

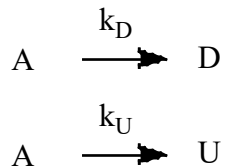
# Lecture 12

**Chemical Reaction Engineering (CRE)** is the field that studies the rates and mechanisms of chemical reactions and the design of the reactors in which they take place.

# Lecture 12 – Tuesday

- Multiple Reactions

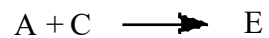
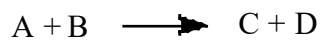
- Selectivity and Yield



- Series Reactions



- Complex Reactions



# 4 Types of Multiple Reactions

- Series:  $A \rightarrow B \rightarrow C$
- Parallel:  $A \rightarrow D$   
 $A \rightarrow U$
- Independent:  $A \rightarrow B$   
 $C \rightarrow D$
- Complex:  $A + B \rightarrow C + D$   
 $A + C \rightarrow E$

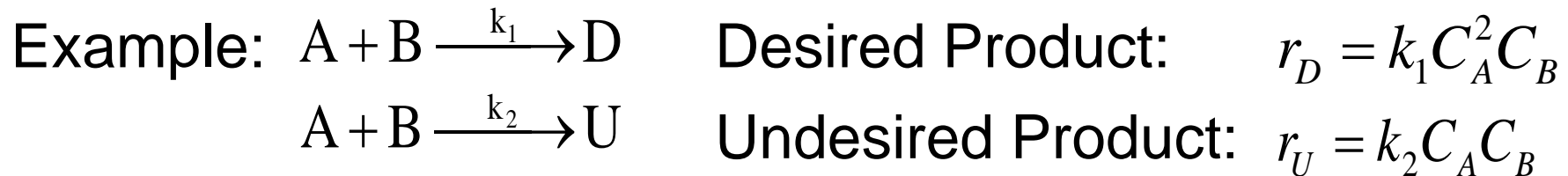
With multiple reactors, either molar flow or number of moles must be used (no conversion!)

# Selectivity and Yield

There are two types of selectivity and yield:  
Instantaneous and Overall.

	Instantaneous	Overall
Selectivity	$S_{DU} = \frac{r_D}{r_U}$	$\tilde{S}_{DU} = \frac{F_D}{F_U}$
Yield	$Y_D = \frac{r_D}{-r_A}$	$\tilde{Y}_D = \frac{F_D}{F_{A0} - F_A}$

# Selectivity and Yield



$$S_{D/U} = \frac{r_D}{r_U} = \frac{k_1 C_A^2 C_B}{k_2 C_A C_B} = \frac{k_1}{k_2} C_A$$

To maximize the selectivity of D with respect to U run at high concentration of A and use **PFR**.

# Gas Phase Multiple Reactions



Following the Algorithm

**Number all reactions**

**Mole balances:**

Mole balance on each and every species

PFR 
$$\frac{dF_j}{dV} = r_j$$

CSTR 
$$F_{j0} - F_j = -r_j V$$

Batch 
$$\frac{dN_j}{dt} = r_j V$$

Membrane ("i" diffuses in) 
$$\frac{dF_i}{dV} = r_i + R_i$$

Liquid-semibatch 
$$\frac{dC_j}{dt} = r_j + \frac{v_0(C_{j0} - C_j)}{V}$$

**Rates:**

Laws 
$$r_{ij} = k_{ij} f_i(C_j, C_n)$$

Relative rates 
$$\frac{r_{iA}}{-a_i} = \frac{r_{iB}}{-b_i} = \frac{r_{iC}}{c_i} = \frac{r_{iD}}{d_i}$$

Net rates 
$$r_j = \sum_{i=1}^q r_{ij}$$

**Stoichiometry:**

*Gas phase*

$$C_j = C_{T0} \frac{F_j P T_0}{F_T P_0 T} = C_{T0} \frac{F_j T_0}{F_T T} y$$

$$y = \frac{P}{P_0}$$

$$F_T = \sum_{j=1}^n F_j$$

$$\frac{dy}{dW} = -\frac{\alpha}{2y} \left( \frac{F_T}{F_{T0}} \right) \frac{T}{T_0}$$

*Liquid phase*

$$v = v_0$$

$$C_A, C_B, \dots$$

**Combine:**

Polymath will combine all the equations for you. Thank you, !

