

# Intrinsic Conductivity in Magnesium–Oxygen Battery Discharge Products: MgO and MgO<sub>2</sub>

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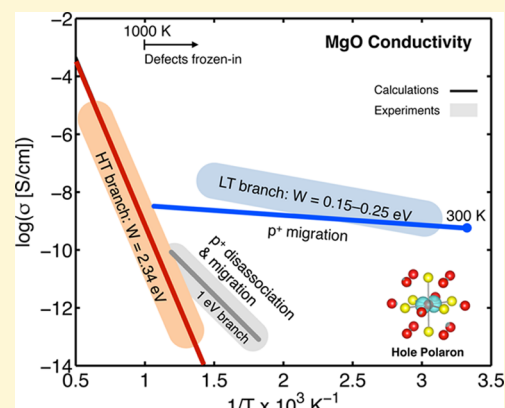
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**ABSTRACT:** Nonaqueous magnesium–oxygen (or “Mg-air”) batteries are attractive next generation energy storage devices due to their high theoretical energy densities, projected low cost, and potential for rechargeability. Prior experiments identified magnesium oxide, MgO, and magnesium peroxide, MgO<sub>2</sub>, as the primary discharge products in a Mg/O<sub>2</sub> cell. Charge transport within these nominally insulating compounds is expected to limit battery performance; nevertheless, these transport mechanisms either are incompletely understood (in MgO<sub>2</sub>) or remain a matter of debate (in MgO). The present study characterizes the equilibrium conductivity associated with intrinsic (point) defects within both compounds using first-principles calculations. For MgO, negative Mg vacancies and hole polarons—the latter localized on oxygen anions—were identified as the dominant charge carriers. However, the large formation energies associated with these carriers suggest low equilibrium concentrations. A large asymmetry in the carrier mobility is predicted: hole polarons are highly mobile at room temperature, while Mg vacancies are essentially immobile. Accounting for nonequilibrium effects such as frozen-in defects, the calculated conductivity data for MgO is shown to be in remarkable agreement with the three “Arrhenius branches” observed in experiments, thus clarifying the long-debated transport mechanisms within these regimes. In the case of MgO<sub>2</sub>, electronic charge carriers alone—electron and hole polarons—are the most prevalent. Similar to MgO, the equilibrium concentration of carriers in MgO<sub>2</sub> is low, and moderate-to-poor mobility further limits conductivity. If equilibrium behavior is realized, then we conclude that (i) sluggish charge transport in MgO or MgO<sub>2</sub> will limit battery performance when these compounds cover the cathode support and (ii) what little conductivity exists in these phases is primarily electronic in nature (i.e., polaron hopping). Artificially increasing the carrier concentration via monovalent substitutions is suggested as a strategy for overcoming transport limitations.

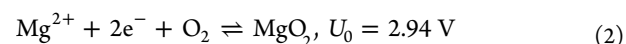
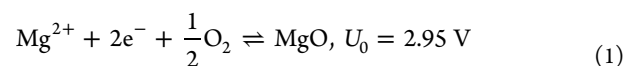


## I. INTRODUCTION

The demand for energy-dense batteries suitable for electric vehicle propulsion has sparked interest in metal–oxygen electrochemistry. For example, a rechargeable battery based on a multivalent Mg/O<sub>2</sub> couple that discharges to magnesium oxide has a theoretical energy density that is nearly seven times (3.9 kW h/kg) that of conventional Li-ion batteries (0.57 kW h/kg) and even surpasses that of a “Li–air” cell (3.5 kW h/kg, assuming discharge to Li<sub>2</sub>O<sub>2</sub>).<sup>1–5</sup> Additional advantages of magnesium-based systems compared to Li analogues include an anode with higher volumetric capacity (3832 mA h cm<sup>-3</sup> Mg vs 2062 mA h cm<sup>-3</sup> Li), suppressed dendrite formation, and lower cost.<sup>6,7</sup>

As a result of the nearly identical formation energies of magnesium oxide,  $\Delta G_f^0(\text{MgO}) = -568.9 \text{ kJ/mol}$ ,<sup>8</sup> and magnesium peroxide,  $\Delta G_f^0(\text{MgO}_2) = -567.8 \text{ kJ/mol}$ ,<sup>9</sup> both compounds may be expected to participate in the cycling of

nonaqueous Mg/O<sub>2</sub> cells. By analogy with Li–air cells that reversibly cycle Li<sub>2</sub>O<sub>2</sub>, these MgO<sub>x</sub> compounds could appear as a solid discharge product within the cathode during discharge and subsequently decompose during charging, according to



Mg/O<sub>2</sub> batteries using nonaqueous electrolytes have recently been reported.<sup>10–13</sup> In the experiments performed by Shiga et al. it was concluded that the discharge product was MgO; furthermore, it was demonstrated that MgO was not

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rechargeable at moderate voltages unless a redox mediator was present.<sup>10,11</sup> In a later study employing a different electrolyte, Vardar et al. found the discharge product to comprise a mixture of MgO and MgO<sub>2</sub>, with the peroxide component exhibiting more facile decomposition during recharge.<sup>12</sup> In agreement with these experimental observations, ab initio calculations of the theoretical limiting voltage in Mg/O<sub>2</sub> batteries predicted low voltaic efficiency (~30%) for cells that cycle MgO. In contrast, cells that discharge to MgO<sub>2</sub> were predicted to achieve much higher efficiencies, up to approximately 90%.<sup>9</sup>

To realize high energy densities it is desirable to maximize the quantity of the MgO<sub>x</sub> discharge product formed within the cathode (i.e., maximize discharge capacity). Unfortunately, maximizing capacity is likely at odds with achieving efficient battery operation, as “sudden death” and high charging voltages in analogous Li/O<sub>2</sub> batteries have both been attributed to the insulating nature of the discharge product.<sup>1,4,14–20</sup> In these systems lithium peroxide (Li<sub>2</sub>O<sub>2</sub>) forms on the cathode during discharge, impeding charge migration from the cathode support to the electrolyte/Li<sub>2</sub>O<sub>2</sub> interface. One may speculate that sluggish charge transport through the discharge product is a universal limitation that must be circumvented to cycle any metal–oxygen battery at high capacity, regardless of the anode composition. These considerations suggest that understanding transport mechanisms in metal–oxygen compounds—including oxides, peroxides, and superoxides—is a prerequisite for the rational design of efficient metal–air batteries.

In the case of MgO, most experimental measurements of transport have been conducted at temperatures exceeding 1000 K.<sup>21–32</sup> Of course, for battery applications it is the low-temperature conductivity that is most relevant. The paucity of measurements near room temperature is presumably due to difficulties associated with measuring the (very low) conductivity of nominally insulating MgO. These measurements are further compounded by the presence of impurities, variations in sample preparation, and sensitivity to the temperature history of the sample, the latter being indicative of a failure to achieve equilibrium at low temperatures.<sup>31,32</sup>

Despite these complications, clear trends in the experimental data for MgO have emerged. Most notable among these is the observation of three distinct “Arrhenius branches” associated with the conductivity as a function of temperature.<sup>33</sup> The conductivity is conventionally described in terms of the Arrhenius energy,  $W$ , according to the expression  $\sigma = \sigma_0 \exp(-W/k_B T)$ . Here  $W$  is the sum ( $E_f + E_b$ ) of the formation and migration energies of a particular charge carrier. Unusually, the conductivity data for MgO exhibits three distinct  $W$  values of approximately 2.4, 1, and 0.2 eV.<sup>21,31–33</sup> These differing values suggest the transport mechanism in MgO varies as a function of temperature. Although speculation regarding the different mechanisms has been offered,<sup>32–35</sup> little direct evidence exists.

Transport in MgO has also been studied using a variety of computational methods. These studies typically focus on intrinsic ionic (point) defects such as vacancy–interstitial pairs (i.e., Frenkel defects) or vacancies involving both cations and anions (i.e., Schottky defects). Early calculations from Catlow<sup>36</sup> and Mackrodt<sup>37,38</sup> using model potentials found that the formation energies were large for both Schottky (~7 eV) and Frenkel (~12–15 eV) defects, precluding any sizable concentration. Subsequent studies<sup>39–49</sup> have reaffirmed these high formation energies using more sophisticated methods that range from first-principles DFT<sup>41,46–48</sup> to quantum Monte

Carlo.<sup>44</sup> In contrast to the large number of studies on Schottky and Frenkel defects in MgO, little effort has been devoted to characterizing electronic carriers such as polarons.

