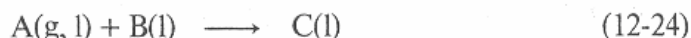


R12.2 Trickle Bed Reactors

In a trickle bed reactor the gas and liquid flow (trickle) cocurrently downward over a packed bed of catalyst particles. Industrial trickle beds are typically 3 to 6 m deep and up to 3 m in diameter and are filled with catalyst particles ranging from $\frac{1}{8}$ to $\frac{1}{32}$ in. in diameter. The pores of the catalyst are filled with liquid. In petroleum refining, pressures of 34 to 100 atm and temperatures of 350 to 425°C are not

Characteristics and
uses of a trickle
bed reactor

uncommon. A pilot-plant trickle bed reactor might be about 1 m deep and 4 cm in diameter. Trickle beds are used in such processes as the hydrodesulfurization of heavy oil stocks, the hydrotreating of lubricating oils, and reactions such as the production of butynediol from acetylene and aqueous formaldehyde over a copper acetylide catalyst. It is on this latter type of reaction, i.e.,



that we focus in this section. In a few cases, such as Fischer-Tropsch synthesis, the liquid is inert and acts as a heat-transfer medium.

R12.2A Fundamentals

The basic reaction and transport steps in trickle bed reactors are similar to slurry reactors. The main differences are the correlations used to determine the mass-transfer coefficients. In addition, if there is more than one component in the gas phase (e.g., liquid has a high vapor pressure or one of the entering gases is inert), there is one additional transport step in the gas phase. Figure 12-7 shows the various transport steps in trickle bed reactors. Following our analysis for slurry reactors we develop the equations for the rate of transport of each step.

1. **Transport from the bulk gas phase to the gas-liquid interface:** The rate of transport per mass of catalyst is

$$-r'_A = k_g a_i \frac{1}{(1 - \epsilon_b) \rho_p} [C_A(g) - C_{A_i}(g)] \quad \frac{\text{mol}}{\text{g cat} \cdot \text{s}} \quad (12-25)$$

where

- a_i = interfacial area per volume of bed, m^2/m^3
- k_g = gas-phase mass-transfer coefficient, m/s
- ρ_p = density of catalyst pellet, kg/m^3
- $1 - \epsilon_b$ = volume of solids/volume of bed (voids + solids)
- $C_A(g)$ = bulk gas-phase concentration of A, kmol/m^3
- $C_{A_i}(g)$ = concentration of A at interface, kmol/m^3

2. **Equilibrium at gas-liquid interface:**

$$C_{A_i} = \frac{C_{A_i}(g)}{H} \quad (12-26)$$

where C_{A_i} = concentration of A in liquid at the interface
 H = Henry's constant

3. **Transport from interface to bulk liquid:**

$$-r'_A = k_l a_i \frac{1}{(1 - \epsilon_b) \rho_p} (C_{A_i} - C_{Ab}), \quad \frac{\text{mol}}{\text{g cat} \cdot \text{s}} \quad (12-27)$$

where

- k_l = liquid-phase mass-transfer coefficient, m/s
- C_{A_i} = concentration of A in liquid at interface
- C_{Ab} = bulk liquid concentration of A

Transport
from bulk gas
to gas-liquid
interface
to bulk liquid
to solid-liquid
interface
Diffusion and
reaction in catalyst
pellet

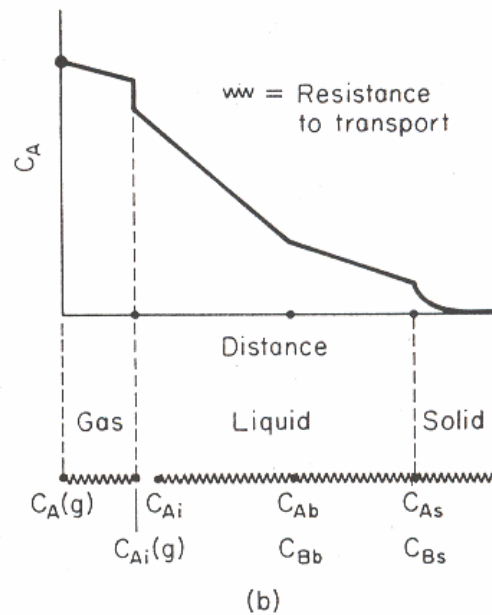
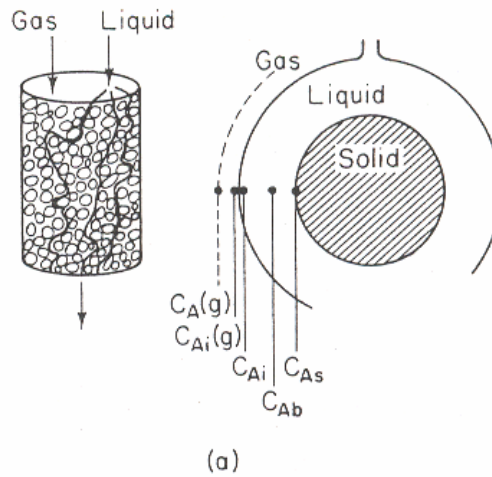