# Multiple Reactions

**A) Mole Balance** of each and every species

Flow

$$\frac{dF_A}{dV} = r_A$$

$$\frac{dF_B}{dV} = r_B$$

Batch

$$\frac{dN_A}{dt} = r_A V$$

$$\frac{dN_B}{dt} = r_B V$$

# Multiple Reactions

## B) Rates

a) **Rate Law** for each reaction:

$$\begin{aligned} -r_{1A} &= k_{1A} C_A C_B \\ -r_{2A} &= k_{2A} C_C C_A \end{aligned}$$

b) Net **Rates**:

$$r_A = \sum_{i=1} r_{iA} = r_{1A} + r_{2A}$$

c) Relative **Rates**:

$$\frac{r_{iA}}{-a_i} = \frac{r_{iB}}{-b_i} = \frac{r_{iC}}{c_i} = \frac{r_{iD}}{d_i}$$

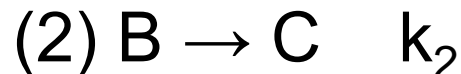
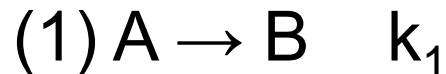


# Multiple Reactions

## C) Stoichiometry

**Gas:** 
$$C_A = C_{T0} \frac{F_A}{F_{A0}} \left( \frac{P}{P_0} \right) \left( \frac{T_0}{T} \right)$$

**Liquid:** 
$$C_A = F_A / v_0$$



# Batch Series Reactions

## 1) Mole Balances

$$\frac{dN_A}{dt} = r_A V$$

$$\frac{dN_B}{dt} = r_B V$$

$$\frac{dN_C}{dt} = r_C V$$

$$V = V_0 \text{ (constant batch)}$$

$$\frac{dC_A}{dt} = r_A \quad \frac{dC_B}{dt} = r_B \quad \frac{dC_C}{dt} = r_C$$

# Batch Series Reactions

## 2) Rate Laws

$$-r_{1A} = k_{1A} C_A$$

$$-r_{1B} = k_{1B} C_B$$

Laws

$$r_A = r_{1A}$$

$$r_B = r_{1B} + r_{2B}$$

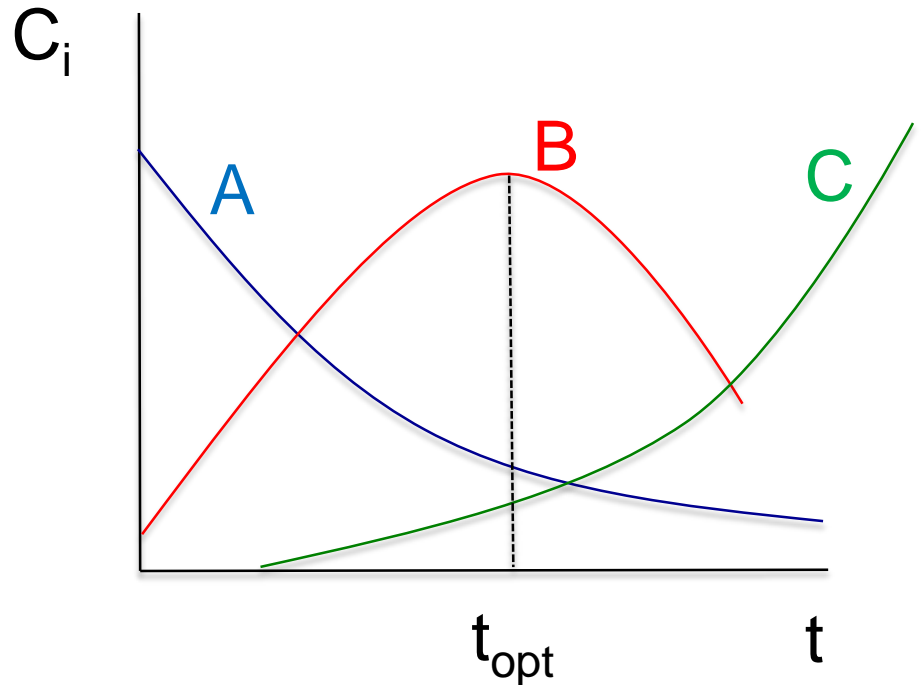
Net rates

$$\frac{r_{1A}}{-1} = \frac{r_{1B}}{1}$$

$$\frac{r_{2B}}{-1} = \frac{r_{2C}}{1}$$

Relative rates

# Example: Batch Series Reactions



## 1) Mole Balances $V = V_o$

$$\frac{dC_A}{dt} = r_A$$

$$\frac{dC_B}{dt} = r_B$$

$$\frac{dC_C}{dt} = r_C$$

# Example: Batch Series Reactions

## 2) Rate Laws

Laws:  $r_{1A} = -k_1 C_A$

$$r_{2B} = -k_2 C_B$$

Relative:  $\frac{r_{1A}}{-1} = \frac{r_{1B}}{1}$        $\frac{r_{2B}}{-1} = \frac{r_{2C}}{1}$

# Example: Batch Series Reactions

## 3) Combine

Species A: 
$$-\frac{dC_A}{dt} = -r_A = k_1 C_A$$

$$C_A = C_{A0} \exp(-k_1 t)$$

Species B: 
$$\frac{dC_B}{dt} = r_B$$

$$r_B = r_{B \text{ NET}} = r_{1B} + r_{2B} = k_1 C_A - k_2 C_B$$

$$\frac{dC_B}{dt} + k_2 C_B = k_1 C_{A0} \exp(-k_1 t)$$

# Example: Batch Series Reactions

Using the integrating factor,  $I.F. = \exp\left(\int k_2 dt\right) = \exp(k_2 t)$

$$d \frac{[C_B \exp(k_2 t)]}{dt} = k_1 C_{A0} \exp(k_2 - k_1)t$$

at  $t = 0$ ,  $C_B = 0$

$$C_B = \frac{k_1 C_{A0}}{k_2 - k_1} \left[ \exp(-k_1 t) - \exp(-k_2 t) \right]$$

$$C_C = C_{A0} - C_A - C_B$$

$$C_C = \frac{C_{A0}}{k_2 - k_1} \left[ k_2 (1 - e^{-k_1 t}) - k_1 (1 - e^{-k_2 t}) \right]$$

# Example: **CSTR** Series Reactions



What is the optimal  $\tau$ ?

