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Retrospective-Cost-Based Adaptive State Estimation and Input Reconstruction for a Maneuvering Aircraft with Unknown Acceleration

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A method is presented to obtain state estimates for a possibly nonminimum-phase system in the presence of unknown harmonic inputs. The method estimates the states and reconstructs the unknown harmonic input. An adaptive feedback model injects an input into the estimator such that the error between the estimator output and the actual output converges to zero despite the presence of the unknown harmonic input. Using input reconstruction based on a retrospective cost, the unknown harmonic input is reconstructed. Using the reconstructed input, the parameters of the adaptive feedback system are updated using recursive least squares. Results are presented for a rigid body, a damped rigid body, and a 2D missile with a three-loop autopilot topology.

I. Introduction

In the traditional formulation of state estimation, the Kalman filter uses measurements to recursively refine state estimates. In effect, the Kalman filter uses a model of the system to filter measurements of states that are measured and to observe states that are not measured.[3, 7, 10, 14] The input to the system is typically modeled as a combination of an unknown stochastic signal and a known deterministic signal. When the Kalman filter is used within the context of LQG control, the deterministic signal is injected numerically into the Kalman filter in order to take advantage of the separation principle. In practice, however, the deterministic input may not be precisely known, and treating this signal as part of the stochastic input may or may not violate the zero-mean assumption of the process noise and, in either case, may yield poor state estimates due to the modeling mismatch. Consequently, extensive research has been devoted to developing extensions of the Kalman filter that are either insensitive to knowledge of the deterministic input or that attempt to estimate this signal in addition to the states. These techniques are referred to as unbiased Kalman filters, unknown input observers, and state estimators with input reconstruction.[4, 5, 9, 11, 12, 15]

Aside from state estimation, the goal of input reconstruction is to estimate the input of a system based on its output. These techniques depend on model inversion and thus must pay careful attention to the presence of zeros in the system, especially nonminimum-phase zeros that preclude stable inversion.[2] The starting point for the present paper is the technique of adaptive state estimation.[13] This approach uses an adaptive input reconstruction technique to asymptotically estimate the unknown input to the system. A regularization technique is used in the case where the transfer function from the disturbance to the measurement is nonmiminum phase, in which case the Kalman filter is unable to achieve asymptotically exact estimation. The goal of the present paper is to investigate the performance of the adaptive state estimation technique of[13] for aerospace applications. In particular, we consider state and input estimation

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for a rigid body and a damped rigid body with unknown inputs. For realism, we apply the discrete-time adaptive state estimation technique of[13] to a sampled-data model that exhibits nonminimum-phase zeros due to sampling. The phenomenon of sampling zeros is discussed in.[2] We then apply this technique to the linearized missile model given in[8] and demonstrate the ability to estimate the unknown acceleration. This technique thus provides an alternative for estimating unknown acceleration.[6]

II. Problem Formulation

Consider the linear-time-invariant system

$$x(k+1) = Ax(k) + Bu(k) + Bw(k), \tag{1}$$
$$y(k) = Cx(k), \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the unknown state, $u(k) \in \mathbb{R}^m$ is an unknown input, $w(k) \in \mathbb{R}^m$ is unknown zero-mean Gaussian white noise, and $y(k) \in \mathbb{R}^p$ is the measured output, which is assumed to be bounded. The matrices $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{p\times n}$ are known, and (A, C) is observable. Furthermore we assume that $u(k)$ is the output of a Lyapunov-stable, linear system.

In order to obtain an estimate $\hat{x}(k) \in \mathbb{R}^n$ of the state $x(k)$, we construct an adaptive state estimator of the form

$$\hat{x}(k+1) = A\hat{x}(k) + B\hat{u}(k), \tag{3}$$
$$\hat{y}(k) = C\hat{x}(k), \tag{4}$$
$$z(k) = y(k) - \hat{y}(k), \tag{5}$$

where $\hat{y}(k) \in \mathbb{R}^p$ is the estimated output, $\hat{u}(k) \in \mathbb{R}^m$ is the estimator input, and $z(k) \in \mathbb{R}^p$ is the measured output error. Furthermore, the reconstructed input $\hat{u}(k)$ is the output of the strictly proper adaptive feedback system of order n_c, with input $z(k)$ given by

$$\hat{u}(k) = \sum_{i=1}^{n_c} M_i(k)\hat{u}(k-i) + \sum_{i=1}^{n_c} N_i(k)z(k-i), \tag{6}$$

where, for all $i = 1, \ldots, n_c$, $M_i(k) \in \mathbb{R}^{m\times m}$ and $N_i(k) \in \mathbb{R}^{m\times p}$. The goal is to update $M_{i,k}$ and $N_{i,k}$ using the measured output error $z(k)$. Figure 1 shows the adaptive input reconstruction and state estimation (AIRSE) architecture.

III. Adaptive Input Reconstruction and State Estimation Using a Retrospective Cost

For $i \geq 1$, define the Markov parameter H_i of (A, B, C) given by

$$H_i \stackrel{\triangle}{=} CA^{i-1}B. \tag{7}$$

For example, $H_1 = CB$ and $H_2 = CAB$. Let r be a positive integer. Then, for all $k \geq r$,

$$\hat{x}(k) = A^r\hat{x}(k-r) + \sum_{i=1}^{r} A^{i-1}B\hat{u}(k-i), \tag{8}$$

and thus

$$z(k) = CA^r\hat{x}(k-r) + y(k) + \bar{H}\hat{\bar{U}}(k-1), \tag{9}$$

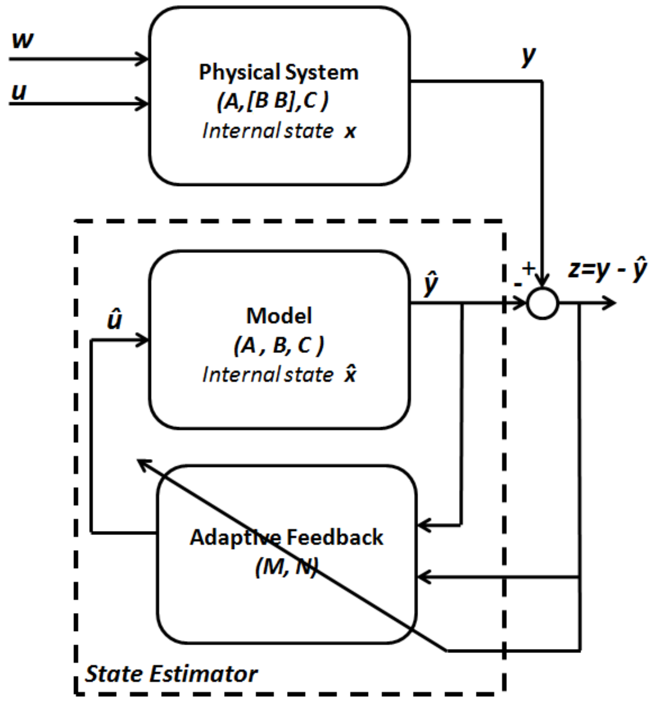

Figure 1. Architecture for Adaptive Input Reconstruction and State Estimation

where

$$\bar{H} \triangleq \begin{bmatrix} H_1 & \cdots & H_r \end{bmatrix} \in \mathbb{R}^{p\times rm}$$

and

$$\hat{\bar{U}}(k-1) \triangleq \begin{bmatrix} \hat{u}(k-1) \\ \vdots \\ \hat{u}(k-r) \end{bmatrix}.$$

