OPTIMAL TAYLOR RULES WHEN TARGETS ARE UNCERTAIN*

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ABSTRACT

We analyze the optimal Taylor rule in the standard New Keynesian model when output and inflation are imperfectly observed. When the central bank observes inflation and the output gap with error, the optimal Taylor rule features tempered responses so as not to impart unnecessary volatility to the economy. If the Taylor rule is expressed in terms of *estimated* output and inflation, it is optimal to respond infinitely strongly to estimated deviations from the targets. Because filtered estimates are based on current and past observations, such Taylor rules appear to exhibit interest rate smoothing even though the monetary authority has no explicit preference for smooth interest rates. Under such a Taylor rule, the estimates of inflation and the output gap are perfectly negatively correlated. In the data, these gaps are slightly positively correlated, suggesting that the central bank is systematically underreacting to estimated inflation and the output gap.

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1. Introduction

A key challenge for monetary policy is that the current state of the economy is uncertain. Consequentially, policy makers face a tradeoff between aggressively responding to changes in economic conditions on the one hand, and imparting unnecessary volatility to the economy due to policy errors on the other. This paper seeks to analyze such uncertainty in the standard New Keynesian model when monetary policy is governed by a Taylor rule – a simple linear relationship between a central bank's choice of a target interest rate, observed output (or the "output gap") and observed inflation (Taylor, 1993). The Taylor rule is a natural rule to study since it is a common feature of macroeconomic models and, suitably parameterized, is a reasonable description of how actual central banks set interest rates.

This paper examines three cases. First, we consider the optimal Taylor rule without target uncertainty. In this case, the optimal Taylor rule coefficients are infinite. That is, if there is no measurement error, the optimal Taylor rule requires the central bank to respond infinitely strongly to deviations in inflation and the output gap. These extreme responses arise because, in the absence of measurement error, the central bank faces no costs of responding arbitrarily strongly. In contrast, estimated Taylor rules indicate that central banks' actual responses to output and inflation are relatively modest (e.g. Judd and Rudebusch, 1998, Bogdanova and Hoffman, 2012). This apparent under-reaction to measured inflation and output is often attributed to the fact that the central bank observes output and inflation with error (Orphanides 2001, 2003).

We next analyze the optimal Taylor rule when central banks react to the noisy measures of output and inflation. If inflation and output are both measured with error, then the Taylor coefficients are finite. As the variance of measurement error grows, the optimal coefficients fall and the central bank reacts less and less to current measures of economic activity.

Finally, we consider the case in which monetary policy responds to optimal estimates ("nowcasts") of output and inflation. In this case, the central bank first solves a signal extraction problem to estimate the output gap and inflation and subsequently sets the interest rate as a function of these estimates. In this setting, it is optimal for the central bank to respond infinitely strongly to any deviation of estimated inflation or the estimated output gap from their targets – a result that echoes our earlier finding in the model without measurement error. A central bank that adopts such a rule will often appear to smooth interest rates. Indeed, there are special cases in which a Taylor rule with interest rate smoothing can duplicate the policy of responding to estimated inflation and the output gap. The intuition that signal extraction leads to interest rate smoothing appears to align with actual experiences of policymakers. In 2004, Ben Bernanke, then member of the Board of Governors, noted: "[G]iven the highly uncertain environment in which policy operates, a gradual adjustment of rates has the advantage of allowing the FOMC to monitor the evolution of the economy […] making adjustments along the way as needed" (Bernanke, 2004a).[1]

Under an optimal Taylor rule, estimated deviations of output and inflation from their targets should be perfectly negatively correlated. Intuitively, the optimal Taylor rule eliminates the effects of estimated demand shocks on estimated inflation and the output gap. At the optimum, only variation due to estimated cost-push shocks remains, which implies that estimated inflation and output move in opposite directions. Actual data on estimates of inflation and the gap are not strongly correlated, suggesting that the central bank is not reacting to demand shocks as aggressively as it should.

Our work is related to a large literature on optimal monetary policy and instrument rules (e.g. Giannoni and Woodford 2003a, 2003b, Woodford 2003, Ch. 7 and Giannoni, 2014).[2] Unlike we do in this paper, Giannoni and Woodford (2003b) and Giannoni (2014) assume that the central bank has an explicit preference for smooth interest rates. Some of the papers in this literature focus on the role of measurement error in tempering the central bank's reactions. Orphanides (2001, 2003) shows that real-time measures of inflation and the output gap are sufficiently noisy to justify relatively small Taylor rule coefficients. Similarly, Rudebusch (2001), Smets (2002) and Billi (2012) all conclude that measurement error naturally encourages central banks to adopt less aggressive policy reaction rules.[3] While we confirm this finding in the canonical New Keynesian model when the central bank responds to noisy measures, we also demonstrate that the optimal Taylor rule coefficients remain infinite if the central bank solves a signal extraction problem and the Taylor rule is expressed in terms of estimated output and inflation.

Our paper is also related to the literature on signal extraction and optimal monetary policy. Aoki (2003) considers an environment similar to ours, but without shocks to the New Keynesian Phillips Curve. Thus, in Aoki's work, the "divine coincidence" holds: The central bank can eliminate the output gap by successfully stabilizing prices (Blanchard and Galí, 2007). Further, while Aoki discusses optimal policy under discretion, we intentionally restrict attention to the optimal Taylor

[1] Throughout this paper we assume that the private sector has full information. This assumption is common in the literature (e.g. Svensson and Woodford, 2004). It simplifies the model solution and focuses attention on the central banks uncertainty about the true state of the economy. As we discuss below, a number of alternative informational assumptions are plausible. Lubik, Matthes, and Mertens (2016) show that informational assumptions can matter for equilibrium determinacy.

[2] While we restrict our attention to Taylor rules in this paper, it is well-known that optimal monetary policy is rarely given by a Taylor rule (see Woodford, 1999). See Svensson (2003) for a more general criticism of Taylor rules and see Taylor and Williams (2011) for a literature review.

[3] Both Rudebusch (2001) and Smets (2002) numerically analyze optimal policy based on estimated New Keynesian systems. Billi (2012) numerically analyzes optimal monetary policy at the zero lower bound. Both Billi and Smets assume that the central bank wants to minimize interest rate variation in addition to variation in output and inflation. Cateau (2007) considers the implications of model uncertainty on the optimal Taylor rule. Gorodnichenko and Shapiro (2007) argue that price-level targeting is preferable to inflation targeting if the central bank faces uncertainty about the output gap. See Taylor (1999) for additional work on optimal monetary policy rules.

rule – a form of commitment. Finally, when formulating the signal extraction problem, Aoki assumes that the central bank learns the true values of output and inflation with a one period lag. In our formulation, these values are never fully revealed. Methodologically, we draw on results in Svensson and Woodford (2003, 2004). Although they do not consider restricted instrument rules such as the Taylor rule, several of their findings continue to hold in our setting.

2. Baseline Model and Equilibrium

Our analysis focuses on a version of the standard New Keynesian framework in which the output gap and inflation are both measured with error. The model is described by a Phillips curve and a New Keynesian IS curve.[4] The Phillips curve relates inflation, π_t, to the output gap, y_t, expected future inflation, and a cost-push shock, u_t,

$$\pi_t = \kappa y_t + \beta E_t[\pi_{t+1}] + u_t, \tag{1}$$

where $\kappa > 0$. The New Keynesian IS curve is

$$y_t = E_t[y_{t+1}] - \frac{1}{\sigma}(i_t - \rho - r_t^e - E_t[\pi_{t+1}]). \tag{2}$$

Here $\rho + r_t^e$ is the efficient rate of interest – the interest rate consistent with the level of output that would prevail under perfect price flexibility in the absence of all other distortions. We express this rate as the sum of the rate of time preference ρ and a shock r_t^e which is centered at zero. The remaining terms are i_t, the nominal interest rate, and σ, the coefficient of relative risk aversion (equivalently, the inverse of the intertemporal elasticity of substitution).