## 1) **Mole Balances**

$$\mathbf{A:} \quad F_{A0} - F_A + r_A V = 0$$

$$C_{A0} v_0 - C_A v_0 + r_A V = 0$$

$$C_{A0} - C_A + r_A \tau = 0$$

$$\mathbf{B:} \quad 0 - v_0 C_B + r_B V = 0$$

$$-C_B + r_B \tau = 0$$



# Example: CSTR Series Reactions



## 2) Rate Laws

Laws:  $r_{1A} = -k_1 C_A$

$$r_{2B} = -k_2 C_B$$

Relative:  $\frac{r_{1A}}{-1} = \frac{r_{1B}}{1} \quad \frac{r_{2B}}{-1} = \frac{r_{2C}}{1}$

Net:  $r_A = r_{1A} + 0 = -k_1 C_A$

$$r_B = -r_{1A} + r_{2B} = k_1 C_A - k_2 C_B$$

# Example: CSTR Series Reactions



**3) Combine**

$$C_{A0} - C_A - k_1 C_A t = 0$$

$$C_A = \frac{C_{A0}}{1 + k_1 t}$$

$$-C_B + (k_1 C_A - k_2 C_B) t = 0$$

$$C_B = \frac{k_1 C_A t}{1 + k_2 t}$$

$$C_B = \frac{k_1 C_{A0} t}{(1 + k_2 t)(1 + k_1 t)}$$

# Example: CSTR Series Reactions



Find  $\tau$  that gives maximum concentration of B

$$C_B = \frac{k_1 C_{A0} \tau}{(1 + k_2 \tau)(1 + k_1 \tau)}$$

$$\frac{dC_B}{d\tau} = 0$$

$$\tau_{\max} = \frac{1}{\sqrt{k_1 k_2}}$$



## Following the Algorithm

### Number all reactions

#### Mole balances:

Mole balance on each and every species

$$\text{PFR} \quad \frac{dF_j}{dV} = r_j$$

$$\text{CSTR} \quad F_{j0} - F_j = -r_j V$$

$$\text{Batch} \quad \frac{dN_j}{dt} = r_j V$$

$$\text{Membrane ("i" diffuses in)} \quad \frac{dF_i}{dV} = r_i + R_i$$

$$\text{Liquid-semibatch} \quad \frac{dC_j}{dt} = r_j + \frac{v_0(C_{j0} - C_j)}{V}$$

#### Rates:

$$\text{Laws} \quad r_{ij} = k_{ij} f_i(C_j, C_n)$$

$$\text{Relative rates} \quad \frac{r_{iA}}{-a_i} = \frac{r_{iB}}{-b_i} = \frac{r_{iC}}{c_i} = \frac{r_{iD}}{d_i}$$

$$\text{Net rates} \quad r_j = \sum_{i=1}^q r_{ij}$$

#### Stoichiometry:

*Gas phase*

$$C_j = C_{T0} \frac{F_j P T_0}{F_T P_0 T} = C_{T0} \frac{F_j T_0}{F_T T} y$$

$$y = \frac{P}{P_0}$$

$$F_T = \sum_{j=1}^n F_j$$

$$\frac{dy}{dW} = -\frac{\alpha}{2y} \left( \frac{F_T}{F_{T0}} \right) \frac{T}{T_0}$$