Figure 12-7 (a) Trickle bed reactor; (b) reactant concentration profile.

4. **Transport from bulk liquid to external catalyst surface:**

$$-r'_A = k_c a_c (C_{A_b} - C_{A_s}), \quad \text{mol/g cat} \cdot \text{s} \quad (12-28)$$

where a_c [equation (12-18)] and C_{A_b} are defined as before

k_c = liquid-solid mass-transfer coefficient, m/s

C_{A_s} = concentration of A at solid-liquid interface

5. **Diffusion and reaction in the pellet:** If we assume first-order reaction in dissolved gas A and in liquid B, we have

$$-r'_A = \eta k C_{A_s} C_{B_s} \quad \frac{\text{mol}}{\text{g cat} \cdot \text{s}} \quad (12-29)$$

where η and C_{As} are defined as before

k = specific reaction rate, m^6 of liquid/mol·g cat·s

Combining equations (12-25) through (12-29) and rearranging in an identical manner to that leading to the development of equation (12-8) for slurry reactors, we have

The overall rate equation for A

$$-r'_A = \frac{1/H}{\frac{(1-\epsilon_b)\rho_p}{Hk_g a_i} + \frac{(1-\epsilon_b)\rho_p}{k_l a_i} + \frac{1}{k_c a_c} + \frac{1}{\eta k C_{Bs}}} C_A(g) \quad \frac{\text{mol}}{\text{g cat}\cdot\text{s}}$$

(12-30)

i.e.,

$$-r'_A = k_{vg} C_A(g) \quad (12-31)$$

where k_{vg} is the overall transfer coefficient for the gas into the pellet (m^3 of gas/g cat·s). A mole balance on species A gives

Mole balance on A

$$\frac{dF_A}{dW} = r'_A = -k_{vg} C_A(g) \quad (12-32)$$

We next consider the transport and reaction of species B, which does not leave the liquid phase.

6. Transport of B from bulk liquid to solid catalyst interface:

$$-r'_B = k_c a_c (C_B - C_{Bs}) \quad \frac{\text{mol}}{\text{g cat}\cdot\text{s}} \quad (12-33)$$

where C_B and C_{Bs} are the concentrations of B in the bulk fluid and at the solid interface, respectively.

7. Diffusion and reaction of B inside the catalyst pellet:

$$-r'_B = \eta k C_{As} C_{Bs} \quad (12-34)$$

Combining equations (12-33) and (12-34) and rearranging, we have

The overall rate equation for B

$$-r'_B = \frac{1}{\frac{1}{k_c a_c} + \frac{1}{\eta k C_{As}}} C_B \quad \frac{\text{mol}}{\text{g cat}\cdot\text{s}} \quad (12-35)$$

$$-r'_B = k_{vl} C_B \quad (12-36)$$

A mole balance on species B gives

Mole balance on B

$$\frac{dF_B}{dW} = v_l \frac{dC_B}{dW} = r'_B = -k_{vl} C_B \quad (12-37)$$

One notes that the surface concentrations of A and B, $C_{A,s}$ and $C_{B,s}$, appear in the denominator of the overall transport coefficients k_{vg} and k_{vl} . Consequently, equations (12-30), (12-32), (12-35), and (12-37) must be solved simultaneously. In some cases analytical solutions are available, but for complex rate laws, one resorts to numerical solutions.[†] However, we shall consider some limiting situations.