The efficient rate shock and the cost-push shock are assumed to follow the AR(1) processes

$$r_{t+1}^e = \varrho_r \, r_t^e + \varepsilon_{t+1}^r, \qquad \varrho_r \in [0,1), \tag{3}$$

$$u_{t+1} = \varrho_u \, u_t + \varepsilon_{t+1}^u. \qquad \varrho_u \in [0,1). \tag{4}$$

We close the model by assuming that the monetary authority commits to a Taylor rule,

$$i_t = \rho + \phi_\pi \pi_t^m + \phi_y y_t^m. \tag{TR1}$$

Importantly, we distinguish between the actual output gap and the output gap observed by the monetary authority, and similarly between actual inflation and measured inflation. Because the central bank can respond only to measured output and inflation, π_t^m and y_t^m in (TR1) denote

[4] The standard New Keynesian model abstracts from investment in physical capital and durable goods. While this assumption is common, it has important consequences for the analysis of the model and for optimal policy. See Barsky et al. (2007) and Barsky et al. (2015) for a more detailed discussion of the consequences of this assumption.

measured inflation and *measured* output.[5] We assume that $\pi_t^m = \pi_t + m_t^\pi$ and $y_t^m = y_t + m_t^y$, where m_t^π and m_t^y denote the respective measurement errors. Both types of measurement error follow AR(1) processes

$$m_{t+1}^\pi = \varrho_{m\pi}\, m_t^\pi + \varepsilon_{t+1}^{m\pi}, \qquad \varrho_{m\pi} \in [0,1), \tag{5}$$

$$m_{t+1}^y = \varrho_{my}\, m_t^y + \varepsilon_{t+1}^{my}. \qquad \varrho_{my} \in [0,1). \tag{6}$$

All error terms are uncorrelated. While the central bank observes noisy measures of output and inflation, we assume for simplicity that households and firms have full information.

We further assume that the well-known condition for a unique equilibrium,

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0 \tag{7}$$

holds at all times.[6]

We characterize the model's solution in the following lemma. Proofs of all results are in the appendix.

Lemma 1: *The unique locally stable competitive equilibrium of the model is given by the equations*

$$\begin{aligned}\pi_t = {} & \frac{\kappa}{\Phi_r + \phi_y(1-\beta\varrho_r) + \kappa\phi_\pi} r_t^e + \frac{\phi_y + (1-\varrho_u)\sigma}{\Phi_u + \phi_y(1-\beta\varrho_u) + \kappa\phi_\pi} u_t \\ & - \frac{\kappa\phi_\pi}{\Phi_{m\pi} + \phi_y(1-\beta\varrho_{m\pi}) + \kappa\phi_\pi} m_t^\pi - \frac{\kappa\phi_y}{\Phi_{my} + \phi_y\left(1-\beta\varrho_{my}\right) + \kappa\phi_\pi} m_t^y,\end{aligned} \tag{8}$$

$$\begin{aligned}y_t = {} & \frac{1-\beta\varrho_r}{\Phi_r + \phi_y(1-\beta\varrho_r) + \kappa\phi_\pi} r_t^e + \frac{\varrho_u - \phi_\pi}{\Phi_u + \phi_y(1-\beta\varrho_u) + \kappa\phi_\pi} u_t \\ & - \frac{(1-\beta\varrho_{m\pi})\phi_\pi}{\Phi_{m\pi} + \phi_y(1-\beta\varrho_{m\pi}) + \kappa\phi_\pi} m_t^\pi - \frac{\left(1-\beta\varrho_{my}\right)\phi_y}{\Phi_{my} + \phi_y\left(1-\beta\varrho_{my}\right) + \kappa\phi_\pi} m_t^y,\end{aligned} \tag{9}$$

where $\Phi_j = \sigma\left(1-\varrho_j\right)\left(1-\beta\varrho_j\right) - \kappa\varrho_j$, $j \in \{r, u, m\pi, my\}$, *are constants that are independent of monetary policy.*

Both sources of measurement error reduce inflation and the output gap. A positive innovation to m_t^π makes inflation appear higher than actual inflation. In response, the central bank raises interest rates causing both output and inflation to fall. Similar reasoning applies to measurement error in the output gap.

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[5] Bernanke (2004b) described measurement error as creating a "foggy windshield" through which the monetary authority sees the economy. For additional discussion, see Orphanides (2001).

[6] The argument in Bullard and Mitra (2002) continues to hold in our setting.

3. Optimal Taylor Rules

The central bank seeks to minimize an expected discounted sum of weighted squared inflation and the output gap (see Rotemberg and Woodford, 1999, and Woodford, 2003),

$$(1-\beta)E\left[\sum_{t=0}^{\infty}\beta^t(\alpha\pi_t^2+y_t^2)\right]=E[\alpha\pi_t^2+y_t^2]. \quad (10)$$

Here, α denotes the relative weight that the central bank places on inflation and $E[\cdot]$ is the unconditional expectations operator. The optimal policy problem is then to choose ϕ_y and ϕ_π to minimize (10) subject to (8) and (9).

Monetary Policy without Uncertainty

We begin by considering the optimal Taylor rule in the absence of uncertainty. This is a useful benchmark against which to compare the optimal Taylor rule when the central bank faces uncertainty about output and inflation. The following proposition characterizes the optimal Taylor rule and the equilibrium paths of output and inflation in the standard model.

Proposition 1

Suppose the central bank observes output and inflation without error.

(i) The optimal Taylor rule coefficients in the standard model are given by ϕ_y^* and ϕ_π^* which satisfy

$$\phi_\pi^*=\varrho_u+\frac{\alpha\kappa\sigma(1-\varrho_u)}{1-\beta\varrho_u}+\frac{\alpha\kappa}{1-\beta\varrho_u}\phi_y^*. \quad (11)$$

with $\phi_y^*\to\infty$.

(ii) Under the optimal Taylor rule, the equilibrium satisfies

$$\pi_t=\frac{1-\beta\varrho_u}{\alpha\kappa^2+(1-\beta\varrho_u)^2}u_t, \qquad y_t=-\frac{\alpha\kappa}{\alpha\kappa^2+(1-\beta\varrho_u)^2}u_t.$$

We make three observations. First, when the central bank observes output and inflation without error, it is optimal for the central bank to respond infinitely strongly to deviations of inflation and the output gap relative to their targets. Reviewing expressions (8) and (9), it is clear that arbitrarily aggressive reactions to either inflation or output allow the central bank to completely eliminate the effects of shocks to the efficient rate of interest r_t^e. We emphasize that while the optimal Taylor rule coefficients given in Proposition 1 are infinite, the nominal interest rate i_t remains finite in equilibrium.

Second, in the limit as ϕ_y^* and ϕ_π^* become arbitrarily large, the ratio ϕ_π^*/ϕ_y^* approaches $\alpha\kappa/1-\beta\varrho_u$. This ratio depends on the relative tradeoff between inflation versus output stabilization (α), the slope of the Phillips curve (κ), and the persistence of the cost-push shock (ϱ_u). Unlike shocks to r_t^e, cost-push shocks cannot be eliminated through monetary policy. Instead, the central bank trades off output stability versus inflation stability (Clarida, Galí, and Gertler, 1999). The optimal tradeoff implies a specific ratio of the Taylor rule coefficients.

