbances, captures the effect emphasized by Bernanke and Gertler (1989).[6] The magnitude of the agency cost effect depends on the absolute values of $\frac{\partial \Delta}{\partial w}$ and ε_{IR}. If investment is very sensitive to the interest rate and if the interest rate is very sensitive to internal funds, then the agency cost effect will contribute significantly to dynamics. There is no reason to believe, a priori, that the agency cost effect is large relative to the other effects. In fact, since empirical estimates of ε_{IR} are often low, the agency cost effect may not be very strong in reality.

[6] The decline in the premium is due to different reasons, however. In Bernanke and Gertler (1989), the premium falls because it is less likely for the bank to monitor so average monitoring costs fall. In the adverse selection model, the premium falls because risky firms leave the loan pool while safe ones enter.

The third term in (15) gives the direct effect of additional internal funds on investment holding R constant. This term is called the *Investment Effect* since it is the dynamic analog of the effect that internal funds had on investment in the static model. The sign of this term is the same as the sign of $\frac{\partial \Delta}{\partial \bar{\rho}}$. In the static models $\frac{\partial \Delta}{\partial \bar{\rho}}$ differentiates overinvestment equilibria from underinvestment equilibria. When there is underinvestment $\frac{\partial \Delta}{\partial \bar{\rho}} > 0$. When there is overinvestment $\frac{\partial \Delta}{\partial \bar{\rho}} < 0$. Thus for models like the SW model, this channel imparts additional accelerator dynamics to the system. In models like DW this effect causes entrepreneurs to reduce investment and has a stabilizing effect on the economy.

The fourth term, called the *Efficiency Gain*, quantifies the change in K_{t+1} due to changes in the composition of investment. More precisely, one can show that if the change in investment is zero (i.e., if $\frac{\partial I}{\partial w} = 0$), the elasticity of K_{t+1} with respect to w_t is given by the efficiency gain coefficient. Like the agency cost effect, the efficiency gain depends on $\frac{\partial \Delta}{\partial w}$. Since $\frac{\partial \Delta}{\partial w}$ is always negative, this effect is positive and amplifies shocks. Thus, even if there is no change in the level of investment, more internal funds amplify shocks by improving the allocation of investment.

The final term, the *Payoff Effect*, represents adverse selection effects due to expected changes in the future marginal product of capital. The net effect is determined by $\frac{\partial \Delta}{\partial r}$ and $\frac{\partial \Delta}{\partial \bar{\rho}}$. Like changes in w, changes in r affect dynamics by changing the level and composition of investment.

The total effect is the sum of these components. Previous models emphasize the agency cost channel which is always positive. In the adverse selection model, the other components in equation (15) may work to further magnify a shock or to dampen the effect of a shock. It is possible to have a stabilizing effect that is so strong that the overall adverse selection effect from a positive shock is negative. One example of such a case is a pure DW distribution (i.e., all projects have the same payoff x).[7] In this case, the impulse response functions for the capital stock and output are below those of the full information economy.

For illustration, Table 1 shows the model's reaction to a one percent productivity shock for two distributions of (p, k). One distribution corresponds to an accelerator equilibrium while the other is a stabilizer equilibrium. The table gives the effects of the shock on impact, i.e., the immediate response of the economy. The first two columns show the impact effects under symmetric information. In each case, K increases while the risky interest rate R is unchanged.[8] The next

[7] It is not difficult to verify this mathematically; a proof is available from the author by request.

[8] If there is no adverse selection problem, then either there is only one type of project (and

two columns show the equilibrium with adverse selection. In the accelerator model the increase in K is twenty percent greater than without adverse selection. For the "stabilizer" model the increase in K is roughly twenty percent less. Because increases in internal funds always reduce the default rate, the interest rate spread is always countercyclical.

Table 1 also decomposes the change in K into the five components in (15). To calculate each component, the equilibrium realizations of $\tilde{w}_t$ and $\tilde{r}_t$ are multiplied by the coefficients in (15). The table also reports $\frac{\partial \Delta}{\partial \bar{\rho}}$, $\frac{\partial \Delta}{\partial w}$ and ε_{IR}. In the accelerator model $\frac{\partial \Delta}{\partial \bar{\rho}} > 0$ and the investment effect is positive. For the stabilizer model $\frac{\partial \Delta}{\partial \bar{\rho}} < 0$ and the investment effect is negative. This effect accounts for most of the difference in the behavior of the models.[9]

It is important to emphasize that the possibility of a financial stabilizer is general and does not depend on adverse selection per se. Two conditions are required for stabilization. First, market imperfections must be able to cause overinvestment. Second, the behavior of the entrepreneurs must "improve" when their net worth rises. Most existing financial accelerator models have the second property but not the first (costly-state-verification always causes underinvestment). As a result, these models always amplify shocks.

3.3. Equity Financing and Optimal Contracting

One objection to the results is that, so far, contracts have been restricted to standard debt contracts. Allowing agents to issue equity or to write optimal contracts changes the equilibrium. This subsection analyzes equity financing and optimal contracting in the adverse selection model.