*Liquid phase*

$$v = v_0$$

$$C_A, C_B, \dots$$

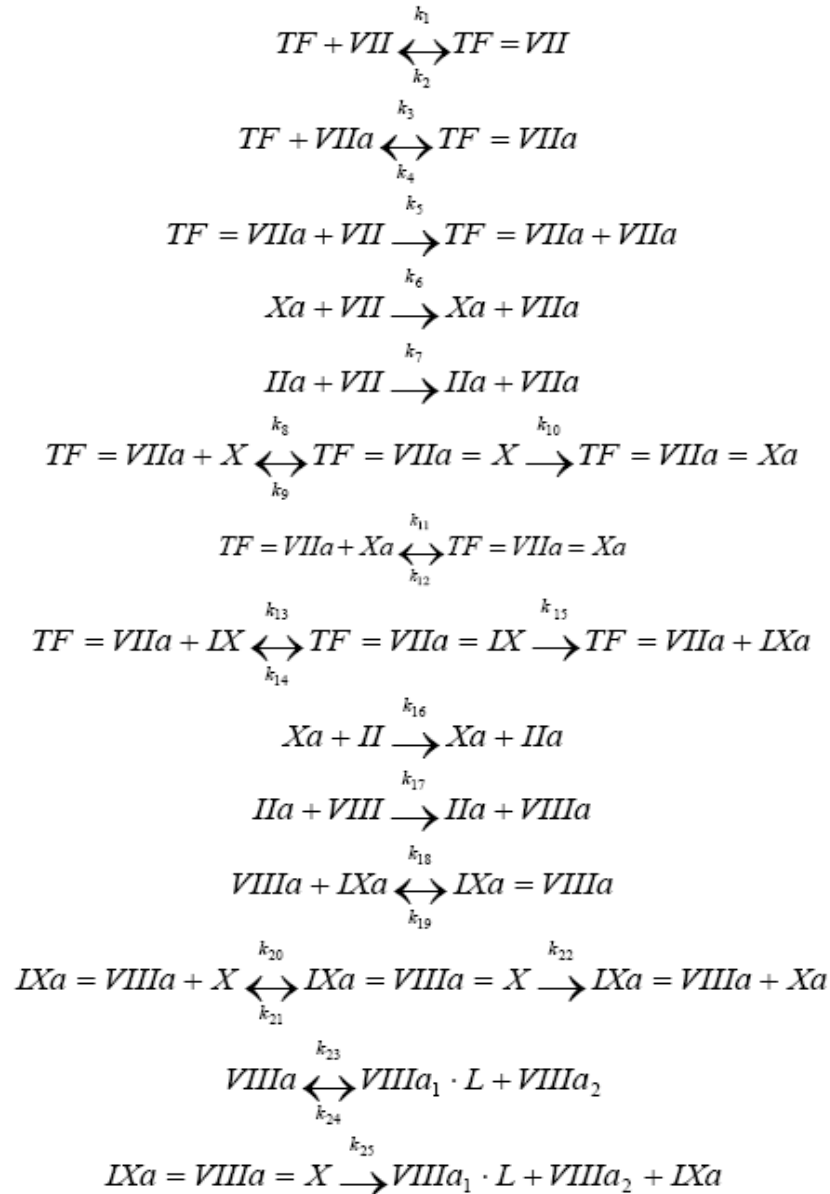
#### Combine:

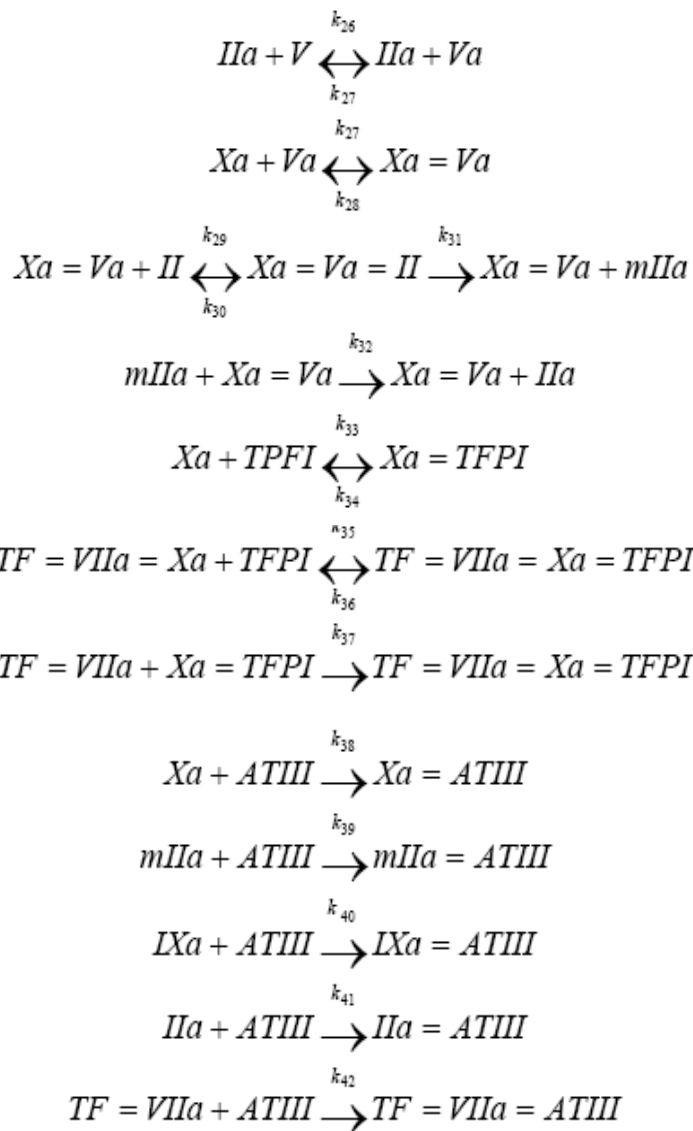
Polymath will combine all the equations for you. Thank you, .