Next, we rearrange the columns of $\bar{H}$ and the components of $\hat{\bar{U}}(k-1)$ and partition the resulting matrix and vector so that

$$\bar{H}\hat{\bar{U}}(k-1) = \mathcal{H}'\hat{U}'(k-1) + \mathcal{H}\hat{U}(k-1), \tag{10}$$

where $\mathcal{H}' \in \mathbb{R}^{p\times(rm-l_{\hat{U}})}$, $\mathcal{H} \in \mathbb{R}^{p\times l_{\hat{U}}}$, $\hat{U}'(k-1) \in \mathbb{R}^{rm-l_{\hat{U}}}$, and $\hat{U}(k-1) \in \mathbb{R}^{l_{\hat{U}}}$. Then, we can rewrite (9) as

$$z(k) = \mathcal{S}(k) + \mathcal{H}\hat{U}(k-1), \tag{11}$$

where

$$\mathcal{S}(k) \triangleq CA^r\hat{x}(k-r) + y(k) + \mathcal{H}'\hat{U}'(k-1). \tag{12}$$

Note that the decomposition of $\bar{H}\hat{\bar{U}}(k-1)$ in (10) is not unique. Let s be a positive integer. Then, for $i = 1,\ldots,s$, we replace $\mathcal{H}$, $\hat{U}(k-1)$, $\mathcal{H}'$, and $\hat{U}'(k-1)$ in (10) with $\mathcal{H}_j \in \mathbb{R}^{p\times l_{\hat{U}_j}}$, $\hat{U}_j(k-1) \in \mathbb{R}^{l_{\hat{U}_j}}$,

$\mathcal{H}'_j \in \mathbb{R}^{p\times(rm-l_{\hat{U}_j})}$, and $\hat{U}'_j(k-1) \in \mathbb{R}^{rm-l_{\hat{U}_j}}$, respectively, such that (10) becomes

$$\bar{H}\hat{\hat{U}}(k-1) = \mathcal{H}'_j\hat{U}'_j(k-1) + \mathcal{H}_j\hat{U}_j(k-1). \tag{13}$$

Therefore, for $j = 1,\ldots,s$, we can rewrite (11) as

$$z(k) = \mathcal{S}_j(k) + \mathcal{H}_j\hat{U}_j(k-1), \tag{14}$$

where

$$\mathcal{S}_j(k) \triangleq CA^r\hat{x}(k-r) + y(k) + \mathcal{H}'_j\hat{U}'_j(k-1). \tag{15}$$

Next, let $0 \le k_1 \le k_2 \le \cdots \le k_s$. Replacing k by $k-k_j$ in (14) yields

$$z(k-k_j) = \mathcal{S}_j(k-k_j) + \mathcal{H}_j\hat{U}_j(k-k_j-1). \tag{16}$$

Now, by stacking $z(k-k_1),\ldots,z(k-k_s)$, we define the *extended performance*

$$Z(k) \triangleq \begin{bmatrix} z(k-k_1) \\ \vdots \\ z(k-k_s) \end{bmatrix} \in \mathbb{R}^{sp}. \tag{17}$$

Therefore,

$$Z(k) \triangleq \tilde{\mathcal{S}}(k) + \tilde{\mathcal{H}}\hat{\hat{U}}(k-1), \tag{18}$$

where

$$\tilde{\mathcal{S}}(k) \triangleq \begin{bmatrix} \mathcal{S}_1(k-k_1) \\ \vdots \\ \mathcal{S}_s(k-k_s) \end{bmatrix} \in \mathbb{R}^{sp} \tag{19}$$

and $\hat{\hat{U}}(k-1)$ has the form

$$\hat{\hat{U}}(k-1) \triangleq \begin{bmatrix} \hat{u}(k-q_1) \\ \vdots \\ \hat{u}(k-q_g) \end{bmatrix} \in \mathbb{R}^{gm}, \tag{20}$$

where $k_1 \le q_1 < q_2 < \cdots < q_g \le k_s + r$. The vector $\hat{\hat{U}}(k-1)$ is formed by stacking $\hat{U}_1(k-k_1-1),\ldots,\hat{U}_s(k-k_s-1)$ and removing copies of repeated components, and $\tilde{\mathcal{H}} \in \mathbb{R}^{sp\times gm}$ is constructed according to the structure of $\hat{\hat{U}}(k-1)$.

Next, we define the *retrospective performance*

$$\hat{z}(k-k_j) \triangleq \mathcal{S}_j(k-k_j) + \mathcal{H}_jU^*_j(k-k_j-1), \tag{21}$$

where the past input estimates $\hat{U}_j(k-k_j-1)$ in (16) are replaced by the retrospectively optimized input estimates $U^*_j(k-k_j-1)$, which are determined below. In analogy with (17), the *extended retrospective performance* is defined as

$$\hat{Z}(k) \triangleq \begin{bmatrix} \hat{z}(k-k_1) \\ \vdots \\ \hat{z}(k-k_s) \end{bmatrix} \in \mathbb{R}^{sp} \tag{22}$$

and thus is given by

$$\hat{Z}(k) = \tilde{\mathcal{S}}(k) + \tilde{\mathcal{H}}\tilde{U}^*(k-1), \tag{23}$$

where the components of $\tilde{U}^*(k-1) \in \mathbb{R}^{l_{\hat{U}}}$ are the components of $U_1^*(k-k_1-1), \ldots, U_s^*(k-k_s-1)$ ordered in the same way as the components of $\hat{\hat{U}}(k-1)$. Subtracting (18) from (23) yields

$$\hat{Z}(k) = Z(k) - \tilde{\mathcal{H}}\hat{\hat{U}}(k-1) + \tilde{\mathcal{H}}\tilde{U}^*(k-1). \tag{24}$$

Finally, we define the *retrospective cost function*

$$J(\tilde{U}^*(k-1), k) \triangleq \hat{Z}^{\mathrm{T}}(k)R_1(k)\hat{Z}(k) + \eta(k)\tilde{U}^{*\mathrm{T}}(k-1)R_2(k)\tilde{U}^*(k-1), \tag{25}$$

where $R_1(k) \in \mathbb{R}^{ps\times ps}$ is a positive-definite performance weighting, $R_2(k) \in \mathbb{R}^{gm\times gm}$ is a positive-definite input estimate weighting, and $\eta(k) \geq 0$ is a regularization weighting. The goal is to determine retrospective input estimates $\tilde{U}^*(k-1)$ that would have provided better performance than the estimated inputs $\hat{U}(k-1)$ that were applied to the system. The retrospectively optimized input estimates $\tilde{U}^*(k-1)$ are then used to update the controller. Substituting (24) into (25) yields

$$J(\tilde{U}^*(k-1), k) = \tilde{U}^{*\mathrm{T}}(k-1)\mathcal{A}(k)\tilde{U}^*(k-1) + B\tilde{U}^*(k-1) + \mathcal{C}(k), \tag{26}$$

where

$$\mathcal{A}(k) \triangleq \tilde{\mathcal{H}}^{\mathrm{T}}R_1(k)\tilde{\mathcal{H}} + \eta(k)R_2(k), \tag{27}$$

$$\mathcal{B}(k) \triangleq 2\tilde{\mathcal{H}}^{\mathrm{T}}R_1(k)[Z(k) - \tilde{\mathcal{H}}\hat{\hat{U}}(k-1)], \tag{28}$$

$$\mathcal{C}(k) \triangleq Z^{\mathrm{T}}(k)R_1(k)Z(k) - 2Z^{\mathrm{T}}(k)R_1(k)\tilde{\mathcal{H}}\hat{\hat{U}}(k-1) + \hat{\hat{U}}^{\mathrm{T}}(k-1)\tilde{\mathcal{H}}^{\mathrm{T}}R_1(k)\tilde{\mathcal{H}}\hat{\hat{U}}(k-1). \tag{29}$$

If either $\tilde{\mathcal{H}}$ has full column rank or $\eta(k) > 0$, then $\mathcal{A}(k)$ is positive definite. In this case, $J(\tilde{U}^*(k-1), k)$ has the unique global minimizer

$$\hat{\hat{U}}(k-1) = -\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k), \tag{30}$$

which is the retrospectively optimized input estimates.