Although a large number of studies have been conducted on MgO, charge transport in alkaline earth peroxides such as MgO<sub>2</sub> is essentially unexplored. This also differs from the situation for alkali metal peroxides and superoxides, such as Li<sub>2</sub>O<sub>2</sub>, sodium peroxide (Na<sub>2</sub>O<sub>2</sub>), and sodium superoxide (NaO<sub>2</sub>),<sup>14,15,50–59</sup> for which a number of studies have recently appeared. In the case of Li<sub>2</sub>O<sub>2</sub>, experiments and calculations agree on the identity of the charge carriers as negative Li vacancies and positive hole polarons, although there is some discrepancy in the magnitude of the conductivity.<sup>15,52</sup> Calculations on these compounds predict low electronic conductivities in the range of approximately 10<sup>–20</sup> to 10<sup>–19</sup> S/cm.<sup>15,56</sup> Likewise, ionic conductivity was also predicted to be low in the peroxides—10<sup>–19</sup> S/cm in Li<sub>2</sub>O<sub>2</sub> and Na<sub>2</sub>O<sub>2</sub>—and several orders of magnitude higher, 10<sup>–10</sup> S/cm, for the superoxide NaO<sub>2</sub>.<sup>15,56</sup> Regarding experiments, the electrical conductivity of Li<sub>2</sub>O<sub>2</sub> at 100 °C was measured at 10<sup>–12</sup> to 10<sup>–11</sup> S/cm.<sup>52</sup> Measurements on the alkali–metal superoxides (KO<sub>2</sub>, RbO<sub>2</sub>, and CsO<sub>2</sub>) reported values in the same range as for Li<sub>2</sub>O<sub>2</sub>.<sup>57</sup> The difference between theory and experiment can arise from the presence of impurities or from nonequilibrium defect concentrations.<sup>60–62</sup> The latter effect results in higher-than-expected concentrations at low temperatures, due to the freeze-in of defects upon cooling of the sample.<sup>61</sup> To put these values in context, the electrical conductivity of common Li-ion cathode materials falls in the range of 10<sup>–5</sup> to 10<sup>–9</sup> S/cm.<sup>63</sup>

In the present study ab initio calculations at the hybrid level of theory and beyond (GW method) are employed to identify the concentrations and mobilities of intrinsic (point) defects in MgO and MgO<sub>2</sub>. In so doing, we shed light on potential performance limitations in Mg/O<sub>2</sub> batteries arising from sluggish transport through these phases. In addition, a comparison between our calculations and the experimental literature allow us to clarify the elusive conduction mechanisms associated with the three Arrhenius branches observed for MgO. Formation energies and concentrations are calculated for several varieties of vacancies, interstitials, and polarons. Subsequently, the mobility of the dominant (i.e., highest-concentration) defects are calculated with the nudged elastic band method.<sup>64–66</sup> The resulting conductivity data are compared to that of the discharge products of other metal–oxygen systems, namely, Li/O<sub>2</sub> and Na/O<sub>2</sub>.

Many-body perturbation theory calculations performed at the GW level of theory reveal MgO and MgO<sub>2</sub> to be insulators with large bandgaps approaching 8 eV. For MgO we find the dominant point defects to be hole polarons localized on the oxygen sublattice and negative Mg vacancies ( $V_{\text{Mg}}^{2-}$ ) with a formal charge of –2. The formation energy for both defects is high, 2.2 eV, indicative of low concentrations. The calculated mobility of  $V_{\text{Mg}}^{2-}$ ,  $4 \times 10^{-38}$  cm<sup>2</sup>/(V s), is very low, but in excellent agreement with experimental data,  $1.5 \times 10^{-37}$  cm<sup>2</sup>/(V s).<sup>29</sup> In contrast, the mobility of hole polarons is much higher,  $6 \times 10^{-3}$  cm<sup>2</sup>/(V s). In the case of MgO<sub>2</sub>, electronic charge carriers alone—electron and hole polarons—are the most prevalent. Similar to MgO, the absolute concentration of carriers in MgO<sub>2</sub> is low, and moderate-to-poor mobility further limits conductivity.

Our calculations indicate that at room temperature the equilibrium conductivity for both MgO and MgO<sub>2</sub> is low (~10<sup>–36</sup> S/cm), even when compared to that of other

insulating metal/O<sub>2</sub> discharge products such as Li<sub>2</sub>O<sub>2</sub> and NaO<sub>2</sub>. Consequently, sluggish charge transport will limit the performance of Mg/O<sub>2</sub> batteries by restricting the effective thicknesses and, thus, the amount of discharge product formed (i.e., capacity), during discharge. Nevertheless, the observation of moderate-to-high mobility for hole polarons in both compounds suggests a strategy for improving battery performance: artificially enhancing the hole polaron concentration via introduction of monovalent dopants.

## II. METHODOLOGY

Defect formation energy and mobility calculations were performed using the Vienna ab initio simulation package (VASP code).<sup>67–70</sup> Blochl's projector augmented wave (PAW) method<sup>71</sup> was used to treat core–valence electron interactions, with valence states of 3s adopted for magnesium and 2s2p for oxygen. Many-body perturbation theory (GW method) was used to predict the bandgap of MgO and MgO<sub>2</sub>.<sup>72,73</sup> The screened hybrid functional expressed with the formulation of Heyd–Scuseria–Ernzerhof (HSE)<sup>74,75</sup> was used with the fraction of exact exchange tuned ( $\alpha = 0.42$ ) to reproduce the bandgap from GW calculations. Finite-size energy corrections for charged systems were included via the Makov–Payne method.<sup>76,77</sup> The dielectric constants of MgO and MgO<sub>2</sub> were calculated using density functional perturbation theory;<sup>78</sup> values of 10.7 and 6.8 were obtained, respectively. The calculated value for MgO is in good agreement with the experimental dielectric constant, which falls in the range of 9 to 10.<sup>35</sup> To our knowledge the dielectric constant of MgO<sub>2</sub> has not been reported.

For calculations involving conventional (bulk) unit cells, the Brillouin zone was sampled with a Gamma-centered  $k$ -point mesh of density  $4 \times 4 \times 4$  for oxides/peroxides and  $16 \times 16 \times 16$  for Mg. Conventional cells were expanded into a  $2 \times 2 \times 2$  supercell for defect and mobility calculations on MgO and MgO<sub>2</sub>. In these cases  $k$ -point sampling was performed only at the Gamma point. The plane-wave cutoff energy was set to 400 eV; all atom positions were relaxed until the forces were less than 0.04 eV/Å. The convergence criterion for electronic self-consistency was set to  $10^{-6}$  eV. All calculations were spin polarized to account for the unpaired electron configurations preferred by some defects.

The formation energy,  $E_b$ , of a point defect X in charge state  $q$  was calculated according to the following equation:

$$E_f(X^q) = E(X^q) - E_0 - \sum_i n_i \mu_i + q\epsilon_f + E_{\text{MPI}} \quad (3)$$

Here,  $E_0$  represents the energy of a defect-free cell,  $n_i$  and  $\mu_i$  denote the number and chemical potential of a species  $i$  that has been added to or removed from the cell in the course of creating the defect,  $\epsilon_f$  is the Fermi level (referenced to the valence band maximum), and  $E_{\text{MPI}}$  is the Makov–Payne correction for charged systems.<sup>76,77</sup> Composite defects comprising more than a single point defect can also be studied with this formalism. For example, the formation energy of a Schottky defect can be evaluated by taking the sum of the energies required to form a stoichiometric defect involving vacancies on both cation and anion sites. Similarly, a Frenkel defect is evaluated as the sum of formation energies for a vacancy and an interstitial.

