CALCULATING AND PRESENTING SPATIO-TEMPORAL PATTERNS OF DIFFUSION AND INTERDEPENDENCE

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ABSTRACT: Spatial and spatio-temporal lag models have become popular statistical tools in empirical studies of policy diffusion and strategic policy interdependence. While this is undoubtedly a positive development, too often researchers have ignored what is arguably the most important and substantively interesting information contained in their results: the spatial and spatio-temporal patterns implied by their estimates. In empirical models containing "spatial dynamics", as in those having only temporal dynamics, the coefficients on explanatory variables give only the pre-dynamic impetuses to the outcome variable. In this paper, we discuss how to calculate and present the spatial and spatio-temporal multipliers, which allow one to express the spatial and spatio-temporal responses across units (spatial) and across spatial units and over time to counterfactual shocks, and how to use the delta method to compute standard errors for these effects. We illustrate our recommendations using a recent study of Active Labor Market policymaking in Europe.

I. INTRODUCTION:

A large body of quantitative empirical work in the social sciences is focused on diffusion and other important kinds of spatial interdependence among political units. The diffusion literature in political science and sociology has blossomed in recent years with studies on the spread of exchange rate regimes, capital account liberalization, structural reform, tax policy, democracy, and human rights protections to name just a few of the topics (e.g., Simmons and Elkins 2004, Swank 2003, Gleditsch and Ward 2003, Wotipka et al. 2003). In economics, there is a related and growing literature on strategic policy interdependence that examines a broad range of policy areas from taxation and budgetary expenditures to environmental regulations (e.g., Case et al. 1993, Brueckner and Saavedra 2001, Fredriksson and Millimet 2002, Redoano 2003 and Allers and Elhorst 2005).[1]

Spatial and spatio-temporal lag models are the statistical tools of choice in these two literatures. While the use of these models is undoubtedly a positive development, too often researchers have ignored what is arguably the most important and substantively interesting information contained in their statistical results: the spatial and spatio-temporal patterns implied by their estimates. In empirical models containing "spatial dynamics", as in those having only temporal dynamics, the coefficients on explanatory variables give only the pre-dynamic impetuses to the outcome variable. In this paper, we discuss how to calculate and present the spatial and spatio-

[1] Diffusion implies the spread of policies and institutions. Strategic interdependence may cause political units to adopt similar policies and institutions, for example, in the case where economic competition leads to "race to the bottom" dynamics, but it can lead polities to change policy in opposite directions as well when externalities lead to free-riding behavior (see Franzese and Hays 2006a). For conceptual treatments of diffusion, see Elkins and Simmons (2005), Simmons et al. (2006), and Braun and Gilardi (2006).

temporal multipliers, which allow one to express the spatial and spatio-temporal effects across units (spatial) and across spatial units and over time of counterfactual shocks, and how to use the delta method to compute standard errors for these effects. We illustrate our recommendations using an analysis in another paper (Franzese and Hays 2006a) where we estimated spatio-temporal-lag models to evaluate empirically the strategic interdependence among European countries in active-labor-market policy, specifically labor-market-training (LMT) expenditures.

The rest of the paper is organized as follows. We begin in the next section by describing the basic spatio-temporal regression model—a model used for analyzing time-series-cross-section (TSCS) data—and how to estimate it using maximum likelihood. We then show, in section three, how to calculate and present substantively interesting spatial and spatio-temporal effects. We conclude in section four.

II. Spatial-Temporal Models and Their Estimation[2]:

One common way to model diffusion processes and spatial interdependence in TSCS data is with *spatio-temporal-lagged-dependent-variable, STLDV* models, or just *spatio-temporal-lag* models, written:

$$\mathbf{y_t} = \phi\mathbf{y_{t-1}} + \rho\mathbf{W}\mathbf{y_t} + \mathbf{X_t}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t} \qquad \textbf{(1)}$$

where $\mathbf{y_t}$ is a *Nx1* vector of observations; $\mathbf{y_{t-1}}$ is a one-period *temporal lag* of the dependent

[2] Much of the methodological literature on purely spatial dependence focuses on cross-sections of data (*T=1*). For a comprehensive treatment of spatial econometrics, see Anselin (1988), and for new developments, see Anselin (2001, 2006).

variable and ϕ is the temporal-autoregressive coefficient; ρ is the spatial-autoregressive coefficient; **W** is an *NxN* spatial-weighting matrix, with zeros along the diagonal and elements w_{ij} reflecting the relative degree of connection from unit *j* to *i*. **Wy** is thus the *spatial lag*; i.e., for each observation on y_{it}, **Wy** gives a weighted sum of the y_{jt}, with weights given by the relative connectivity from *j* to *i*. **X** is an *NxK* matrix of observations on *K* independent variables; **β** is a *Kx1* vector of coefficients on **X**; **ε** is a *Nx1* residual vector. Note also that, as the w_{ij} elements of **W** reflect the relative connectivity from unit *j* to *i*, **W** may not be symmetric. Finally, ρ gives the impact of the outcomes in all the other ($j \neq i$) spatial units, each weighted by w_{ij}, on the outcome in *i*.

