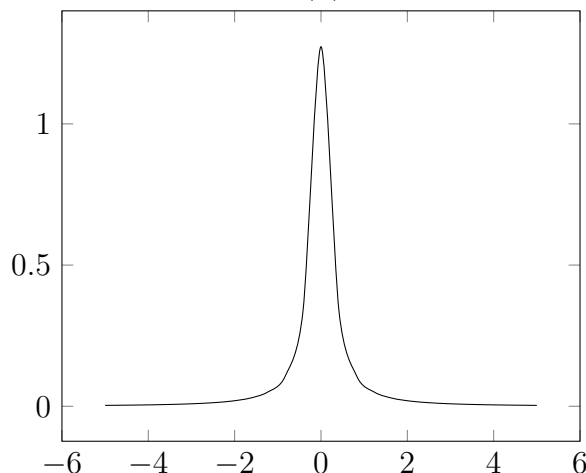


1. False. The mean has to exist first.
2. True. We computed the mean as $\frac{1}{c}$ and the median as $\frac{\ln(2)}{c}$, and $\ln(2) < 1$.
3. (a) Here is the graph of $f(x)$.



- (b) $f(x) \geq 0$ since x^2 is always positive, so we only need to check that $\int_{-\infty}^{\infty} f(x)dx = 1$.

$$\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{\infty} \frac{4}{\pi} \frac{1}{1 + 16x^2} dx$$

Use the substitution $u = 4x$. Then $du = 4dx$, so the integral becomes:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1 + u^2} du$$

and this is number 14 on the homework, so it is a pdf. If you don't remember number 14, do the following:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \frac{1}{1 + u^2} du = \frac{1}{\pi} \left(\lim_{t \rightarrow -\infty} \int_t^0 \frac{1}{1 + u^2} du + \lim_{t \rightarrow \infty} \int_0^t \frac{1}{1 + u^2} du \right)$$

The antiderivative of $\frac{1}{1+u^2}$ is $\arctan(u)$, which has horizontal asymptotes at $y = \frac{\pi}{2}$ and $y = \frac{-\pi}{2}$, so the previous integral is:

$$\begin{aligned} \frac{1}{\pi} \left(\lim_{t \rightarrow -\infty} (\arctan(0) - \arctan(t)) + \lim_{t \rightarrow \infty} (\arctan(t) - \arctan(0)) \right) \\ = \frac{1}{\pi} \left(0 - \frac{-\pi}{2} + \frac{\pi}{2} - 0 \right) = \frac{\pi}{\pi} = 1 \end{aligned}$$

- (c) Since the function is symmetric about $x = 0$, the median is 0. The mean does not exist (see number 14 on the homework, it's almost the same computation).