The chemical potential of magnesium was determined using the calculated energy,  $g$  of MgO <sub>$x$</sub>  ( $x = 1, 2$ ) and the chemical potential of oxygen gas ( $\mu_{\text{O}_2}$ ),  $\mu_{\text{Mg}} = g_{\text{MgO}_x} - (x/2)\mu_{\text{O}_2}$ . Following earlier studies, corrections based on the experimental formation enthalpies were applied to both compounds to account for the combined effects of overbinding within O<sub>2</sub> gas and for oxidation state errors in the solid phase.<sup>9,79</sup>

The equilibrium concentration  $c$  of a defect can be expressed as  $c = N \exp(-E_f/k_B T)$ , where  $N$  is the number of (symmetry-equivalent) available defect sites per unit volume,  $k_B$  is the Boltzmann constant, and  $T$  is the absolute temperature.<sup>80</sup> The mobility of a defect, defined as  $\mu$ , is expressed as<sup>81</sup>

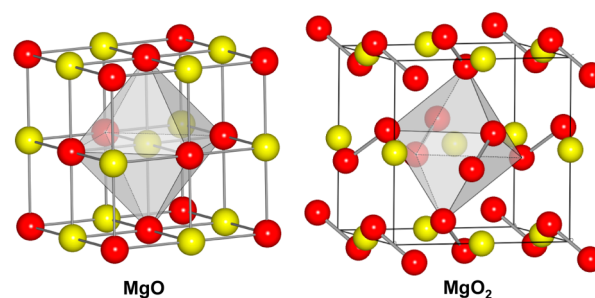
$$\mu = \frac{\nu q a^2}{k_B T} \exp(-E_b/k_B T) \quad (4)$$

where  $\nu$  is the attempt frequency ( $10^{13} \text{ s}^{-1}$ ) for migration,<sup>82</sup>  $q$  is the charge on the defect,  $a$  is the distance between neighboring defect sites, and  $E_b$  is the migration barrier. The total conductivity is expressed as a sum of the product of concentration and mobility contributions from all charge carriers  $i$ :

$$\sigma = \sum_i |q_i| c_i \mu_i \quad (5)$$

## III. RESULTS AND DISCUSSION

**Lattice Geometry.** MgO adopts the rocksalt structure with an experimental lattice constant of  $a = 4.21 \text{ \AA}$ .<sup>83</sup> Vannerburg reported that MgO<sub>2</sub> crystallizes in the cubic pyrite structure with a lattice constant of  $a = 4.84 \text{ \AA}$ .<sup>84</sup> These crystal structures are illustrated in Figure 1. Our calculations employing the PBE-



**Figure 1.** Crystal structure of MgO and MgO<sub>2</sub>. Magnesium ions are octahedrally coordinated by oxygen ions in MgO and by oxygen dimers in MgO<sub>2</sub>. Red spheres represent oxygen atoms, yellow spheres are magnesium, and the oxygen octahedron is shown in gray.

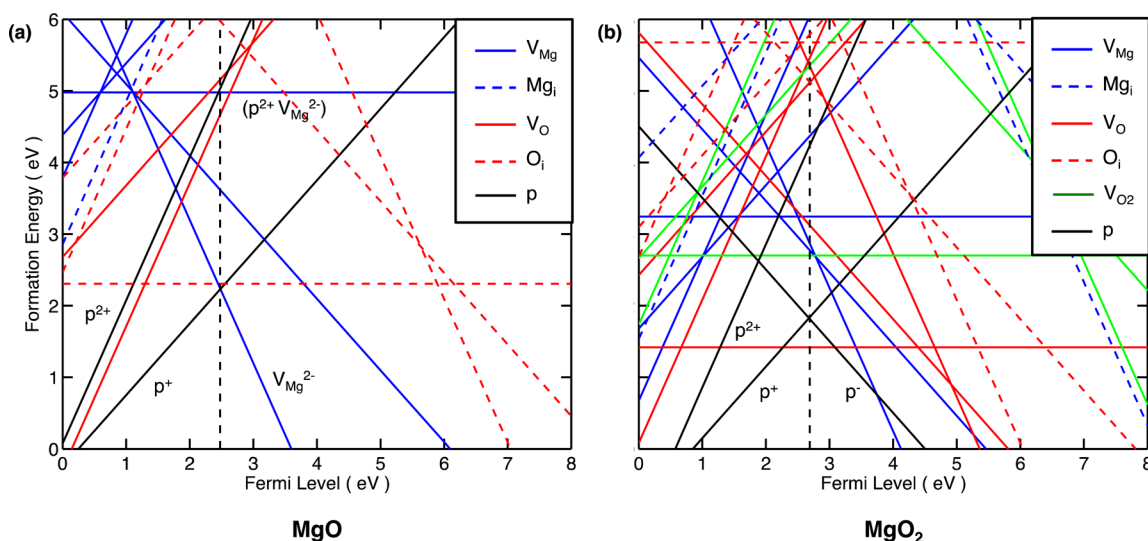
GGA functional result in a slight overprediction of the lattice constants: 4.24 Å for MgO and 4.88 Å for MgO<sub>2</sub>.<sup>9</sup> On the other hand, the HSE06 functional slightly under-predicts the lattice constants: 4.19 and 4.78 Å, respectively. Consequently, we adopted the experimental lattice constants, which fall between our GGA and HSE values, for subsequent GW and defect calculations.<sup>85</sup>

**Bandgaps.** The bandgaps of MgO and MgO<sub>2</sub> were calculated using five different levels of theory: GGA, HSE $\alpha$ , non-self-consistent GW ( $G_0W_0$ ), partially self-consistent GW ( $GW_0$ ), and self-consistent GW (GW) methods. The GW-based methods used the GGA wave functions as input. The calculated bandgaps are summarized in Table 1. The  $GW_0$  method has been shown to best describe the band gap of semiconductors and insulators.<sup>85</sup> Our calculated  $GW_0$  value of 7.5 eV for MgO closely matches that of an earlier GW study (7.7 eV),<sup>85</sup> as well as the experimental band gap of  $\sim 7.8 \text{ eV}$ .<sup>86</sup> Our calculations indicate that MgO has a direct band gap at  $\Gamma$ , while MgO<sub>2</sub> has an indirect band gap. All levels of theory except the GGA predict a slightly larger bandgap for MgO<sub>2</sub> compared to MgO. On the basis of the GW calculations, the HSE functional was tuned to reproduce the bandgaps by mixing a fraction ( $\alpha = 0.42$ ) of exact exchange with semilocal exchange, resulting in HSE $\alpha$  bandgaps of 7.8 and 7.9 eV for MgO and MgO<sub>2</sub>, respectively.

**Defect Concentrations.** Figure 2 shows the formation energies of 51 intrinsic defects evaluated as a function of the Fermi level for MgO and MgO<sub>2</sub>. Five distinct charge states ( $-2$ ,  $-1$ ,  $0$ ,  $+1$ , and  $+2$ ) were considered, with specific defect types including magnesium vacancies (blue lines), magnesium

**Table 1.** Calculated Band Gap for MgO and MgO<sub>2</sub> from Different Levels of Theory and Experimental Optical Band Gaps of MgO<sup>86,98</sup>

	bandgap (eV)					Expt.
	GGA	HSE $\alpha$	GGA+G <sub>0</sub> W <sub>0</sub>	GGA+GW <sub>0</sub>	GW	
MgO	4.5	7.8	7.1	7.5	8.2	7.4, 7.69, 7.7, 7.8
MgO (ref 85)	–	–	7.3	7.7	8.5	–
MgO <sub>2</sub>	3.9	7.9	7.5	8.2	9.1	–

**Figure 2.** Formation energies of intrinsic defects in (a) MgO and (b) MgO<sub>2</sub> calculated using the HSE $\alpha$  functional. Magnesium vacancies ( $V_{Mg}$ ) are depicted using blue lines, magnesium interstitials ( $Mg_i$ ) with blue dashed lines, oxygen vacancies ( $V_O$ ) with red lines, oxygen interstitials ( $O_i$ ) with red dashed lines, and dioxygen vacancies ( $V_{O_2}$ ) with green lines. Hole and electron polarons are shown in black. The slope of each line corresponds to its respective charge state; values of  $-2$ ,  $-1$ ,  $0$ ,  $+1$ , and  $+2$  were considered. The dashed line indicates the position of the Fermi level.

interstitials (blue dashed lines), oxygen vacancies (red lines), oxygen interstitials (red dashed lines), and dioxygen vacancies (green lines); hole and electron polarons are shown in black. The slope of each line corresponds to charge state of that defect.

The vertical dashed line in Figure 2 indicates the position of the Fermi level. Under equilibrium conditions the Fermi level is set by the condition of charge neutrality,  $\sum q_i c_i = 0$ . This condition places the Fermi level somewhat below the center of the band gap, or 2.48 eV above the valence band maximum (VBM) for MgO and 2.69 eV above the VBM for MgO<sub>2</sub>.

Figure 2a shows the formation energies for intrinsic defects in MgO. The defects having the lowest formation energies are the hole polaron ( $p^+$ ) and magnesium vacancy ( $V_{Mg}^{2-}$ ). The formation energies for these defects is high, 2.23 eV, resulting in low equilibrium concentrations for both defects of approximately  $1 \times 10^{-15} \text{ cm}^{-3}$ . For comparison, this is many orders of magnitude less than the intrinsic carrier concentration of silicon ( $\sim 10^{10} \text{ cm}^{-3}$ )<sup>87</sup> and for Li<sub>2</sub>O<sub>2</sub> and Na<sub>2</sub>O<sub>2</sub> ( $\sim 10^7 \text{ cm}^{-3}$ ).<sup>15,56</sup> The defect with the next-lowest formation energy is the neutral oxygen interstitial at 2.3 eV.