R12.2.2 Limiting situations

Mass Transfer of the Gaseous Reactant Limiting For this situation we assume that either the first three terms in the denominator of equation (12-30) are dominant, or that the liquid-phase concentration of species B does not vary significantly through the trickle bed. For these conditions k_{vg} is constant, and we can integrate the mole balance. For negligible volume change $\epsilon = 0$, then

Catalyst weight
necessary to
achieve a
conversion X_A of
gas-phase reactant

$$W = \frac{v_g}{k_{vg}} \ln \frac{C_{A(\text{in})}}{C_{A(\text{out})}} = \frac{v_g}{k_{vg}} \ln \frac{1}{1 - X_A} \quad (12-38)$$

Mass Transfer and Reaction of Liquid Species Limiting Here we assume that the liquid phase is entirely saturated with gas throughout the column. As a result, $C_{A,s}$ is a constant and therefore so is k_{vl} . Consequently, we can integrate the combined mole balance and rate law to give

Catalyst weight
necessary to
achieve a
conversion X_B of
liquid-phase
reactant

$$W = \frac{v_l}{k_{vl}} \ln \frac{C_{B(\text{in})}}{C_{B(\text{out})}} = \frac{v_l}{k_{vl}} \ln \frac{1}{1 - X_B} \quad (12-39)$$

12.2.3 Evaluating the transport coefficients

The mass-transfer coefficients, k_g , k_l , and k_c depend on a number of variables, such as type of packing, flow rates, wetting of particle, and geometry of the column, and as a result the correlations vary significantly from system to system. Consequently, we will not give all the correlations here but instead will give correlations for particular systems and refer the reader to four specific references

[†]A number of worked example problems for three-phase reactors can be found in the article "Predicting performance of three phase catalytic reactors, by P. A. Ramachandran and R. V. Chaudhari, *Chem. Eng.*, 87(24), 74 (1980).

where other correlations for trickle bed reactors may be found.[†] Typical correlations are given in Table 12-3. Note that the correlation for organic particles tends to underpredict the transport coefficient.

The representative correlations given in Table 12-3 assume complete wetting of the catalyst particles. Corrections for incomplete wetting as well as flow regimes, pressure-drop equations, and other mass-transfer correlations can be found in the reviews by Shah, by Smith, and by Satterfield.[‡]

The plug-flow design equation may be applied successfully provided the ratio of reactor length L to particle diameter d_p satisfies the criteria (Satterfield, 1975)

Criteria for
assuming that plug
flow is valid

$$\frac{L}{d_p} > \frac{20}{Pe} n \ln \frac{1}{1-X} \quad (12-40)$$

where $Pe = \text{Péclet number} = d_p U_l / D_{Ax}$
 $D_{Ax} = \text{axial dispersion coefficient}$
 $n = \text{reaction order}$

The CSTR design equations apply to the trickle bed when[§]

$$\frac{L}{d_p} < \frac{4}{Pe} \quad (12-41)$$

Techniques for determining the single-phase axial dispersion coefficient are given in Chapter 14.

Example 12-3 Trickle Bed Reactor

The hydrogenation of an unsaturated organic is to be carried out in a trickle bed reactor packed with 0.20-cm-diameter spherical catalyst particles.



The reaction in the pellet is first order in both hydrogen and the organic. Hydrogen and nitrogen are fed in equimolar portions at a total pressure of 20 atm and total molar rate of 10 mol/s. The reactor diameter is to be 1.0 m. The superficial liquid mass velocity is 5.0 kg/m²·s. The corresponding pressure gradient through the bed is 25 kPa/m. As a first approximation, assume that the concentration of organic is constant and the pseudo-first-order specific reaction rate is $3 \times 10^{-5} \text{ m}^3/\text{kg cat} \cdot \text{s}$ at 400 K.

(a) For each transport step, determine its fraction of the total resistance to mass transport and reaction.

[†]C. N. Satterfield, "Trickle bed reactors" (journal review), *AIChE J.*, 21, 209 (1975); M. Herskowitz and J. M. Smith, "Trickle bed reactors, a review," *AIChE J.*, 29, 1 (1983); G. A. Hughmark, "Heat and mass transfer for spherical particles in a fluid field," *Ind. Eng. Chem. Fundam.*, 19, 198 (1980); Y. T. Shah, B. G. Kelkar, S. P. Godbole, and W. D. Deckwer, "Design parameters estimations for bubble column reactors" (journal review), *AIChE J.*, 28, 353 (1982).

[‡]Ibid.

[§]P. A. Ramachandran and R. V. Chaudhari *Chem. Eng.*, 87(24), 74 (1980).