3.3.1. Equity

Suppose entrepreneurs finance projects with equity rather than debt. Let π_t be the share price. The agent needs to raise $1 - w_t$ external funds to finance the project so he must sell $\zeta_t = \frac{1-w_t}{\pi_t}$ shares. The payoff to investing in a project

thus no selection is possible) or there are many different but observable, types. In the first instance $pR_t = \bar{\rho}$; in the second, there are separate interest rates for each project, $pR(p) = \bar{\rho}$. In either case, the risky interest rate(s) is constant.

[9] The reader will notice that the symmetric information response is different from the perfect information effect. The perfect information effect is calculated from the equilibrium path of $\tilde{r}_t$ generated by the adverse selection model. The symmetric information response is solved separately and has its own equilibrium path for $\tilde{r}_t$.

(k,p) is $r_{t+1}k\,[1-\zeta_t]$, which is independent of p. The payoff is clearly increasing in k so any $k>\hat{k}_t$ will issue shares. The critical $\hat{k}_t$ satisfies

$$r_{t+1}\hat{k}_t\left[1-\frac{1-w_t}{\pi_t}\right]=w_t\bar{\rho}. \tag{16}$$

The term in brackets must be positive otherwise no entrepreneur will invest. The efficient cutoffs are $\frac{\bar{\rho}}{r_{t+1}}$, which require $\pi_t=1$.

In equilibrium, the return on a diversified portfolio of stocks must be

$$\bar{\rho}=\frac{r_{t+1}}{\pi_t}\frac{\int_{\hat{k}}^{\infty}k\xi(k)dk}{1-\Xi(\hat{k})}. \tag{17}$$

Here $\xi(k)$, and $\Xi\,(k)$ are, respectively, the marginal density of k and the cumulative marginal distribution of k implied by $f\,(k,p)$, i.e., $\xi\,(k)=\int_0^1 f\,(k,p)\,dp$, and $\Xi\,(k)=\int_0^k\left\{\int_0^1 f\,(k,p)\,dp\right\}dk$.

If there is more than one type then (17) implies that $\bar{\rho}$ is greater than the return on the marginal project.

$$\bar{\rho}>\frac{r_{t+1}}{\pi_t}\hat{k}_t. \tag{18}$$

Using the definition of $\hat{k}_t$, (18) can be rearranged to get

$$\bar{\rho}\,[\pi_t-(1-w_t)]>w_t\bar{\rho}.$$

This inequality implies that $\pi_t>1$ and therefore $\hat{k}<\frac{\bar{\rho}}{r_{t+1}}$. Thus with equity financing there is always overinvestment in equilibrium. Intuitively, the marginal project $\hat{k}$ is the least productive project. Because the price of equity reflects the average return on investment, the equity issued by the marginal project is sold at a price that is too high. At the margin, projects are able to raise funds too easily.

Because it generates overinvestment in equilibrium, equity financing causes the model to be excessively stable compared with the symmetric information equilibrium. Differentiating (16) and using the inequality in (18), it is straightforward to show that

$$\frac{\partial\hat{k}_t}{\partial w_t}=\frac{\hat{k}_t}{w_t\rho}\left[\bar{\rho}-\frac{r_{t+1}\hat{k}_t}{\pi_t}\right]>0.$$

Given r_{t+1} and π_t, an increase in internal funds reduces investment. Thus with equity financing there is overinvestment in the steady state and the adverse selection equilibrium is more stable than the equilibrium with symmetric information.

3.3.2. Optimal Contracts

Neither equity nor standard debt contracts are optimal in this model. This section analyzes optimal contracts in the adverse selection model. Since project outcomes x are costlessly observable, an optimal contract will condition interest payments on the ex post realization of a project. Instead of repaying R if a project succeeds, the borrower will repay $R(x)$. In addition, banks may attempt to screen borrowers ex ante. Consider contracts of the form $\{R(x), c\}$ where $c \leq w$ is the entrepreneur's contribution to his own project.

Because there are separate contracts for each outcome, we can restrict attention to a single outcome x. For any x, the payoff to accepting contract $\{R, c\}$ is $p[x - R(1-c)] + \bar{\rho}(w-c)$. This payoff can be written as

$$px - \bar{\rho}(1-w) + (\bar{\rho} - pR)(1-c). \tag{19}$$

The payoff to not investing is simply $w\bar{\rho}$.

Entrepreneurs choose contracts that maximize their utility. An entrepreneur accepts a contract only if no other contract offers a higher payoff. Banks offer contracts that maximize their profits. Banks are competitive so every active contract must earn the safe rate of return $\bar{\rho}$ per unit lent.

While it is possible for banks to screen borrowers by offering contracts with differing contributions, this does not occur in the equilibrium. Instead, regardless of p, projects with the same x all get the same standard debt contract if they invest. The reason that banks do not offer contracts with different contributions c is that competitive banks always have an incentive to offer contracts with greater contributions. Because these contracts attract profitable borrowers, in equilibrium, all contracts simply require $c = w$. Since there is no variation in c, it cannot be used as a selection device.