Finally, under the optimal Taylor rule, Proposition 1 implies that

$$\pi_t = -\frac{1-\beta\varrho_u}{\alpha\kappa}y_t = -\frac{\phi_y^*}{\phi_\pi^*}y_t.$$

Hence, for the optimal Taylor rule, equilibrium output and inflation are perfectly negatively correlated.[7]

MONETARY POLICY WITH UNCERTAIN TARGETS

The simple New Keynesian framework captures many realistic features of monetary policy. The model embodies a tradeoff between inflation and output and suggests that the central bank has a particular advantage in minimizing economic instabilities that arise from "demand shocks" (shocks to the IS-Curve). Despite these attractive features, the model does not entail any costs to excessively strong reactions on the part of the central bank. In stark contrast to the modest empirical estimates of actual Taylor rules (see Judd and Rudebusch, 1998, and more recently Hofmann and Bogdanova, 2012), the optimal Taylor rule coefficients are typically infinite.

Measurement error – uncertainty about actual output and inflation – is a natural candidate for why central banks do not respond more to observed changes in GDP and inflation. This concern was emphasized by Friedman (1953) who pointed out that activist policies might be destabilizing if policy actions were not sufficiently correlated with the true policy targets. From (9) and (10) it is clear that, for any fixed coefficients ϕ_y and ϕ_π, greater measurement error reduces the correlation between the policy instrument and the targets and thus entails greater unwanted variation in output and inflation. Indeed, if measurement error were sufficiently high, it would be optimal not to respond to observed variations in inflation and output at all.[8]

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[7] Note that under the optimal policy, estimation of a New Keynesian Phillips curve will be particularly problematic. Typically, the structural shock u_t is correlated with the regressors y_t and $E_t[\pi_{t+1}]$ but under optimal policy, both regressors are functions *only* of u_t and are therefore *perfectly* correlated with the error. The optimal Taylor rule eliminates all variation other than variation associated with u_t making the bias particularly pronounced.

[8] This might present a problem for equilibrium determinacy. It is well known that determinacy requires that the central bank responds sufficiently strongly to inflation and output. If measurement error is large however, the Fed

With arbitrary variation in the efficient rate, the cost-push shock, and both types of measurement error, an analytical solution of the optimal policy problem is generally not feasible and so we instead use numerical methods to characterize the optimal Taylor rule. To build intuition, however, we first consider a special case in which an analytical characterization is possible.

Proposition 2: *Suppose all shocks are white noise, that is,* $\varrho_r = \varrho_u = \varrho_{m\pi} = \varrho_{my} = 0$. *Then the minimization of (10) subject to (8) and (9) yields the following optimal Taylor rule coefficients*

$$\phi_y^* = \frac{1}{\sigma} \cdot \frac{V[r_t^e]}{V[m_t^y]}, \tag{12}$$

$$\phi_\pi^* = \frac{\kappa}{\sigma} \cdot \frac{(\alpha\kappa^2 + 1)V[m_t^y] + \alpha V[u_t]}{(\alpha\kappa^2 + 1)V[m_t^\pi] + V[u_t]} \cdot \frac{V[r_t^e]}{V[m_t^y]} + \frac{\alpha\kappa\sigma V[u_t]}{(\alpha\kappa^2 + 1)V[m_t^\pi] + V[u_t]}, \tag{13}$$

where $V[\cdot]$ *denotes the unconditional variance operator.*

There are several striking features of the optimal Taylor rule in this setting. First, the optimal Taylor coefficients in this model are finite. The central bank avoids aggressive reactions to measured inflation and output because it knows that its actions would cause excessive fluctuations in actual output and inflation. As measurement error decreases, the central bank can adopt more and more aggressive reactions to inflation and output. It is also worth noting that only measurement error in the output gap is necessary for finite Taylor coefficients. Assuming that $V[u_t] > 0$, measurement error in inflation is neither necessary nor sufficient for finite coefficients.

Second, the optimal choice of ϕ_y^* depends neither on α, nor on κ, nor on $V[u_t]$. Instead, ϕ_y^* depends only on the ratio of the variance of shocks to the efficient rate, $V[r_t^e]$, to the variance of measurement error in the output gap, $V[m_t^y]$, together with the coefficient of relative risk aversion, σ. The reader might find the result in (12) somewhat counterintuitive. If risk aversion is relatively high, then the household will strongly dislike output variability and presumably prefer a stronger output reaction. In contrast, the optimal reaction is decreasing in σ. The reason for this apparent contradiction is that we chose to specify the IS shocks as shocks to the efficient rate of interest itself (as is common in the literature). If we instead stated the shocks in terms of exogenous changes in the efficient growth rate of output, we would have $r_t^e = \sigma E_t[\Delta y_{t+1}^e] = -\sigma y_t^e$ (the second equality uses the assumption that the autocorrelation of shocks is zero). In this case, the central bank's choice of ϕ_y^* can be written as

could be forced to choose between a locally indeterminate equilibrium on the one hand and destabilization resulting from reactions to erroneous signals on the other.

$$\phi_y^* = \sigma \frac{V[y_t^e]}{V[m_t^y]},$$

which is increasing in σ.

The optimal response to measured inflation (equation 13) is somewhat more complex. We start with its more intuitive properties. First, as with ϕ_y^*, a larger variance of the efficient rate $V[r_t^e]$ implies a stronger response to inflation. Second, greater measurement error in inflation requires more attenuated responses. Third, one can show that $\partial\phi_\pi^*/\partial\alpha > 0$ for *any* choice of model parameters so a stronger preference for price stability always implies stronger reactions to measured inflation.

The relationship of ϕ_π^* with the remaining parameters is less clear. To see how this coefficient depends on the shock variances consider the following limiting cases. Suppose first that cost-push shocks are dominant – that is, consider the behavior of ϕ_π^* as $V[u_t] \to \infty$. In this case, the optimal reaction to inflation approaches

$$\phi_\pi^* = \frac{\kappa\alpha}{\sigma} \cdot \frac{V[r_t^e]}{V[m_t^y]} + \alpha\kappa\sigma = \kappa\alpha\sigma\left(\frac{V[y_t^e] + V[m_t^y]}{V[m_t^y]}\right),$$

where we have again used the relationship $r_t^e = -\sigma y_t^e$. The inflation response is increasing in the signal-to-noise ratio for the output gap, the weight the central bank places on inflation stability, the slope of the IS curve and the macroeconomic rate of price adjustment. Notice also that for large $V[u_t]$ neither ϕ_y^* nor ϕ_π^* depend on measurement error in inflation.

Alternatively, suppose there are no cost-push shocks at all. In this case,

$$\phi_\pi^* = \frac{\kappa}{\sigma} \cdot \frac{V[r_t^e]}{V[m_t^\pi]} = \kappa\sigma \frac{V[y_t^e]}{V[m_t^\pi]}.$$

Analogous to equation (12), the ratio of the variance of the efficient rate of output to that of measured inflation governs the strength of the reaction. In particular, as $V[u_t]$ approaches zero, ϕ_π^* becomes independent of measurement error in the output gap. Finally, a larger macroeconomic rate of price adjustment, κ, raises the policy response.

