51st IEEE Conference on Decision and Control
December 10-13, 2012. Maui, Hawaii, USA

Sensor-to-Sensor Identification of Hammerstein Systems

Khaled Aljanaideh[1], Asad A. Ali[1], M. S. Holzel[1], Sunil L. Kukreja[2], and Dennis S. Bernstein[3]

***Abstract*— Traditional system identification uses measurements of the inputs, but when these measurements are not available, alternative methods, such as blind identification, output-only identification, or operational modal analysis, must be used. Yet another method is sensor-to-sensor identification (S2SID), which estimates pseudo transfer functions whose inputs are outputs of the original system. A special case of S2SID is transmissibility identification. Since S2SID depends on cancellation of the input, this approach does not extend to nonlinear systems. However, in the present paper we show that, for the case of a two-output Hammerstein system, the least-squares estimate of the PTF is consistent, that is, asymptotically correct, despite the presence of the nonlinearities.**

I. Introduction

Traditional linear system identification uses measurements of inputs and outputs to construct a dynamic model. These measurements may be noisy, and thus the challenge is to construct the most accurate model possible with a limited amount of imperfect data and without knowledge of the order of the true system and the statistical properties of the noise. The system may also be subjected to additional inputs that are not measured; in this case, the effect of the unmeasured inputs can be viewed as noise corrupting the output.

In many applications, however, measurements of the inputs are not available. In this case, there are several possible approaches to system identification. One approach is to use statistical knowledge about the input signal and then estimate the model based on the signal characteristics; this approach is called blind identification, output-only identification, or operational modal analysis [1–6].

A second approach to identification when input measurements are not available is *sensor-to-sensor identification* (S2SID). In S2SID, one sensor signal is viewed as the input of a *pseudo transfer function* (PTF) whose output is another sensor signal. The sensor signals are thus called the *pseudo input* and *pseudo output*, respectively, of the PTF. It is easy to see that, if the same set of states is observable from both output channels, then the denominators of the "original" transfer functions cancel, and the PTF retains information about only the zeros of the transfer functions from the input to the sensor. Most importantly, the contribution of the unmeasured input signal also cancels, and therefore the resulting estimate of the PTF is independent of the detailed history of the excitation signal.

A version of S2SID is routinely performed in the special case of modal vibration analysis, where ratios of FFT's of accelerometer signals are used to obtain frequency-domain estimates of transmissibilities [7–11]. This frequency-domain approach, however, does not address issues that arise in time-domain identification, namely, the order of the PTF, the effect of the initial condition, the persistency of the input, and the impact of sensor noise.

A time-domain approach to S2SID was proposed in [12–15], where the order of the resulting PTF was analyzed in the presence of a nonzero initial condition. In [13], it was shown, somewhat surprisingly, that the denominator dynamics cancel despite the presence of nonzero initial conditions. The effect of sensor noise was considered in [13] within the context of least squares identification, which also reveals the level of persistency in terms of the condition number of the regressor matrix. S2SID with more than two sensors is addressed in [14].

As noted above, the usefulness of S2SID depends on the ability to estimate a transfer function independently of the details of the excitation signal. This ability depends on the fact that the input signal is cancelled in the construction of the PTF. As expected, however, this cancellation does not occur in the case of nonlinear systems, which suggests that S2SID is confined to linear systems. However, in the present paper we consider the case of a Hammerstein system, and we estimate the Markov parameters of a *linear* PTF between the pseudo input and pseudo output despite the fact that these signals are not linearly related. Under these conditions we show that, despite the presence of the input nonlinearities, the estimates of the Markov parameters of the identified PTF are semi-consistent, that is, up to a uniform scale factor, they are asymptotically correct estimates of the Markov parameters of the corresponding PTF of the system in the absence of the input nonlinearities. This statement holds for the case in which both input nonlinearities are nonzero, but otherwise arbitrary.

The contents of the paper are as follows. In Section II, we formulate the problem. In section III, we define the identification architecture. In section IV we analyze the consistency of the Markov parameters obtained from the proposed method. In section V we show the numerical examples. We give conclusions in section VI.

II. Problem Formulation

Consider the block diagram shown in Figure 1, where u is the input, $\mathcal{N}_1 : \mathbb{R} \to \mathbb{R}$ and $\mathcal{N}_2 : \mathbb{R} \to \mathbb{R}$ are memoryless

[1] Graduate student, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109.
[2] Aerospace Engineer, Structural Dynamics Group, NASA Dryden Flight Research Center.
[3] Professor, Department of Aerospace Engineering, The University of Michigan, Ann Arbor, MI 48109.
This research was supported by NASA under grants NNX11AN93A and NNX09A055H.

nonlinearities, $\mathcal{N}_1(u)$ and $\mathcal{N}_2(u)$ are the intermediate signals, and y_1 and y_2 are the output signals of the asymptotically stable, SISO, linear, time-invariant, causal, discrete-time systems G_1 of order n_1 and G_2 of order n_2, respectively.