Thus, ρ gauges overall interdependence-strength, and the w_{ij} describe the relative magnitudes of the specific interdependences paths between units. Typically, the set of w_{ij} are determined by theoretical and substantive argumentation as to which units will have greatest affect on outcomes in which other units; ρ is the coefficient on these pre-specified spatial-weights, reflecting the general strength of interdependence along these pre-specified **W** paths. The spatial weights matrix **W**, is frequently used to specify diffusion via contiguity (borders), leader-emulation, or cultural-connection mechanisms, for example, is to consider outcomes from some unit or set of units {*j*}, but not the outcomes from other units, to diffuse to the outcome in *i*. This implies the weights are *1* or $1/(n_{\{j\}}-1)$, for sums or averages respectively, for those *ij* where *i* and *j* both belong to some group (e.g., they share a border, or a language, or membership in an institution or any other group) and 0 for all others. Such *co-membership* interdependence either occurs from some *j* to *i* (e.g., if they use a common language) or does not (if they do not), so the weights in this sum/average are equal (*1* or $(n_{\{j\}}-1)^{-1}$) for all co-members and zero for all outside the group.

There are a number of ways to estimate the coefficients in equation **(1)**. Under many circumstances, *OLS* can be used, but the spatial lag **Wy** is endogenous (even when the errors are spherical), and therefore, this estimator suffers from simultaneity bias (Franzese and Hays 2006b).[3] A second strategy instruments for the endogenous spatial lag **Wy** using **X** and **WX**. This *instrumental-variables by two-stage-least-squares* approach, produces consistent and asymptotically efficient estimates, like all properly specified IV estimators do, provided its necessary conditions are met: namely, that the **X** are indeed exogenous but related to **Y**.[4] *Maximum-likelihood* (*ML*) offers a third strategy of estimating the coefficients in **(1)** with a model that specifies the joint likelihood of **y**, fully reflecting the spatial interdependence (Ord 1975).[5] *ML* estimation is more computationally intense than the other estimators and can be difficult to implement in models with more than the simplest forms of spatial dependence, but its parameter estimates will be consistent and asymptotically efficient if the model, including the interdependence pattern, is correctly specified.[6] We use the *ML* estimator in the example provided below and, therefore, describe this particular

[3] Franzese and Hays (2006b) provide an evaluation of several estimators using Monte Carlo simulations. Their main conclusions are that spatial OLS, despite its bias, performs acceptably (in mean squared error terms) under moderate interdependence-strength and reasonable sample-dimensions and is virtually always in all ways preferable to non-spatial OLS. Spatial two-stage-least squares or spatial maximum-likelihood may be recommended for other conditions.

[4] See Bartels (1991) for a treatment of these issues.

[5] This list of estimators is hardly exhaustive. For a more complete one, see Kelejian et al. (2003). See Kelejian and Robinson (1993) for a technical treatment of the spatial two-stage least squares estimator.

[6] More precisely, as in all ML, such estimates would, if the model is correctly specified, be *BANC*: "best asymptotic-normal and consistent", i.e., asymptotically most efficient among estimators that are consistent and asymptotically normally distributed.

estimation procedure in more detail.

Typically, the routine for implementing the *ML* estimator relies upon an approximation to the determinant of (**I**-ρ**W**). To see why, start by writing the spatio-temporal-lag model with the stochastic component on the left:

$$\mathbf{y_t} = \phi \mathbf{y_{t\text{-}1}} + \rho \mathbf{W} \mathbf{y_t} + \mathbf{X_t} \boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t} \Rightarrow \boldsymbol{\varepsilon}_\mathbf{t} = \left(\mathbf{I} - \rho \mathbf{W} \right) \mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X}\boldsymbol{\beta} \equiv \mathbf{A} \mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X_t} \boldsymbol{\beta} \qquad \textbf{(2)}$$