Much of the optimal monetary policy literature assumes a quadratic objective function and linear constraints. In these settings the globally optimal policy exhibits certainty equivalence. That is, the presence and nature of additive stochastic disturbances does not affect optimal policy (see, e.g. Sargent and Ljungqvist, 2004, Ch. 5). However, when optimal policy is restricted to a Taylor rule of the form (TR1), equations (9) and (10) show that the constraints are no longer linear in the choice variables. Hence, it is not surprising that certainty equivalence breaks down in our setting. This result is consistent with earlier findings (see, e.g. Smets, 2002).

In Figure 1 we use a calibrated version of the model to illustrate how the optimal Taylor coefficients change as we vary the degree of measurement error. For these numerical illustrations we make the following parametric choices: Each time period is a quarter. We assume logarithmic utility ($\sigma = 1$). The discount factor β is set to 0.99 and the effective rate of price adjustment is $\kappa = 0.34$.[9] We choose an autoregressive parameter of the efficient rate ϱ_r equal to 0.9, similar to calibrations of trend stationary productivity shocks in the real business cycle literature. Following Galí (2008, Ch. 5) we set the persistence of the cost-push shocks ϱ_u to 0.5. The variances of the innovations to the efficient rate and the cost-push shocks are chosen so that the annual unconditional variances of r_t^e and u_t are 1.00.

Orphanides (2003) provides estimates for the measurement error processes. Based on his calculations, measurement error in inflation is best approximated by a white noise process with a quarterly standard deviation of roughly 0.5. In contrast, measurement error in the output gap has a quarterly autoregressive coefficient of rougly 0.95 and an innovation standard deviation of 0.66.

Finally, we assume that the monetary authority dislikes inflation and the output gap equally so $\alpha = 1$. Table 1 summarizes the baseline calibration. Under these parameter values, the optimal Taylor rule coefficients are $\phi_\pi^* = 2.00$ and $\phi_y^* = 0.61$.[10]

Figure 1 plots the coefficients for the optimal Taylor rule as we vary the amount of measurement error in the model. The figure shows that as uncertainty about current inflation (the left panel) and uncertainty about the output gap (the right panel) increase, optimal policy is less aggressive. It is worth noting that, at least for our baseline calibration, measurement error in the output gap is substantially more influential than measurement error in inflation. This is consistent with the analytical result in Proposition 2.

MONETARY POLICY WITH SIGNAL EXTRACTION

To this point, we have assumed that the central bank directly responds to current *measured* inflation π_t^m and the *measured* gap y_t^m. Here, we consider a modified Taylor rule in which the central bank sets the interest rate as a function of *estimated* inflation and the *estimated* output gap.[11]

[9] This value of κ can be derived from a Calvo model with a probability of price rigidity of 2/3 per quarter, together with our calibrated value of β, a Frisch labor supply elasticity of 1.00 and a linear production function.

[10] Whenever we report numerical values for the coefficient on the output gap, we annualize it by multiplying the quarterly value by four.

[11] Among others, Orphanides (2001) has advocated this specification. A number of researchers have examined signal extraction problems of central banks. Our approach draws on the results of Svensson and Woodford (2003, 2004). See also Swanson (2000), Smets (2002), Aoki (2003).

We assume the central bank uses the Kalman filter to estimate the output gap and the inflation rate. For a generic variable x_t, we let $x_{t|t}$ denote the central bank's estimate of the variable given the information available at date t.[12] Having solved the signal extraction problem, the central bank sets the interest rate according to the modified Taylor rule

$$i_t = \rho + \psi_\pi \pi_{t|t} + \psi_y y_{t|t} \ . \tag{TR2}$$

ψ_π and ψ_y are the Taylor rule coefficients which operate on the "nowcast" estimates $\pi_{t|t}$ and $y_{t|t}$. We list this Taylor rule as (TR2) to distinguish it from the more conventional Taylor rule (TR1). The remaining model equations are unchanged.

Lemma 2 characterizes the estimates $\pi_{t|t}$ and $y_{t|t}$. This lemma is analogous to Lemma 1 when there is no measurement error.

Lemma 2: *The central bank's estimates of inflation and the output gap satisfy*

$$\pi_{t|t} = \frac{\kappa}{\Phi_r + \psi_y(1-\beta\varrho_r) + \kappa\psi_\pi} r^e_{t|t} + \frac{\psi_y + (1-\varrho_u)\sigma}{\Phi_u + \psi_y(1-\beta\varrho_u) + \kappa\psi_\pi} u_{t|t}, \tag{14}$$

$$y_{t|t} = \frac{1-\beta\varrho_r}{\Phi_r + \psi_y(1-\beta\varrho_r) + \kappa\psi_\pi} r^e_{t|t} + \frac{\varrho_u - \psi_\pi}{\Phi_u + \psi_y(1-\beta\varrho_u) + \kappa\psi_\pi} u_{t|t}, \tag{15}$$

where Φ_r and Φ_u are defined as in Lemma 1.

The lemma shows that, with a suitable reinterpretation of the shocks, the equilibrium paths of the filtered variables $y_{t|t}$ and $\pi_{t|t}$ obey the same equilibrium conditions as the actual underlying variables y_t and π_t (see equations 8 and 9).

The optimal coefficients $\{\psi^*_y, \psi^*_\pi\}$ minimize (10) subject to (1) - (6), (TR2), and the central bank's informational constraints. The following proposition presents a result for the i.i.d. case in which we can obtain a closed-form solution.

Proposition 3: *Suppose all shocks are contemporaneously uncorrelated with each other and i.i.d over time. Then the coefficients $\{\psi^*_y, \psi^*_\pi\}$ satisfying*

$$\psi^*_\pi = \alpha\kappa(\psi^*_y + \sigma) + (1+\alpha\kappa^2)\frac{Cov[r^e_{t|t}, u_{t|t}]}{V[u_{t|t}]} \tag{11'}$$

and $\psi^*_y \to \infty$ are optimal.

[12] Formally, the central bank's information set is $I^{CB}_t = \{\Theta, \pi^m_{t-j}, y^m_{t-j} \colon j \geq 0\}$ where Θ is a vector of all model parameters. Then, for any variable x_t, the central bank's date t estimate is $x_{t|t} = E[x_t | I^{CB}_t]$. The observation equations are $\pi^m_t = \pi_t + m^\pi_t$ and $y^m_t = y_t + m^y_t$.

Proposition 3 shows that, there is an optimal Taylor rule in terms of *filtered* output $y_{t|t}$ and inflation $\pi_{t|t}$ which embodies similar properties as Proposition 1 with $\varrho_u = 0$. In particular, the optimal Taylor rule coefficients are again infinitely large and ψ_π^*/ψ_y^* converges to $\alpha\kappa$. Notice the following subtle difference: Since we assumed that all exogenous shocks are uncorrelated, there is no covariance term in equation (11) of Proposition 1. Here, in equation (11'), the correlation between the estimates $r_{t|t}^e$ and $u_{t|t}$ in Proposition 3 arises endogenously even though the underlying shocks r_t^e and u_t are uncorrelated.

The correlation between $r_{t|t}^e$ and $u_{t|t}$ comes from the central bank's effort to infer the true shocks. For example, suppose the central bank observes positive inflation and a negative output gap. A negative supply shock ($u_t > 0$) is a natural candidate for such observations. Another possibility is the occurrence of a positive demand shock together with a negative innovation to measurement error in the output gap. Because the central bank attaches positive probability to many potential combinations of shocks, its estimates $r_{t|t}^e$ and $u_{t|t}$ will typically be correlated. For the optimal Taylor rule, the induced correlation is immaterial. Equations (14) and (15) with $\varrho_u = \varrho_r = 0$ imply that only the limit of ψ_π^*/ψ_y^* affects the central bank's estimates of inflation and the output gap.