The regularization weighting $\eta(k)$ can be used to bound the retrospectively optimized input estimates $\tilde{U}^*(k-1)$ and thus indirectly bound the estimated inputs $\hat{\hat{U}}(k)$. For example, $\eta(k)$ may be performance based

$$\eta(k) = \eta_0||Z(k)||_2^2 \tag{31}$$

or error based

$$\eta(k) = \eta_0(k)||\tilde{U}^*(k-2) - \hat{\hat{U}}(k-2)||_2^2, \tag{32}$$

where $\eta_0(k) \geq 0$. Alternatively, the estimated inputs can be bounded directly by using a saturation function, where $\eta(k) \equiv 0$ in (27) and (30) is replaced by

$$\tilde{U}^*(k-1) \triangleq \mathrm{sat}_{[a,b]}[-\frac{1}{2}\mathcal{A}^{-1}(k)\mathcal{B}(k)], \tag{33}$$

where $\mathrm{sat}_{[a,b]}(\zeta)$ is the component-wise saturation function defined for scalar arguments by

$$\mathrm{sat}_{[a,b]}(\zeta) \triangleq \begin{cases} b, & \text{if } \zeta \geq b, \\ \zeta, & \text{if } a < \zeta < b, \\ a & \text{if } \zeta \leq a, \end{cases} \tag{34}$$

where $a < b$ are the component-wise saturation levels.

IV. Adaptive Feedback Construction and Update

The estimated input $\hat{u}(k)$ given by (6) can be expressed as

$$\hat{u}(k) = \theta(k)\phi(k-1), \tag{35}$$

where

$$\theta(k) \triangleq [M_1(k) \ \cdots \ M_{n_c}(k) \ N_1(k) \ \cdots \ N_{n_c}(k)] \in \mathbb{R}^{m \times n_c(m+p)} \tag{36}$$

and

$$\phi(k-1) \triangleq \begin{bmatrix} \hat{u}(k-1) \\ \vdots \\ \hat{u}(k-n_c) \\ y(k-1) \\ \vdots \\ y(k-n_c) \end{bmatrix} \in \mathbb{R}^{n_c(m+p)}. \tag{37}$$

IV.A. Recursive Least Squares Update of $\theta(k)$

We define the cumulative cost function

$$J_R(\theta(k)) \triangleq \sum_{i=q_g+1}^{k} \lambda^{k-i} \|\phi^T(i-q_g-1)\theta^T(k) - u^{*T}(i-q_g)\|^2 + \lambda^k(\theta(k)-\theta(0))P^{-1}(0)(\theta(k)-\theta(0))^T, \tag{38}$$

where $\|\cdot\|$ is the Euclidean norm and, for some $\varepsilon \in (0,1)$, $\lambda(k) \in (\varepsilon, 1]$ is the forgetting factor, and $P^{-1}(0) \in \mathbb{R}^{n_c(m+p)\times n_c(m+p)}$ is the initial covariance matrix. Minimizing (38) yields

$$\begin{aligned}\theta^T(k) \triangleq{}& \theta^T(k-1) + \beta(k)P(k-1)\phi(k-q_g-1)[\phi^T(k-q_g-1)P(k-1)\phi(k-q_g-1)+\lambda(k)]^{-1} \\ &\cdot [\theta(k-1)\phi(k-q_g-1) - u^*(k-q_g)]^T,\end{aligned} \tag{39}$$

where $\beta(k)$ is either 0 or 1. When $\beta(k) = 1$, the controller is allowed to adapt, whereas, when $\beta(k) = 0$, the adaptation is off. The covariance is updated by

$$\begin{aligned}P(k) \triangleq{}& (1-\beta(k))P(k-1) + \beta(k)\lambda^{-1}(k)P(k-1) - \beta(k)\lambda^{-1}(k)P(k-1)\phi(k-q_g-1) \\ &\cdot [\phi^T(k-q_g-1)P(k-1)\phi(k-q_g-1)+\lambda(k)]^{-1}\phi^T(k-q_g-1)P(k-1).\end{aligned} \tag{40}$$

We initialize the covariance matrix as $P(0) = \gamma I$, where $\gamma > 0$. Furthermore, the updates (39) and (40) are based on the g^{th} component of $\tilde{U}^*(k-1)$. However any or all of the components of $\tilde{U}^*(k-1)$ may be used in the update of $\theta(k)$ and $P(k)$.

V. Examples

In this section, we apply AIRSE to a rigid body, a damped rigid body, and a linearized missile longitudinal autopilot. We also apply AIRSE to a damped oscillator with unknown damping coefficient for the following two cases:

1. The unknown input is zero, and we estimate the damping coefficient.
2. The unknown input is not zero and we estimate the input.

V.A. Example 1: Rigid Body, Unstable and Nonminimum-Phase

Consider a rigid body for which the equation of motion is given by

$$\dot{x} = A_c x + B_c u, \tag{41}$$

where

$$x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}, \quad A_c = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}, \tag{42}$$

$m = 1$ kg is the mass of the rigid body. Sampling (41), with $T_s = 1$ sec yields

$$x(k+1) = Ax(k) + Bu(k), \tag{43}$$

where

$$A = e^{A_c T_s} = \begin{bmatrix} 1 & T_s \\ 0 & 1 \end{bmatrix}, \quad B = \int_0^{T_s} e^{A_c \tau} d\tau B_c = \begin{bmatrix} \frac{T_s^2}{2m} \\ \frac{T_s}{m} \end{bmatrix}. \tag{44}$$

Since only the position is measured, the output matrix is

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{45}$$

Although the continuous-time system does not have zeros, the discretized system has a nonminimum-phase sampling zero at -1. This makes the problem challenging since the discretized system is both unstable and nonminimum-phase. Let $u(k) = 0.6\sin(0.2k)$ be the unknown input, $\eta_0 = 0.01$, $n_c = 15$, $P(0) = 0.1 I_{30\times 30}$, and $\tilde{\mathcal{H}} = CB$. Figure 2 shows the performance and estimator parameters. Figure 3 shows the actual and reconstructed input. Figure 4 shows the actual and estimated states for the rigid body.

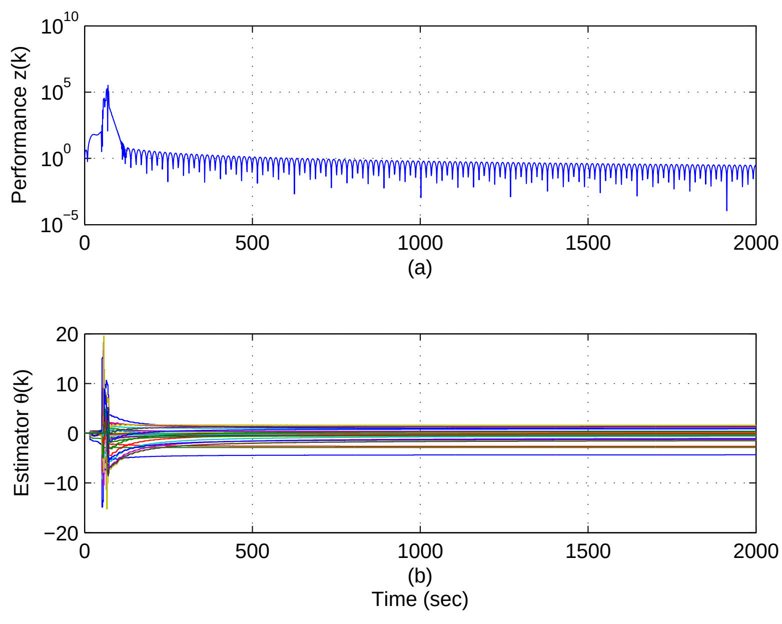

Figure 2. (a) Performance and (b) Estimator Parameters for the Discretized Rigid Body with an Unknown Harmonic Input