A polaron consists of a localized charge (electron or hole) and an induced local lattice distortion (i.e., polarization) in the crystal. In MgO, O<sup>2-</sup> anions are octahedrally coordinated by Mg<sup>2+</sup> cations with an oxygen–Mg distance of 2.1 Å. The absence of one electron on an oxygen site results in the formation of a hole polaron ( $p^+$ ). The resulting oxygen ion has an electronic configuration of  $2s^2 2p^5$  and a formal charge O<sup>1-</sup> and hosts an unpaired electron.

The magnetization density of MgO in the vicinity of a hole polaron is illustrated in Figure 3a. The density isosurface has a shape consistent with that of an oxygen 2p orbital and is aligned along the direction where the Mg ions are furthest from the hole ( $d = 2.25 \text{ Å}$ ). The oxygen–Mg distance along the other

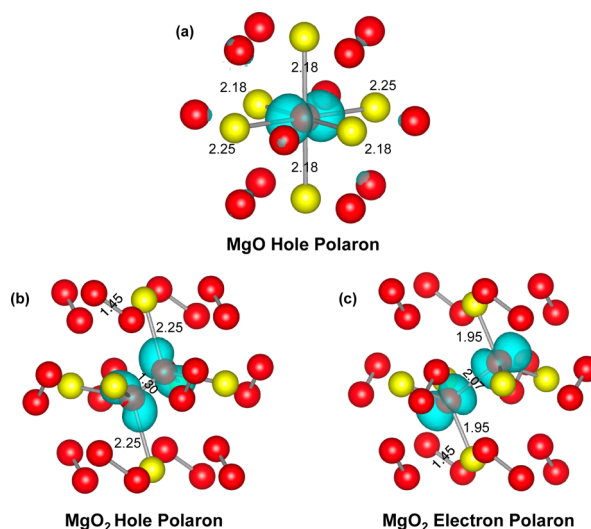
**Figure 3.** Magnetization density for (a) hole polaron in MgO, (b) hole polaron in MgO<sub>2</sub>, and (c) electron polaron in MgO<sub>2</sub>. The iso-surface is plotted at  $0.01 \text{ e}/\text{Å}^3$ . The numbers in the figure indicate bond length in Å. The oxygen to Mg bond distance in bulk is 2.1 Å for both MgO and MgO<sub>2</sub>.

Table 2. Formation Energies and Migration Barriers for Various Defects in MgO and MgO<sub>2</sub><sup>a</sup>

	formation energy ( $E_f$ )				migration barrier ( $E_b$ )	
	Schottky	cation Frenkel	anion Frenkel	selected point defects	polaron	$V_{\text{Mg}}^{2-}$
MgO (this study)	6.91	10.05	13.8	2.23 ( $V_{\text{Mg}}^{2-}$ or $p^+$ )	0.11 ( $p^+$ )	2.20
MgO (prior studies)	7.5–7.9 <sup>b</sup> , 7.5 <sup>c</sup> , 7.72 <sup>d</sup> , 8.2 <sup>e</sup> , 6.88 <sup>f</sup> , 7.53 <sup>g</sup> , 8.44 <sup>h</sup> , 7.5 <sup>i</sup> , 6.0 <sup>k</sup> , 8.8 <sup>k</sup> , 5.79 <sup>j</sup> , 5.05 <sup>m</sup> , 7.22 <sup>n</sup> [5–7 <sup>89</sup> ]	11.9 <sup>c</sup> , 12.43 <sup>d</sup> , 13.35 <sup>h</sup> , 14.1 <sup>i</sup> , 10.3 <sup>k</sup> , 14.1 <sup>k</sup> , 10.41 <sup>m</sup>	15.2 <sup>c</sup> , 12.33 <sup>d</sup> , 13.81 <sup>h</sup> , 13.57 <sup>i</sup> , 12.2 <sup>k</sup> , 13.6 <sup>k</sup> , 13.32 <sup>m</sup>	–	–	1.9–2.2 <sup>b</sup> , 2.16 <sup>c</sup> , 2.07 <sup>d</sup> , 1.93 <sup>g</sup> , 2.08 <sup>h</sup> , 2.10 <sup>i</sup> , [2.28 <sup>29</sup> ]
MgO <sub>2</sub> (this study)	8.33	9.78	<sup>o</sup> 10.24, <sup>p</sup> 12.16	1.82 ( $p^+$ or $p^-$ )	0.56 ( $p^+$ ) 1.76 ( $p^-$ )	–

<sup>a</sup>Schottky and Frenkel defects are comprised of a pair of point defects; their formation energies are reported on a per pair basis. Experimental data appears in brackets. <sup>b</sup>Catlow et al. (1976).<sup>36</sup> <sup>c</sup>Mackrodt et al. (1979).<sup>38</sup> <sup>d</sup>Sangster et al. (1981).<sup>39</sup> <sup>e</sup>Grimes et al. (1990).<sup>40</sup> <sup>f</sup>De Vita et al. (1992).<sup>41</sup> <sup>g</sup>Vočadlo et al. (1995).<sup>42</sup> <sup>h</sup>Busker et al. (2000).<sup>43</sup> <sup>i</sup>Alfè et al. (2005).<sup>44</sup> <sup>j</sup>Uberuaga et al. (2005).<sup>45</sup> <sup>k</sup>Gilbert et al. (2007).<sup>46</sup> <sup>l</sup>Runevall et al. (2011).<sup>47</sup> <sup>m</sup>Mulroue et al. (2011).<sup>48</sup> <sup>n</sup>Grüneis (2015).<sup>49</sup> <sup>o</sup>( $V_{\text{O}}^+$ ,  $\text{O}_1^-$ ). <sup>p</sup>( $V_{\text{O}}^{2+}$ ,  $\text{O}_1^{2-}$ ).

two directions is elongated to a smaller degree,  $d = 2.18$  Å, but is still larger than the bulk Mg–O nearest-neighbor distance (2.1 Å). The general elongation of these bonds relative to bulk is consistent with a weaker electrostatic interaction between the  $\text{Mg}^{2+}$  sublattice and the less-negatively charged  $\text{O}^{1-}$  (resulting from the presence of  $p^+$ ). We note that the hole polaron forms only from a starting configuration wherein the symmetry of the MgO crystal is broken along one of the O–Mg bond directions. Without this distortion, the charge density relaxes to a local minimum in which the hole is delocalized. The stabilization energy for  $p^+$ , defined as the energy difference between the localized and delocalized state, is  $-0.24$  eV. Although hole polarons can be localized on oxygen sites in MgO, we were unable to localize electrons on Mg cations.

Our prediction that doubly negative Mg vacancies,  $V_{\text{Mg}}^{2-}$ , and holes,  $p^+$ , comprise the dominant defects in MgO is consistent with experiments.<sup>33–35,88</sup> For example, magnetic susceptibility measurements indicate the presence of paramagnetic species in MgO,<sup>34</sup> a feature which could be explained by a nonzero concentration of  $p^+$ . Also, abrupt changes in the magnetic properties, charge distribution, and conductivity in MgO with respect to temperature were attributed to holes localized on the oxygen sublattice that are formed upon dissociation of a so-called “peroxy defect”.<sup>88</sup> This composite defect is comprised of a peroxide ion ( $\text{O}_2^{2-}$ ) and  $V_{\text{Mg}}^{2-}$  and is therefore neutral and nonmagnetic. Its formation energy is shown in Figure 2a as ( $p^{2+} V_{\text{Mg}}^{2-}$ ).