Since the input u is not measured, it is not possible to identify the SISO Hammerstein systems ($\mathcal{N}_1, G_1$) and ($\mathcal{N}_2, G_2$). Furthermore, because of the presence of the nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$, the relationship between y_1 and y_2 is not linear. Nevertheless, for reasons explained in subsequent sections, we identify a linear model whose input and output are the signals y_1 and y_2, respectively, see Figure 2. This linear model, which we call a pseudo-transfer-function (PTF), has the form

$$\mathcal{G}(\mathbf{q}) = \frac{B(\mathbf{q})}{A(\mathbf{q})}, \tag{1}$$

where $\mathcal{G}$ is the PTF, $\mathbf{q}$ is the forward shift operator, and A and B are polynomials in $\mathbf{q}$. For simplicity, we assume that $\mathcal{G}$ is a finite impulse response (FIR) model, and thus $A(\mathbf{q}) = \mathbf{q}^\mu$ and $B(\mathbf{q}) = \sum_{i=0}^{\mu} H_i \mathbf{q}^i$, where μ is the model order. Consequently, the FIR PTF model $\mathcal{G}$ that relates the pseudo input y_1 to the pseudo output y_2 has the form

$$y_2(k) = \sum_{j=0}^{\mu} H_j y_1(k-j), \tag{2}$$

where $H_0, \ldots, H_{\mu-1}$ are the Markov parameters of (1).

In order for the PTF to be causal, the relative degree of G_2 must be greater than or equal to the relative degree of G_1. If this is not the case then we delay the pseudo output y_2 as needed.

III. Least Squares Identification of the PTF

The FIR model (2) can be expressed as

$$y_2(k) = \theta_\mu \phi_\mu(k), \tag{3}$$

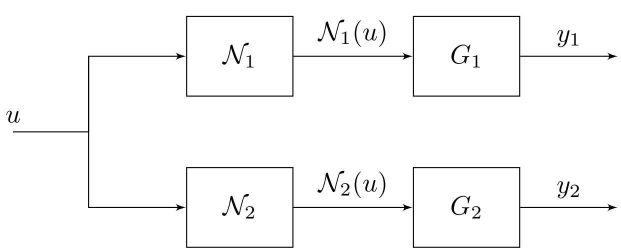

Fig. 1. SIMO Hammerstein system, $\mathcal{N}_1$ and $\mathcal{N}_2$ represent memoryless nonlinearities, and y_1 and y_2 represent outputs of the linear transfer functions G_1 and G_2, respectively.

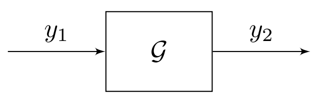

Fig. 2. The pseudo-transfer function $\mathcal{G}$ is a linear model that is identified based on the input and output signals y_1 and y_2, respectively. This identification does not assume that the relationship between y_1 and y_2 is linear.

where

$$\theta_\mu \triangleq \begin{bmatrix} H_0 & \cdots & H_{\mu-1} \end{bmatrix},$$
$$\phi_\mu(k) \triangleq \begin{bmatrix} y_1(k) & \cdots & y_1(k-\mu+1) \end{bmatrix}^{\mathrm{T}}.$$

The least squares estimate $\hat{\theta}_{\mu,\ell}$ of θ_μ is given by

$$\hat{\theta}_{\mu,\ell} = \arg\min_{\bar{\theta}_\mu} \left\| \Psi_{y_2,\ell} - \bar{\theta}_\mu \Phi_{\mu,\ell} \right\|_{\mathrm{F}}, \tag{4}$$

where $\bar{\theta}_\mu$ is a variable of appropriate size, $||\ .\ ||_{\mathrm{F}}$ denotes the Frobenius norm,

$$\Psi_{y_2,\ell} \triangleq \begin{bmatrix} y_2(\mu) & \cdots & y_2(\ell) \end{bmatrix},$$
$$\Phi_{\mu,\ell} \triangleq \begin{bmatrix} \phi_\mu(\mu) & \cdots & \phi_\mu(\ell) \end{bmatrix},$$

and ℓ is the number of samples. It follows from (4) that

$$\Psi_{y_2,\ell} \Phi_{\mu,\ell}^{\mathrm{T}} = \hat{\theta}_{\mu,\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^{\mathrm{T}}. \tag{5}$$

Next, consider the system in Figure 3, which represents the system in Figure 1 without the Hammerstein nonlinearities $\mathcal{N}_1$ and $\mathcal{N}_2$. Note that y_1' and y_2' represent the outputs of G_1 and G_2, respectively.