TABLE 12-3. SAMPLE MASS-TRANSFER CORRELATIONS FOR TRICKLE BEDS^a

Transport Step	Correlation	Typical Values	Reference	Eq. No.
Gas-to-gas interface	$k_g a_i = 2 + 0.91 E_g^{2/3}$ E_g is in $\text{ft} \cdot \text{lb}_f / \text{ft}^3 \cdot \text{s}$ $E_g = \left(\frac{\Delta P}{\Delta L} \right) U_g$ $k_g a_i = 2 + 0.12 E_g^{2/3}$ $E_g = \text{kW}/\text{m}^3$	$k_g a_i = 7.4 \text{ s}^{-1}$ for $U_g = 10 \text{ ft/s}$ $\frac{\Delta P}{\Delta L} = 10^{-2} \text{ psi/ft}$	IEC Proc. Des. Dev., 6, 486 (1967)	(12-40)
Liquid interface to bulk liquid				(12-41)
Aqueous	$\frac{k_l a_i}{D_l} = 8.08 \left(\frac{G_l}{\mu} \right)^{0.41} \text{ Sc}^{1/2}$	$k_l a_i \sim 0.01 \text{ s}^{-1}$ for $U_l = 0.2 \text{ cm/s}$	Chem. Eng. Sci., 34, 1425 (1979)	(12-42)
Organic ^b	$\frac{k_l a_i}{D_l} = 16.8 \frac{\text{Re}_l^{1/4}}{\text{Ga}^{0.22}} \text{ Sc}^{1/2} \text{ cm}^{-2}$ $k_l a_i$ in s^{-1} , D_l in cm^2/s		Chem. Eng. Sci., 36, 569 (1981)	(12-43)
Bulk liquid-to- solid interface				
For $\text{Re} < 60$	$\text{Sh}' = 0.815 \text{Re}_l^{0.822} \text{Sc}^{1/3}$	$k_c a_c \sim 0.2 \text{ s}^{-1}$ for $\text{Re}_l = 50$ and $d_p = 0.5 \text{ cm}$	AIChE J., 24, 709 (1978)	(12-44)
For $\text{Re} < 20$	$\text{Sh}' = 0.266 \text{Re}_l^{1.15} \text{Sc}^{1/3}$			(12-45)
	$\text{Ga} = \text{Galileo number} = \frac{d_p^3 \rho_l^2 g}{\mu_l^2}$ $g = 9.8 \text{ m/s}^2$ $G_l = \text{superficial mass velocity of liquid, g/cm}^2 \cdot \text{s}$ $U_l = \text{superficial velocity of liquid } (v_0/A_c), \text{ cm/s}$ $\text{Sh}' = \frac{k_c d_p \alpha}{D_l}$, $\alpha = \text{fraction of external surface that is wetted}$ $a_s = \frac{6}{d_p} = \frac{\text{interfacial area}}{\text{volume of pellet}}$, $a_c = \frac{6}{\rho_p d_p} = \frac{\text{interfacial area}}{\text{mass of pellet}}$ $a_i = \frac{6(1 - \epsilon_b)}{d_p} = \frac{\text{interfacial area}}{\text{volume of reactor}}$, $\epsilon_b = \text{bed porosity}$	$\text{Re}_l = \frac{G_l d_p}{\mu_l}$		

^a Also see N. Midoux, B. I. Morsi, M. Purwasasmita, A. Laurent, and J. C. Charpentier, *Chem. Eng. Sci.* 39, 781 (1984), for a comprehensive list of correlations.

^b In some cases this gives a low estimate of $k_l a_i$; see M. Herskowitz and J. M. Smith, *AIChE J.*, 29, 1 (1983); F. Turek and R. Lange, *Chem. Eng. Sci.*, 36, 569 (1981).

(b) Calculate the catalyst weight necessary to achieve 55% conversion of the hydrogen.

Additional data:

Liquid viscosity: $1.8 \text{ Cp} = 0.0018 \text{ kg/m} \cdot \text{s}$

Liquid density: 840 kg/m^3

Hydrogen liquid diffusivity in oil: $2.4 \times 10^{-9} \text{ m}^2/\text{s}$

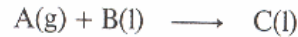
Organic diffusivity in organic product: $1.2 \times 10^{-9} \text{ m}^2/\text{s}$

Molecular weight of organic: 168 daltons
 Hydrogen solubility: $0.008 \text{ kmol/m}^3 \cdot \text{atm}$
 Pellet porosity: 0.3
 Pellet density: 1500 kg/m^3
 Bed porosity: 0.4

Assume that tortuosity = 1.5 and constriction = 0.8.