Assuming *i.i.d.* normality, the likelihood function for $\boldsymbol{\varepsilon}_\mathbf{t}$ is then the typical linear one:

$$L(\boldsymbol{\varepsilon}_\mathbf{t}) = \left(\frac{1}{\sigma^2 2\pi} \right)^{\frac{NT}{2}} \exp\left(-\frac{\boldsymbol{\varepsilon}_\mathbf{t}' \boldsymbol{\varepsilon}_\mathbf{t}}{2\sigma^2} \right) \qquad \textbf{(3)},$$

which, in this case, will produce the following likelihood and log-likelihood functions in terms of joint vector of observations $\mathbf{y_t}$:

$$L(\mathbf{y_t}) = |\mathbf{A}| \left(\frac{1}{\sigma^2 2\pi} \right)^{\frac{NT}{2}} \exp\left(-\frac{1}{2\sigma^2} \left(\mathbf{A}\mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X}\boldsymbol{\beta} \right)' \left(\mathbf{A}\mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X}\boldsymbol{\beta} \right) \right) \qquad \textbf{(4a)}$$

$$\ln L(\mathbf{y_t}) = \ln |\mathbf{A}| - \left(\frac{NT}{2} \right) \ln \pi - \left(\frac{NT}{2} \right) \ln \sigma^2 - \left(\frac{1}{2\sigma^2} \left(\mathbf{A}\mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X}\boldsymbol{\beta} \right)' \left(\mathbf{A}\mathbf{y_t} - \phi \mathbf{y_{t\text{-}1}} - \mathbf{X}\boldsymbol{\beta} \right) \right) \qquad \textbf{(4b)}$$

These likelihoods still resemble the typical linear-normal likelihood, except that the transformation from $\boldsymbol{\varepsilon}_\mathbf{t}$ to $\mathbf{y_t}$, is not by the usual factor, 1, but by |**A**|=|**I**- ρ**W**|. Since |**A**| depends on *ρ*, each time the maximum-likelihood routine recalculates the likelihood with updated estimates of ρ, it would have to recalculate the determinant for these new ρ-values. Ord's (1975) solution to this computational-intensity issue was to use the approximation $\mathbf{\Pi}_i\lambda_i$ for $|\mathbf{W}|$ because the eigenvector $\boldsymbol{\lambda}$ in this approximation does not depend on ρ. Then $|\mathbf{I}- \rho\mathbf{W}| = \mathbf{\Pi}_i(1-\lambda_i)$, which requires the estimation routine only to recalculate a product, not a determinant, as it updates. The estimated variance-covariances of parameter estimates follow the usual ML formula (negative the inverse of Hessian of the likelihood) and so are also functions of $|\mathbf{A}|$. The same approximation serves there.[7]

We use a two-stage procedure outlined in Anselin to maximize the log-likelihood function (1988, 181-182). [8] Anselin shows that solving the relevant first order conditions leads to a concentrated likelihood in the spatial parameter. In particular, the solution to $\frac{d}{d\boldsymbol{\beta}}[\ln L(y)] = 0$ gives

$$\begin{aligned}\hat{\boldsymbol{\beta}} &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{A}\mathbf{y} \\ &= (\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{y} - \rho(\mathbf{X}'\mathbf{X})^{-1}\mathbf{X}'\mathbf{W}\mathbf{y} \\ &= \hat{\boldsymbol{\beta}}_{\mathbf{O}} - \rho.\hat{\boldsymbol{\beta}}_{\mathbf{L}}\end{aligned} \qquad \textbf{(5)}$$

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[7] This likelihood is for the conditional ML estimator, which treats the first observation y_1 as non-stochastic. The conditional ML estimator may be undesirable when T is small and the initial observation contributes significantly to the likelihood. Elhorst (2005) discusses these issues in detail and provides an unconditional estimator.

[8] With respect to software and code, we used MATLAB and the demoFE-SEM.m routine available from Paul Elhorst's webpage http://www.eco.rug.nl/~elhorst/ (accessed 8/29/2005), which builds on LeSage's code written earlier http://www.spatial-econometrics.com/.

where $\hat{\boldsymbol{\beta}}_{\mathbf{O}}$ and $\hat{\boldsymbol{\beta}}_{\mathrm{L}}$ are the OLS estimates obtained from regressing **X** on **y** and **Wy** respectively. The residuals from these regressions,

$$\hat{\boldsymbol{\varepsilon}}_{\mathbf{O}} = \mathbf{y} - \mathbf{X}\boldsymbol{\beta}_{\mathbf{O}} \quad \text{and} \quad \hat{\boldsymbol{\varepsilon}}_{\mathrm{L}} = \mathbf{Wy} - \mathbf{X}\boldsymbol{\beta}_{\mathrm{L}}, \tag{6}$$

can be used to calculate the error variance:

$$\hat{\sigma}^2 = (1/NT)(\hat{\boldsymbol{\varepsilon}}_{\mathbf{O}} - \rho\hat{\boldsymbol{\varepsilon}}_{\mathrm{L}})'(\hat{\boldsymbol{\varepsilon}}_{\mathbf{O}} - \rho\hat{\boldsymbol{\varepsilon}}_{\mathrm{L}}) \tag{7}$$

Substituting the formulas for $\hat{\boldsymbol{\beta}}$ and $\hat{\sigma}^2$ into **(4b)** gives the concentrated likelihood function

$$\ln L_C(\mathbf{y_t}) = -\left(\frac{NT}{2}\right)\ln\pi + \ln|\mathbf{A}| - \frac{NT}{2}\ln\left(\frac{1}{NT}(\boldsymbol{\varepsilon}_{\mathbf{O}} - \rho\boldsymbol{\varepsilon}_{\mathrm{L}})'(\boldsymbol{\varepsilon}_{\mathbf{O}} - \rho\boldsymbol{\varepsilon}_{\mathrm{L}})\right), \tag{8}$$

which contains a single coefficient ρ. The *ML* estimate for ρ is the value that maximizes equation **(8)**. Once $\hat{\rho}$ is obtained, it is substituted into **(5)** to get $\hat{\boldsymbol{\beta}}$.