End of Lecture 12

# Supplementary Slides

# Blood Coagulation





Courtesy of Hockin, M.F., Jones, K.C., Everse, S.J. and Mann, K.G. (2002). A model for the stoichiometric regulation of blood coagulation. *The Journal of Biological Chemistry* 277 (21), 18322-18333.



# Notations

Species symbol	Nomenclature
TF	Tissue factor
VII	proconvertin
TF=VIIa	factor TF=VIIa
VIIa	factor novoseven
TF=VIIa	factor TF=VIIa complex
Xa	Stuart prower factor activated
IIa	thrombin
X	Stuart Prower factor
TF=VIIa=X	TF=VIIa=X complex
TF=VIIa=X	TF=VIIa=X complex
IX	Plasma Thromboplastin Component
TF=VIIa=IX	TF=VIIa=IX complex
IXa	factor IXa
II	prothrombin
VIII	antihemophilic factor
VIIIa	antihemophilic factor activated
IXa=VIIIa	IXa=VIIIa complex
IXa=VIIIa=X	IXa=VIIIa=X complex

# Notations

VIIIa <sub>1</sub> L	factor VIIIa <sub>1</sub> L
VIIIa <sub>2</sub>	factor VIIIa <sub>2</sub>
V	proaccelerin
Va	factor Va
Xa=Va	Xa=Va complex
Xa=Va=II	Xa=Va=II complex
mIIa	meizothrombin
TFPI	tissue factor pathway inhibitor
Xa=TFPI	Xa=TFPI complex
TF=VIIa=Xa=TFPI	TF=VIIa=Xa=TFPI complex
ATIII	antithrombin
Xa=ATIII	Xa=ATIII complex
mIIa=ATIII	mIIa=ATIII complex
IXa=ATIII	IXa=ATIII complex
TF=VIIIa=ATIII	TF=VIIIa=ATIII complex
IIa=ATIII	IIa=ATIII complex

# Mole Balances

$$\frac{dC_{TF}}{dT} = k_2 \cdot C_{TFVII} - k_1 \cdot C_{TF} \cdot C_{VII} - k_3 \cdot C_{TF} \cdot C_{VIIa} + k_4 \cdot C_{TFVIIa}$$

$$\frac{dC_{VII}}{dt} = k_2 \cdot C_{TFVII} - k_1 \cdot C_{TF} \cdot C_{VII} - k_6 \cdot C_{Xa} \cdot C_{VII} - k_7 \cdot C_{IIa} \cdot C_{VII} - k_5 \cdot C_{TFVIIa} \cdot C_{VII}$$

$$\frac{dC_{TFVII}}{dt} = -k_2 \cdot C_{TFVII} + k_1 \cdot C_{TF} \cdot C_{VII}$$

$$\frac{dC_{VIIa}}{dt} = k_4 \cdot C_{TFVIIa} - k_3 \cdot C_{TF} \cdot C_{VIIa} + k_5 \cdot C_{TFVIIa} \cdot C_{VII} + k_6 \cdot C_{Xa} \cdot C_{VII} + k_7 \cdot C_{IIa} \cdot C_{VII}$$

$$\frac{dC_{TFVIIa}}{dt} = -k_4 \cdot C_{TFVIIa} + k_3 \cdot C_{TF} \cdot C_{VIIa} + k_9 \cdot C_{TFVIIaX} - k_8 \cdot C_{TFVIIa} \cdot C_X - k_{11} \cdot C_{TFVIIa} \cdot C_{Xa} +$$

$$k_{12} \cdot C_{TFVIIaXa} - k_{13} \cdot C_{TFVIIa} \cdot C_{IX} + k_{14} \cdot C_{TFVIIaIX} + k_{15} \cdot C_{TFVIIaIX} - k_{37} \cdot C_{TFVIIa} \cdot C_{XaTFPI} -$$

$$k_{42} \cdot C_{TFVIIa} \cdot C_{ATIII}$$

$$\frac{dC_{Xa}}{dt} = k_{11} \cdot C_{TFVIIa} \cdot C_{Xa} + k_{12} \cdot C_{TFVIIaXa} + k_{22} \cdot C_{IXaVIIIaX} + k_{28} \cdot C_{XaVa} - k_{27} \cdot C_{Xa} \cdot C_{Va} +$$

$$k_{34} \cdot C_{XaTFPI} - k_{33} \cdot C_{Xa} \cdot C_{TFPI} - k_{38} \cdot C_{Xa} \cdot C_{ATIII}$$

$$\frac{dC_{IIa}}{dt} = k_{16} \cdot C_{Xa} \cdot C_{II} + k_{32} \cdot C_{mIIa} \cdot C_{XaVa} - k_{41} \cdot C_{IIa} \cdot C_{ATIII}$$

$$\frac{dC_X}{dt} = -k_8 \cdot C_{TFVIIa} \cdot C_X + k_9 \cdot C_{TFVIIaX} - k_{20} \cdot C_{IXaVIIIa} \cdot C_X + k_{21} \cdot C_{IXaVIIIaX} + k_{25} \cdot C_{IXaVIIIaX}$$

$$\frac{dC_{TFVIIaX}}{dt} = k_8 \cdot C_{TFVIIa} \cdot C_X - k_9 \cdot C_{TFVIIaX} - k_{10} \cdot C_{TFVIIaX}$$