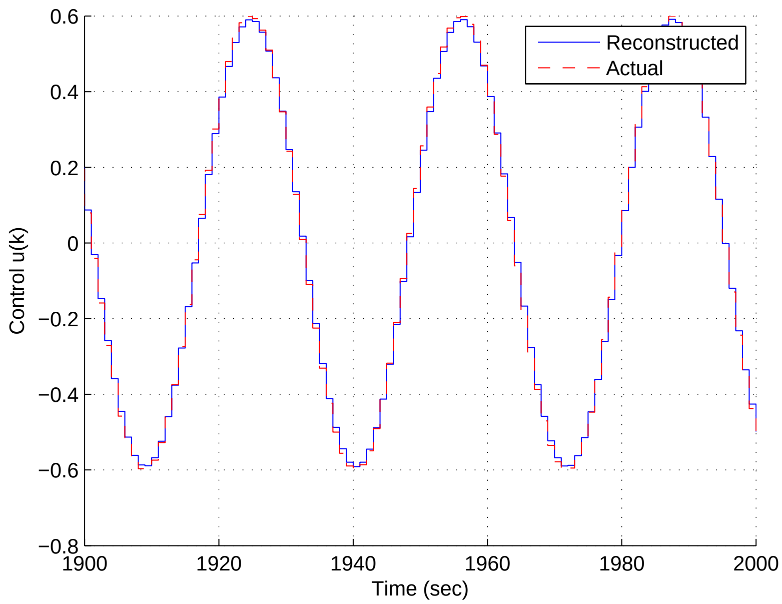

Figure 3. Actual and Reconstructed Input u for the Discretized Rigid Body with an Unknown Harmonic Input

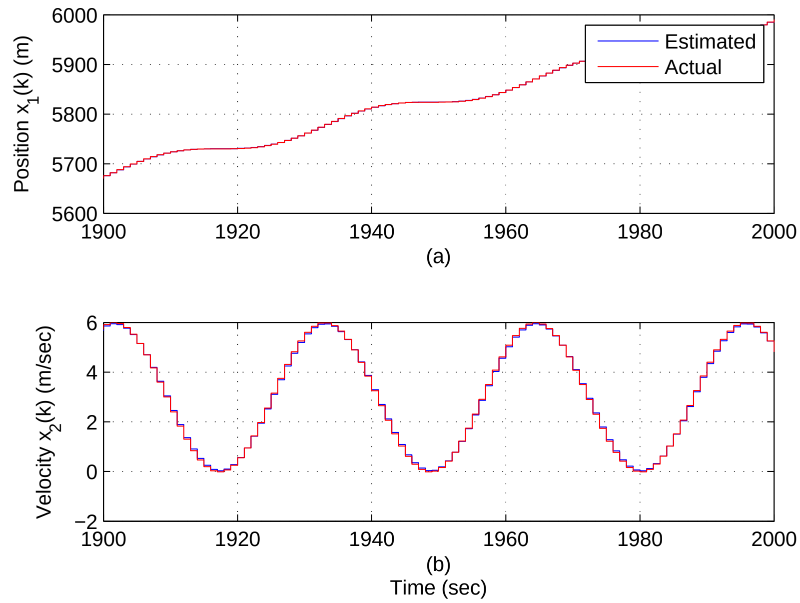

Figure 4. Actual and Estimated States for the Discretized Rigid Body with an Unknown Harmonic Input (a) Position and (b) Velocity

V.B. Example 2: Damped Rigid Body, Semistable and Minimum-Phase

Consider a damped rigid body for which the equation of motion is given by (41), with

$$A_c = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{c}{m} \end{bmatrix}, \tag{46}$$

$m = 1$ kg is the mass, and $c = 5$ N-sec/m is the damping coefficient for the damped rigid body. The system (41) is sampled with $T_s = 1$ sec, which yields

$$A = e^{A_c T_s} = \begin{bmatrix} 1 & \frac{m}{c} - \frac{m}{ce^{\frac{cT_s}{m}}} \\ 0 & \frac{1}{e^{\frac{cT_s}{m}}} \end{bmatrix}, \quad B = \int_0^{T_s} e^{A_c \tau} d\tau B_c = \begin{bmatrix} \frac{m}{c^2 e^{\frac{cT_s}{m}}} - \frac{m}{c^2} + \frac{T_s}{c} \\ \frac{-1}{ce^{\frac{cT_s}{m}}} + \frac{1}{c} \end{bmatrix}. \tag{47}$$

Since only the position is measured, the output matrix is

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{48}$$

Let $u(k) = 0.6\sin(0.2k)$ be the unknown input, $\eta_0 = 0$, $n_c = 10$, $P(0) = I_{20\times 20}$, and $\tilde{\mathcal{H}} = CB$. Figure 5 shows the performance and estimator parameters. Figure 6 shows the actual and reconstructed input. Figure 7 shows the actual and estimated states for the damped rigid body.

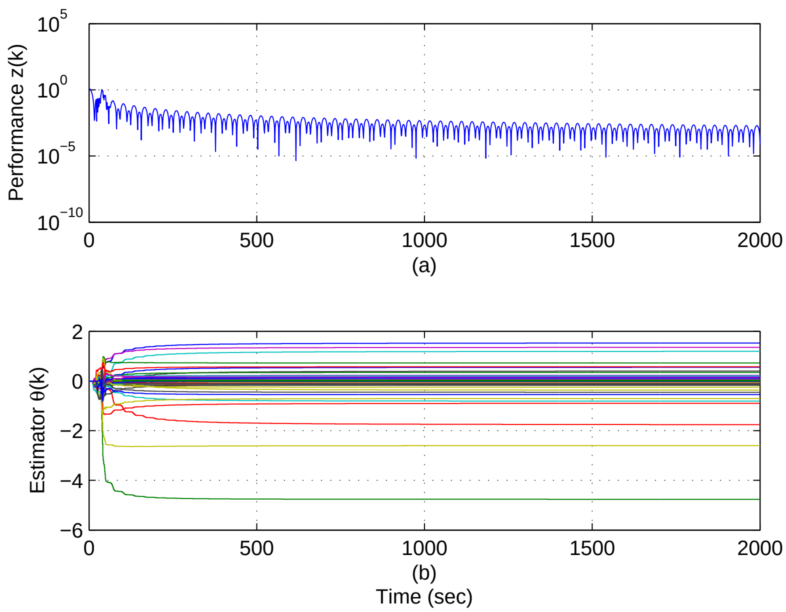

Figure 5. (a) Performance and (b) Estimator Parameters for the Discretized Damped Rigid Body with an Unknown Harmonic Input

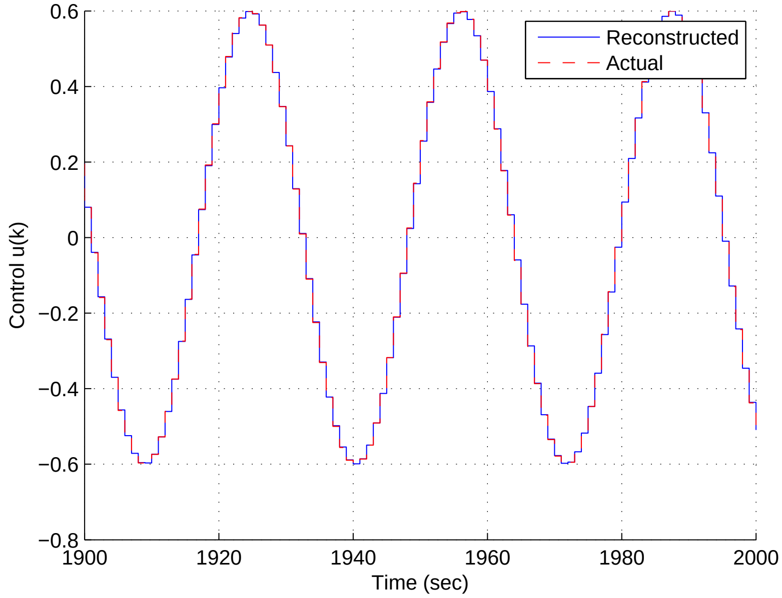

Figure 6. Actual and Reconstructed Input u for the Discretized Damped Rigid Body with an Unknown Harmonic Input