Define $H_0', \ldots, H_{\mu-1}'$ to be Markov parameters of the PTF $\mathcal{G}'$ constructed by y_1' and y_2', see Figure 4. It follows that

$$\Psi_{y_2',\ell} = \theta_\mu' \Phi_{\mu,\ell}', \tag{6}$$

where

$$\Psi_{y_2',\ell} \triangleq \begin{bmatrix} y_2'(\mu) & \cdots & y_2'(\ell) \end{bmatrix}, \tag{7}$$
$$\theta_\mu' \triangleq \begin{bmatrix} H_0' & \cdots & H_{\mu-1}' \end{bmatrix}, \tag{8}$$
$$\Phi_{\mu,\ell}' \triangleq \begin{bmatrix} \phi_\mu'(\mu) & \ldots & \phi_\mu'(\ell) \end{bmatrix}, \tag{9}$$
$$\phi_\mu'(k) \triangleq \begin{bmatrix} y_1'(k) & \cdots & y_1'(k-\mu+1) \end{bmatrix}^{\mathrm{T}}. \tag{10}$$

Although the PTF $\mathcal{G}'$ is unknown and cannot be identified, the goal is to compare the Markov parameters of the identified FIR PTF $\mathcal{G}$ relating y_1 and y_2 to the Markov parameters of the PTF $\mathcal{G}'$ relating y_1' to y_2'.

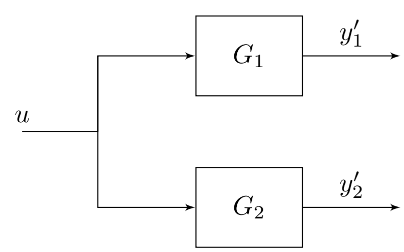

Fig. 3. y_1' and y_2' are the outputs of the linear transfer functions G_1 and G_2, respectively, with input u. This system does not exist and is used only for analysis

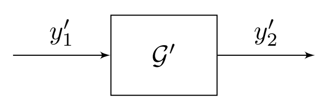

Fig. 4. The pseudo-transfer function $\mathcal{G}'$ is a linear model that is identified based on the input and output signals y_1' and y_2', respectively.

IV. Consistency Analysis

Assumption 4.1: u is a realization of a stationary white random process U, and y_1, y_2, y_1', and y_2' are realizations of stationary random processes Y_1, Y_2, Y_1', and Y_2', respectively.

Assumption 4.2: For all $k \geq 0$, $\mathcal{N}_1\big(U(k)\big)$, $\mathcal{N}_2\big(U(k)\big)$, $\mathcal{N}_1^2\big(U(k)\big)$, and $\mathcal{N}_1\big(U(k)\big)\mathcal{N}_2\big(U(k)\big)$ have finite mean and variance.

Assumption 4.3: For all $k \geq 0$, $\mathbb{E}\Big[\mathcal{N}_1\Big(U(k)\Big)\Big] = 0$, $\mathbb{E}\Big[\mathcal{N}_1\Big(U(k)\Big)\mathcal{N}_2\Big(U(k)\Big)\Big] \neq 0$, and $\mathbb{E}\Big[\mathcal{N}_1^2\Big(U(k)\Big)\Big] \neq 0$.

Assumption 4.4: θ_μ is not zero.

Definition 4.1: The least squares estimator $\hat{\theta}_{\mu,\ell}$ of θ_μ is a *semi-consistent* estimator of θ'_μ if there exists nonzero $\gamma \in \mathbb{R}$ such that

$$\lim_{\ell\to\infty} \hat{\theta}_{\mu,\ell} \overset{\text{wp1}}{=} \gamma\theta'_\mu.$$

Theorem 4.1: Let assumptions 4.1-4.4 hold. Then $\hat{\theta}_{\mu,\ell}$ is a semi-consistent estimator of θ'_μ.

Proof 4.1: Note that,

$$y_1'(k) = \big(u * h_1\big)(k) = \sum_{i=-\infty}^{k} u(i)h_1(k-i), \tag{11}$$

$$y_2'(k) = \big(u * h_2\big)(k) = \sum_{i=-\infty}^{k} u(i)h_2(k-i), \tag{12}$$

$$y_1(k) = \Big(\mathcal{N}_1\big(u\big) * h_1\Big)(k) = \sum_{i=-\infty}^{k} \mathcal{N}_1\big(u(i)\big)h_1(k-i), \tag{13}$$

$$y_2(k) = \Big(\mathcal{N}_2\big(u\big) * h_2\Big)(k) = \sum_{i=-\infty}^{k} \mathcal{N}_2\big(u(i)\big)h_2(k-i), \tag{14}$$

where h_1 and h_2 are the impulse response sequences of G_1 and G_2, respectively. Furthermore,

$$y_2(k) = \frac{\alpha(k)}{\beta(k)} y_2'(k), \tag{15}$$

where

$$\begin{aligned} \alpha(k) &\triangleq \Big(\mathcal{N}_2(u) * h_2\Big)(k), \\ \beta(k) &\triangleq \big(u * h_2\big)(k). \end{aligned}$$