III. EMPIRICAL ILLUSTRATION: CALCULATING SPATIAL AND SPATIO-TEMPORAL EFFECTS:

In most of the empirical work on diffusion and spatial interdependence, the analysis is limited to testing the null hypothesis that the spatial lag coefficient ρ is zero. To be fair, these tests are extremely important and often used to identify the mechanisms underlying the observed interdependence. Nevertheless, researchers who focus exclusively on the statistical significance of ρ ignore what is arguably the most important and substantively interesting information contained in their results: the spatial and spatio-temporal patterns implied by their estimates. Calculation and presentation of *effects* in empirical models with spatial interdependence, as in any model beyond the purely linear-additive, involve more than simply considering coefficient estimates.[9] In empirical models containing *spatial dynamics*, as in those with only temporal dynamics, coefficients on explanatory variables give only the *pre-dynamic* impetuses to the outcome variable from increases in those explanatory variables. This represents the pre-interdependence impetus, which, incidentally, is unobservable if spatial dynamics are instantaneous (i.e., incur within observation period). This section discusses calculation of spatial and spatio-temporal multipliers, which allow expression of the *effects* across units or across spatial units and over time of counterfactual shocks, and it applies the delta-method to compute standard errors for these *effects*.[10]

We illustrate using an analysis in another paper (Franzese and Hays 2006a) where we estimated spatio-temporal-lag models to evaluate empirically the interdependence among European countries in active-labor-market policy, specifically labor-market-training (LMT) expenditures. With positive employment spillovers across borders, countries have incentives to free-ride on neighbors' LMT expenditures. If so empirically, the estimated coefficient on the spatial lag $(\hat{\rho})$ should be statistically significantly negative. In a sample of annual data from

[9] For example, even in models differing from the purely linear-additive only in containing (time) dynamics (see, e.g., DeBoef and Keele 2005) or multiplicative interaction terms (see, e.g., Kam and Franzese 2005), *coefficients* and *effects* are different things.

[10] For an excellent discussion of spatial multipliers, see Anselin (2003).

1987-98 in 15 European countries,[11] the dependent variable $\mathbf{Y_t}$ was LMT expenditures per unemployed worker (2000 $, PPP) or the ratio LMT expenditures to GDP. In addition to spatial and temporal lags of LMT spending, we included controls for macroeconomic performance (GDP per capita, unemployment), labor-market characteristics (deindustrialization, union density), exposure to external shocks (trade openness, foreign direct-investment), and domestic politics (left cabinet-seats, Christian-Democratic cabinet-seats, left-libertarian vote, government consumption). All regressions included country and year dummies to account for unit- and period-heterogeneity, a strategy that, given the tendency for inadequate modeling of such heterogeneity to induce overestimation of spatial interdependence (Franzese and Hays 2006b), errs conservatively against the hypothesis of interdependent ALM policymaking.

We calculated the spatial lag, $\mathbf{WY_t}$, as a standardized *binary contiguity-weights matrix*, which first codes w_{ij}=1 for countries *i* and *j* that share borders and w_{ij}=0 for pairs *ij* that do not.[12] Then, we *row-standardized* (as common in spatial econometrics) the resulting matrix by dividing each cell in a row by that row's sum. We also considered an alternative spatial-weights matrix that substituted shared-border length for the ones in the *binary contiguity-weights matrix*. We estimated six models in all by spatial *ML*. Table 1 presents the results. In all models, the estimated spatial-lag coefficients are statistically significantly negative. These coefficients give some indication of short-run effect on country *i*'s LMT expenditures of a one-unit positive shock to all of its neighbors' spending. These results imply that an increase in neighbor spending induces a decrease in domestic

[11] The countries are Austria, Belgium, Denmark, Germany, Greece, Finland, France, Ireland, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the UK.

[12] France, Belgium, and the Netherlands are "contiguous" with Britain, and Denmark with Sweden, in our coding.

spending, suggesting some free-riding among European countries in LMT spending.