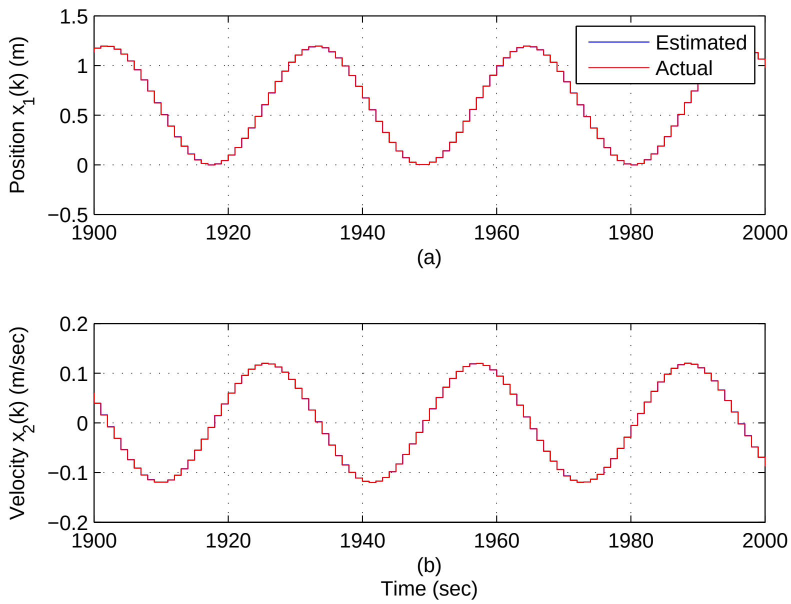

Figure 7. Actual and Estimated States for the Discretized Damped Rigid Body with an Unknown Harmonic Input (a) Position and (b) Velocity

V.C. Example 3: Damped Oscillator, Damping Coefficient Estimation, Stable and Minimum-Phase

Consider a damped oscillator with zero input but with unknown damping coefficient c. The equation of motion is given by

$$m\ddot{q} + c\dot{q} + kq = 0. \tag{49}$$

Equation (49) can be rewritten as

$$m\ddot{q} + \hat{c}_0\dot{q} + kq = \hat{c}_0\dot{q} - c\dot{q}, \tag{50}$$

where $\hat{c}_0$ is an initial estimate of c. Equation (50) can be written as (41), where

$$A_c = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\hat{c}_0}{m} \end{bmatrix}. \tag{51}$$

The system (41) is sampled with $T_s = 0.1$ sec, where

$$A = e^{A_c T_s}, \quad B = \int_0^{T_s} e^{A_c \tau} d\tau B_c. \tag{52}$$

Since only the position is measured, the output matrix is

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{53}$$

The mass is $m = 1$ kg, and $k = 2$ N-m is the spring stiffness. Futhermore, $c = 5$ N-sec/m is the true damping coefficient and $\hat{c}_0 = 3$ N-sec/m is the initial estimate of c.

The term $u_{\text{est}} = \hat{c}_0\dot{q} - c\dot{q}$ can be considered as an unknown input. After the unknown input is reconstructed, c can be estimated using (54), where $\dot{q}$ is the estimated velocity.

$$\hat{c}(k) = \frac{\hat{c}_0\dot{q}(k) - u_{\text{est}}(k)}{\dot{q}(k)}. \tag{54}$$

Let $u(k) = 0$, $\eta_0 = 0$, $n_c = 1$, $P(0) = I_{2\times 2}$, and $\tilde{\mathcal{H}} = CB$. Figure 8 shows the performance and estimator parameters. Figure 9 shows the actual and estimated states of the damped oscillator. Figure 10 shows the the actual and estimated damping coefficient of the damped oscillator. Note that the accuracy of the damping coefficient depends on the sampling time.

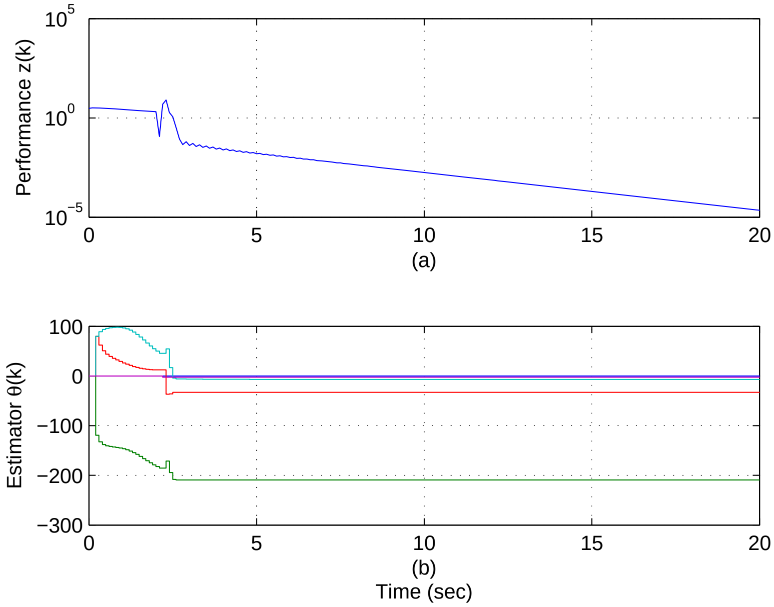

Figure 8. (a) Performance and (b) Estimator Parameters of the Discretized Damped Oscillator with Zero Input

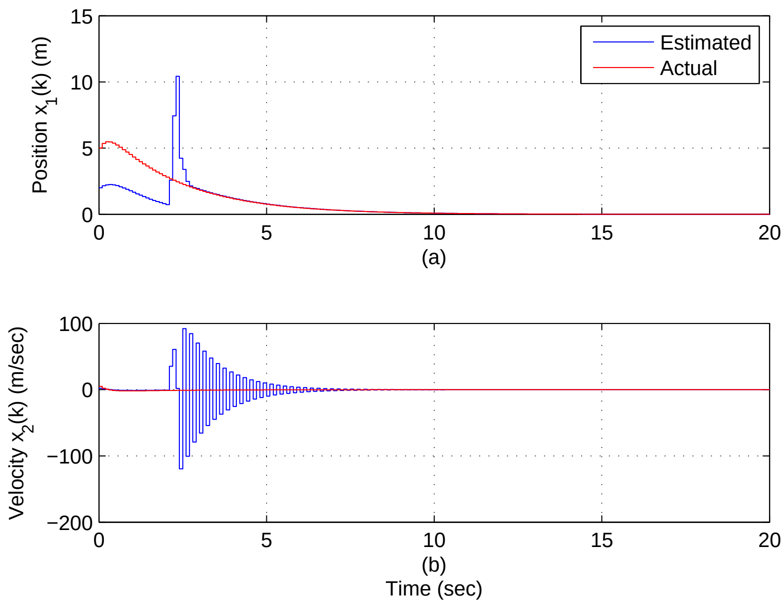

Figure 9. Actual and Estimated States of the Discretized Damped Oscillator with Zero Input (a) Position and (b) Velocity

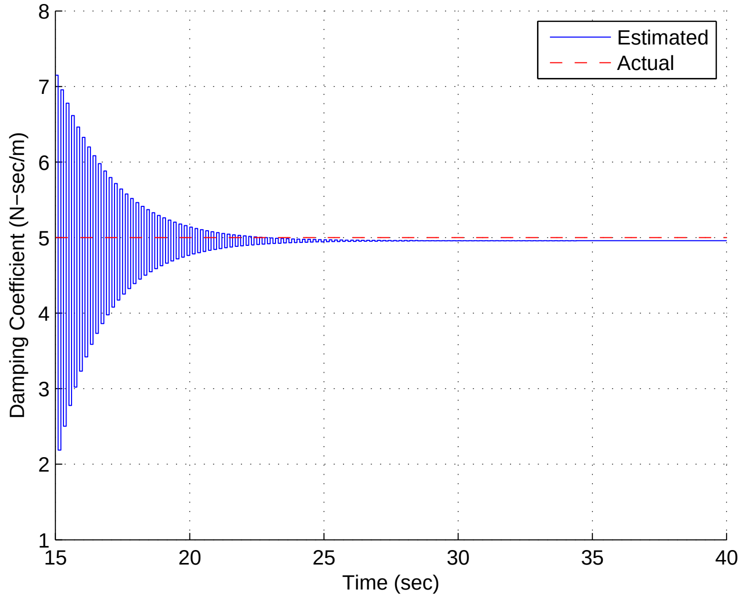

Figure 10. Actual and Estimated Damping Coefficient $\hat{c}(k)$ given by (54) of the Discretized Damped Oscillator with Zero Input

