Why std::binary_search of std::list Works, But You Shouldn't Use It!

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A common error made by beginning users of the Standard Library containers and algorithms is to write code whose run-time complexity (big-O) is a *flaming disaster*. These facilities were invented by algorithm and data structure fanatics who took big-O *really* seriously; their goal was to make it possible for you to use "best of breed" data structures and algorithms very easily. They certainly did not intend to help you write *awful* code easily!

In fact, in most cases, the Standard Library tries to keep you out of trouble by making it inconvenient to use in a really inefficient way. But thanks to the great generality and power of the C++ templates used in the Standard Library, there are some loopholes that allow you to write hopelessly inefficient code as easily as very efficient code. One such loophole is that you can easily write Standard Library code that does a binary search on a linked list, which is so ridiculously inefficient that saying "binary search on a linked list" is actually a geeky programming joke! Linked-lists are supposed to be searched linearly! The purpose of this document is to explain why the Standard Library makes telling this joke so easy to do, and demonstrate with some run-time comparisons why it is so bad.

The Standard Library allows you to apply the `binary_search` and `lower_bound` algorithms to any *sorted sequence container*, including `std::list`, and it will produce a correct result. The following works for any sequence container:

```
binary_search(container.begin(), container.end(), probe)
```

However, if you look at any normal code for binary search (e.g. as in Kernighan and Ritchie), it is written to use array subscripting. Array subscripting, a so-called *random-access* mechanism, runs in constant time, regardless of the size of the array or value of the subscript. In contrast, a linked list has the property that the only way you can find a particular node is to start at one end of the list and follow the links from one node to the next, checking them one at a time. Unlike with array subscripting, there is no way to compute the location of a list node directly from its numerical position in the list - it could be anywhere in memory. So how is it that you can do a `binary_search` on a `std::list`?

How binary_search is implemented. Below is a somewhat simplified copy of the Metrowerks Standard Library version of `lower_bound`; the `binary_search` algorithm just calls `lower_bound` and checks the result (other implementations might differ, but only in details).

```
template <class ForwardIterator, class T>
ForwardIterator
lower_bound(ForwardIterator first, ForwardIterator last, const T& value)
{
   typedef typename iterator_traits<ForwardIterator>::difference_type difference_type;
   difference_type len = distance(first, last);
   while (len > 0)
   {
      ForwardIterator i = first;
      difference_type len2 = len / 2;
      advance(i, len2);
      if (*i < value)
      {
         first = ++i;
         len -= len2 + 1;
      }
      else
         len = len2;
   }
   return first;
}
```

First, see how this algorithm is written in terms of iterators, so that it can apply to any sequence container that supports the standard iterator interface. The two *input iterator*s are *first* and *last*, marking the beginning and end of the range to be searched. Among the iterator types, input iterators can be iterators pointing into any type of container.

The basic binary search algorithm involves calculating the midpoint of a range of values, and then checking the value at that midpoint. The code does this by calling `std::distance`, which returns the numerical distance between the first and last iterators. This distance is divided by two, and then `std::advance` is called to move the first iterator forward by that amount to get to the midpoint of the range. The `distance` and `advance` functions are also function templates that are defined so that they work with iterators into any type of container. Template magic is used to specialize them for different iterator types. For *random-access iterator*s (which behave like pointers or subscripts, supplied by `std::vector` and `std::deque`), the definition of `distance` that is used is:

```
template <class RandomAccessIterator>
inline
typename iterator_traits<RandomAccessIterator>::difference_type
__distance(RandomAccessIterator first, RandomAccessIterator last, random_access_iterator_tag)
{
    return last - first;
}
```

The subtraction operator is defined for these iterators because the internal pointers can simply be subtracted to get the distance directly via pointer arithmetic. This is exactly what subtracting the indices would do in the array form of the binary search algorithm.

However, for the more general input iterators, which can only move forward or back by one step at a time, the definition used to implement `distance` is:

```
template <class InputIterator>
inline
typename iterator_traits<InputIterator>::difference_type
__distance(InputIterator first, InputIterator last, input_iterator_tag)
{
    typename iterator_traits<InputIterator>::difference_type result = 0;
    for (; first != last; ++first)
       ++result;
    return result;
}
```

In other words, compute the distance between two general input iterators by incrementing the first until it equals the last, and count how many increments are required.

The `advance` function has a similar pair of specializations. Advancing a random-access iterator simply adds the number of steps to the iterator, corresponding directly to pointer arithmetic, but advancing an input iterator requires incrementing the iterator the supplied number of times.

When these functions are applied to a built-in array or a `std::vector` container, the `distance` and `advance` functions will compile down to simple subtraction and addition of the subscript values/pointers, taking almost no time. But if applied to a list, whose iterators support only moving forward or back by one step at a time, then the binary search will require using the `distance` function to repeatedly count the nodes between the ends of the narrowing range and then `advance` with increment and count again to position at the midpoint. Surely all this link-following will add up to a substantial amount of time!

The C++98 Standard in fact states that `lower_bound` and `binary_search` will run in *O(log n)* time when applied to a container with random access iterators. When applied to a container that lacks random-access iterators, like `std::list`, the Standard states that the search will be logarithmic with the number of comparisons, but linear with the number of nodes visited. So whether it runs faster than a linear search depends on how much time it takes to do the comparisons compared to counting the nodes over and over again.