Therefore,

$$\begin{aligned} \Psi_{y_2,\ell} &= \begin{bmatrix} \frac{\alpha(\mu)}{\beta(\mu)} y_2'(\mu) & \cdots & \frac{\alpha(\ell)}{\beta(\ell)} y_2'(\ell) \end{bmatrix} \\ &= \Psi_{y_2',\ell} A_\ell, \end{aligned} \tag{16}$$

where

$$A_\ell \triangleq \begin{bmatrix} \frac{\alpha(\mu)}{\beta(\mu)} & & 0 \\ & \ddots & \\ 0 & & \frac{\alpha(\ell)}{\beta(\ell)} \end{bmatrix}. \tag{17}$$

Therefore, (6) and (16) imply that

$$\Psi_{y_2,\ell} = \theta'_\mu \Phi'_{\mu,\ell} A_\ell. \tag{18}$$

It follows from (5) and (18) that $\hat{\theta}_{\mu,\ell}$ satisfies

$$\theta'_\mu \Phi'_{\mu,\ell} A_\ell \Phi_{\mu,\ell}^{\mathrm{T}} = \hat{\theta}_{\mu,\ell} \Phi_{\mu,\ell} \Phi_{\mu,\ell}^{\mathrm{T}}. \tag{19}$$

Note that,

$$\Phi'_{\mu,\ell} A_\ell \Phi_{\mu,\ell}^{\mathrm{T}} =$$

$$\begin{bmatrix} y_1'(\mu) & \cdots & y_1'(\ell) \\ \vdots & & \vdots \\ y_1'(1) & \cdots & y_1'(\ell-\mu+1) \end{bmatrix} \begin{bmatrix} \frac{y_2(\mu)}{y_2'(\mu)} & & 0 \\ & \ddots & \\ 0 & & \frac{y_2(\ell)}{y_2'(\ell)} \end{bmatrix} \begin{bmatrix} y_1(\mu) & \cdots & y_1(\ell) \\ \vdots & & \vdots \\ y_1(1) & \cdots & y_1(\ell-\mu+1) \end{bmatrix}^{\mathrm{T}}$$

$$= \begin{bmatrix} \sum_{i=\mu}^{\ell} \frac{y_1'(i)y_2(i)y_1(i)}{y_2'(i)} & \cdots & \sum_{i=\mu}^{\ell} \frac{y_1'(i)y_2(i)y_1(i-\mu+1)}{y_2'(i)} \\ \vdots & \ddots & \vdots \\ \sum_{i=\mu}^{\ell} \frac{y_1'(i-\mu+1)y_2(i)y_1(i)}{y_2'(i)} & \cdots & \sum_{i=\mu}^{\ell} \frac{y_1'(i-\mu+1)y_2(i)y_1(i-\mu+1)}{y_2'(i)} \end{bmatrix}. \tag{20}$$

Since Y_1, Y_2, Y_1', and Y_2' are stationary random processes, it follows that for all $k \geq 0$ we can calculate

$$\lim_{\ell\to\infty} \frac{1}{\ell} \Phi'_{\mu,\ell} A_\ell \Phi_{\mu,\ell}^{\mathrm{T}} \overset{\text{wp1}}{=} \tag{21}$$

$$\mathbb{E}\begin{bmatrix} \frac{Y_1'(k)Y_2(k)Y_1(k)}{Y_2'(k)} & \cdots & \frac{Y_1'(k)Y_2(k)Y_1(k-\mu+1)}{Y_2'(k)} \\ \vdots & \ddots & \vdots \\ \frac{Y_1'(k-\mu+1)Y_2(k)Y_1(k)}{Y_2'(k)} & \cdots & \frac{Y_1'(k-\mu+1)Y_2(k)Y_1(k-\mu+1)}{Y_2'(k)} \end{bmatrix},$$

where (21) is independent of k. Moreover, note that,

$$\begin{aligned} &\mathbb{E}\left[\frac{Y_1'(k)Y_2(k)Y_1(k)}{Y_2'(k)}\right] \\ &= \mathbb{E}\left[\frac{\sum_{i=-\infty}^{k} U(i)h_1(k-i) \sum_{j=-\infty}^{k} \mathcal{N}_2(U(j))h_2(k-j) \sum_{r=-\infty}^{k} \mathcal{N}_1(U(r))h_1(k-r)}{\sum_{q=-\infty}^{k} U(q)h_2(k-q)}\right] \\ &= \mathbb{E}\left[\frac{\sum_{i=-\infty}^{k}\sum_{j=-\infty}^{k}\sum_{r=-\infty}^{k} U(i)\mathcal{N}_2(U(j))\mathcal{N}_1(U(r))h_1(k-i)h_2(k-j)h_1(k-r)}{\sum_{q=-\infty}^{k} U(q)h_2(k-q)}\right]. \end{aligned} \tag{22}$$