V.D. Example 4: Damped Oscillator, Unknown Input Reconstruction with an Unknown Damping Coefficient, Asymptotically Stable and Minimum-Phase

Consider a damped oscillator with nonzero unknown input and unknown damping coefficient c. The equation of motion is given by

$$m\ddot{q} + c\dot{q} + kq = u. \tag{55}$$

Equation (55) can be rewritten as

$$m\ddot{q} + \hat{c}_0\dot{q} + kq = (\hat{c}_0\dot{q} - c\dot{q}) + u, \tag{56}$$

where $\hat{c}_0$ is an initial estimate of c. Equation (56) can be written as (41), where

$$A_c = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{\hat{c}_0}{m} \end{bmatrix}. \tag{57}$$

The system (41) is sampled with $T_s = 0.1$ sec, where

$$A = e^{A_c T_s}, \quad B = \int_0^{T_s} e^{A_c \tau} d\tau B_c. \tag{58}$$

Since only the position is measured, the output matrix is

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}. \tag{59}$$

The mass is $m = 1$ kg, and the spring stiffness is $k = 2$ N-m. Furthermore, $c = 5$ N-sec/m is the true damping coefficient. It is seen that the AIRSE algorithm reconstructs the unknown input u correctly if c is close enough to $\hat{c}_0$. Let $u(k) = 0.6\sin(0.2k)$ be the unknown input, $\eta_0 = 0$, $n_c = 5$, $P(0) = I_{10\times 10}$, and $\tilde{\mathcal{H}} = CB$. Figure 11 shows the performance and estimator parameters for $\hat{c}_0 = 4$ N-sec/m. Figure 12 shows the actual and reconstructed input for $\hat{c}_0 = 4$ N-sec/m. Figure 13 shows the actual and estimated states of the damped oscillator for $\hat{c}_0 = 4$ N-sec/m. Figure 14 shows the performance and estimator parameters for $\hat{c}_0 = 10$ N-sec/m. Figure 15 shows the actual and reconstructed input for $\hat{c}_0 = 10$ N-sec/m. Figure 16 shows the actual and estimated states of the damped oscillator for $\hat{c}_0 = 10$ N-sec/m.

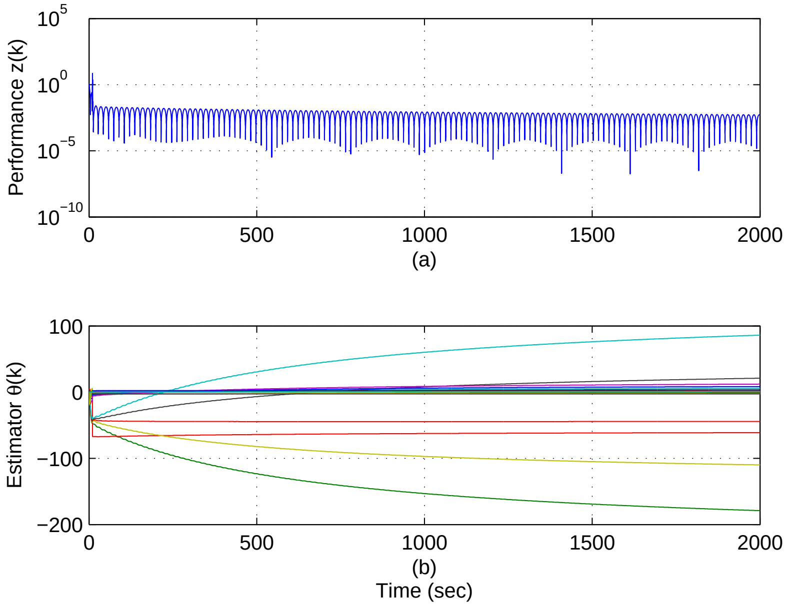

Figure 11. (a) Performance and (b) Estimator Parameters of the Discretized Damped Oscillator with an Unknown Harmonic Input ($c = 5$ N-sec/m, $\hat{c}_0 = 4$ N-sec/m)

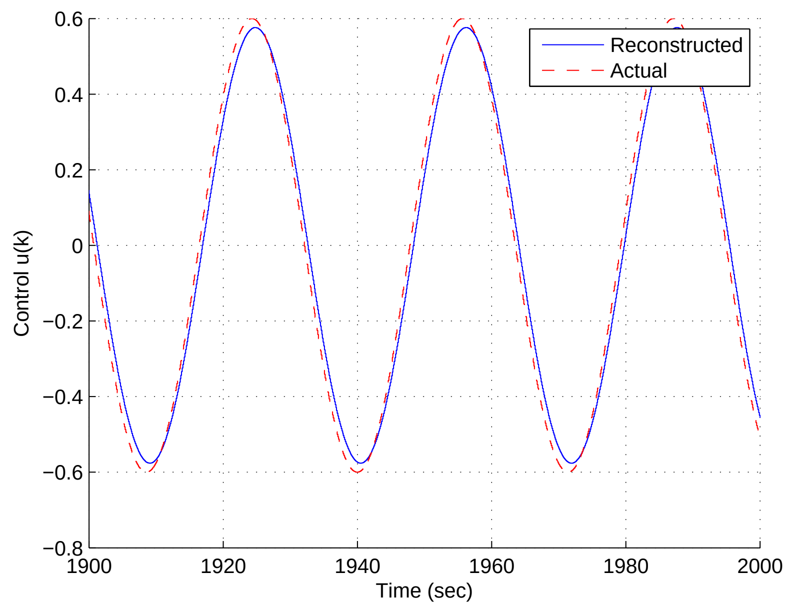

Figure 12. Actual and Reconstructed Input u of the Discretized Damped Oscillator with an Unknown Harmonic Input ($c = 5$ N-sec/m, $\hat{c}_0 = 4$ N-sec/m)

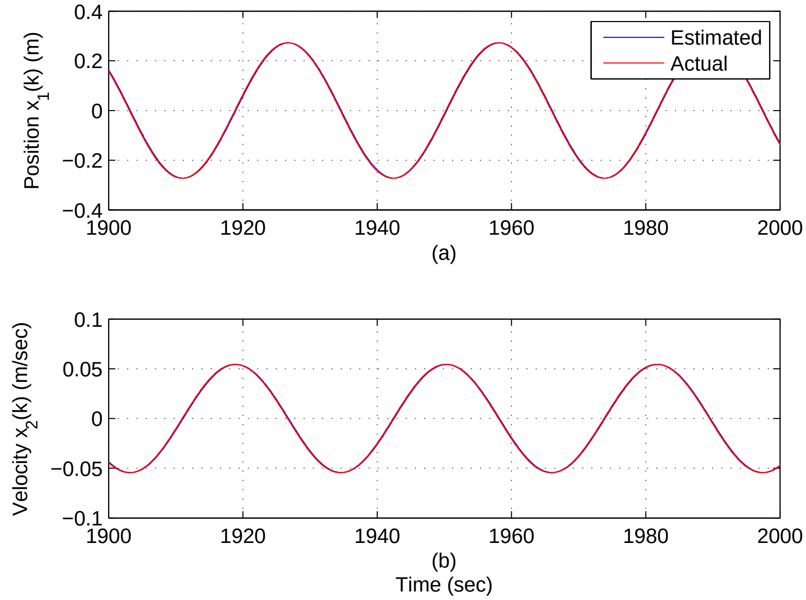

Figure 13. Actual and Estimated States of the Discretized Damped Oscillator with an Unknown Harmonic Input (a) Position and (b) Velocity ($c = 5$ N-sec/m, $\hat{c}_0 = 4$ N-sec/m)