To see this, first notice that if more than one type accepts contract $\{R, c\}$ in equilibrium then it must be that $c = w$. If this were not the case – i.e., if $c < w$ – then there is a feasible contract $\{R, c'\}$ with $c' > c$. Any entrepreneur for whom $pR > \bar{\rho}$ prefers $\{R, c'\}$ to $\{R, c\}$. Because more than one type applied, there are at least some p's for which $pR > \bar{\rho}$. Put differently, the alternative contract will attract all of the profitable loans from the original contract at the same interest rate. It would therefore be profitable for a bank to offer $\{R, c'\}$. Since deviations are inconsistent with equilibrium, if more than one type accepts $\{R, c\}$, then $c = w$.

To see that there is at most one active contract in equilibrium, suppose that there were two active contracts $\{R, c\}$ and $\{R', c'\}$. Without loss of generality let $R' < R$. If a single type applied at $\{R, c\}$, then $pR = \bar{\rho}$. This type will prefer $\{R', c'\}$ regardless of c'. If more than one type applies at $\{R, c\}$, then $c = w$. Because more than one type accepts $\{R, c\}$, there are some p's for which $pR < \bar{\rho}$. For these types $0 < \bar{\rho} - pR < \bar{\rho} - pR'$. Now, $1 - w \leq 1 - c'$ implies that $(\bar{\rho} - pR)(1 - w) < (\bar{\rho} - pR')(1 - c')$. Thus (19) says that types for which $pR < \bar{\rho}$ will deviate to contract $\{R', c'\}$, which is inconsistent with an equilibrium. Thus, if $\{R, c\}$ is active, there are no contracts with $R' < R$. This implies that, for each outcome x, there is at most one contract. If there is more than one p with outcome x, then $c = w$.

To summarize, projects with outcome x all get standard debt contracts with a common interest payment $R(x)$. Thus, for any joint density $f(r, p) = f(\frac{x}{p}, p)$, the markets effectively separate the distribution into a set of distinct markets $\{f(p|x)\}_x$ where each univariate distribution $f(p|x)$ is the conditional density of p given x implied by $f(\frac{x}{p}, p)$. Optimal contracting breaks the original adverse selection problem into a set of De Meza-Webb economies each with a univariate distribution $f(p|x)$. Moreover, the DW economies are stabilizers in the dynamic model.

3.3.3. Discussion

If entrepreneurs use equity financing or if they write optimal contracts, then, for any distribution of projects, adverse selection makes the economy excessively stable. Put differently, both equity financing and optimal contracting eliminate the accelerator equilibria. Thus, to an extent, stabilizer equilibria are more robust than accelerator equilibria.

In reality, many small business loans have both debt and equity features. If the business fails, the lenders are still the residual claimants. If the business succeeds, the financiers are paid according to the degree of success. These contracts are similar to the optimal contracts in the adverse selection model. The contracts here differ from equity because payoffs may be non-increasing in the outcome x. If the default rate for group x is less than x', then $R(x) < R(x')$ even if $x' > x$.

In addition to the screening contracts considered here, one could consider a signaling game in which entrepreneurs choose their collateral and then the banks offer interest rates $R(x)$ once they have observed the collateral. This setup, in which the more informed parties act first, also eliminates the accelerator equilibria.[10]

[10] See Spence (1973), (1974). Rothschild and Stiglitz (1976) give the original formulation and

4. Empirical Implications

Ultimately, whether credit market imperfections stabilize or destabilize the economy is an empirical question. This section reviews three types of empirical evidence: interest rate spreads, access to bond markets, and cash-flow sensitivity.

Interest rate spreads are countercyclical, which researchers argue is a sign of a financial accelerator mechanism.[11] Internal funds fall during a recession, magnifying agency costs and adverse selection problems. As a result, interest rates rise causing a further reduction in economic activity. In the adverse selection model however, interest rate spreads are countercyclical even when credit market imperfections stabilize the economy. This is true even though the change in the spread is due entirely to changes in market distortions. Because interest rate spreads are countercyclical whether or not credit market distortions destabilize the economy, they provide very limited evidence of a financial accelerator mechanism.

Large firms often have access to bond markets while small firms are more dependent on bank loans. This may be because asymmetric information is more severe for small firms. Gertler and Gilchrist (1994) show that sales for small firms are more volatile than sales for large firms and suggest that the difference is due to credit market imperfections. However, since it is possible that credit market distortions stabilize activity, one cannot conclude that the small firms are excessively volatile. It could be instead that the large firms are excessively stable since they have access to an anonymous bond market which may induce them to make inefficient investment decisions. In contrast, intermediated loans, may result in efficient though volatile behavior.

In principle, a cash-flow sensitivity regression is a direct test of the financial accelerator hypothesis. These regressions test whether investment reacts to exogenous changes in internal funds. There are well-known problems with such tests, however. Chief among these is the endogeneity of cash-flow. Endogenous increases in cash-flow have two effects on investment. First, cash-flow signals future profitability and thus provides a rational incentive to invest. Second, cash-flow affects investment through credit market imperfections. In practice, it is very difficult to

analysis of screening problems like the one considered in this paper. See also chapter 13 in Mas-Colell, Whinston and Green (1995).

[11] See Gertler, Hubbard, and Kashyap (1991). Bacchetta and Caminal (2000) argue that the cyclicality of the external finance premium is sufficient to tell whether the financial markets are accelerating or stabilizing. In his comments on Fuerst's (1995) paper, Gertler (1995) suggested that if the spread moved in the right direction, Fuerst's model would exhibit a financial accelerator. See also Azariadis and Chakraborty (1999).

separate these effects.[12] Furthermore, empirical studies often present conflicting results regarding the sensitivity of investment to cash-flow.[13] As a result, one cannot rely on cash-flow sensitivity tests to distinguish between stabilizing and destabilizing credit market distortions.