# Mole Balances

$$\frac{dC_{TFVNaXa}}{dt} = k_{10} \cdot C_{TFVNaX} + k_{11} \cdot C_{TFVNa} \cdot C_{Xa} - k_{12} \cdot C_{TFVNaXa} + k_{36} \cdot C_{TFVNaXaTFPI} - k_{35} \cdot C_{TFVNaXa} \cdot C_{TFPI}$$

$$\frac{dC_{IX}}{dt} = k_{14} \cdot C_{TFVNaIX} - k_{13} \cdot C_{TFVNa} \cdot C_{IX}$$

$$\frac{dC_{TFVNaIX}}{dt} = -k_{14} \cdot C_{TFVNaIX} + k_{13} \cdot C_{TFVNa} \cdot C_{IX} - k_{15} \cdot C_{TFVNaIX}$$

$$\frac{dC_{IXa}}{dt} = k_{15} \cdot C_{TFVNaIX} - k_{18} \cdot C_{VIIIa} \cdot C_{IXa} + k_{19} \cdot C_{IXaVIIIa} + k_{25} \cdot C_{IXaVIIIaX} - k_{40} \cdot C_{IXa} \cdot C_{ATIII}$$

$$\frac{dC_{II}}{dt} = -k_{16} \cdot C_{Xa} \cdot C_{II} + k_{30} \cdot C_{XaVIIaI} - k_{29} \cdot C_{XaVIIa} \cdot C_{II}$$

$$\frac{dC_{VIII}}{dt} = -k_{17} \cdot C_{IIa} \cdot C_{VIII}$$

$$\frac{dC_{VIIIa}}{dt} = k_{17} \cdot C_{IIa} \cdot C_{VIII} - k_{18} \cdot C_{VIIIa} \cdot C_{IXa} + k_{19} \cdot C_{IXaVIIIa} - k_{23} \cdot C_{VIIIa} + k_{24} \cdot C_{VIIIa,I}$$

$$\frac{dC_{IXaVIIIa}}{dt} = k_{18} \cdot C_{VIIIa} \cdot C_{IXa} - k_{19} \cdot C_{IXaVIIIa} + k_{21} \cdot C_{IXaVIIIaX} - k_{20} \cdot C_{IXaVIIIa} \cdot C_X + k_{22} \cdot C_{IXaVIIIaX}$$

$$\frac{dC_{IXaVIIIaX}}{dt} = -k_{21} \cdot C_{IXaVIIIaX} + k_{20} \cdot C_{IXaVIIIa} \cdot C_X - k_{22} \cdot C_{IXaVIIIaX} - k_{25} \cdot C_{IXaVIIIaX}$$

$$\frac{dC_{VIIIa,I}}{dt} = k_{23} \cdot C_{VIIIa} - k_{24} \cdot C_{VIIIa,I} \cdot C_{VIIIa} + k_{25} \cdot C_{IXaVIIIaX}$$

$$\frac{dC_{VIIIa_2}}{dt} = k_{23} \cdot C_{VIIIa} - k_{24} \cdot C_{VIIIa,I} \cdot C_{VIIIa_2} + k_{25} \cdot C_{IXaVIIIaX}$$

# Mole Balances

$$\frac{dC_V}{dt} = -k_{26} \cdot C_{IIa} \cdot C_V$$

$$\frac{dC_{Va}}{dt} = k_{26} \cdot C_{IIa} \cdot C_V + k_{28} \cdot C_{Xa} \cdot C_{Va} - k_{27} \cdot C_{Xa} \cdot C_{Va}$$

$$\frac{dC_{XaVa}}{dt} = -k_{28} \cdot C_{Xa} \cdot C_{Va} + k_{27} \cdot C_{Xa} \cdot C_{Va} - k_{29} \cdot C_{IIaVa} \cdot C_{II} + k_{30} \cdot C_{XaValI} + k_{31} \cdot C_{XaValII}$$

$$\frac{dC_{XaValI}}{dt} = k_{29} \cdot C_{IIaVa} \cdot C_{II} - k_{30} \cdot C_{XaValI} - k_{31} \cdot C_{XaValI}$$

$$\frac{dC_{mIIa}}{dt} = k_{31} \cdot C_{XaValI} - k_{32} \cdot C_{mIIa} \cdot C_{XaVa} - k_{39} \cdot C_{mIIa} \cdot C_{ATIII}$$

$$\frac{dC_{TFPI}}{dt} = k_{34} \cdot C_{XaTFPI} - k_{33} \cdot C_{Xa} \cdot C_{TFPI} + k_{36} \cdot C_{TFVIIaXaTFPI} - k_{35} \cdot C_{TFVIIaXa} \cdot C_{TFPI}$$