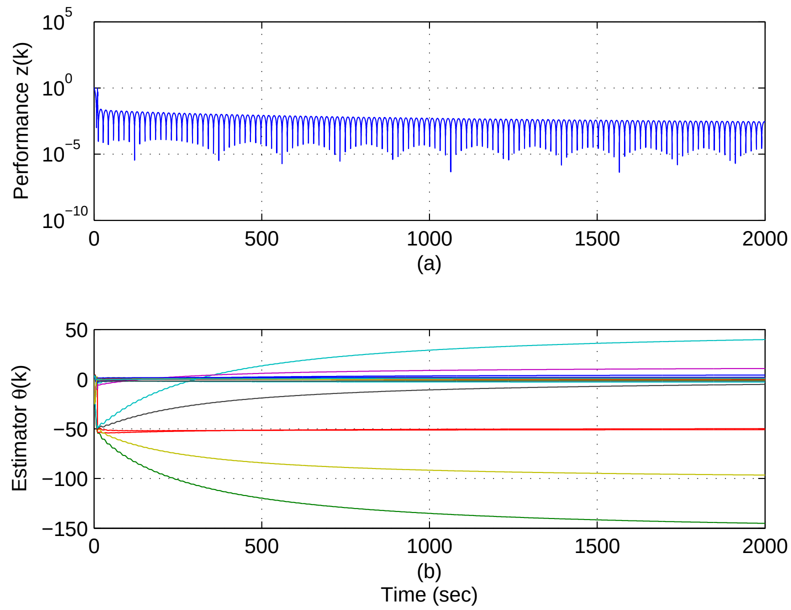

Figure 14. (a) Performance and (b) Estimator Parameters of the Discretized Damped Oscillator with an Unknown Harmonic Input ($c = 5$ N-sec/m, $\hat{c}_0 = 10$ N-sec/m)

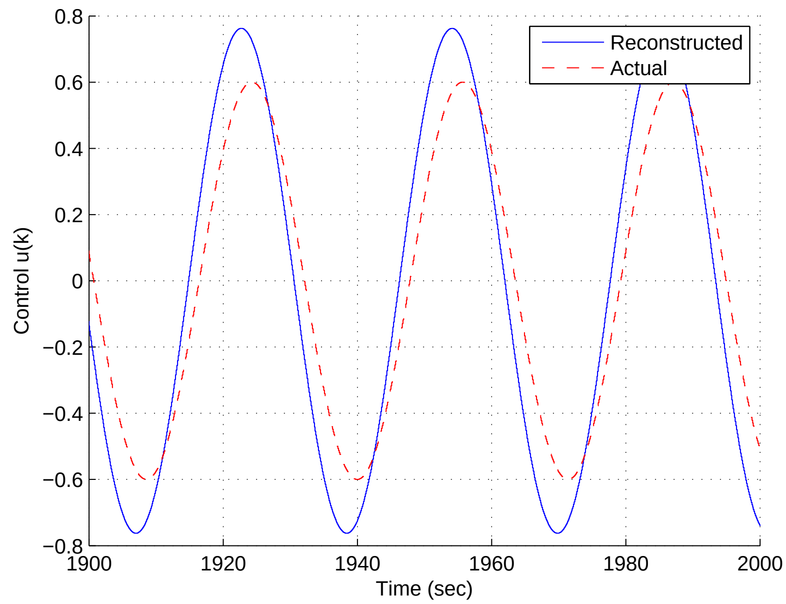

Figure 15. Actual and Reconstructed Input u of the Discretized Damped Oscillator with an Unknown Harmonic Input ($c = 5$ N-sec/m, $\hat{c}_0 = 10$ N-sec/m)

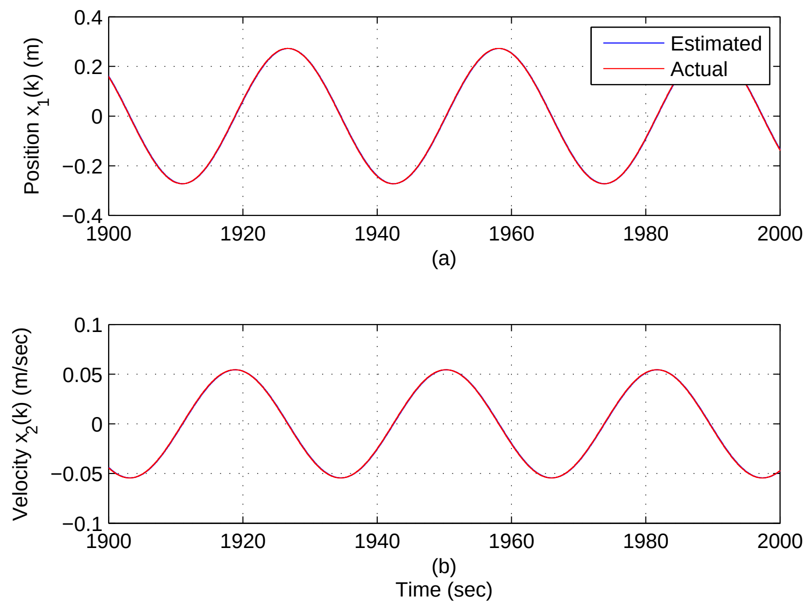

Figure 16. Actual and Estimated States of the Discretized Damped Oscillator with an Unknown Harmonic Input (a) Position and (b) Velocity ($c = 5$ N-sec/m, $\hat{c}_0 = 10$ N-sec/m)

V.E. Example 5: Missile Longitudinal Autopilot, Stable and Minimum-Phase

Consider a three-loop autopilot topology used as a missile longitudinal autopilot,[8] for which the equations of motion are given by

$$\dot{x} = A_1 x + B_1 u, \tag{60}$$
$$\dot{y} = C_1 x + D_1 u - \tilde{K}_{\rm ss} r, \tag{61}$$
$$z = H_1 x + L_1 u - K_{\rm ss} r, \tag{62}$$

where

$$x = \begin{bmatrix} \alpha \\ q \\ \delta_p \end{bmatrix}, \quad u = \dot{\delta}_p, \quad y = \begin{bmatrix} A_{\rm zm} - K_{\rm ss} r \\ q_m \\ \delta_p \end{bmatrix}, \tag{63}$$

$$A_1 = \begin{bmatrix} A & B \\ 0 & 0 \end{bmatrix}, \quad B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad \tilde{K}_{\rm ss} = \begin{bmatrix} K_{\rm ss} \\ 0 \end{bmatrix}, \tag{64}$$

$$C_1 = \begin{bmatrix} C & D \\ 0 & 1 \end{bmatrix}, \quad D_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \tag{65}$$

$$H_1 = \left[\frac{\bar{Q}SC_{\rm z\alpha0}}{mg} - \frac{\bar{Q}SdC_{\rm m\alpha0}\bar{x}}{gI_{\rm YY}} \quad 0 \quad \frac{\bar{Q}SC_{\rm z\delta p0}}{mg} - \frac{\bar{Q}SdC_{\rm m\delta p0}\bar{x}}{gI_{\rm YY}} \right], \quad L_1 = 0, \tag{66}$$

$$A = \begin{bmatrix} \frac{1}{V_{\rm m0}}(\frac{\bar{Q}SC_{\rm z\alpha0}}{m} - A_{\rm X0}) & 1 \\ \frac{\bar{Q}SdC_{\rm m\alpha0}}{I_{\rm YY}} & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{\bar{Q}SC_{\rm z\delta p0}}{mV_{\rm m0}} \\ \frac{\bar{Q}SdC_{\rm m\delta p0}}{I_{\rm YY}} \end{bmatrix}, \tag{67}$$

$$C = \begin{bmatrix} \frac{\bar{Q}SC_{\rm z\alpha0}}{mg} - \frac{\bar{Q}SdC_{\rm m\alpha0}\bar{x}}{gI_{\rm YY}} & 0 \\ 0 & 1 \end{bmatrix}, \quad D = \begin{bmatrix} \frac{\bar{Q}SC_{\rm z\delta p0}}{mg} - \frac{\bar{Q}SdC_{\rm m\delta p0}\bar{x}}{gI_{\rm YY}} \\ 0 \end{bmatrix}. \tag{68}$$