On the whole, the available empirical evidence suggests that credit market imperfections play a role in investment and production decisions. There is, however, insufficient evidence to conclude that such imperfections tend to destabilize the economy.

5. Related Literature

The term *financial accelerator* was coined by Bernanke and Gertler (1989). In their model, costly state verification problems cause distortions in the credit market. Although Bernanke and Gertler emphasized that costly state verification is only one possible source of credit market failure, most of the subsequent literature has continued to focus on this case.[14]

The literature on credit market frictions in dynamic settings has grown significantly since the paper by Bernanke and Gertler (1989). Fuerst (1995) was an early attempt to quantify the financial accelerator. Carlstrom and Fuerst (1997) expanded on this work by allowing the entrepreneurs in their model to be long-lived. This modification introduced a positive autocorrelation to output growth not usually found in business cycle models but was incapable of generating much amplification. Kiyotaki and Moore (1997) and Bernanke, Gertler and Gilchrist (1999) show that leverage plays a crucial role in ability of the financial accelerator to magnify shocks.

In addition to the theoretical literature there is also a large and growing body

[12] Fazzari, Hubbard and Peterson (1988) use Tobin's Q to control for the efficient level of investment in cash-flow regressions and find positive cash-flow effects. See also Kashyap, Lamont and Stein (1994), Hubbard, Kashyap, and Whited (1995), and Calomiris and Hubbard (1995).

[13] Gilchrist and Himmelberg (1995) show that financially constrained firms have high cash flow sensitivities. In contrast, Kaplan and Zingales (1997) show that financially unconstrained firms actually have the highest cash-flow sensitivities. Erickson and Whited (2000) argue that cash-flow is statistically insignificant after correcting for measurement error in Q. While many studies find a positive relationship between cash-flow and investment, Kaplan and Zingales (1997), Erickson and Whited (2000) and Cleary (1999) obtain several negative cash-flow estimates.

[14] See Townsend (1979) and Gale and Hellwig (1985) for the original analysis of the costly state verification model. Bernanke and Gertler (1990) and Kiyotaki and Moore (1997) are important departures from the costly state verification framework.

of empirical work on credit markets and business cycles. Good summaries are found in Bernanke, Gertler, and Gilchrist (1996) and Gertler (1988). Kashyap, Stein and Wilcox (1993) show that following a monetary contraction, the ratio of commercial paper issuances to bank loans rises (see also Calomiris, Himmelberg and Wachtel (1995)). Lang and Nakamura (1992) show that the ratio of low risk loans to high risk loans moves countercyclically.

6. Conclusion

Credit market distortions arising from adverse selection have several effects on the economy's dynamic response to shocks. The total effect of the adverse selection problem can be broken down into components that have economic interpretations. This decomposition provides a window into the workings of dynamic models of financial market imperfections. The decomposition establishes a direct connection between overinvestment and underinvestment in static models and stabilizers and accelerators in dynamic models. The possibility that credit market imperfections inefficiently stabilize the economy is not present in existing dynamic models of credit market imperfections. Equity financing or optimal contracting between borrowers and lenders eliminate all of the accelerator equilibria for any specification of the model. In these cases, stabilizer outcomes are the only possible equilibria. While there is evidence that suggests that loan markets are distorted, the data do not show that credit market imperfections destabilize the economy. Much of the empirical evidence is consistent with a model in which financial market failures cause the economy to be excessively stable.

The potential for credit market imperfections to inefficiently stabilize the economy is present in any setting with overinvestment in the steady state. The fact that it is difficult to distinguish the stabilizers from the accelerators in the data suggests that more care should be taken before concluding that credit market frictions are a destabilizing feature of the economy.

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Appendices

Appendix A.1 : Deriving Equation (15)

Combining equation (10) with (11) we get:

$$\begin{aligned}\tilde{K}_{t+1} &= \frac{-1}{K}\int_0^1 \hat{k}(p) f(\hat{k}(p),p)\left[\frac{\partial \hat{k}}{\partial R_t} R \frac{1}{\rho'}\left(w\frac{\partial \Delta}{\partial w}\tilde{w}_t + r\frac{\partial \Delta}{\partial w}\tilde{r}_{t+1}\right)\right. \\ &\quad \left. + r\frac{\partial \hat{k}}{\partial r_{t+1}}\tilde{r}_{t+1} + w\frac{\partial \hat{k}}{\partial w_t}\tilde{w}_t\right] dp\end{aligned}$$