Let's find out what happens. Sometimes you need *benchmarks* to see how theory works out in practice. I defined two classes of objects which contain an ID value used in `operator<` and `operator==`, and with constructors that give each object a unique value. One class, Cheap, uses a single integer for the ID, so comparisons that should be very fast. The other,

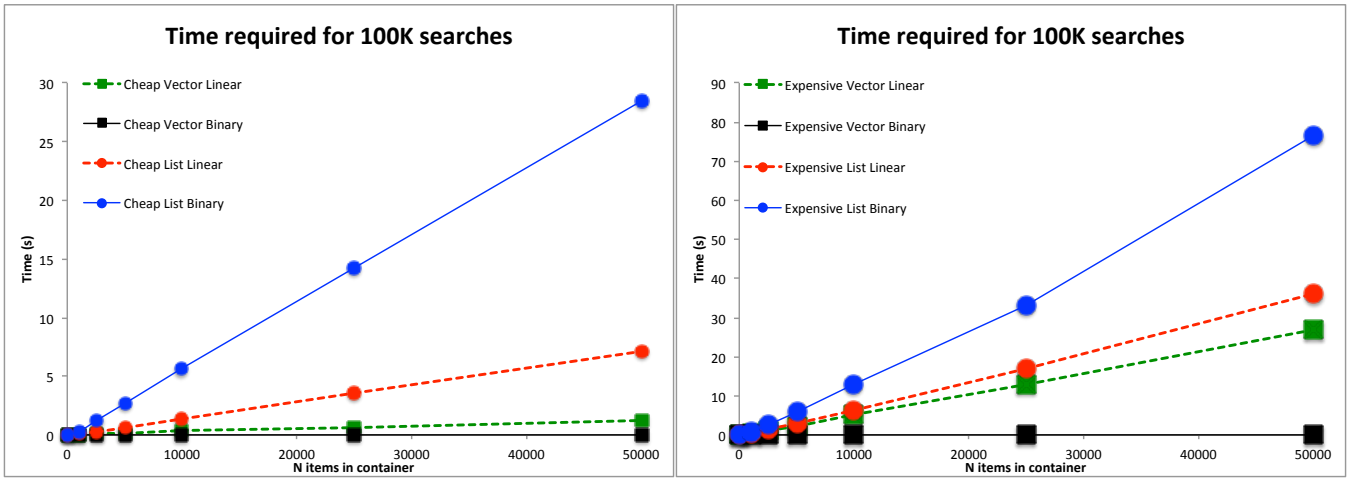
Expensive, uses an ID `std::string` containing 33 characters, the first 28 of which are identical, and the remainder are digits corresponding to the Cheap ID number value. The comparison is done with `std::string`'s < and = operators, which will have to test the first 28 characters before finding the different ones.

My benchmark code filled containers with 100, 1000, 2500, 5000, 10000, 25000, or 50000 objects, in order by ID, and then did 100,000 searches of each container size, using `std::find` (which does a linear search) or `std::binary_search` to search for each one of the objects, so each possible position in the container was searched for. This search sequence was repeated more for shorter containers so that the final results show the total time for 100,000 searches distributed evenly over all the positions in the container, giving reasonably stable average run times. I compared run times for the eight combinations of `std::vector` vs. `std::list` containers, linear search with `std::find` vs. `std::binary_search`, and Cheap vs. Expensive objects. The code was compiled under gcc 4.7 with `g++` 4.7 with `-std=c++11` and with a high level of optimization specified by `-Os`. The runs were done on a CAEN remote login workstation; the complete run was done twice; there was very little difference, but the times were averaged for the two runs.

The results. As shown in the graphs below, the Expensive object searches (right panel) required about three times longer than the corresponding Cheap object searches (left panel), so the difference in the two classes is significant. The clear winner is `binary_search` on the `vector`, whose logarithmic search time (black squares) is so fast that it looks flat and essentially zero when plotted on this scale for both Cheap and Expensive objects! Clearly, this combination is the best solution, and it should be used unless there is some *compelling* reason to use something else. Using a linear search with a list container (red circles) or with a vector (green squares) looks very poor by comparison. As you might expect, the vector version of linear search is faster, especially for the Cheap objects. The two containers aren't that different in linear search when the comparisons are expensive and dominate the run time.

The *big loser* is the binary search on the inked list; its performance is awful for both Cheap and Expensive objects. You would expect to see some benefit with Expensive objects because the search time would be logarithmic with the number of comparisons, but notice how the actual time still looks linear - this is what big-O tells us; the time is dominated by the fastest-increasing time component of the algorithm, which would be the linear time required to count through the list to find the middle element. Maybe if the comparisons were much more expensive, some benefit would appear, but based on these results, the comparisons would have to take a very long time to make up for the node-counting time.

Conclusion: Is there ever a reason to do a binary search of a linked list? Based on these results, the costs of the comparison would have to be *gigantic*, much more than in this comparison, to even get into the running. You should only consider it if for some reason you *have* to use a linked-list container. Even so, you should construct and run a relevant benchmark to be sure. It certainly looks like doing a binary search on a linked list is a joke after all.



Time (in seconds) required for 100K searches as a function of container size, for Cheap- and Expensive-comparison objects, for list vs vector containers and linear vs binary searches. Cheap is shown with small points in the left panel, Expensive with large points in the right panel.Vector container is shown with square plotting points; list with circles. Linear is with dotted lines, binary with solid lines.