Since U is a stationary white random process and $\mathcal{N}_2$ and $\mathcal{N}_1$ are memoryless nonlinearities, it follows that the expectation in (22) is nonzero when the arguments i, j, and r are equal and zero otherwise. Therefore, (22) can be also written as

$$\begin{aligned} &\mathbb{E}\left[\frac{Y_1'(k)Y_2(k)Y_1(k)}{Y_2'(k)}\right] \\ &= \mathbb{E}\left[\frac{\sum_{i=-\infty}^{k} U(i)h_2(k-i) \sum_{j=-\infty}^{k}\sum_{r=-\infty}^{k} \mathcal{N}_2\big(U(j)\big)\mathcal{N}_1\big(U(r)\big)h_1(k-j)h_1(k-r)}{\sum_{q=-\infty}^{k} U(q)h_2(k-q)}\right] \\ &= \mathbb{E}\left[\sum_{j=-\infty}^{k}\sum_{r=-\infty}^{k} \mathcal{N}_2\big(U(j)\big)\mathcal{N}_1\big(U(r)\big)h_1(k-j)h_1(k-r)\right] \\ &= \sum_{j=-\infty}^{k}\sum_{r=-\infty}^{k} \mathbb{E}\left[\mathcal{N}_2\big(U(j)\big)\mathcal{N}_1\big(U(r)\big)\right] h_1(k-j)h_1(k-r) \\ &= \sum_{j=-\infty}^{k} \mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right] h_1^2(k-j). \end{aligned} \tag{23}$$

Since $\mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]$ is a nonzero constant for all

$k \geq 0$ and independent of j in (23), it follows that

$$\mathbb{E}\left[\frac{Y_1'(k)Y_2(k)Y_1(k)}{Y_2'(k)}\right]$$
$$= \mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right] \sum_{j=-\infty}^{k} h_1^2(k-j)$$
$$= \mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right] \sum_{i=0}^{\infty} h_1^2(i), \tag{24}$$

for all $k \geq 0$.

Using the same procedure we obtain

$$\lim_{\ell\to\infty} \frac{1}{\ell}\Phi'_{\mu,\ell} A_\ell \Phi_{\mu,\ell}^{\mathrm{T}} \overset{\text{wp1}}{=} \mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]\Gamma, \tag{25}$$

where

$$\Gamma \triangleq \begin{bmatrix} \sum_{i=0}^{\infty} h_1^2(i) & \cdots & \sum_{i=0}^{\infty} h_1(i)h_1(\mu-1+i) \\ \vdots & \ddots & \vdots \\ \sum_{i=0}^{\infty} h_1(\mu-1+i)h_1(i) & \cdots & \sum_{i=0}^{\infty} h_1^2(i) \end{bmatrix} \in \mathbb{R}^{\mu\times\mu}. \tag{26}$$

Likewise,

$$\lim_{\ell\to\infty} \frac{1}{\ell}\Phi_{\mu,\ell}\Phi_{\mu,\ell}^{\mathrm{T}} \overset{\text{wp1}}{=} \mathbb{E}\left[\mathcal{N}_1^2\big(U(k)\big)\right]\Gamma. \tag{27}$$

Dividing (19) by ℓ and using (25) and (27) yields

$$\mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]\theta'_\mu\Gamma \overset{\text{wp1}}{=} \lim_{\ell\to\infty}\mathbb{E}\left[\mathcal{N}_1^2\big(U(k)\big)\right]\hat{\theta}_{\mu,\ell}\Gamma. \tag{28}$$

That is,

$$\Big(\mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]\theta'_\mu - \mathbb{E}\left[\mathcal{N}_1^2\big(U(k)\big)\right]\lim_{\ell\to\infty}\hat{\theta}_{\mu,\ell}\Big)\Gamma \overset{\text{wp1}}{=} 0_{1\times\mu}. \tag{29}$$

Since Γ is nonsingular, it follows that

$$\mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]\theta'_\mu \overset{\text{wp1}}{=} \mathbb{E}\left[\mathcal{N}_1^2\big(U(k)\big)\right]\lim_{\ell\to\infty}\hat{\theta}_{\mu,\ell}.$$

Finally,

$$\lim_{\ell\to\infty}\hat{\theta}_{\mu,\ell} \overset{\text{wp1}}{=} \frac{\mathbb{E}\left[\mathcal{N}_2\big(U(k)\big)\mathcal{N}_1\big(U(k)\big)\right]}{\mathbb{E}\left[\mathcal{N}_1^2\big(U(k)\big)\right]}\theta'_\mu, \tag{30}$$

for all $k \geq 0$. Thus, $\hat{\theta}_{\mu,\ell}$ is a semi-consistent estimator of θ'_μ. □

