

1. True. The Taylor series of any polynomial at any point is the polynomial itself.
2. True, Newton's method can "get stuck" sometimes.
3. (a) The Taylor series for $g(x) = \frac{1}{1-x}$ is $T_g(x) = 1 + x + x^2 + \cdots + x^i + \cdots$ so to find the Taylor series for $f(x) = g(x^2)$, plug in x^2 for x into $T_g(x)$.
Thus, the Taylor series for $f(x) = \frac{1}{1-x^2}$ is $T_f(x) = 1 + x^2 + x^4 + \cdots + x^{2i} + \cdots$.
In sum notation, this is $T_f(x) = \sum_{i=0}^{\infty} x^{2i}$.
(b) We want to find $\sqrt{101}$, so we want to find the positive root of $f(x) = x^2 - 101$.
The formula for one iteration of Newton's method is $x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$.
Thus, $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 10 - \frac{-1}{2 \cdot 10} = 10 + \frac{1}{20} = \frac{201}{20}$.