$$\frac{dC_{XaTFPI}}{dt} = -k_{34} \cdot C_{XaTFPI} + k_{37} \cdot C_{Xa} \cdot C_{TFPI} - k_{37} \cdot C_{TFVIIa} \cdot C_{XaTFPI}$$

$$\frac{dC_{TFVIIaXaTFPI}}{dt} = -k_{36} \cdot C_{TFVIIaXaTFPI} + k_{35} \cdot C_{TFVIIaXa} \cdot C_{TFPI} + k_{37} \cdot C_{TFVIIa} \cdot C_{XaTFPI}$$

$$\frac{dC_{ATIII}}{dt} = -k_{38} \cdot C_{Xa} \cdot C_{ATIII} - k_{39} \cdot C_{mIIa} \cdot C_{ATIII} - k_{40} \cdot C_{IXa} \cdot C_{ATIII} - k_{41} \cdot C_{IIa} \cdot C_{ATIII} - k_{42} \cdot C_{TFVIIa} \cdot C_{ATIII}$$

$$\frac{dC_{XaATIII}}{dt} = k_{38} \cdot C_{Xa} \cdot C_{ATIII}$$

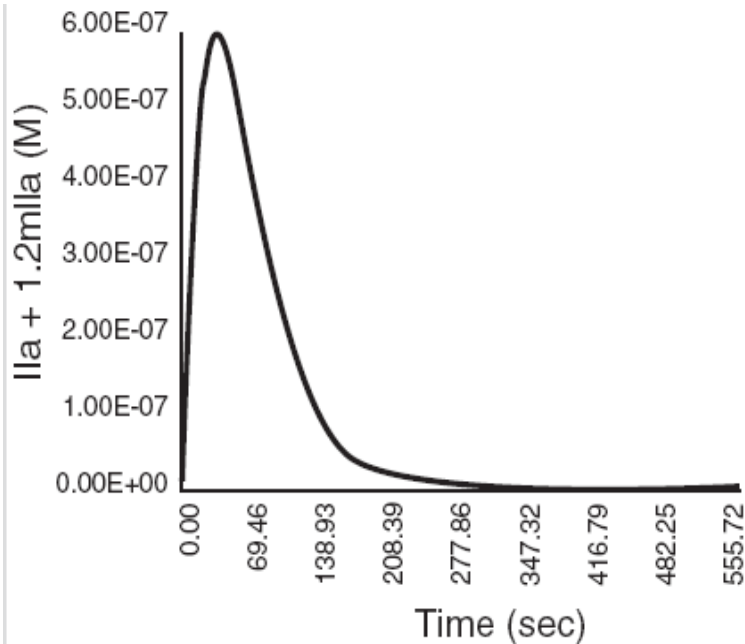
$$\frac{dC_{mIIaATIII}}{dt} = k_{39} \cdot C_{mIIa} \cdot C_{ATIII}$$

$$\frac{dC_{IXaATIII}}{dt} = k_{40} \cdot C_{IXa} \cdot C_{ATIII}$$

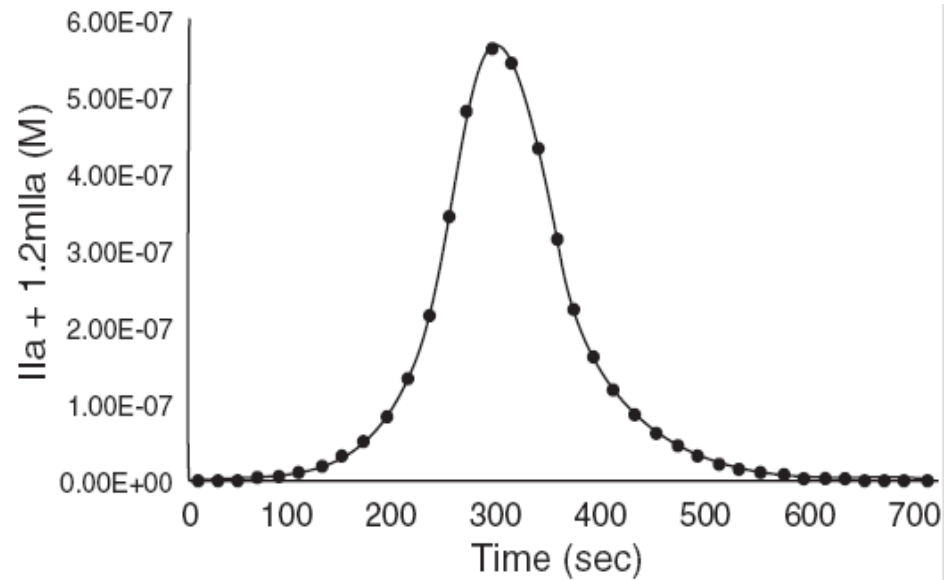
$$\frac{dC_{TFVIIaATIII}}{dt} = k_{42} \cdot C_{TFVIIa} \cdot C_{ATIII}$$

$$\frac{dC_{IIaATIII}}{dt} = k_{41} \cdot C_{IIa} \cdot C_{ATIII}$$

# Results



**Figure D.** Total thrombin as a function of time with an initiating TF concentration of 25 pM (after running Polymath) for the abbreviated blood clotting cascade.