Other defects of relevance in MgO include Schottky (a vacancy pair on anion and cation sublattices) and Frenkel (a vacancy–interstitial pair) defects, which are often present in ionic materials;<sup>81</sup> the formation energies of these composite defects are summarized in Table 2. Cation and anion Frenkel defects have a calculated formation energy of 10.1 and 13.8 eV, respectively, similar in magnitude to the result obtained by Mackrodt<sup>38</sup> (11.9 and 15.2 eV) using interatomic potentials. Regarding Schottky defects, experimental measurements of the formation energy range from 5 to 7 eV.<sup>89</sup> Table 2 shows our calculated Schottky defect formation energy, 6.91 eV, and a comparison with prior calculations and experiments. Our value falls near the middle of the range of calculated values,<sup>36–49</sup> which span 5.1–8.8 eV. A recent study<sup>49</sup> compared the calculated Schottky defect formation energies in MgO, as evaluated using the LDA, GGA, Hartree–Fock theory, and many electron perturbation theory. They concluded that the formation energy was within the range of 6.9 to 7.22 eV, in good agreement with our value. To our knowledge the present

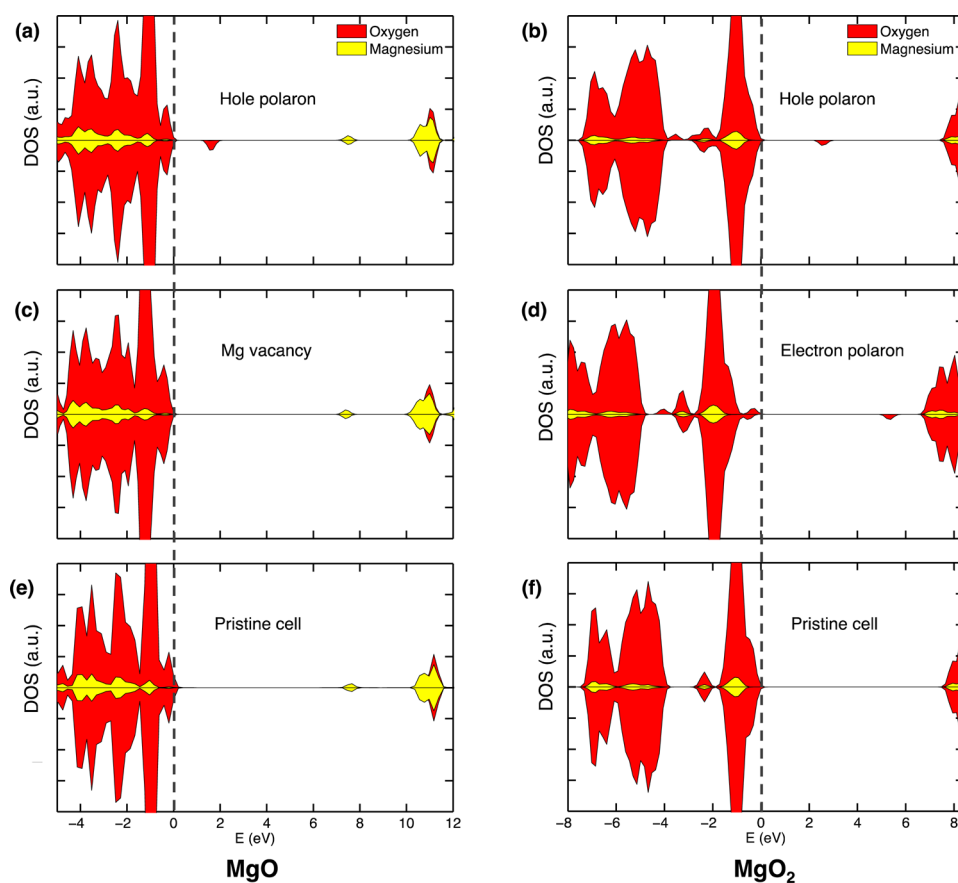
calculations are the first to employ a tuned hybrid functional (HSE $\alpha$ ) to evaluate defect formation energies in MgO.

Figure 2b shows the formation energies of intrinsic defects in MgO<sub>2</sub>. The defect with the lowest formation energy overall is the neutral oxygen vacancy. However, since this is a neutral species a contribution to charge transport is not expected. The dominant charged defects are hole ( $p^+$ ) and electron polarons ( $p^-$ ). The formation energies of both defects is 1.82 eV, and their concentrations are  $6 \times 10^{-9}$  cm<sup>-3</sup>. The prediction that both charged defects in MgO<sub>2</sub> are electronic in nature (rather than ionic) differs from the behavior of Li- and Na-peroxide, where the lowest-energy carriers are hole polarons and negatively charged vacancies on the metal sublattice. In contrast to these other peroxides, the formation energy of a Mg cation vacancy in MgO<sub>2</sub> (assuming it is charge-compensated by a hole polaron) is high, 2.18 eV. An additional feature distinguishing MgO<sub>2</sub> from the alkali–metal peroxides is its relatively high formation energies: the most populous carriers in MgO<sub>2</sub> have formation energies (1.82 eV) that are approximately double those reported for Na<sub>2</sub>O<sub>2</sub> and Li<sub>2</sub>O<sub>2</sub>.<sup>15,56</sup>

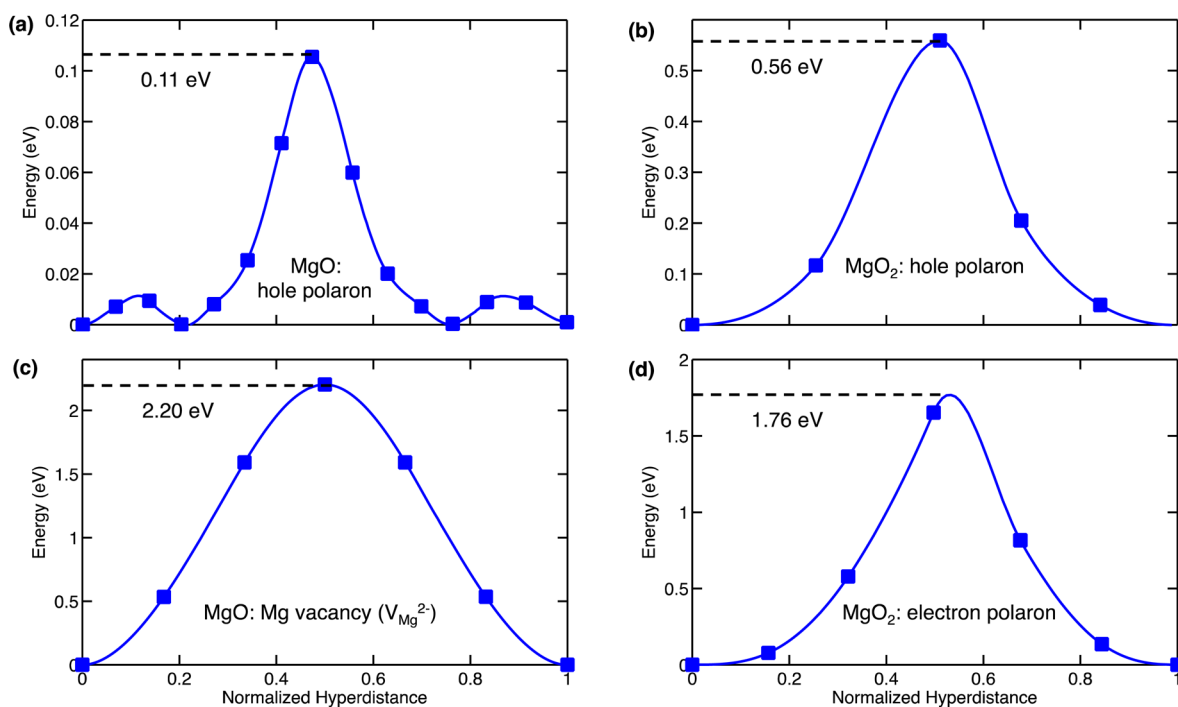
In MgO<sub>2</sub>, oxygen dimers have a formal charge of  $2-$ , i.e.,  $\text{O}_2^{2-}$ , and are octahedrally coordinated by  $\text{Mg}^{2+}$  ions. The hole and electron polarons are localized on these dimers.  $p^+$  is formed by removing an electron from the dimer, resulting a half-filled  $\pi^*$  molecular orbital with superoxide-like character. This orbital is illustrated using magnetization density isosurfaces in Figure 3b. The shortened O–O bond distance of 1.3 Å in the presence of  $p^+$  is consistent with the bond length in magnesium superoxide.<sup>90</sup> In addition, the neighboring Mg ions closest to the  $p^+$  slightly increase their distance to the dimer to 2.25 Å. All bond lengths beyond the nearest neighbor bonds are similar to those in bulk MgO<sub>2</sub>, 2.1 Å. The stabilization energy for  $p^+$  is  $-0.84$  eV.