<Table 1 About Here>

The substantive magnitude of this free-riding interdependence, i.e., of the estimated effects of individual EU countries' ALM policies on their neighbors', however, is not understood immediately and fully from the $\hat{\rho}$ estimate alone. To see these *effects*, we need to calculate the *spatial multiplier* in the model from **(1)**:

$$\mathbf{Y_t} = \rho\mathbf{W}\mathbf{Y_t} + \phi\mathbf{Y_{t-1}} + \mathbf{X_t}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t} \Rightarrow \mathbf{Y_t}(\mathbf{I} - \rho\mathbf{W}) = \phi\mathbf{Y_{t-1}} + \mathbf{X_t}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t}$$
$$\Rightarrow \mathbf{Y_t} = (\mathbf{I} - \rho\mathbf{W})^{-1}(\phi\mathbf{Y_{t-1}} + \mathbf{X_t}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t}) \qquad \textbf{(9)}$$

The spatial multiplier, $(\mathbf{I}\text{-}\rho\mathbf{W})^{-1}$, captures the feedback from, say, Belgium on France and other contiguous countries, from those on their contiguous countries, including back on Belgium, and so on recursively. We can calculate time-*t* effects on the vector of policy-outcomes in all 15 countries, $\mathbf{Y_t}$, including this recursive feedback, with the spatial multiplier by considering certain counter-factual shocks to the rest of **(9)**'s right-hand side. For example, multiplying $(\mathbf{I}\text{-}\rho\mathbf{W})^{-1}$ by an *Nx1* column-vector $\mathbf{S_i}$ with 1 in row *i* and 0 elsewhere gives the immediate effect of a unit-shock to country *i* on policies in the other *(N-1)* countries *j*. Multiplying $(\mathbf{I}\text{-}\rho\mathbf{W})^{-1}$ by a *15x1* column-vector with 0 in all rows except Austria's, which gets a 1, e.g., yields a *15x1* column-vector containing the estimated effects of a unit-shock in Austria on the other 14 countries in their respective rows.[13] To

[13] In Austria's row will be the estimated effect after feedback of a unit-shock to the rest of Austria's

simplify exposition, define these spatial-multiplier effects: $\mathbf{M} \equiv (\mathbf{I}-\rho\mathbf{W})^{-1}\mathbf{S_i}$.

Note that spatial-multiplier effects are nonlinear functions of the $\hat{\rho}$ estimates. The delta method uses linear (Taylor-series) approximations of such nonlinear functions of parameter estimates to derive estimated asymptotic standard errors. The linearization makes it easy to calculate standard errors since the variance of a linear function of a random variable X is simply $\text{var}(aX+b)=a^2\,\text{var}(X)$. It follows that the estimated (asymptotic and approximated) variance of $\mathbf{M}(\hat{\rho})$ by the delta-method is

$$\widehat{V(\mathbf{M}(\hat{\rho}))}=\left(\frac{\partial\mathbf{M}(\hat{\rho})}{\partial\hat{\rho}}\right)\widehat{V(\hat{\rho})}\left(\frac{\partial\mathbf{M}(\hat{\rho})}{\partial\hat{\rho}}\right)' \qquad \mathbf{(10)},$$

which gives an *NxN* variance-covariance matrix, the diagonal elements of which are the estimated variances of the effects of a unit-shock to country *i* on policies in the other *(N-1)* countries *j*.[14]

The off-diagonal cells of Table 2 illustrate calculations of **(9)** and **(10)** using the fourth-column estimates from Table 1. This regression has LMT expenditures per unemployed as its dependent variable, uses the binary contiguity matrix for the spatial lag, and includes the full set of control variables. The first number in each cell is the immediate effect of a unit-increase (+$1) in the

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right-hand side on Austria itself, which, in this case, will be somewhat more than the original unit because an increase in Austria's LMT spending induces other EU members to cut theirs which induces Austria to raise its further and so on, recursively.

[14] We use the central-difference formula (i.e., calculate changes in the function, here $\mathbf{M}(\hat{\rho})$, for small changes in its arguments, centered on some point, here $\hat{\rho}$) to calculate the vector of derivatives numerically (see Mathews and Fink 2004, 323-342). We are not aware of any studies that evaluate the small-sample performance of these standard errors.

column country's LMT expenditures per unemployed on its row counterparts. The number in parentheses is the standard error of this immediate spatial-effect. Immediate spatial-effects exceeding twice their standard errors are starred and highlighted. Though we have yet to prove it rigorously, we suspect that the Law of Large Numbers, the Central Limit Theorem, and Slutsky's Theorem imply that the sampling distribution for the multiplier is asymptotically normal.[15]