Solution

(a) Let A = H_2 , B = unsaturated organic, and C = saturated organic:



1. **Mole balance on H_2 (A):**

$$\frac{dF_A}{dW} = r'_A \quad (\text{E12-3.1})$$

2. **Rate law:** Assuming a constant liquid reactant concentration for low conversion of B,

$$-r'_A = k_{vg} C_A(\text{g}) \quad (\text{E12-3.2})$$

with

$$k_{vg} = \frac{1/H}{\frac{(1 - \epsilon_b)\rho_p}{Hk_g a_i} + \frac{(1 - \epsilon_b)\rho_p}{k_l a_i} + \frac{1}{k_c a_c} + \frac{1}{\eta k}} \quad (\text{E12-3.3})$$

$\epsilon_b = \text{bed porosity} = 0.4$

3. **Stoichiometry:** The isothermal gas-phase concentration is

$$C_A = C_{A0} \left(\frac{1 - X}{1 + \epsilon X} \right) \frac{P}{P_0} \quad (\text{E12-3.4})$$

$$\epsilon = y_{A0} \delta = (0.5)(0 - 0 - 1) = -0.5$$

4. **Pressure drop:**

$$\frac{dP}{dL} = -\alpha = -25 \frac{\text{kPa}}{\text{m}} = -0.246 \text{ atm/m}$$

$$\frac{d(P/P_0)}{dW} = \frac{-\alpha}{P_0 A_c (1 - \epsilon_b) \rho_p} = -\beta \quad (\text{E12-3.5})$$

$$\frac{P}{P_0} = 1 - \beta W \quad (\text{E12-3.6})$$

5. **Combining yields**

$$F_{A0} \frac{dX}{dW} = k_{vg} C_{A0} \left(\frac{1 - X}{1 + \epsilon X} \right) (1 - \beta W)$$

Integrating gives us

$$(1 + \epsilon) \ln \frac{1}{1 - X} - \epsilon X = \frac{k_{vg}}{v_0} \left(W - \frac{\beta}{2} W^2 \right) \quad (\text{E12-3.7})$$

6. **Evaluating** the parameters:

A. *Solubility*

$$H' = 0.008 \frac{\text{kmol}}{\text{m}^3 \cdot \text{atm}}$$

$$P_{A_i} H' = C_{A_i} \quad \frac{C_{A_i}(\text{g})}{H} = C_{A_i}$$

$$H = \frac{1}{RTH'} = \frac{1}{\left(0.082 \frac{\text{m}^3 \cdot \text{atm}}{\text{kmol} \cdot \text{K}}\right) (400 \text{ K}) \left(\frac{0.008 \text{ kmol}}{\text{m}^3 \cdot \text{atm}}\right)} = 3.81 \quad (\text{E12-3.8})$$

B. *Internal diffusion and reaction*

$$\text{resistance} = R_R = \frac{1}{\eta k} \quad (\text{E12-3.9})$$

$$\phi = R \sqrt{\frac{k \rho_p}{D_e}} \quad (\text{E12-3.10})$$

$$D_e = D_L \frac{\sigma \epsilon_p}{\tau} = 2.4 \times 10^{-9} \frac{\text{m}^2}{\text{s}} \times \frac{(0.8)(0.3)}{1.5} = 3.84 \times 10^{-10} \frac{\text{m}^2}{\text{s}} \quad (\text{11-1})$$

$$\phi = 10^{-3} \text{ m} \sqrt{\frac{(3 \times 10^{-5} \text{ m}^3/\text{kg} \cdot \text{s})(1500 \text{ kg}/\text{m}^3)}{3.84 \times 10^{-10} \text{ m}^2/\text{s}}} = 10.8$$

For large values of the Thiele modulus,

$$\eta = \frac{3}{\phi} = \frac{3}{10.8} = 0.277$$

$$R_R = \frac{1}{(0.277)(3 \times 10^{-5} \text{ m}^3/\text{kg} \cdot \text{s})} = 1.2 \times 10^5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3} \quad (\text{E12-3.11})$$