Example 4.1: Let $\mathcal{N}_1(U) = U^3$, $\mathcal{N}_2(U) = U^7$, and let $U(k)$ be uniformly distributed with the density function

$$p(u) = \begin{cases} \frac{1}{2a}, & |u| \leq a, \\ 0, & |u| > a. \end{cases} \tag{31}$$

Then,

$$\mathbb{E}\Big[\mathcal{N}_2\Big(U(k)\Big)\mathcal{N}_1\Big(U(k)\Big)\Big] = \frac{1}{2a}\int_{-a}^{a} U^{10}(k)\,dU(k) = a^{10}/11,$$

$$\mathbb{E}\Big[\mathcal{N}_1^2\Big(U(k)\Big)\Big] = \frac{1}{2a}\int_{-a}^{a} U^6(k)\,dU = a^6/7.$$

Finally, it follows from (30) that

$$\lim_{\ell\to\infty}\hat{\theta}_{\mu,\ell} \overset{\text{wp1}}{=} \frac{7a^4}{11}\theta'_\mu.$$

V. Numerical Examples

Consider the transfer functions

$$G_1(\mathbf{q}) = \frac{4\mathbf{q}+1}{(\mathbf{q}-0.6)(\mathbf{q}+0.8)(\mathbf{q}-0.9)}, \tag{32}$$

$$G_2(\mathbf{q}) = \frac{2\mathbf{q}+5}{(\mathbf{q}-0.55)(\mathbf{q}+0.6)(\mathbf{q}-0.4)}. \tag{33}$$

Then, the PTF is

$$\mathcal{G}(\mathbf{q}) = \frac{G_2(\mathbf{q})}{G_1(\mathbf{q})}, = \frac{(\mathbf{q}-0.6)(\mathbf{q}+0.8)(\mathbf{q}-0.9)(2\mathbf{q}+5)}{(\mathbf{q}-0.55)(\mathbf{q}+0.6)(\mathbf{q}-0.4)(4\mathbf{q}+1)}. \tag{34}$$

It follows from (1) that

$$B(\mathbf{q}) = (\mathbf{q}-0.6)(\mathbf{q}+0.8)(\mathbf{q}-0.9)(2\mathbf{q}+5),$$
$$A(\mathbf{q}) = (\mathbf{q}-0.55)(\mathbf{q}+0.6)(\mathbf{q}-0.4)(4\mathbf{q}+1).$$

Define the normalized Markov parameters of the PTF constructed from y_1' and y_2' by

$$H_i'^{n} \triangleq \frac{H_i'}{H_d'},$$

where H_d' is the first nonzero Markov parameter of the PTF. The estimated Markov parameters $\hat{H}_i$, obtained from $\hat{\theta}_{\mu,\ell}$, are normalized by $\hat{H}_d$ to obtain $\hat{H}_i^n$. The least squares estimates are computed for 200 independent realizations of U. We also define the error metric

$$\varepsilon = \frac{1}{200}\sum_{i=0}^{200}\frac{|H_i'^{n} - \hat{H}_i^n|}{|H_i'^{n}|}. \tag{35}$$

In the following we show five examples involving both odd, even, and neither odd nor even nonlinearities in both cases of zero mean and nonzero mean for $\mathcal{N}_2(u)$. In example 5.2 the term $M(u^2)$ denotes the mean of the realization of the random process U^2 and in example 5.5 the term $M(u^2e^u)$ denotes the mean of the realization of the random process U^2e^U.

Example 5.1: $\mathcal{N}_1(u) = \mathrm{sign}(u)$, $\mathcal{N}_2(u) = \sin(u)$

Consider the transfer functions G_1 in (32) and G_2 in (33), and let U be white and have the uniform pdf (31) with $a = 5$. Figure 5 indicates that the estimates of the Markov parameters H_2, H_3, H_4, and H_5 are semi-consistent.

Example 5.2: $\mathcal{N}_1(u) = u^2 - M(u^2)$, $\mathcal{N}_2(u) = \cos(u)$

Consider the transfer functions G_1 in (32) and G_2 in (33), and let U be white and have the Gaussian pdf $N(0,1)$. Figure 6 indicates that the estimates of the Markov parameters H_2, H_3, H_4, and H_5 are semi-consistent.

Example 5.3: $\mathcal{N}_1(u) = \sinh(u)$, $\mathcal{N}_2(u) = u^3$

Consider the transfer functions G_1 in (32) and G_2 in (33), and let U be white and have the Gaussian pdf $N(0,1)$. Figure

7 indicates that the estimates of the Markov parameters H_2, H_3, H_4, and H_5 are semi-consistent.

Example 5.4: $\mathcal{N}_1(u) = \text{sign}(u)$, $\mathcal{N}_2(u) = e^u$

Consider the transfer functions G_1 in (32) and G_2 in (33), and let U be white and have the uniform pdf (31) with $a = 5$. Figure 8 indicates that the estimates of the Markov parameters H_2, H_3, H_4, and H_5 are semi-consistent.