Gathering the coefficients on $\tilde{r}_{t+1}$ and $\tilde{w}_t$ gives:

$$\begin{aligned}\tilde{K}_{t+1} &= \left[\frac{-1}{K}\int_0^1 \hat{k}(p) f(\hat{k}(p),p)\left(\frac{\partial \hat{k}}{\partial R_t} R \frac{1}{\rho'} r \frac{\partial \Delta}{\partial r} + r\frac{\partial \hat{k}}{\partial r_{t+1}}\right) dp\right]\tilde{r}_{t+1} \\ &\quad + \left[\frac{-1}{K}\int_0^1 \hat{k}(p) f(\hat{k}(p),p)\left(\frac{\partial \hat{k}}{\partial R_t} R \frac{1}{\rho'} w \frac{\partial \Delta}{\partial w} + w\frac{\partial \hat{k}}{\partial w_t}\right) dp\right]\tilde{w}_t\end{aligned}$$

Notice that for any variable x:

$$\int_0^1 \hat{k}(p) f(\hat{k}(p),p)\frac{\partial \hat{k}}{\partial x} dp = \int_0^1 \left[\frac{\bar{\rho}}{r} + \frac{(1-w)}{r}(pR-\bar{\rho})\right] f(\hat{k}(p),p)\frac{\partial \hat{k}}{\partial x} dp$$

and

$$\frac{\partial \Delta}{\partial x} = \frac{1}{IR}\int_0^1 [Rp-\bar{\rho}]\, f(\hat{k})\left[\frac{\partial \hat{k}}{\partial x}\right] dp$$

$$\frac{\partial I}{\partial x} = -\int_0^1 f(\hat{k}(p),p)\frac{\partial \hat{k}}{\partial x} dp$$

so that:

$$\int_0^1 \hat{k}(p) f(\hat{k}(p),p)\frac{\partial \hat{k}}{\partial x} dp = -\frac{\bar{\rho}}{r}\frac{\partial I}{\partial x} + \frac{(1-w)}{r} IR\frac{\partial \Delta}{\partial x}$$

Then, the coefficient on $\tilde{r}_{t+1}$ is given by:

$$\frac{-1}{K}\left[R\frac{r}{\rho'}\frac{\partial \Delta}{\partial r}\left(-\frac{\bar{\rho}}{r}\frac{\partial I}{\partial R_t} + \frac{(1-w)}{r} IR\frac{\partial \Delta}{\partial R_t}\right) + r\left(-\frac{\bar{\rho}}{r}\frac{\partial I}{\partial r_{t+1}} + \frac{(1-w)}{r} IR\frac{\partial \Delta}{\partial r_{t+1}}\right)\right]$$

and the coefficient on $\tilde{w}_t$ is:

$$\frac{-1}{K}\left[R\frac{w}{\rho'}\frac{\partial \Delta}{\partial w}\left(-\frac{\bar{\rho}}{r}\frac{\partial I}{\partial R_t} + \frac{(1-w)}{r} IR\frac{\partial \Delta}{\partial R_t}\right) + w\left(-\frac{\bar{\rho}}{r}\frac{\partial I}{\partial w_t} + \frac{(1-w)}{r} IR\frac{\partial \Delta}{\partial w_t}\right)\right]$$

Combining these and factoring out I, gives:

$$\begin{aligned}\tilde{K}_{t+1} &= \frac{I}{K}\left[\frac{R}{\rho'}\frac{\partial \Delta}{\partial r}\left((1-\Delta)\varepsilon_{IR} - (1-w)R\frac{\partial \Delta}{\partial R_t}\right) + \left(\frac{\bar{\rho}}{I}\frac{\partial I}{\partial r_{t+1}} - (1-w)R\frac{\partial \Delta}{\partial r_{t+1}}\right)\right]\tilde{r}_{t+1} \\ &\quad + \frac{I}{K}\frac{w}{r}\left[\frac{R}{\rho'}\frac{\partial \Delta}{\partial w}\left((1-\Delta)\varepsilon_{IR} - (1-w)R\frac{\partial \Delta}{\partial R_t}\right) + \left(\frac{\bar{\rho}}{I}\frac{\partial I}{\partial w_t} - (1-w)R\frac{\partial \Delta}{\partial w_t}\right)\right]\tilde{w}_t\end{aligned}$$

using the fact that $\rho' = \left[1-\Delta-R\frac{\partial \Delta}{\partial R_t}\right]$ gives:

$$\begin{aligned}\tilde{K}_{t+1} &= \frac{\bar{\rho} I}{K}\left[\frac{1}{\rho'}\frac{\partial \Delta}{\partial r}\left(\varepsilon_{IR} - (1-w)\right) + \frac{1}{I}\frac{\partial I}{\partial r_{t+1}}\right]\tilde{r}_{t+1} \\ &\quad + \frac{\bar{\rho} I}{K}\frac{w}{r}\left[\frac{1}{\rho'}\frac{\partial \Delta}{\partial w}\left(\varepsilon_{IR} - (1-w)\right) + \frac{1}{I}\frac{\partial I}{\partial w_t}\right]\tilde{w}_t\end{aligned}$$

Note that:

$$\frac{\partial I}{\partial r_{t+1}} = \int_0^1 f(\hat{k}(p),p)\frac{\hat{k}(p)}{r}dp = \int_0^1 f(\hat{k}(p),p)\frac{\bar{\rho}}{r}dp + (1-w)\int_0^1 f(\hat{k}(p),p)\frac{[Rp-\bar{\rho}]}{r}dp$$

and

$$\frac{\partial I}{\partial w_t} = \int_0^1 f(\hat{k}(p),p)\frac{(pR-\bar{\rho})}{r}dp$$