We next turn to the general case in which all shocks have arbitrary autocorrelation. Since the analytical solution of the optimal policy problem is difficult, we use numerical methods to characterize the optimal Taylor rule.

Figure 2 shows level curves of the central bank's objective function when all parameters are set to the values of our baseline calibration summarized in Table 1. As we have seen in Proposition 3, stronger responses to both expected inflation and the expected output gap reduce the central bank's loss function (10). This continues to be the case when there are persistent innovations. Again, the optimal Taylor rule coefficients are infinitely large.[13]

As the Taylor rule coefficients approach infinity, (14) and (15) imply that expected inflation and the expected output gap depend only on the estimated supply shock. In particular, $\pi_{t|t} = Au_{t|t}$ and $y_{t|t} = -Bu_{t|t}$ for some strictly positive constants A and B. Eliminating $u_{t|t}$ yields

$$y_{t|t} = -BA^{-1}\pi_{t|t}.$$

The central bank acts to neutralize the effect of estimated demand shocks on $\pi_{t|t}$ and $y_{t|t}$ so that under the optimal Taylor rule, only the effects of cost-push shocks (i.e., "supply shocks") remain. As a consequence, the central bank's best estimates of current inflation and output should be perfectly negatively correlated.

[13] We have experimented with alternative calibrations. In all cases the optimal Taylor rule coefficients approach infinity.

There is a close connection between the Taylor rule given by (TR2) and interest rate smoothing. Interest rate smoothing can be captured by a policy rule of the form

$$i_t = \rho + \phi_\pi \pi_t^m + \phi_y y_t^m + \nu i_{t-1}\,. \qquad \text{(TR3)}$$

In this specification the central bank sets the interest rate as a function of measured inflation, measured output, and the lagged interest rate.[14] The parameter ν governs the extent to which the central bank anchors its current policy with the interest rate from the previous quarter.

Taylor rules with interest rate smoothing are similar to Taylor rules based on filtered output and inflation. In fact, there are special cases in which the two rules exactly coincide. To see this, consider the model in the previous section given by equations (1) to (6) and (TR2), in which the central bank observes $\pi_t^m = \pi_t + m_t^\pi$ and $y_t^m = y_t + m_t^y$ and uses the Kalman filter to estimate the output gap and inflation. Suppose further that there are no cost-push shocks ($V[u_t] = 0$) and that the persistence of the measurement error shocks is zero ($\varrho_{my} = \varrho_{m\pi} = 0$). For this special case, we have the following result.

Proposition 4: *Given the assumptions above, for any Taylor rule* (*TR2*) *with coefficients* ψ_y *and* ψ_π*, there exist coefficients* $\{\tilde{\phi}_y, \tilde{\phi}_\pi, \tilde{\nu}\}$ *such that the policy rule* (*TR3*) *generates the same equilibrium paths for all variables.*

Thus, if there are no cost-push shocks and measurement error is i.i.d., the Taylor rule in which the central bank responds to the estimates $y_{t|t}$ and $\pi_{t|t}$ can be implemented exactly by a Taylor rule with interest rate smoothing of the form (TR3).[15]

While exact equivalence between interest rate smoothing and Taylor rules of the form (TR2) requires strong assumptions, the spirit of Proposition 4 extends to very general settings. As long as the central bank follows Taylor rule (TR2) and forms its estimates $y_{t|t}$ and $\pi_{t|t}$ using the Kalman filter, the interest rate will change gradually because the estimates of the output gap and inflation change gradually. In such a setting, an econometrician would continue to conclude that lagged interest rates play an important role in shaping policy. To demonstrate this claim quantitatively, we again consider the model consisting of equations (1) to (6), the central bank's informational constraints described above and Taylor rule (TR2). When we estimate the interest rate smoothing rule (TR3) on data simulated from this model using the benchmark calibration summarized in Table 1, we obtain a smoothing parameter ν of about 0.7.

[14] See Sack and Wieland (2000), Williams (2003), and Rudebusch (2006) for discussions of interest rate smoothing.
[15] Aoki (2003) reaches a similar conclusion in a setting where the central bank optimizes under discretion.

COMPARING THE TAYLOR RULES

Table 2 reports the value of the objective function (equation 8), as well as the associated variance of inflation and the output gap for the three Taylor rules (TR1), (TR2), and (TR3). For rules (TR1) and (TR3) the optimal coefficients are shown in the bottom rows of the table. Not surprisingly, the simple Taylor rule in which the central bank responds to measured inflation and measured output (equation TR1) is inferior to both alternatives. For our baseline calibration, Taylor rule (TR2) outperforms the interest rate smoothing rule (TR3) though we note that it is possible to construct cases in which optimal interest rate smoothing outperforms Taylor rule (TR2).[16]

Figures 3 and 4 report impulse response functions for the four structural shocks ($r_t^e, u_t, m_t^y, m_t^\pi$). The top panels of Figure 3 show the impulse response functions of the output gap, inflation, and the interest rate to a shock to the efficient rate r_t^e (a 'demand' shock). When the central bank follows the simple Taylor rule (TR1), the output gap is remarkably close to zero. However, inflation jumps up substantially and only slowly converges back to its steady state value of zero. The history dependent rules (TR2) and (TR3) are superior in this regard.

Impulse response functions for the cost-push shock are shown in the bottom panels of Figure 4. On impact, the output gap is largest for the simple Taylor rule (TR1). No further striking differences of the paths of output and inflation are revealed for the different policy rules.

As in our baseline calibration, Orphanides (2003) estimated that measurement error in the output gap is highly persistent. The top panels of Figure 4 show that this persistence is reflected in the dynamic reaction to a noise shock. When the central bank solves a signal extraction problem and sets the interest rate according to Taylor rule (TR2), it learns gradually that the shock must have been measurement error. Although the output gap and inflation drop immediately after the shock, optimal state estimation soon reveals that the shock is likely noise and the central bank guides the economy back to steady state. When the central bank follows rules (TR1) or (TR3) instead, the persistence of the measurement error shock causes the economy to go through a protracted period of low inflation.

Not surprisingly, signal extraction is particularly important when measurement error is persistent. Because our baseline calibration features no persistence for measurement error in inflation,

[16] Consider a calibration in which all of the structural shocks have no persistence. In this case, the filtered estimates $\pi_{t|t}$ and $y_{t|t}$ are functions of only current observations π_t^m and y_t^m. As a consequence, the optimized value of the objective is the same for the simple Taylor rule (TR1) and for the Taylor rule with filtered data (TR2). However, since a central bank following (TR3) can choose an additional parameter it must be able to weakly improve on the restricted rules. The interest rate smoothing parameter allows the central bank some ability to commit to future actions which is often a feature of the globally optimal policy. See, e.g., Clarida, Galí and Gertler (1999), Woodford (1999) and Rotemburg and Woodford (1999).

when the economy experiences such a shock, there are only slight differences in output and inflation across the policies. This is shown in the bottom panels of Figure 4. While the impact response of the output gap is largest for the simple Taylor rule (TR1), the lack of history dependence implies a return to the steady state in a single period. Rules (TR2) and (TR3) show a somewhat slower convergence rate.

Does the Federal Reserve Follow an Optimal Taylor Rule?