Example 5.5: $\mathcal{N}_1(u) = u^2 e^u - M(u^2 e^u)$, $\mathcal{N}_2(u) = \sin(u) + 10$

Consider the transfer functions G_1 in (32) and G_2 in (33), and let U be white and have the uniform pdf (31) with $a = 5$. Figure 9 indicates that the estimates of the Markov parameters H_2, H_3, H_4, and H_5 are semi-consistent.

VI. Conclusions

We used least squares with an FIR model structure to identify a pseudo transfer function for a two-output Hammerstein system. Only output signals of the Hammerstein system were used since the intermediate signals were inaccessible. Despite the presence of the input nonlinearities, we proved that, under certain assumptions, the least squares estimate of the Markov parameters of the PTF is semi-consistent. This method was demonstrated on several numerical examples including odd, even, and neither odd nor even nonlinearities in both cases of zero mean and nonzero mean for the output channel Hammerstein nonlinearity, where the input channel Hammerstein nonlinearity should be of zero mean.

References

[1] L. Tong, G. Xu, and T. Kailath, "Blind identification and equalization of multipath channels," in *IEEE Int. Conf. on Commun.*, Chicago, IL, Jun 1992, pp. 1513–1517.

[2] W. Zhou and D. Chelidze, "Blind source separation based vibration mode identification," *Mechanical Sys. and Signal Processing*, vol. 21, no. 8, pp. 3072–3087, 2007.

[3] M. Basseville, A. Benveniste, M. Goursat, and L. Hermans, "Output-only subspace-based structural identification: From theory to industrial testing practice," *ASME J. Dyn. Sys. Meas. Contr.*, vol. 123, no. 4, pp. 668–676, 2001.

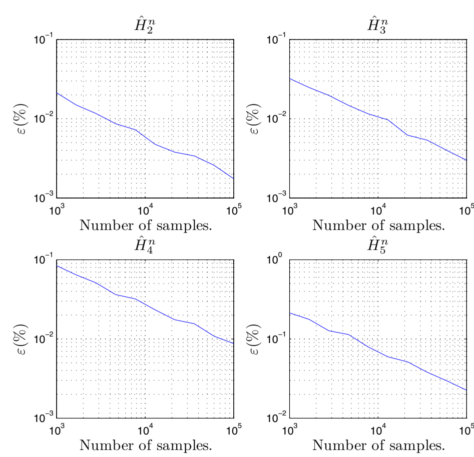

Fig. 5. Semi-consistency of the estimates of the Markov parameters obtained from the FIR model with the Hammerstein nonlinearities $\mathcal{N}_1(u) = \text{sign}(u)$ and $\mathcal{N}_2(u) = \sin(u)$.

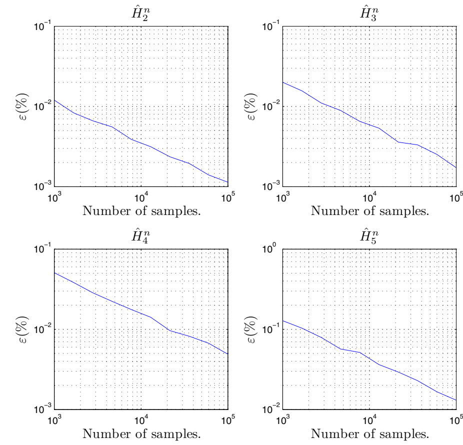

Fig. 6. Semi-consistency of the estimates of the Markov parameters obtained from the FIR model with the Hammerstein nonlinearities $\mathcal{N}_1(u) = u^2 - M(u^2)$ and $\mathcal{N}_2(u) = \cos(u)$.

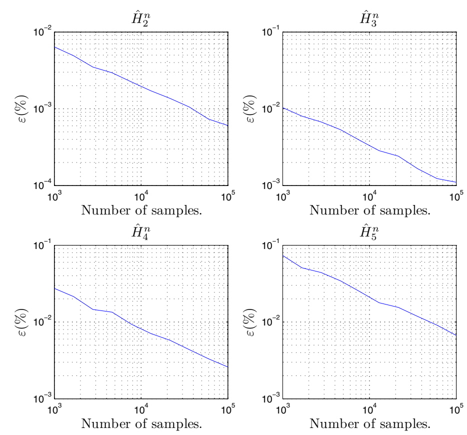

Fig. 7. Semi-consistency of the estimates of the Markov parameters obtained from the FIR model with the Hammerstein nonlinearities $\mathcal{N}_1(u) = \sinh(u)$ and $\mathcal{N}_2(u) = u^3$.