C. *Gas absorption*

$$\text{resistance} = R_g = \frac{(1 - \epsilon_b) \rho_p}{H k_g a_i} \quad (\text{E12-3.12})$$

$$k_g a_i = 2 + 0.91 E_g^{2/3}$$

$$E_g = \frac{\Delta P}{\Delta L} U_g$$

$$v_0 = 10 \frac{\text{mol}}{\text{s}} \times 0.0224 \frac{\text{m}^3}{\text{mol}} \times \frac{1 \text{ atm}}{20 \text{ atm}} \times \frac{400 \text{ K}}{273 \text{ K}} = 0.0164 \frac{\text{m}^3}{\text{s}}$$

$$A_c = \frac{\pi D^2}{4} = 0.785 \text{ m}^2$$

$$U_g = \frac{v_0}{A_c} = \frac{0.0164 \text{ m}^3/\text{s}}{0.785 \text{ m}^2} = 0.021 \text{ m/s}$$

$$E_g = 25 \frac{\text{kPa}}{\text{m}} \left(0.021 \frac{\text{m}}{\text{s}} \right) = 0.52 \frac{\text{kPa}}{\text{s}} = 10.9 \frac{\text{ft} \cdot \text{lb}_f}{\text{ft}^3 \cdot \text{s}}$$

$$k_g a_i = 2 + 0.91(10.9)^{2/3} = 6.47 \text{ s}^{-1}$$

$$R_g = \frac{(1 - 0.4)(1500 \text{ kg/m}^3)}{(3.8)(6.47 \text{ s}^{-1})} = 36.5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3}$$

D. Transport from gas-liquid interface to bulk liquid

$$R_l = \frac{(1 - \epsilon_b) \rho_p}{k_l a_i} \quad (\text{E12-3.13})$$

$$\text{Re}_l = \frac{G d_p}{\mu_l} = \frac{(5 \text{ kg/m}^2 \cdot \text{s})(2 \times 10^{-3} \text{ m})}{0.0018 \text{ kg/m} \cdot \text{s}} = 5.55$$

$$\text{Ga} = \frac{d_p^3 \rho_l^2 g}{\mu_l^2} = \frac{(2 \times 10^{-3} \text{ m})^3 (840 \text{ kg/m}^3)^2 (9.8 \text{ m/s}^2)}{(0.0018 \text{ kg/m} \cdot \text{s})^2}$$

$$= 17,073$$

$$\text{Sc} = \frac{\mu_l}{\rho_l D_L} = \frac{0.0018 \text{ kg/m} \cdot \text{s}}{(840 \text{ kg/m}^3)(2.4 \times 10^{-9} \text{ m}^2/\text{s})} = 893$$

From the correlation for organic liquids,

$$k_l a_i = D_L \left(16.8 \frac{\text{Re}_l^{1/4} \text{Sc}^{1/2}}{\text{Ga}^{0.22}} \text{ cm}^{-2} \right) \quad (\text{E12-3.14})$$

It has been noted[†] that this correlation gives a mass-transfer coefficient that is too low.

$$k_l a_i = 2.4 \times 10^{-5} \frac{\text{cm}^2}{\text{s}} \left[\frac{16.8(5.55)^{1/4}(893)^{1/2}}{(17,073)^{0.22}} \text{ cm}^{-2} \right]$$

$$= 2.17 \times 10^{-3} \text{ s}^{-1}$$

$$R_l = \frac{(1 - 0.4)(1500 \text{ kg/m}^3)}{2.17 \times 10^{-3} \text{ s}^{-1}} = 4.15 \times 10^5 \frac{\text{kg} \cdot \text{s}}{\text{m}^3}$$

[†]M. Herskowitz and J. M. Smith, op. cit.; F. Turek and R. Lange, *Chem. Eng. Sci.*, 36, 569 (1981).