If the central bank followed an optimal Taylor rule, then expected inflation and the expected output gap should be perfectly negatively correlated. To see whether this prediction holds in the data, we examine the Fed's real time forecasts of current quarter inflation and the output gap. These forecasts are produced by the Federal Reserve staff before every meeting of the Federal Open Market Committee and made public with a five year lag by Federal Reserve Bank of Philadelphia. We take the Fed's contemporaneous estimate of Core CPI inflation as the empirical analogue of $\pi_{t|t}$.

Figure 5 plots the current-quarter forecasts of Core CPI inflation and the output gap from 1987Q3 - 2007Q4. In the figure, 'initial' refers to the forecast of the output gap produced for the first quarterly meeting and 'revised' refers to the forecast for the second meeting. Panel A shows the unfiltered series. There is no obvious relationship between the two series in the figure. The sample correlation between the initial output gap estimate and the estimate of Core CPI inflation is -0.09 (the correlation changes to -0.1 if we use the revised estimate). In contrast, under the optimal Taylor rule, the correlation between the two series should be -1.00.

One concern with using the raw series to compute this correlation is that much of the variation in inflation is due to a steady declining trend since the mid 1980's. In Panel B, we show HP-filtered time series for both the estimated output gap and estimated inflation. (We use the standard quarterly smoothing parameter of 1600). In this figure, the estimates of inflation and the output gap have a modest positive correlation. The correlation coefficient is 0.35 for the initial output gap estimate and 0.29 for the revised estimate.

In neither case is the correlation close to the prediction of negative one. Interpreted through the lense of our model the Fed seems to underreact to its own estimates of inflation and output.

4. Conclusion

In this paper, we analyzed optimal Taylor rules in New Keynesian models when the monetary authority's targets are uncertain. In the absence of measurement error, activist monetary policy is costless and the optimal Taylor rule coefficients are infinite. When the central bank responds to noisy measures of inflation and output, the optimal Taylor rule coefficients are finite. If the central bank sets the interest rate as a function of *estimated* output and inflation, then the optimal coefficients on estimated inflation and the estimated gap are again infinite. Signal extraction on the part of the central bank also introduces behavior which mimics history dependence in monetary policy. Because the central bank's beliefs about the state of the economy are updated gradually, it will appear that the central bank is practicing interest rate smoothing even though it is reacting only to current estimates of inflation and the output gap.

Optimal monetary policy in the model with signal extraction implies a robust negative correlation between estimated inflation and estimated output. In contrast, data on the Federal Reserve's estimates of current inflation and the output gap exhibit either zero correlation or a modest positive correlation depending on whether or not they are detrended. When interpreted through the lens of the model, this suggests that the Fed is insufficiently aggressive in responding to deviations from its targets.

Our model assumes that the central bank faces uncertainty about fundamentals and learns about the true state gradually over time while the private sector knows the true fundamentals. In practice, it is likely that the private sector also has imperfect information, and potentially less information than the central bank. Indeed, Nakamura and Steinsson (2018) find evidence for an "information effect" in which increases in the federal funds rate signal higher growth in the future. This partially offsets the contractionary effects of higher interest rates and suggests that the private sector has less information than the central bank. While a full discussion of this alternative informational structure is beyond the scope of this paper, we speculate that the central bank may change its optimal policy in two ways. First, the central bank may try to counteract the effects of noisy signals received by households and firms. Second, to the extent that the private sector draws inferences about fundamentals from policy actions, the central bank may have an incentive to temper its reactions to accurate information. More generally, a number of alternative informational assumptions appear plausible.[17] We leave such extensions for future research.

[17] Svensson and Woodford (2003) study a case with symmetric information in a model in which potential output and hence the output gap is unobserved, while inflation is observed without error. Although they do not discuss commitment to a Taylor rule, several of their findings are similar to ours.

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TABLE 1: BASELINE CALIBRATION		
Parameter	Value	Description
β	0.99	Discount factor
σ	1	Coefficient of relative risk aversion
θ	2/3	Poisson rate of price stickiness
η	1	Labor supply elasticity
$(\varrho_r, \varrho_u, \varrho_{m\pi}, \varrho_{my})$	$(0.9, 0.5, 0, 0.95)$	Persistence of shock processes
$sd[\varepsilon^r]$	0.22	Standard deviation (SD) of innovation to efficient rate
$sd[u]$	0.43	SD of innovation to the cost-push shock
$sd[\varepsilon^{m\pi}]$	0.50	SD of innovation to measurement error in inflation
$sd[\varepsilon^{my}]$	0.66	SD of innovation to measurement error in the gap
α	1	Weight on inflation in the policy objective function

TABLE 2: EVALUATION OF POLICY RULES			
	Standard Taylor Rule (TR1)	Taylor Rule with signal extraction (TR2)	Interest Rate Smoothing (TR3)
Criterion (10)	1.56	1.03	1.16
$V[y]$	1.11	0.69	0.86
$V[\pi]$	0.45	0.34	0.30
ϕ_y^*	0.61	-	0.27
ϕ_π^*	2.00	-	0.42
ν^*	-	-	1.08

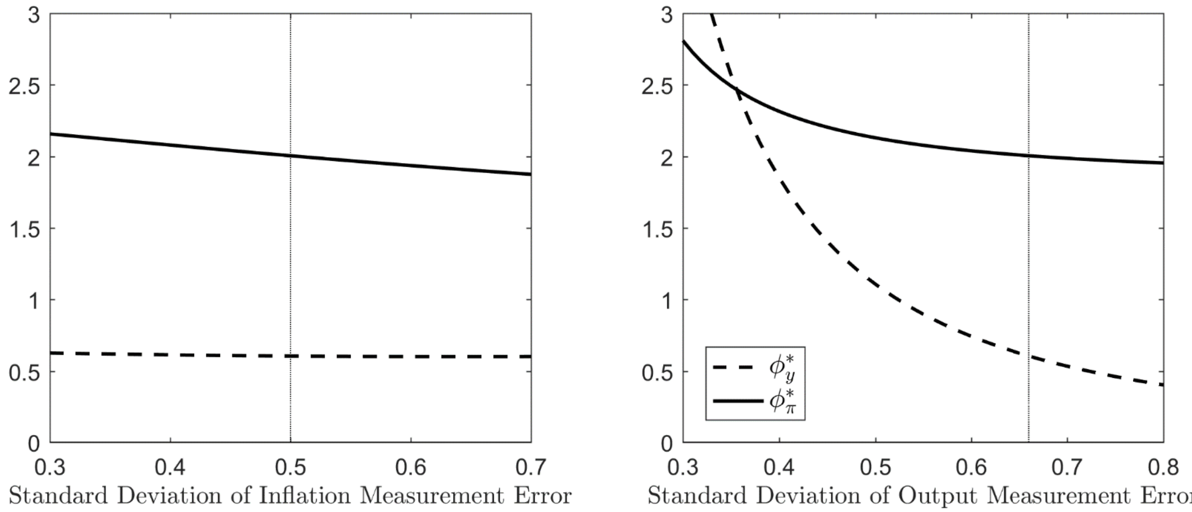

Figure 1: Parametric Variations in Shock Variance

Notes: Each panel plots the optimal Taylor rule coefficients for inflation (heavy solid line) and the output gap (heavy dashed line). The vertical dashed line indicates the parameter value in the baseline calibration. All other parameters are held constant at the baseline level.

