HOMEWORK 4 – MATH 632.

1. Let $0 \leq m \in \mathbb{Z}$. Use Čech cohomology to compute $H^1(\mathbb{P}^1, \Omega_{\mathbb{P}^1}(-m))$. Give a perfect pairing

$$H^0(\mathbb{P}^1, \mathcal{O}(m)) \times H^1(\mathbb{P}^1, \Omega_{\mathbb{P}^1}(-m)) \to H^1(\mathbb{P}^1, \Omega_{\mathbb{P}^1}).$$

2. Let $\mathcal{F}$ and $\mathcal{G}$ be quasicoherent sheaves on a separated Noetherian scheme X. Using Čech cocycles show there is a bilinear map:

$$H^i(X, \mathcal{F}) \times H^j(X, \mathcal{G}) \to H^{i+j}(X, \mathcal{F} \otimes \mathcal{G}).$$

Conclude that $H^*(X, \mathcal{O}_X) = \oplus H^i(X, \mathcal{O}_X)$ is a graded ring and $H^*(X, \mathcal{F})$ is a module over that ring.

the Hartshornes

3. Hartshorne III.2.3.

4. Hartshorne III.4.1.

5. Hartshorne III.4.3.

6. Hartshorne III.4.5.

7. Hartshorne III.4.6.

8. Hartshorne III.5.5.

9. Let C be a smooth projective curve, let L be a very ample line bundle on C, and let $h_L(m)$ be the Hilbert polynomial — i.e. the linear function that asymptotically computes $h^0(C, L^{\otimes m})$. Show that

$$h_L(m) = \chi(C, \mathcal{L}^{\otimes m})$$

and conclude that the arithmetic genus p_a of C equals $h^1(C, \mathcal{O}_C)$.

10. Let C be a smooth projective curve. Riemann-Roch says that for any line bundle L on C:

$$\chi(C, L) = \deg(L) + \chi(C, \mathcal{O}_C).$$

Let $\mathcal{E}$ be a rank r vector bundle on C. Find and prove an expression for $\chi(C, \mathcal{E})$ involving r and $\deg(\det(\mathcal{E}))$ (hint: use Hartshorne II.8.2.).