The electron polaron is formed by adding an electron to  $\text{O}_2^{2-}$ , resulting in a half-filled  $\sigma^*$  molecular orbital with a single, unpaired electron. The shape of the magnetization density for  $p^-$ , shown in Figure 3c, also reflects this orbital topology. The presence of  $p^-$  increases the O–O bond length significantly, to 2.1 Å, compared to 1.45 Å in the bulk. Similarly, formation of  $p^-$  contracts the Mg–O nearest-neighbor bond distance from 2.1 to 1.95 Å. The stabilization energy for  $p^-$  is  $-3.4$  eV.

Regarding other defect types in MgO<sub>2</sub>, the formation energies for Schottky and Frenkel defects are displayed in Table 2. For Schottky defects, the anion is defined as the oxygen dimer (i.e.,  $\text{O}_2^{2-}$ ) resulting in a vacancy pair of ( $V_{\text{Mg}}^{2-}$ ,  $V_{\text{O}_2}^{2+}$ ). The formation energy of Frenkel defects involving



**Figure 4.** Density of states (DOS) calculated with the HSE $\alpha$  functional ( $\alpha = 0.42$ ). [Left panel] (a) Hole polaron, (c) magnesium vacancy, and (e) pristine cell in MgO. [Right panel] (b) Hole polaron, (d) electron polaron, and (f) pristine cell in MgO<sub>2</sub>. The states above the midpoint horizontal axis are spin up, and the states below are spin down. The energies are given with respect to the top of the valence band.



**Figure 5.** Migration energy barriers of dominant defects calculated using the NEB method. (a) Hole polaron and (c) magnesium vacancy ( $V_{\text{Mg}}^{2-}$ ) in MgO. (b) Hole polaron and (d) electron polaron in MgO<sub>2</sub>.

cations was evaluated as the sum of a  $V_{\text{Mg}}^{2-}$  and  $\text{Mg}_\text{I}^{2+}$ . Two types of anion Frenkel defects were considered:  $(V_{\text{O}}^+, \text{O}_\text{I}^-)$  and  $(V_{\text{O}}^{2+}, \text{O}_\text{I}^{2-})$ . As shown in Table 2, all of these defects have formation energies ( $\sim 8$ – $12$  eV) similar to the analogous defects in MgO. The extremely high formation energies of all Schottky and Frenkel defects considered suggest that they will not be present in meaningful concentrations, nor will they contribute significantly to charge transport.

**Electronic Structure.** The density of states (DOS) for defect-free (pristine) MgO and  $\text{MgO}_2$  are shown in Figure 4e,f, respectively. In both cases the DOS reflects the large bandgap of approximately 8 eV predicted by our GW calculations. Also, both compounds exhibit a valence band comprised of oxygen 2p states. The conduction band character differs, however, with MgO exhibiting primarily Mg-based states, while in  $\text{MgO}_2$  the conduction band is comprised of  $\sigma^*$  orbitals on the peroxide units.

Our discussion thus far has used changes in bond lengths and magnetization densities to support the presence of polarons in MgO and  $\text{MgO}_2$ . Additional evidence can be seen in the DOS. The hole polaron in MgO (Figure 4a) and in  $\text{MgO}_2$  (Figure 4b) introduces localized oxygen 2p states in the gap closest to the valence band maximum (VBM). These empty states emerge from the VBM upon removal of electrons from the oxygen  $\pi^*$  orbital during hole creation. Similarly, the electron polaron in  $\text{MgO}_2$  (Figure 4d) introduces new 2p states near the conduction band minimum and also slightly alters the DOS just below the VBM. For comparison, the negative magnesium vacancy ( $V_{\text{Mg}}^{2-}$ ) in MgO is shown in Figure 4c. In this case the DOS is mostly unchanged, indicating that the extraction of a  $\text{Mg}^{2+}$  cation has relatively minor impact on the electronic structure.

**Mobility and Conductivity.** The migration barriers for the highest concentration defects in MgO and  $\text{MgO}_2$  were calculated using the nudged elastic band (NEB) method.<sup>64–66</sup> Both compounds possess high-symmetry (i.e., isotropic) cubic crystal structures with only one symmetry-distinct pathway for defect migration between neighboring sites. As previously mentioned, in MgO the two dominant defects are the hole polaron ( $\text{p}^+$ ) and the negative Mg vacancy,  $V_{\text{Mg}}^{2-}$ . The calculated minimum energy pathway for migration of  $V_{\text{Mg}}^{2-}$  is shown in Figure 5c. This process is characterized by a high migration barrier, 2.2 eV, resulting in a low mobility of  $4 \times 10^{-38}$   $\text{cm}^2/(\text{V s})$  at room temperature. The diffusion coefficient of  $V_{\text{Mg}}^{2-}$  is determined using the Nernst–Einstein equation,  $D = \mu k_{\text{B}} T/q$ , yielding a value of  $1 \times 10^{-39}$   $\text{cm}^2/\text{s}$ , shown in Table 3. Previous computational studies of  $V_{\text{Mg}}^{2-}$  migration have

reported similar activation energies in the range of 1.9 to 2.2 eV.<sup>36,38,42,43,47,91</sup> These values are summarized in Table 2 and agree well with the present hybrid-functional-based calculations. Our calculated activation energy and mobility for  $V_{\text{Mg}}^{2-}$  is also in good agreement with experimental measurements of Sempolinski and Kingery,<sup>29</sup> who reported an activation energy of  $2.28 \pm 0.2$  eV and a mobility of  $1.5 \times 10^{-37}$   $\text{cm}^2/(\text{V s})$  at room temperature.

In contrast to the low mobility of  $V_{\text{Mg}}^{2-}$ , hole polaron migration in MgO is relatively fast. Figure 5a plots the minimum energy pathway for  $\text{p}^+$  migration, revealing a low barrier of 0.11 eV. Consequently, a moderately high mobility of  $6.0 \times 10^{-3}$   $\text{cm}^2/(\text{V s})$  is predicted, along with a diffusion coefficient of  $1.5 \times 10^{-4}$   $\text{cm}^2/\text{s}$ . For comparison, the mobility of holes in p-type silicon in the high doping regime is approximately 50  $\text{cm}^2/(\text{V s})$ .<sup>92</sup> We are not aware of any prior calculations or measurements of polaron mobility in MgO.

In  $\text{MgO}_2$  our calculations predict that the dominant defects are both electronic in nature: hole ( $\text{p}^+$ ) and electron ( $\text{p}^-$ ) polarons. The calculated migration barriers for these carriers are shown in Figure 5b,d and demonstrate that the barrier for the migration of electron polarons is roughly three times higher (1.76 eV) than that for hole polarons, 0.56 eV. These barriers result in mobilities of  $6 \times 10^{-31}$  and  $1 \times 10^{-10}$   $\text{cm}^2/(\text{V s})$ , respectively. To place these values in context, in  $\text{Li}_2\text{O}_2$  Radin et al.<sup>15</sup> reported in-plane and out-of-plane migration barriers for  $\text{p}^+$  of 0.42 and 0.71 eV, respectively, using the HSE $\alpha$  functional. Garcia-Lastra et al.<sup>14</sup> found comparable barriers for  $\text{p}^+$  of 0.39 and 0.48 eV using GGA+U. In contrast, much larger barriers were reported for migration of  $\text{p}^-$ : 1.41 and 1.47 eV. Additionally, the migration barrier for hole polaron hopping in  $\text{Na}_2\text{O}_2$  was reported as 0.47 eV (in plane) and 0.62 eV (out-of-plane).<sup>56</sup> Taken together, these data suggest that in peroxides hole polarons are generally much more mobile than electron polarons, with the former exhibiting hopping barriers that are similar ( $\sim 0.5$  eV) regardless of peroxide composition and approximately one-third the size of  $\text{p}^-$ .

Mobilities, diffusion coefficients, and conductivity data for MgO and  $\text{MgO}_2$  are summarized in Table 3, along with data from the literature for other alkali metal peroxides and superoxides. Under equilibrium conditions, the conductivity arising from the migration of charged point defects is the product of the carrier's charge, concentration, and mobility (eq 5). At room temperature our calculations suggest extremely low conductivities for MgO and  $\text{MgO}_2$  of  $3 \times 10^{-36}$  and  $3 \times 10^{-37}$  S/cm, respectively. The similar conductivity of these compounds reflects a trade-off between the mobility and the concentration of the dominant carriers in these phases. In MgO hole polarons are the dominant carriers; the  $\text{p}^+$  have high mobility but very low equilibrium concentrations. In  $\text{MgO}_2$  this trend is reversed:  $\text{p}^+$  remain the dominant carriers, but their mobility is now lower than in MgO, yet their concentrations are proportionally higher, resulting in approximately equal conductivities.