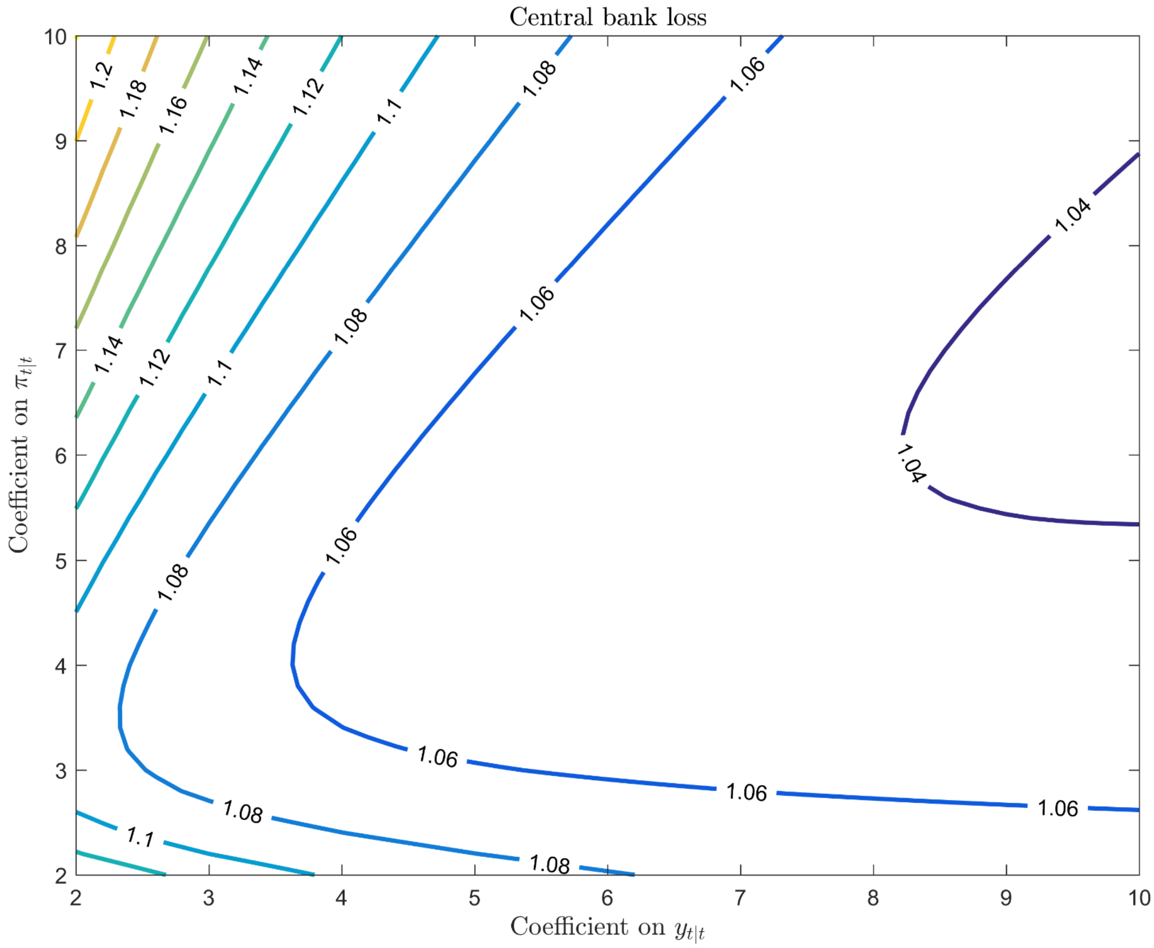

Figure 2: Level curves of the central bank's loss function

Notes: The figure uses the baseline calibration summarized in Table 1.

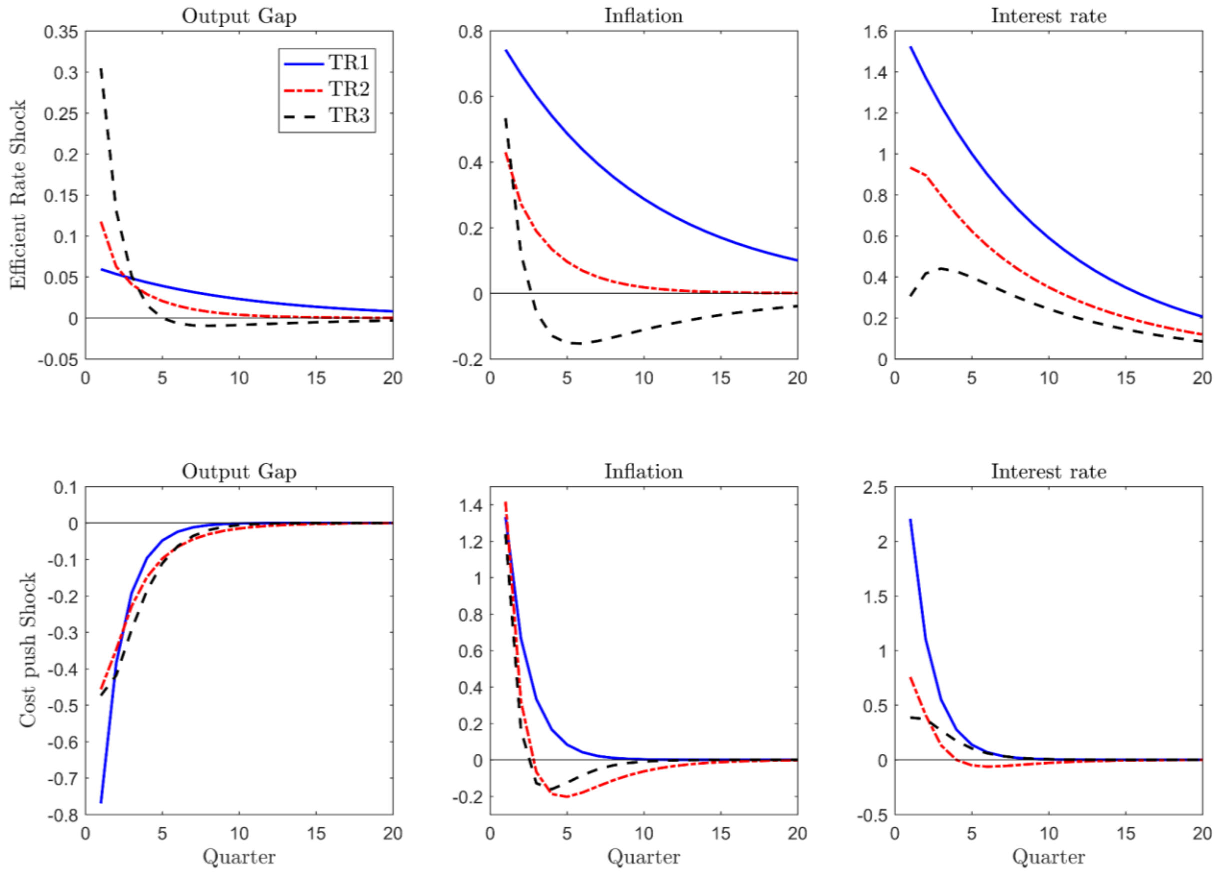

Figure 3: Impulse Response Functions

Notes: All shocks are one standard deviation in size. The unit of the output gap is percentage deviations from its steady state value of zero. The units of inflation and the interest rate are annualized percentage points.