<Table 2 About Here>

In addition to spatial dynamics, our LMT-spending model included a time lag of the dependent variable and corresponding temporal dynamics. Using **(9)**, we could plot the evolution, or time-path, of these responses to counterfactuals from $\mathbf{Y_t}$ to $\mathbf{Y_{t+1}}$, etc., to illustrate their spatio-temporal dynamics. We can also calculate the long-run steady-state effects, including spatial feedback, on all other countries of *permanent* hypothetical shocks to one country (or the long-run steady-state response in one country to a permanent hypothetical shock to other countries). To find these long-run steady-states, set $\mathbf{Y_{t-1}}=\mathbf{Y_t}$ in **(1)**, giving:

$$\mathbf{Y_t} = (\mathbf{I} - (\mathbf{I} - \rho\mathbf{W})^{-1}\phi)^{-1}(\mathbf{I} - \rho\mathbf{W})^{-1}(\mathbf{X_t}\boldsymbol{\beta} + \boldsymbol{\varepsilon}_\mathbf{t}) \qquad \textbf{(11)}.$$

We can write this more compactly by defining a single spatio-temporal weights matrix $\mathbf{Z}$. Again, we begin by setting set $\mathbf{Y_{t-1}}=\mathbf{Y_t}$ in **(1)**,

[15] Asymptotic normality holds for the sampling distribution of the temporal multiplier. We see no reason why it would not apply for the spatial and spatio-temporal multipliers.

$$
\begin{aligned}
\mathbf{y}_t &= \rho \mathbf{W} \mathbf{y}_t + \phi \mathbf{y}_t + \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \\
&= \left(\rho \mathbf{W} + \phi \mathbf{I} \right) \mathbf{y}_t + \mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \\
&= \left[\mathbf{I}_{\mathrm{N}} - \rho \mathbf{W} - \phi \mathbf{I}_{\mathrm{N}} \right]^{-1} \left(\mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \right) \\
&= \begin{bmatrix} 1-\phi & -\rho w_{1,2} & \cdots & \cdots & -\rho w_{1,\mathrm{N}} \\ -\rho w_{2,1} & 1-\phi & & & \vdots \\ \vdots & & \ddots & & \vdots \\ \vdots & & & 1-\phi & -\rho w_{(\mathrm{N}-1),\mathrm{N}} \\ -\rho w_{\mathrm{N},1} & \cdots & \cdots & -\rho w_{\mathrm{N},(\mathrm{N}-1)} & 1-\phi \end{bmatrix}^{-1} \left(\mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \right) \\
&\equiv \mathbf{Z} \left(\mathbf{X}_t \boldsymbol{\beta} + \boldsymbol{\varepsilon}_t \right)
\end{aligned} \qquad \textbf{(12)}
$$

If we define a +1 *shock* to unit *i* in period *t* as $\mathbf{S}_i \equiv d\boldsymbol{\varepsilon}_t^i \equiv \begin{bmatrix} 0 & \cdots & 0 & 1_i & 0 & \cdots & 0 \end{bmatrix}'$. Then, the long-run impact of such a *shock* to *i* on the entire set of units, *I*, is $\mathbf{Z}\mathbf{S}_i$, which is just $\mathbf{z}_i$, the i^{th} column of $\mathbf{Z}$, the estimate of which we denote $\hat{\mathbf{z}}_i$. To obtain the estimated (asymptotic and approximated) standard errors of these estimates, apply the delta method:

$$
\mathbf{V}\left(\hat{\mathbf{z}}_i \right) = \left[\frac{\partial \hat{\mathbf{z}}_i}{\partial \hat{\boldsymbol{\theta}}} \right] \mathbf{V}\left(\hat{\boldsymbol{\theta}} \right) \left[\frac{\partial \hat{\mathbf{z}}_i}{\partial \hat{\boldsymbol{\theta}}} \right]' \text{ , where } \hat{\boldsymbol{\theta}} = \begin{bmatrix} \hat{\rho} \\ \hat{\phi} \end{bmatrix} \qquad \textbf{(13)}
$$

If we are interested in the spatial effects of a one-unit increase in explanatory variable *k* in country *i*, we calculate $\frac{dx_{i,k} \beta_{i,k}}{dx_{i,k}}$. The long run impact of this change on country *i*'s neighbors is $\mathbf{Z} \frac{dx_{i,k} \beta_{i,k}}{dx_{i,k}}$ or simply, $\mathbf{z}_i \beta_k$. The standard errors calculation, using the delta method, is

$$\mathbf{V}\left(\hat{\mathbf{z}}_i\hat{\beta}_k\right)=\left[\frac{\partial\hat{\mathbf{z}}_i\hat{\beta}_k}{\partial\hat{\boldsymbol{\theta}}}\right]\mathbf{V}\left(\hat{\boldsymbol{\theta}}\right)\left[\frac{\partial\hat{\mathbf{z}}_i\hat{\beta}_k}{\partial\hat{\boldsymbol{\theta}}}\right]' \text{, where } \hat{\boldsymbol{\theta}}=\begin{bmatrix}\hat{\rho}\\ \hat{\phi}\\ \hat{\beta}_k\end{bmatrix}. \quad \textbf{(14)}$$

The numbers in Table 3 represent the long-run steady-state responses of the row country to a permanent unit-shock to the column country's LMT. The standard errors of these long-run steady-state spatial-effects are in parentheses. Spatial effects exceeding twice their standard errors are starred and highlighted. Again, we suspect that the sampling distribution for the spatio-temporal multiplier is asymptotically normal.