As mentioned in the Introduction, it is nontrivial to achieve an “apples-to-apples” comparison between the calculated and measured conductivity in nominally insulating compounds at ambient temperatures. These difficulties can be traced to the presence of impurities in the sample and/or the freeze-in of nonequilibrium defect concentrations characteristic of the elevated temperatures at which the sample was prepared or conditioned.<sup>93,94</sup> Contributions from the sample's micro-

**Table 3. Calculated Diffusion Coefficient, Mobility, and Conductivity of Dominant Defects in MgO and  $\text{MgO}_2$ <sup>a</sup>**

compound	diffusion coefficient $D$ ( $\text{cm}^2/\text{s}$ )	mobility $\mu$ ( $\text{cm}^2/(\text{V s})$ )	conductivity $\sigma$ (S/cm)
MgO ( $\text{p}^+$ )	$1 \times 10^{-4}$	$6 \times 10^{-3}$	$3 \times 10^{-36}$
MgO ( $V_{\text{Mg}}^{2-}$ )	$1 \times 10^{-39}$	$4 \times 10^{-38}$	$1 \times 10^{-71}$
$\text{MgO}_2$ ( $\text{p}^+$ )	$4 \times 10^{-12}$	$1 \times 10^{-10}$	$3 \times 10^{-37}$
$\text{MgO}_2$ ( $\text{p}^-$ )	$2 \times 10^{-32}$	$6 \times 10^{-31}$	$7 \times 10^{-58}$
$\text{Li}_2\text{O}_2$ ( $\text{p}^+$ )	$9 \times 10^{-10}$	–	$5 \times 10^{-20}$
$\text{Na}_2\text{O}_2$ ( $\text{p}^+$ )	–	–	$1 \times 10^{-20}$
$\text{NaO}_2$ ( $\text{p}^-$ )	–	–	$1 \times 10^{-19}$
$\text{NaO}_2$ ( $V_{\text{O}_2}^+$ )	$6 \times 10^{-7}$	–	$4 \times 10^{-9}$

<sup>a</sup>Data from literature for  $\text{Li}_2\text{O}_2$ ,  $\text{Na}_2\text{O}_2$ , and  $\text{NaO}_2$  are included for comparison.<sup>15,56</sup>

structure may also be important. All of these effects are ignored in typical single-crystal calculations.

Literature reports have confirmed difficulties in achieving reproducible conductivity values in MgO near room temperature (RT).<sup>21,32</sup> These difficulties are presumably tied to the slow equilibration of charge-carrying defects.<sup>81,93,94</sup> For other oxides, equilibrium is typically achieved above 400–800 °C.<sup>60</sup> Consequently, it is not unexpected that the measured RT conductivity of MgO of  $\sim 10^{-9}$  S/cm<sup>21</sup> is significantly larger than our calculated equilibrium value of  $10^{-36}$  S/cm. We postulate that this discrepancy is an artifact of comparing a nonequilibrium measurement with an equilibrium calculation. As described below, this explanation is bolstered by the good agreement between experiments and calculations at high temperatures, where both systems are in equilibrium.

In the case of MgO<sub>2</sub>, it appears that conductivity measurements have not been reported. Nevertheless, we expect trends similar to those discussed above for MgO to hold for MgO<sub>2</sub>, such as a measured (nonequilibrium) conductivity that is larger than that predicted by calculations at RT.

Of course, in a practical battery RT operation is preferred. The preceding discussion suggests that an equilibrium carrier concentration is unlikely to be achieved at this temperature. Consequently, our calculated RT conductivities should be interpreted as a lower bound to the behavior likely present in an electrochemical cell. In a realistic Mg/O<sub>2</sub> cell there exist two additional, potentially significant contributions to the nonequilibrium nature of the carrier concentration. The first is the rapid growth of the MgO/MgO<sub>2</sub> product during battery discharge. Fast discharge/growth rates should result in substantially higher defect concentrations. Second, as the growth of the discharge product occurs in an (impure) liquid electrolyte, impurities from the electrolyte may be incorporated into this product.

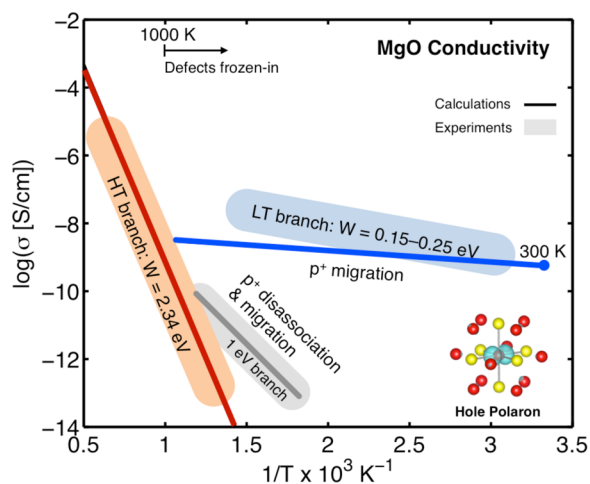
These two effects suggest two strategies for improving Mg/O<sub>2</sub> battery performance. First, since discharging at a higher rate improves the conductivity of the discharge product, performance could in principle be improved by employing pack designs that employ a larger number of reduced-capacity cells. (In these cells the effective discharge rate will be higher.) This assumes other losses due to higher-rate operation can be neglected.

A potentially more promising strategy exploits the incorporation of species from the electrolyte into the discharge product during growth. By intentionally doping the discharge product one may increase the carrier concentration. This could be achieved by substitution of aliovalent dopants on Mg sites. For example, substitution with monovalent impurities such as lithium<sup>95,96</sup> may increase the concentration of hole polarons, which our calculations predict are relatively mobile in both MgO and MgO<sub>2</sub>. Assuming one polaron is created for each monovalent dopant, then a doping concentration of  $10^{12}$  cm<sup>-3</sup> ( $10^{19}$  cm<sup>-3</sup>) would be needed to achieve a conductivity of  $10^{-9}$  S/cm in MgO (MgO<sub>2</sub>). A similar strategy has been proposed to improve the performance of Li/O<sub>2</sub> batteries.<sup>17</sup>

**Explanation for the Three Arrhenius Branches in the Temperature-Dependent Conductivity of MgO.** Measurements of the conductivity of solids such as MgO are conventionally presented as Arrhenius plots of the log of conductivity ( $\log \sigma$ ) vs inverse temperature ( $1/T$ ). The slope of the Arrhenius plot is interpreted as the “Arrhenius energy,”  $W$ , which is the sum of the carrier migration barrier,  $E_b$ , as well as the carrier’s formation energy,  $E_f$  ( $W = E_b + E_f$ ):

$$\sigma(T) = \sigma_0 \exp(-W/k_B T) \quad (6)$$

Most experimental studies of conductivity in MgO focus on high temperatures above approximately 1000 K.<sup>21–30,32</sup> Data in this region, summarized in Figure 6, are in broad agreement



**Figure 6.** Arrhenius plot of the conductivity of MgO, illustrating the three Arrhenius branches reported in experiments: (red) high temperature (HT) branch, (blue) low temperature (LT) branch, and (gray) 1 eV branch.

that the Arrhenius energy is approximately 2.4 eV. Studies below 1000 K<sup>21,32</sup> noted that the conductivity was dependent on the temperature history of the sample; reproducible data could only be obtained during cooling after the sample was conditioned via heat treatment. At these low-to-moderate temperatures two other “Arrhenius branches” in the MgO conductivity data have been reported (Figure 6). These include a branch with a very low Arrhenius energy in the range of 0.15 to 0.25 eV<sup>21,32</sup> and a higher-energy branch with  $W = 1.0$  eV.<sup>32</sup> Although mechanisms responsible for these branches have been proposed, direct evidence linking the measured Arrhenius energies with the identities of specific charge carriers and their respective migration mechanisms does not exist. Below we propose transport mechanisms responsible for all three Arrhenius branches using the catalogue of formation and migration energies evaluated in the present study. Before doing so, we describe the concept of “frozen in” defects in oxides.