Recalling the expression for $\frac{\partial \Delta}{\partial \bar{\rho}}$ implies:

$$\frac{\partial I}{\partial r_{t+1}} = \left.\frac{\partial I}{\partial r_{t+1}}\right|_{\mathrm{PI}} + \frac{(1-w)}{w} I \frac{\bar{\rho}}{1-\Delta}\frac{\partial \Delta}{\partial \bar{\rho}}$$

and

$$\frac{\partial I}{\partial w_t} = \frac{I}{w}\frac{\bar{\rho}}{1-\Delta}\frac{\partial \Delta}{\partial \bar{\rho}}$$

so that:

$$\begin{aligned}\tilde{K}_{t+1} \quad &= \quad \frac{\rho I}{K}\left[\frac{1}{\rho'}\frac{\partial \Delta}{\partial r}\left(\varepsilon_{IR}-(1-w)\right) + \frac{1}{I}\left(\left.\frac{\partial I}{\partial r_{t+1}}\right|_{\mathrm{PI}} + \frac{(1-w)}{w} I \frac{\bar{\rho}}{1-\Delta}\frac{\partial \Delta}{\partial \bar{\rho}}\right)\right]\tilde{r}_{t+1} \\ &\quad + \frac{\rho I}{K}\frac{w}{r}\left[\frac{1}{\rho'}\frac{\partial \Delta}{\partial w}\left(\varepsilon_{IR}-(1-w)\right) + \frac{1}{I}\frac{I}{w}\frac{\bar{\rho}}{1-\Delta}\frac{\partial \Delta}{\partial \bar{\rho}}\right]\tilde{w}_t\end{aligned}$$

which is rearranged to get equation (15).

Appendix A.2. : The Efficiency Gain Effect.

Assume that the increase in internal funds does not increase investment, then:

$$\frac{\partial I}{\partial w} = -\int_0^1 f(\hat{k}(p),p)\left\{\frac{\partial \hat{k}}{\partial w} + \frac{\partial \hat{k}}{\partial R}\frac{\partial R}{\partial w}\right\}dp = 0$$

This implies that:

$$\frac{\partial K_{t+1}}{\partial w} = -\int_0^1 \frac{1}{r}(1-w)\left(Rp-\rho\right) f(\hat{k}(p),p)\left\{\frac{\partial \hat{k}}{\partial w} + \frac{\partial \hat{k}}{\partial R}\frac{\partial R}{\partial w}\right\}dp$$

Note

$$\frac{\partial \hat{k}}{\partial w} = -\frac{1}{r}(pR-\bar{\rho}), \text{ and } \frac{\partial \hat{k}}{\partial R} = \frac{1}{r}p(1-w)$$

Since $\bar{\rho} = R\left[1-\Delta\right]$ we have:

$$\frac{\partial R}{\partial w}\left[1-\Delta\right] - R\left[\frac{\partial \Delta}{\partial w} + \frac{\partial \Delta}{\partial R}\frac{\partial R}{\partial w}\right] = 0$$

so that $\frac{\partial R}{\partial w} = \frac{R}{1-\Delta-R\frac{\partial \Delta}{\partial R}}\frac{\partial \Delta}{\partial w} = \frac{R}{\rho'}\frac{\partial \Delta}{\partial w}$. Then:

$$\frac{\partial K_{t+1}}{\partial w} = -\int_0^1 \frac{1}{r}(1-w)\left(Rp-\bar{\rho}\right) f(\hat{k}(p),p)\left\{-\frac{1}{r}(pR-\bar{\rho}) + \frac{1}{r}p(1-w)\frac{R}{\rho'}\frac{\partial \Delta}{\partial w}\right\}dp$$

For any x we have:

$$\frac{\partial \Delta}{\partial x} = \frac{1}{IR}\int_0^1 \left[Rp-\rho\right] f(\hat{k})\left[\frac{\partial \hat{k}}{\partial x}\right]dp$$

using the expression for $\frac{\partial \hat{k}}{\partial R}$, the second term in the sum is:

$$-\frac{R}{\rho'}\frac{\partial \Delta}{\partial w}(1-w)\frac{1}{r}IR\frac{\partial \Delta}{\partial R}$$

and the first term is:

$$-(1-w)\frac{1}{r}IR\frac{\partial \Delta}{\partial w}$$

Thus:

$$\frac{\partial K_{t+1}}{\partial w} = -(1-w)\frac{1}{r}IR\frac{\partial \Delta}{\partial w}\left[1+\frac{R\frac{\partial \Delta}{\partial R}}{\rho'}\right]$$

Using the definition of ρ',

$$\begin{aligned}\frac{\partial K_{t+1}}{\partial w} &= -(1-w)\frac{1}{r}IR\frac{\partial \Delta}{\partial w}\frac{1}{\rho'}\left[1-\Delta\right] \\ &= -(1-w)\frac{1}{r}I\bar{\rho}\frac{\partial \Delta}{\partial w}\frac{1}{\rho'}\end{aligned}$$

so the percentage change in K due to a percentage change in w conditional on $dI = 0$ is

$$\frac{\partial K_{t+1}}{\partial w}\frac{w}{K} = -(1-w)\frac{I}{K}\frac{\bar{\rho}}{r}\frac{\partial \Delta}{\partial w}\frac{w}{\rho'}$$

which is the efficiency gain effect.