<Table 3 About Here>

All first-order effects are negative; i.e., increased LMT spending in country i's neighbors induces a decrease in domestic LMT. The size of these effects depends on the number of country *i*'s neighbors: the fewer the neighbors, the larger the effect a shock to any one of them has on domestic LMT spending. So, e.g., a permanent +$1 shock to British LMT spending reduces Irish LMT by $0.29 immediately and $1.26 in the long run. The relationship is negative because the UK and Ireland border and large because Britain is Ireland's only neighbor. This steady-state calculation assumes a *permanent* increase in British spending and would require many years to materialize; as such, the calculation likely represents something of an upper bound. Britain would probably not maintain the spending increase permanently, perhaps especially noting the cumulative Irish response over the equally long run, so we would likely never observe this full long-run-equilibrium degree of fiscal free-riding empirically. Few second-order effects (i.e., among non-contiguous countries transmitted by bordering ones) are statistically significant and large. Second-order effects, being how a country responds to its neighbor's neighbors, are all positive here. For example, a permanent +$1 shock to French LMT spending increases Austrian LMT by $0.02 in the short run and $0.16 in the long run, because increased French spending reduces LMT in Germany and Switzerland, Austria's two neighbors, and the negative employment spillovers from Germany and Switzerland then increases Austrian LMT spending.

<Figures 1 and 2 About Here>

These short-run and steady-state spatial effects are shown graphically in Figures 1 and 2 for a one-unit positive shock to German expenditures on ALM policies.[16] Comparison of these two figures shows how the long-run spatial relationships evolve over time. Not surprisingly, the strongest effects concentrate on Germany's small neighbors, particularly Denmark in the north, Austria in the south, and the Netherlands to the west. Again, this is because the magnitude of the spatial effects depends on the number of country *i*'s neighbors: the fewer the neighbors, the larger the effect a shock to any one of them has on domestic LMT spending. For this reason, Germany's largest neighbor, France, is affected less.

IV. CONCLUSION:

We have argued that too often researchers ignore the most important and substantively

[16] Despite the fact that maps are extremely useful for presenting spatial relationships in data, very few scholars studying diffusion and spatial interdependence use them. There are exceptions, of course (e.g., see Gleditsch and Ward, 2000). These days user friendly software makes it easy to generate informative maps. The maps in figures 1 and 2 were produced with very little effort using ArcGIS.

interesting information contained in their statistical results: the spatial and spatio-temporal patterns implied by their estimates. In empirical models containing "spatial dynamics", as in those having only temporal dynamics, the coefficients on explanatory variables give only the pre-dynamic impetuses to the outcome variable. We discussed how to calculate and present the spatial and spatio-temporal multipliers, which allow one to express the spatial and spatio-temporal effects of counterfactual shocks across units (spatial) and across spatial units and over time, and how to use the delta method to compute standard errors for these effects. Maps provide an effective way to present the results of these kinds of counterfactuals. Much work remains to be done. Most importantly, the small sample properties of the delta estimator need to be studied carefully and compared with bootstrap alternatives.

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Table 1. Labor Market Training Expenditures in Europe (1987-1998)

	LMT/Unemp.	LMT/Unemp.	LMT/Unemp.	LMT/Unemp.	LMT/GDP	LMT/Unemp.
Temporal Lag	0.657***	0.553***	0.514***	0.490***	0.691***	0.547***
	(.055)	0.064	(.068)	(.068)	(.054)	(.069)
Spatial Lag	-0.258***	-0.277***	-0.286***	-0.284***	-0.109*	-0.130**
	(.068)	(.066)	(.067)	(.068)	(.065)	(.064)
Real GDP Per Capita		-0.964	-1.203*	-0.863	-0.477**	-0.588
		(.645)	(.655)	(.798)	0.199	(.833)
Unemployment Rate		-0.054***	-0.092***	-0.092***	-0.015**	-0.094***
		(0.019)	(.026)	(.028)	0.006	(.029)
Union Density			0.000	0.000	0.003	0.000
			(.001)	(.001)	(.002)	(.001)
Deindustrialization			0.008	0.011	0.004	0.019**
			(.008)	(.009)	(.007)	(.009)
Trade Openness			0.051*	0.048*	0.001	0.056*
			(.028)	(.029)	(.002)	(.030)
Foreign Direct			0.071	0.027	0.001	0.012
Investment			(.043)	(.050)	(.005)	(.052)
Old Age				0.000	0.008	-0.000
				(.007)	(.012)	(.008)
Left Cabinet Seats				-0.030	0.000	-0.025
				(.021)	(.000)	(.022)
Christian Dem.				-0.002	0.000	-0.012
Cabinet Seats				(.013)	(.001)	(.013)
Left Libertarian				-0.003	-0.008**	-0.002
Vote				(.003)	(.003)	(.003)
Government				0.026	0.024***	0.035
Consumption				(.032)	(.008)	(.033)
Spatial Weights Matrix	Binary Contiguity	Binary Contiguity	Binary Contiguity	Binary Contiguity	Binary Contiguity	Border Length
Log-Likelihood	-27.256	-22.818	-19.626	-17.600	234.573	-23.292
σ^2	0.077	0.074	0.071	0.069	0.004	0.075