As previously mentioned, the dearth of low-temperature conductivity data in MgO can presumably be attributed to difficulties achieving equilibrium at low temperatures. In practice, equilibrium within the crystal and between the crystal and surrounding atmosphere can be reliably obtained only at high temperatures.<sup>93</sup> This is because equilibrium concentrations of ionic defects are established via the transport of ions over potentially long distances, and this transport is often possible only at high temperatures. For example, the creation of a vacancy in the bulk region of a crystallite requires mass transport of Li of ions to a “sink” such as a surface, internal void, dislocation, or grain boundary. As a crystal cools from the temperature of preparation or from an intentional heat treatment, ionic defects quickly become less mobile, and the time to achieve equilibrium increases. Consequently, defect concentrations representative of equilibrium at higher temperatures become “frozen in” with respect to the time scale over which low-temperature measurements are performed. Below these temperatures it is commonly assumed that processes



involving mass transport (i.e., ion migration) are likely to be frozen (i.e., nonequilibrated), while electronic processes such as electron transfer remain equilibrated.<sup>81,93,97</sup>

As described in Table 2, our calculations predict that the predominant charge carriers in MgO are (doubly) negative Mg vacancies,  $V_{\text{Mg}}^{2-}$ , and hole polarons,  $p^+$ . These carriers have identical formation energies of 2.23 eV but dramatically different activation energies for migration,  $E_b$ , of 2.20 and 0.11 eV, respectively. As shown by eq 6, the conductivity will be dominated by the carrier having the smallest Arrhenius energy,  $W$ . For an MgO crystallite in equilibrium, our calculations predict  $W$  values of 4.43 eV for  $V_{\text{Mg}}^{2-}$  and 2.34 eV for  $p^+$ . If equilibrium is not attained, for example, due to rapid cooling resulting in a frozen-in concentration of carriers, then the formation energy contribution to the Arrhenius energy can be neglected. In this case only carrier mobilities contribute to the Arrhenius energy, i.e.,  $W \approx E_b$ .

The conductivity of MgO at high temperatures predicted using the present calculations is shown in Figure 6 with a red line. This data is compared to experimental measurements in the same temperature range, whose spread is depicted using an orange oval.<sup>21–24,27,32</sup> The agreement between the calculations and measurements is very good. At these high temperatures, we expect that thermal equilibrium is achieved for both  $p^+$  and  $V_{\text{Mg}}^{2-}$ . (The calculated diffusivity for  $V_{\text{Mg}}^{2-}$ , the slower-diffusing of the two species, indicates that vacancies can traverse distances of approximately 1  $\mu\text{m}$  at 1000 K over a 24-h period. This distance should be sufficient to establish equilibrium concentrations under these conditions, as diffusing ions would be capable of reaching ion sinks.) Furthermore, the experimental Arrhenius energy,  $W = 2.4$  eV, is in remarkable agreement with the calculated value,  $W = 2.34$  eV, assuming a hole-polaron-dominated conduction mechanism. These data indicate that the mechanism responsible for high temperature conductivity in MgO is the formation and migration of  $p^+$ . In contrast, the large Arrhenius energy ( $W = 4.43$  eV) for ionic conductivity due to  $V_{\text{Mg}}^{2-}$  implies that vacancies have a negligible contribution to conductivity in this temperature range.

The range of experimental data<sup>21,32</sup> for the so-called low-temperature conductivity branch is shown in Figure 6 using a blue oval. These data were obtained by heating the sample to approximately 1000 K and recording the conductivity during cooling. The Arrhenius energy was reported to fall in the range of 0.15 to 0.25 eV. This branch can be rationalized by assuming that the concentration of charge carriers is frozen-in during the time over which measurements were performed. The presence of frozen-in defects has been noted in other oxides as well.<sup>60,61</sup>

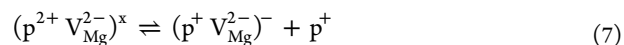
To calculate a conductivity in the low-temperature regime it is necessary to adopt a value for the (nonequilibrium) carrier concentration. Given the low temperature, hole polarons, which possess a very low migration barrier of 0.11 eV, are expected to be the only mobile carriers. We assume the concentration of  $p^+$  to be frozen-in at a value equal to their calculated equilibrium concentration ( $6.2 \times 10^{11} \text{ cm}^{-3}$ ) at 1000 K. This concentration is consistent with experimental measurements<sup>32</sup> that suggest the freezing-in temperature for defects in MgO is near 1000 K. (Samples annealed at room temperature exhibited a continuous decrease in their conductivity, consistent with a decrease in carrier concentration, as equilibrium was slowly approached.)<sup>31,33</sup>

The calculated conductivity of the low-temperature (LT) branch is plotted in Figure 6 using a blue line. These

calculations adopt the frozen concentration of  $p^+$  (at 1000 K) and employ the calculated migration energy. Good agreement is obtained between the calculated and measured (blue oval) data. This agreement reflects the similar values for the Arrhenius energies—0.11 eV calculated vs 0.15 to 0.25 eV measured—and suggests that the conduction mechanism underlying the LT branch is  $p^+$  migration alone.

A third Arrhenius branch in the MgO conductivity data is obtained when heating of the sample is interrupted at approximately 800 K.<sup>31,32</sup> In this case an Arrhenius energy of 1.0 eV was reported. This branch is shown as a gray band in Figure 6 and can be reproduced using a heating and cooling routine where the temperature is progressively raised.<sup>32</sup> This phenomenon, along with anomalies in magnetic susceptibility and charge distribution data, were argued by Batllo and co-workers<sup>34</sup> to arise from the disassociation of holes bound to doubly negative Mg vacancies. These hole–vacancy complexes are created by the incorporation of trace  $\text{H}_2\text{O}$  during crystal growth and are thus of extrinsic origin.<sup>34</sup> The holes were postulated to be localized on the oxygen sublattice, which would make them very similar to the small hole polarons identified in the present study. Taken together, these observations suggest that the conduction mechanism in the 1 eV branch is the dissociative formation, and subsequent migration, of  $p^+$ .

To test this hypothesis we calculated the disassociation energy of a single  $p^+$  bound within a  $2p^+ - V_{\text{Mg}}^{2-}$  complex.<sup>33</sup>



Reaction 7 may be followed by the dissociation of the remaining  $p^+$ , per



The calculated  $p^+$  disassociation energies (eqs 7 and 8) are similar, 0.87 and 0.85 eV, respectively. Taking these dissociation energies as an effective formation energy and combining them with the polaron migration energy, we derive a calculated Arrhenius energy of 0.97 eV. This value is in remarkable agreement with the measured value of 1.0 eV, resulting in the similar slopes for the calculated (gray line) and measured (gray oval) data shown in Figure 6. This observation supports the hypothesis that the 1 eV branch is due to the liberation and migration of  $p^+$  bound to negative vacancies. (Because the carriers in this branch are extrinsic in origin and their concentration is not well-established, the calculated Arrhenius branch in Figure 6 has been positioned (vertically) using the measured conductivity.<sup>32</sup> Therefore, the most meaningful comparison between theory and experiment in this branch is the Arrhenius energy.)

#### IV. CONCLUSIONS

We have presented a comprehensive analysis of charge transport mechanisms within the primary discharge products, MgO and  $\text{MgO}_2$ , of an  $\text{Mg}/\text{O}_2$  battery. While these batteries have extremely high theoretical energy densities, their benefits have yet to be realized in a practical cell that is both reversible and long-lived. Understanding transport within the nominally insulating discharge phases is an important step toward overcoming these performance gaps.

In the case of MgO, our calculations identify doubly negative Mg vacancies and hole polarons as the dominant charge carriers. For  $\text{MgO}_2$ , electronic charge carriers alone—electron

and hole polarons—are the most prevalent. Due to the large formation energies associated with defects in both compounds, the equilibrium concentration of charge carriers is predicted to be low at the near-ambient temperatures expected for battery operation. These low concentrations also offset the moderate (for MgO<sub>2</sub>) to high (in MgO) mobility of hole polarons. Consequently, if equilibrium carrier concentrations are realized, the conductivity for both MgO and MgO<sub>2</sub> will be low and thus likely limit battery performance.

Nevertheless, charge transport within the discharge products of a realistic battery cathode will be strongly influenced by nonequilibrium effects. These effects arise from fast growth rates during discharge, impurity incorporation from the electrolyte, and limited ion mobility at ambient temperatures. In combination with the moderate/high mobilities predicted for hole polarons, these nonequilibrium effects suggest a strategy for improving conductivity: artificially increasing polaron concentrations via the incorporation of monovalent impurities into the discharge product during its growth. In principle, this could be achieved through the intentional addition of small quantities of lithium ions to the electrolyte.

Finally, the calculated conductivity data for MgO are observed to be in remarkable agreement with the three Arrhenius branches reported in experiments and thus clarify the long-debated transport mechanisms within these regimes.

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### Notes

The authors declare no competing financial interest.

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