Appendix B: Numerical Simulations.

The settings for the parameters used in the quantitative model are given in the table below. Keep in mind that this is a bivariate normal in the log odds ratio and log returns : $\left\{\ln\left(\frac{p}{1-p}\right), \ln(k)\right\}$, so that μ_p is the mean of $\ln\left(\frac{p}{1-p}\right)$ rather than the mean of p.

	Distributional parameters				
Model	μ_p	μ_k	σ_p^2	σ_k^2	σ_{pk}
Accelerator:	4	1	1	-.1	-3
Stabilizer:	3.5	1	2.5	.2	3

To solve the model, I employ an ad hoc two dimensional quadrature procedure. Given the parameters of the distribution, the marginal distribution of $\ln\left(\frac{p}{1-p}\right)$ is normally distributed with mean μ_p and variance σ_p^2. I divide this marginal distribution into 20 cross sections or "strips". I take the .05 percentiles of the distribution as the strips and assume that all the projects in a strip all have the same success probability. So for instance, the first strip will be characterized by a number $v = -1.6449$ so that:

$$\frac{\ln\left(\frac{p}{1-p}\right) - \mu_p}{\sigma_p} = -1.6449$$

and the success probability for the strip is:

$$p = (1-p)\left[\exp\left\{\mu_p + \sigma_p v\right\}\right]$$

$$p = \frac{\exp\left\{\mu_p + \sigma_p v\right\}}{1+\exp\left\{\mu_p + \sigma_p v\right\}} = \frac{\exp\left\{\mu_p - \sigma_p 1.6449\right\}}{1+\exp\left\{\mu_p - \sigma_p 1.6449\right\}}$$

The "mass" for each strip is .05 (by construction).

Within each strip $\ln k | \ln\left(\frac{p}{1-p}\right) \sim N(\mu_{k|p}, \sigma^2_{k|p})$ where

$$\mu_{k|p} \equiv \mu_k - \frac{\sigma_{pk}}{\sigma_p^2}\mu_p + \frac{\sigma_{pk}}{\sigma_p^2}\ln\left(\frac{p}{1-p}\right)$$

and

$$\sigma^2_{k|p} \equiv \sigma^2_k - \left(\frac{\sigma_{pk}}{\sigma_p^2}\right)^2 \sigma_p^2$$

Conditional on the success probability p, the conditional distribution obeys:

$$\frac{\ln k - \mu_{k|p}}{\sigma_{k|p}} \sim N(0,1)$$

Since there are 20 cross-sections:

$$\rho(R_t) = R_t\left(\frac{\sum_{t=1}^{20} p_t I_t^t(R_t)}{\sum_{t=1}^{20} I_t^t(R_t)}\right)$$

where

$$I_t^j = I_t(p^j) = .05 \cdot \left[1 - \Phi\left(\frac{\ln(\hat{k}_t^j) - \mu_j}{\sigma_j}\right)\right]$$

where Φ is the standard normal distribution. Then K is governed by:

$$K_{t+1} = .05 \cdot \left[\sum_{j=1}^{20}\left(\int_{\hat{k}_t^j}^{\infty} k f_j(k) dk\right)\right]$$

where f_j is the p.d.f. of the j^{th} log normal (i.e. $f_j(x) = \phi\left(\frac{\ln x - \mu}{\sigma}\right)\frac{1}{x}\frac{1}{\sigma}$ where ϕ is the normal density).

Given R, w, r, ρ one can construct $\hat{k}(p_j)$ from equation (2) in the text. I can then use MATLAB quadrature subroutines to find the 20 I's and the value for K as described above. Clearly w and r (and indirectly R) will all depend on the capital stock. Thus the problem becomes finding a fixed point in K. The solution proceeds as follows.

First guess a value for the steady state capital stock K. This will imply w and r. With these values I perform a grid search to find the associated equilibrium R. Specifically, I pick R and form $\hat{k}(p_j)$ for $j = 1...20$. These imply a real rate of return $\rho(R) = R\left(\frac{\sum_{j=1}^{20} p_j I_j(R)}{\sum_{j=1}^{20} I_j(R)}\right)$. The equilibrium will be the smallest R such that $\rho(R) = \bar{\rho}$ (the exogenously specified safe rate of return). With this R I can construct the an associated I', and K'. If $K' = K$ then we have an equilibrium. If $K' \neq K$ then I revise my initial guess for the steady state capital stock.

With the steady state capital stock, I numerically evaluate the first order components of the function Δ: $\frac{\partial\Delta}{\partial r}$, $\frac{\partial\Delta}{\partial w}$, $\frac{\partial\Delta}{\partial R}$, and $\frac{\partial\Delta}{\partial\bar{\rho}}$. This completes the solution.