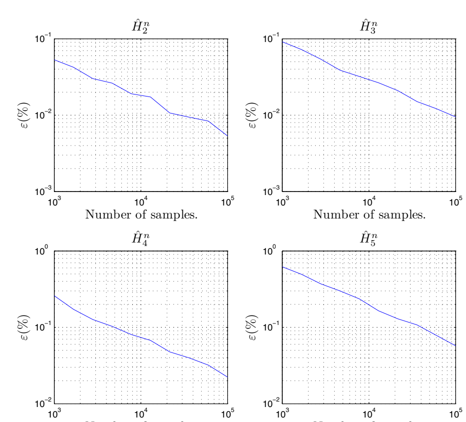

Fig. 8. Semi-consistency of the estimates of the Markov parameters obtained from the FIR model with the Hammerstein nonlinearities $\mathcal{N}_1(u) = \text{sign}(u)$, $\mathcal{N}_2(u) = e^u$.

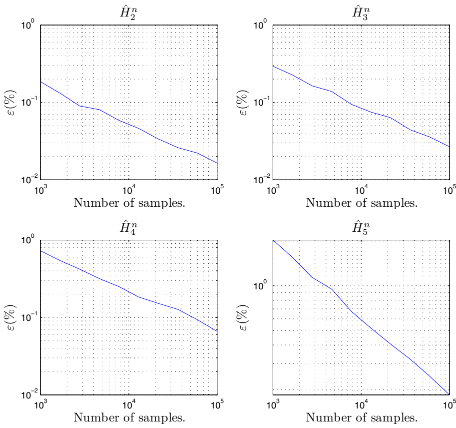

Fig. 9. Semi-consistency of the estimates of the Markov parameters obtained from the FIR model with the Hammerstein nonlinearities $\mathcal{N}_1(u) = u^2e^u - M(u^2e^u)$ and $\mathcal{N}_2(u) = \sin(u) + 10$.

[4] P. G. E. Parloo, S. Vanlanduit and P. Verboven, "Increased reliability of reference-based damage identification techniques by using output-only data," *Sound and Vibration*, vol. 270, pp. 813–832, 2004.

[5] L. Mevel, A. Benveniste, M. Basseville, M. Goursat, B. Peeters, H. V. der Auweraer, and A. Vecchio, "Input/output versus output only data processing for structural identification–application to in-flight data analysis," *Sound and Vibration*, vol. 295, pp. 531–552, 2006.

[6] B. Peeters and G. D. Roeck, "Stochastic system identification for operational modal analysis: A review," *ASME J. Dyn. Sys. Meas. Contr.*, vol. 123, pp. 659–667, 2001.

[7] N. M. M. Maia, J. M. M. Silva, and A. M. Ribeiro, "The transmissibility concept in multi-degree-of-freedom systems," *Mechanical Sys. and Signal Processing*, vol. 15, no. 1, pp. 129–137, 2001.

[8] Y. W. L. M. Haroon, D. E. Adams and A. A. Ferri, "A time and frequency domain approach for identifying nonlinear mechanical system models in the absence of an input measurement," *Sound and Vibration*, vol. 283, pp. 1137–1155, 2005.

[9] C. Devriendt and P. Guillaume, "Identification of modal parameters from transmissibility measurements," *Sound and Vibration*, vol. 314, pp. 343–356, 2008.

[10] C. Devriendt, F. Presezniak, G. D. Sitter, K. Vanbrabant, T. D. Troyer, S. Vanlanduit, and P. Guillaume, "Structural health monitoring in changing operational conditions using transmissibility measurements," *Sound and Vibration*, vol. 17, pp. 651–675, 2010.

[11] C. Devriendt, G. D. Sitter, and P. Guillaume, "An operational modal analysis approach based on parametrically identified multivariable transmissibilities," *Mechanical Sys. and Signal Processing*, vol. 24, no. 5, pp. 1250–1259, 2010.

[12] A. M. D'Amato, A. Brzezinski, M. S. Holzel, J. Ni, and D. S. Bernstein, "Sensor-only noncausal blind identification of pseudo transfer functions," in *Proc. SYSID*, Saint-Malo, France, July 2009, pp. 1698–1703.

[13] A. J. Brzezinski, S. L. Kukreja, J. Ni, and D. S. Bernstein, "Sensor-only fault detection using pseudo-transfer-function identification," in *Proc. Amer. Contr. Conf.*, Baltimore, MD, June 2010, pp. 5433–5438.

[14] ——, "Identification of sensor-only mimo pseudo transfer functions," in *Proc. Conf. Dec. Contr.*, Orlando, FL, December 2011, pp. 1698–1703.

[15] A. A. Ali, A. M. D'Amato, M. S. Holzel, S. L. Kukreja, and D. S. Bernstein, "Consistent identification of hammerstein systems using an ersatz nonlinearity," in *Proc. Amer. Contr. Conf.*, San Francisco, CA, July 2011, pp. 1242–1246.