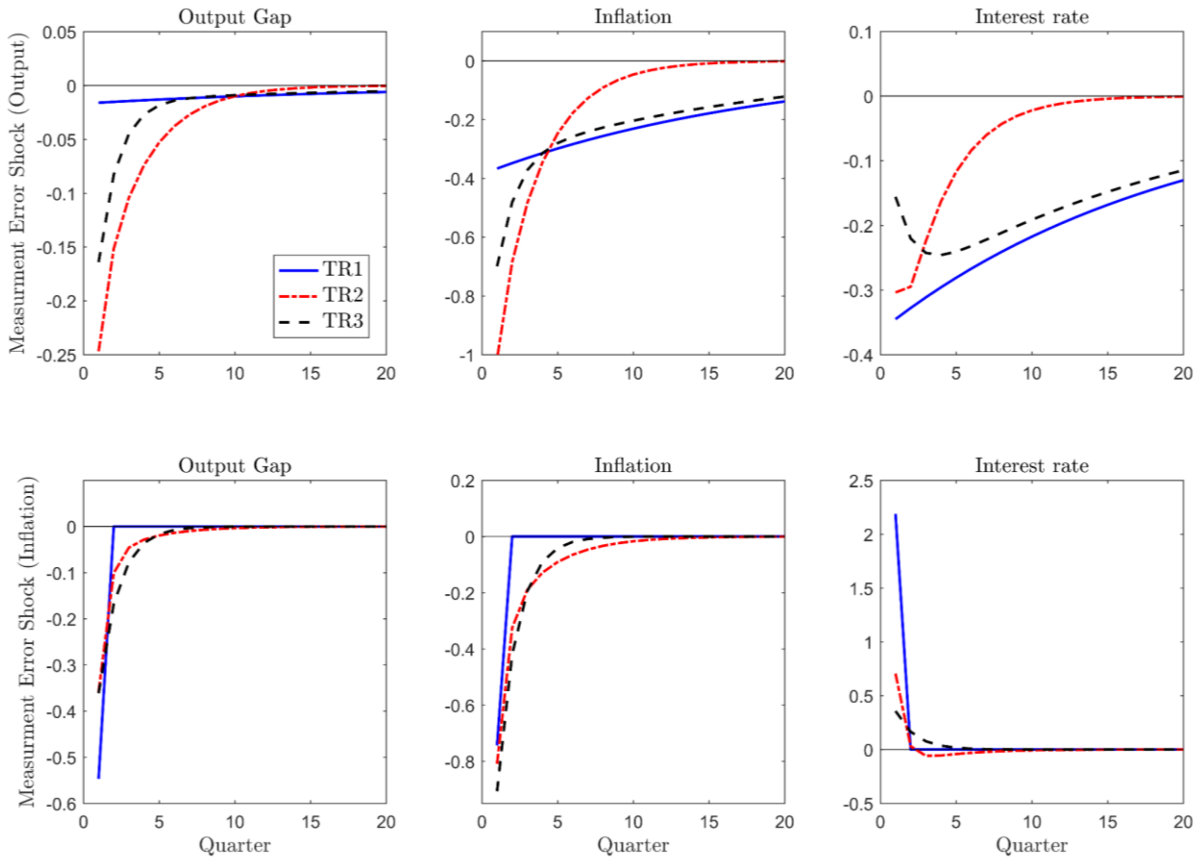

Figure 4: Impulse Response Functions

Notes: All shocks are one standard deviation in size. The unit of the output gap is percentage deviations from its steady state value of zero. The units of inflation and the interest rate are annualized percentage points.

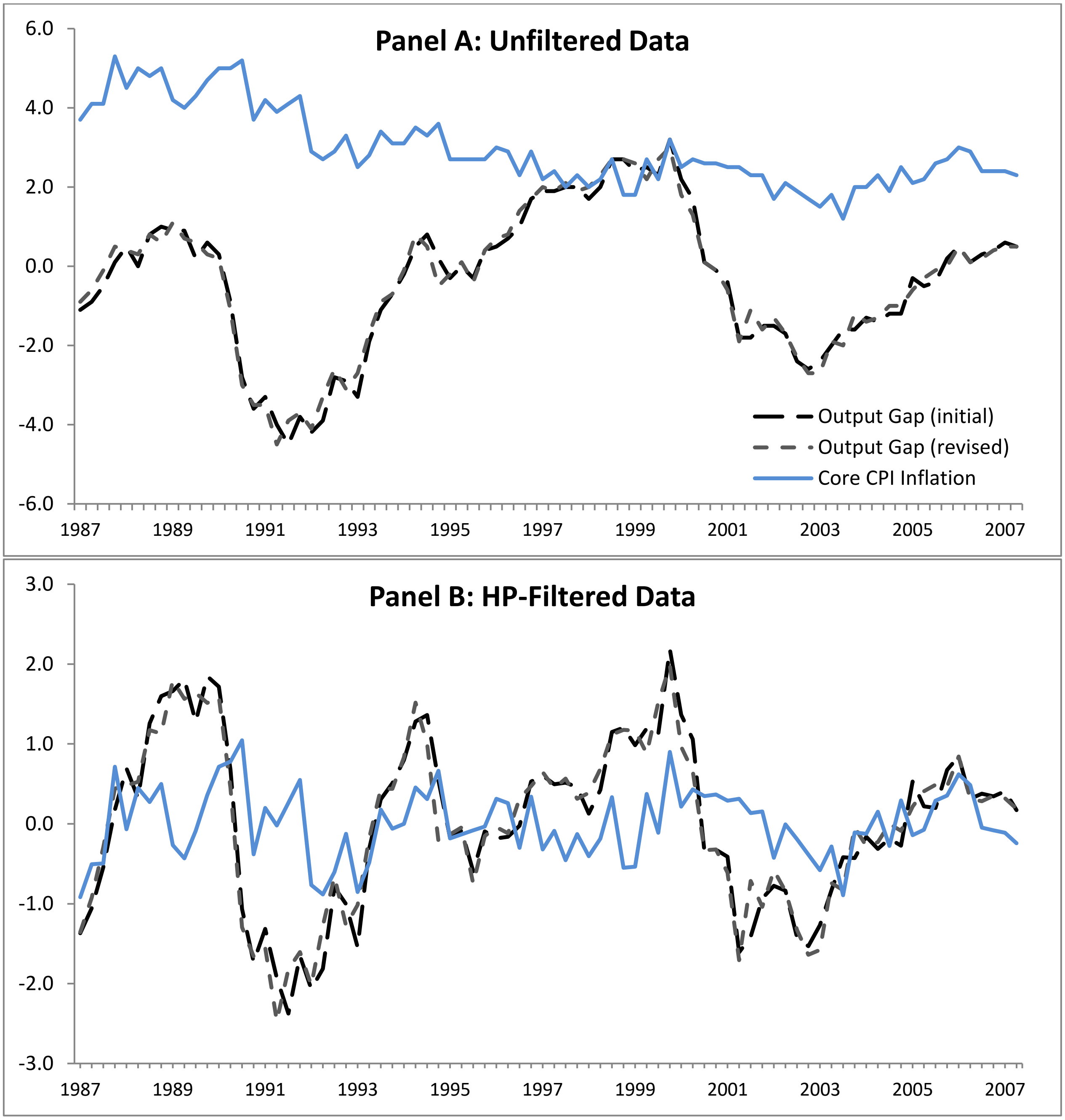

Figure 5: Inflation and output gap estimates of the Fed

Notes: The graphs show the current-quarter estimates of Core CPI inflation and the output gap. All series in Panel B are HP-filtered with a smoothing parameter of 1600. The Federal Open Market Committee meets twice per quarter and two estimates for the output gap are available. “Initial” refers to the estimate for the first and “revised” to the estimate for the second meeting.

Source: Federal Reserve Bank of Philadelphia.