Table 1 : Reaction to a One-Percent Technology Shock [1]

Model	Symmetric Information		Adverse Selection		Decomposition of $\tilde{K}$ in the Adverse Selection Equilibrium (from equation (11))					Numerical Derivatives			
	$\tilde{K}$	$\tilde{R}$	$\tilde{K}$	$\tilde{R}$	Perfect Inf. Effect	Agency Cost Effect	Investment Effect	Efficiency Gain	Payoff Effect	$\frac{\partial \Delta}{\partial \rho}$	$\frac{\partial \Delta}{\partial w}$	ρ'	ε_{IR}
Accelerator	0.99	0.00	1.21	-0.19	0.52	0.10	0.38	0.03	0.17	0.27	-0.08	0.66	-1.04
Stabilizer	0.71	0.00	0.55	-0.22	0.87	0.20	-0.53	0.17	-0.17	-0.34	-0.51	0.90	-0.37

[1] The table gives the impact effects of a one percent productivity shock. For the simulations, I assume that the distribution is defined by a bivariate log-normal

$$\left\{\log\left(\frac{p}{1-p}\right),\log(k)\right\} \sim BVN\left(\mu_1,\mu_2,\sigma_1,\sigma_2,\sigma_{12}\right)$$

The parameters (μ_1, μ_2, σ_1, σ_2, σ_{12}) for the accelerator and stabilizer equilibria are (4.0, 1.0, 1.0, -0.1, -3.0) and (3.5, 1.0, 2.5, 0.2, 3.0) respectively. The remaining parameters are $\rho = 1.01$; $q = 0.95$; and $\alpha = 0.35$. The steady state, and the first-order terms of the function Δ as well as ε_{IR} are computed numerically. For more details of the numerical simulation see House (2004).

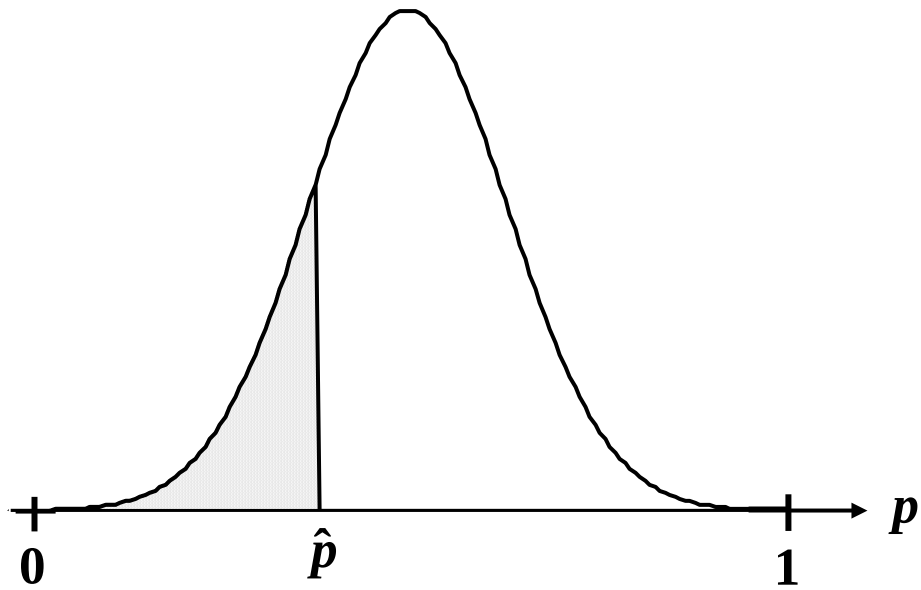

Figure 1.A
Projects undertaken in the Stiglitz and Weiss Model

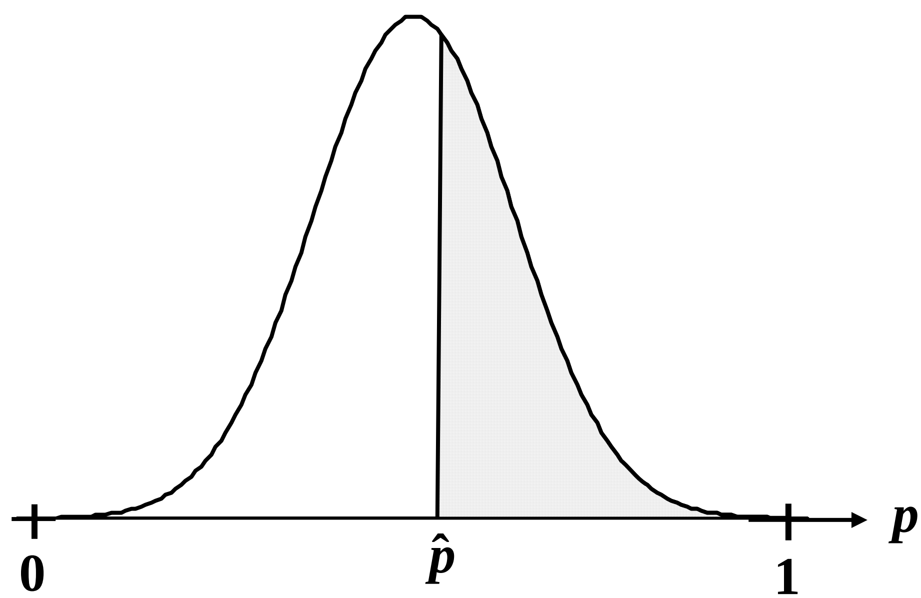

Figure 1.B
Projects undertaken in the De Meza and Webb Model