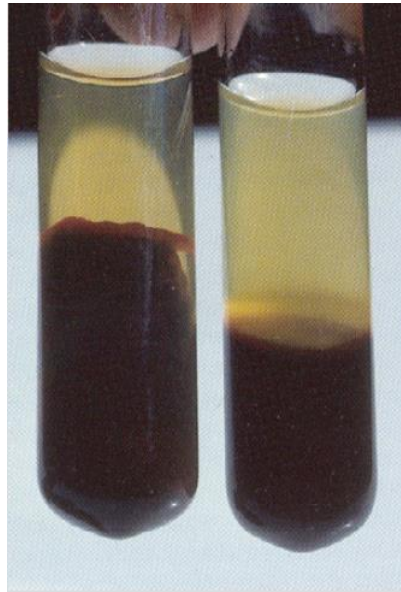


**Figure E.** Total thrombin as a function of time with an initiating TF concentration of 25 pM. [Figure courtesy of M. F. Hockin et al., “A Model for the Stoichiometric Regulation of Blood Coagulation,” *The Journal of Biological Chemistry*, 277[21], pp. 18322–18333 (2002)]. Full blood clotting cascade.

# Blood Coagulation

Many metabolic reactions involve a large number of sequential reactions, such as those that occur in the coagulation of blood.

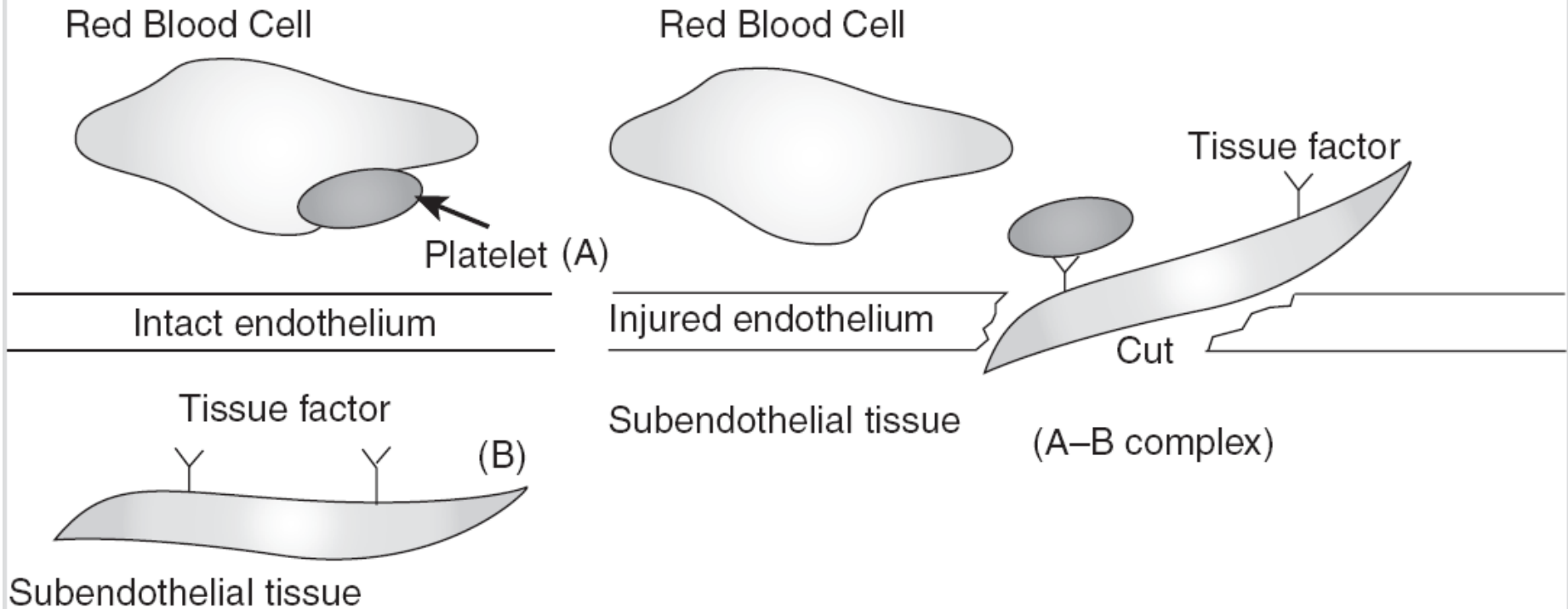
Cut → Blood → Clotting



**Figure A. Normal Clot Coagulation of blood**

(picture courtesy of: Mebs, *Venomous and Poisonous Animals*, Medpharm, Stuttgart 2002, Page 305)

# Schematic of Blood Coagulation



**Figure B.** Schematic of separation of TF (A) and plasma (B) before cut occurs.

**Figure C.** Cut allows contact of plasma to initiate coagulation. (A + B → Cascade)



**Cut**



**A + B**



**C**



**D**



**E**



**F**



**Clot**