The description and the values of the parameters used above are given in.[8] The closed-loop matrices are given by

$$A_c = C_1(A_1 {C_1}^{-1} + B_1 K_{\rm opt}), \tag{69}$$
$$B_c = -C_1 B_1 K_{\rm opt} [K_{\rm ss} \quad 0 \quad 0]^T, \tag{70}$$
$$C_c = H_1 {C_1}^{-1}, \tag{71}$$
$$K_{\rm ss} = [C_c A_c^{-1} (C_1 B_1 K_{\rm opt} [1 \quad 0 \quad 0]^T)]^{-1}, \tag{72}$$
$$K_{\rm opt} = [-2.0740 \quad 11.7514 \quad -119.0269]. \tag{73}$$

These closed-loop matrices are used for the simulation. Let $u(k) = 0.6\sin(0.2k)$ be the unknown input, $\eta_0 = 0$, $n_c = 5$, $P(0) = I_{10\times10}$, and $\tilde{\mathcal{H}} = CB$. Figure 17 shows the performance and estimator parameters. Figure 18 shows the actual and reconstructed input. Figure 19 shows the actual and estimated states for the missile longitudinal autopilot.

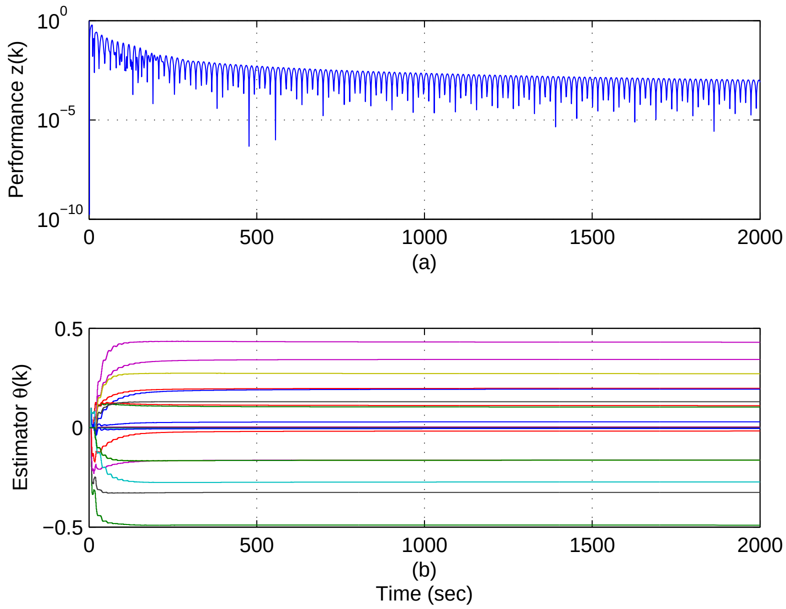

Figure 17. (a) Performance and (b) Estimator Parameters for the Missile Longitudinal Autopilot with an Unknown Harmonic Input

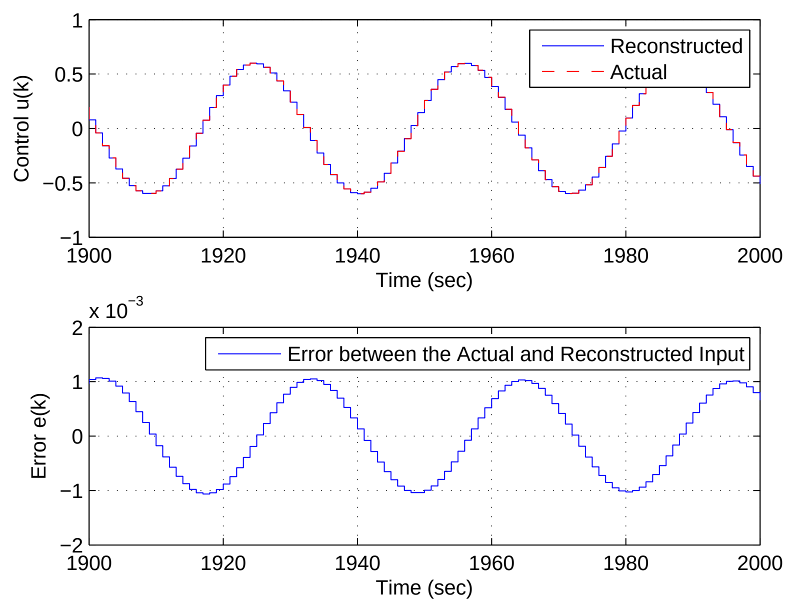

Figure 18. Actual and Reconstructed Input u for the Missile Longitudinal Autopilot with an Unknown Harmonic Input

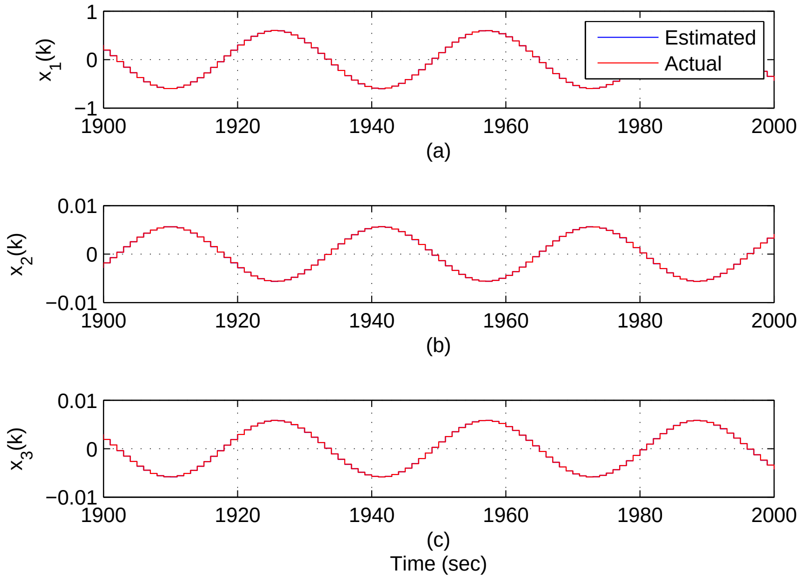

Figure 19. Actual and Estimated States for the Discretized Missile Longitudinal Autopilot with an Unknown Harmonic Input

VI. Conclusions

We presented a method for estimating the states of minimum and nonminimum-phase systems in the presence of unknown harmonic inputs. The estimator uses a system model based on the dynamics of the actual physical system but overall the algorithm does not need the detailed dynamics of the actual physical system. Also, the algorithm reconstructs the unknown harmonic input at each step by minimizing the error between $y(k) - \hat{y}(k)$.

Based on the error between $y(k) - \hat{y}(k)$, an adaptive feedback model is updated, which gives $\hat{u}(k)$ as the output. The output of the feedback model $\hat{u}(k)$ is then used to obtain the state estimates $\hat{x}$ of the system with states $x(k)$.

Finally, the method is demonstrated on minimum and nonminimum-phase linear systems in the presence of an unknown harmonic input. We also show that the method works for the case of a rigid body, which is unstable and has a nonminimum-phase sampling zero.

Future research will compare our results to the technique of[6] and make a connection to alpha-beta filters.[1]

VII. Acknowledgments

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