E. Resistance from bulk liquid to catalyst

$$\text{resistance} = R_c = \frac{1}{k_c a_c} \quad (\text{E12-3.15})$$

$$a_c = \frac{6}{\rho_p d_p} = \frac{6}{(1500 \text{ kg/m}^3)(2 \times 10^{-3} \text{ m})} = 2.0 \frac{\text{m}^2}{\text{kg}}$$

$$\text{Sh} = 0.266 \text{Re}_l^{1.15} \text{Sc}^{1/3} = (0.266)(5.55)^{1.15}(893)^{1/3} = 18.4$$

$$k_c = \frac{\text{Sh} D_L}{d_p} = \frac{(18.4)(2.4 \times 10^{-9} \text{ m}^2/\text{s})}{2 \times 10^{-3} \text{ m}} = 2.2 \times 10^{-5} \text{ m/s}$$

$$R_c = \frac{1}{k_c a_c} = 0.23 \times 10^5 \text{ kg}\cdot\text{s}/\text{m}^3$$

F. Total and percentage resistances

$$R_T = R_c + R_l + R_g + R_R$$

$$= (0.23 \times 10^5 + 4.15 \times 10^5 + 36.5 + 1.2 \times 10^5) \frac{\text{kg}\cdot\text{s}}{\text{m}^3} \quad (\text{E12-3.16})$$

$$R_T = 5.58 \times 10^5 \frac{\text{kg}\cdot\text{s}}{\text{m}^3}$$

Individual resistances:

$$\text{Gas absorption} = 100 \frac{36.5}{5.58 \times 10^5} \approx 0.01\%$$

$$\text{Bulk liquid to catalyst} = 100 \times \frac{0.23}{5.58} = 4.12\%$$

$$\text{Internal diffusion and reaction} = 100 \times \frac{1.2}{5.58} = 21.50\%$$

$$\text{Gas interface to bulk liquid} = 100 \times \frac{4.15}{5.58} = 74.37\%$$

Total	100%
-------	------

(b) Calculate catalyst weight

$$(1 + \epsilon) \ln \frac{1}{1 - X} - \epsilon X = \frac{k_{vg}}{v_0} \left(W - \frac{\beta}{2} W^2 \right) \quad (\text{E12-3.7})$$

$$k_{vg} = \frac{1/H}{R_T} = \frac{1/(3.8)}{5.58 \times 10^5} = 4.6 \times 10^{-7} \frac{\text{m}^3}{\text{kg}\cdot\text{s}} \quad (\text{E12-3.17})$$

$$\frac{k_{vg}}{v_0} = \frac{4.6 \times 10^{-7} \text{ m}^3/\text{kg}\cdot\text{s}}{0.0164 \text{ m}^3/\text{s}} = 2.8 \times 10^{-5} \text{ kg}^{-1}$$

$$\beta = \frac{\alpha}{A_c P_0 \rho (1 - \epsilon_b)} = \frac{0.246 \text{ atm/m}}{(0.785 \text{ m}^2)(20 \text{ atm})(1500 \text{ kg/m}^3)(0.6)}$$

$$= \frac{1.75 \times 10^{-5}}{\text{kg}}$$

Substitution yields

$$(1 - 0.5) \ln \frac{1}{1 - 0.55} - (-0.5)(0.55)$$

$$= 2.8 \times 10^{-5} \left(W - \frac{1.75 \times 10^{-5}}{2} W^2 \right)$$

Solving for W , we obtain

$$W = 34,500 \text{ kg}$$

The reactor volume corresponding to this catalyst weight is

$$V = \frac{W}{\rho_p (1 - \epsilon_b)} = 38.33 \text{ m}^3$$

The total height of the reactor

$$H = \frac{V}{A_c} = 48.83 \text{ m}$$

Four 1-m-diameter towers each 12.2 m in height connected in series will be sufficient.

Checking assumption of constant C_B

$$\text{moles of H}_2 \text{ converted} = F_{A0} X = (0.5)(10 \text{ mol/s})(0.55)$$

$$= 2.75 \text{ mol/s}$$

$$\text{moles of organic leaving} = 5 \frac{\text{kg}}{\text{m}^2 \cdot \text{s}} \times (0.785 \text{ m}^2) \left(\frac{1000 \text{ mol}}{168 \text{ kg}} \right)$$

$$- 2.75 \text{ mol/s}$$

$$= 23.36 - 2.75 = 20.6 \text{ mol/s}$$

Consequently, our assumption that the concentration of organic is essentially constant was valid.

The material presented in this example is meant to serve as an introduction to trickle bed reactors. Other worked-out trickle bed example problems can be found in the article by Ramachandran and Chaudhari.[†] In addition, the hydrodesulfurization of a hydrocarbon in a trickle bed reactor is given in detail by Tarhan.[‡]

[†]Ramachandran and Chaudhari, op. cit.

[‡]M. O. Tarhan, *Catalytic Reactor Design* (New York: McGraw-Hill, 1983), p.189.