# *CD-ROM Appendix D: Using Semilog Plots in Rate Data Analysis*

Semilog graph paper is used when dealing with either exponential growth or decay, such as

$$y = be^{mx} \tag{CDD-1}$$

For the first-order elementary reaction

$$A \longrightarrow \text{Products}$$

which is carried out at constant volume, the rate of the disappearance of A is given by

$$\frac{-dC_A}{dt} = kC_A \tag{CDD-2}$$

When $t = 0$, $C_A = C_{A0}$, where the units of $C_A$ are g mol/dm$^3$; $t$ is expressed in minutes; and $k$ is expressed in reciprocal minutes. Integrating the rate equation, we obtain

$$\ln \frac{C_A}{C_{A0}} = -kt \tag{CDD-3}$$

We wish to determine the specific reaction rate constant, $k$. A plot of ln $C_A$ versus $t$ should produce a straight line whose slope is $-k$. We may eliminate the calculation of the log of each concentration data point by plotting our data on semilog graph paper. The points in Table CDD-1 are plotted on the semilog graph shown in Figure CDD-1.

TABLE CDD-1

| $t$ (min) | 0 | 2 | 4 | 8 | 14 |
|---|---|---|---|---|---|
| $C_A$ (gmol/dm$^3$) | 2.0 | 1.64 | 1.38 | 0.95 | 0.60 |

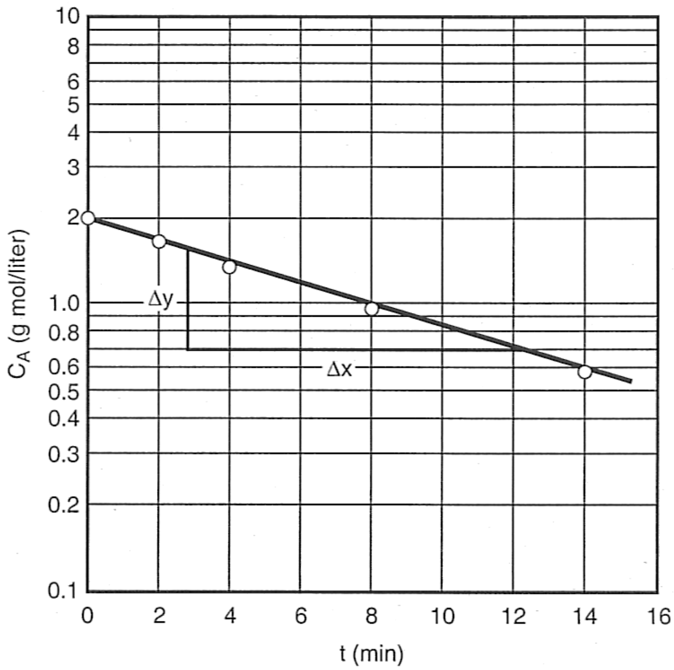


**Figure** CDD-1

***Algebraic Method.*** Draw the best straight line through your data points. Choose two points on this line, $t_1$ and $t_2$, and the corresponding concentrations $C_{A1}$ and $C_{A2}$ at these times:

$$\ln\frac{C_{A1}}{C_{A0}} = -kt_1 \qquad \ln\frac{C_{A2}}{C_{A0}} = -kt_2$$

$$\ln C_{A2} - \ln C_{A1} = -k(t_2 - t_1) \tag{CDD-4}$$

Rearranging yields

$$k = -\frac{\ln C_{A2} - \ln C_{A1}}{t_2 - t_1} = \frac{\ln(C_{A2}/\ln C_{A1})}{t_2 - t_1} \tag{CDD-5}$$

When $t$ = 8, $C_A$ = 1.05; when $t$ = 12, $C_A$ = 0.75. Substituting into Equation (CDD-5) gives us

$$k = \frac{\ln(1.05/0.75)}{12 - 8\text{ min}} = \frac{0.336}{4\text{ min}}$$
$$= (0.084\text{ min})^{-1}$$

***Graphical Technique*** In the preceding example, we had

$$\ln \frac{C_{A1}}{C_{A0}} = -kt_1 \tag{CDD-6}$$

Dividing by 2.3, we convert to log base 10:

$$\frac{\ln(C_A/C_{A0})}{2.3} = \log(C_A/C_{A0}) = \frac{-kt}{2.3}$$

The slope of a plot of log $C_A$ versus time should be a straight line with slope $-k/2.3$. Referring to Figure CDD-1, we draw a right triangle with the acute angles located at points $C_A = 1.6$, $t = 2.8$ and $C_A = 0.7$, $t = 12.8$. Next, the distances $x$ and $y$ are measured with a ruler. These measured lengths in $y$ and $x$ are 1.35 and 4.65 cm, respectively:

$$\Delta y = -1.35 \text{ cm} \times \frac{1 \text{ cycle}}{3.9 \text{ cm}} \ -0.35 \text{ cycle}$$

$$\Delta x = 4.65 \text{ cm} \times \frac{14 \text{ min}}{6.7 \text{ cm}} \ 9.7 \text{ min}$$

$$\text{slope} = \frac{-0.35}{9.7} \ -0.0361$$

$$k = -2.3 \text{ (slope)} = -2.3(-0.0361) \text{ min}^{-1}$$
$$= 0.083 \text{ min}^{-1}$$

A modification of the algebraic method is possible by drawing a line on semilog paper so that the dependent variable changes by a factor of 10. From Equation (CDD-5) in the form

$$k = \frac{\ln(C_{A1}/C_{A2})}{t_2 - t_1} = \frac{2.3 \log (C_{A1}/C_{A2})}{t_2 - t_1} \tag{CDD-7}$$

choose the points ($C_{A1}$, $t_1$) and ($C_{A2}$, $t_2$) so that $C_{A2} = 0.1C_{A1}$:

$$k = \frac{2.3 \log 10}{t_2 - t_1} = \frac{2.3}{t_2 - t_1} \tag{CDD-8}$$

This modification is referred to as the *decade method*.