Notes: All regressions include fixed period and unit effects; those coefficient-estimates suppressed to conserve space. The first five spatial lags are generated with a binary contiguity weighting matrix using shared territorial borders as the criterion, excepting that France, Belgium, and the Netherlands are coded as contiguous with Britain and Denmark as contiguous with Sweden. The last spatial lag is generated using shared border length for the spatial weights. All the spatial weights matrices are row-standardized. The parentheses contain standard errors.

Table 2. Short-Run Spatial Effects of Labor Market Training Expenditures in Europe (Binary Contiguity Weights Matrix)

	AUT	BEL	DEN	FIN	FRA	DEU	IRE	NTH	NOR	PRT	ESP	SWE	CHE	GBR
AUT		0.005* (.0019)	0.006 (.0031)	0.000 (.0001)	0.020* (.0095)	-0.135* (.0317)	0.000 (.0001)	0.006* (.0029)	0.000 (.0001)	0.000 (.0002)	-0.001 (.0009)	-0.001 (.0007)	-0.140* (.0338)	-0.002 (.0015)
BEL	0.002* (.0010)		0.003* (.0014)	0.000 (.0000)	-0.066* (.0152)	-0.064* (.0142)	0.005* (.0021)	-0.065* (.0144)	0.000 (.0000)	-0.001* (.0004)	0.004* (.0019)	0.000 (.0003)	0.006* (.0029)	-0.064* (.0142)
DEN	0.006 (.0031)	0.006* (.0029)		0.012* (.0058)	0.006* (.0030)	-0.148* (.0383)	0.000 (.0001)	0.007* (.0033)	0.012* (.0058)	0.000 (.0001)	0.000 (.0003)	-0.148* (.0387)	0.006* (.0026)	-0.001 (.0011)
FIN	0.000 (.0001)	0.000 (.0083)	0.012* (.0058)		0.000 (.0001)	-0.002 (.0013)	0.000 (.0000)	0.000 (.0001)	-0.134* (.0314)	0.000 (.0000)	0.000 (.0000)	-0.129* (.0294)	0.000 (.0001)	0.000 (.0000)
FRA	0.008* (.0038)	-0.053* (.0122)	0.002 (.0012)	0.000 (.0000)		-0.051* (.0120)	0.004* (.0020)	0.010* (.0049)	0.000 (.0000)	0.009* (.0044)	-0.061* (.0164)	0.000 (.0003)	-0.057* (.0143)	-0.057* (.0144)
DEU	-0.045* (.0106)	-0.043* (.0095)	-0.049* (.0128)	-0.001* (.0004)	-0.043* (.0100)		-0.001 (.0005)	-0.046* (.0114)	-0.001* (.0004)	0.000 (.0003)	0.003* (.0012)	0.007 (.0036)	-0.040* (.0081)	0.010* (.0049)
IRE	0.000 (.0003)	0.018* (.0084)	0.000 (.0002)	0.000 (.0000)	0.020* (.0099)	-0.004 (.0031)		0.020* (.0097)	0.000 (.0000)	0.000 (.0002)	-0.001 (.0009)	0.000 (.0000)	-0.001 (.0007)	-0.294* (.0755)
NTH	0.004* (.0020)	-0.086* (.0192)	0.004 (.0022)	0.000 (.0001)	0.017* (.0082)	-0.093* (.0227)	0.007* (.0032)		0.000 (.0001)	0.000 (.0001)	-0.001 (.0007)	-0.001* (.0005)	0.003* (.0010)	-0.093* (.0231)
NOR	0.000 (.0001)	0.000 (.0001)	0.012* (.0058)	-0.134* (.0314)	0.000 (.0001)	-0.002 (.0013)	0.000 (.0000)	0.000 (.0001)		0.000 (.0000)	0.000 (.0000)	-0.129* (.0294)	0.000 (.0001)	0.000 (.0000)
PRT	0.000 (.0003)	-0.002 (.0016)	0.000 (.0001)	0.000 (.0000)	0.043 (.0220)	-0.002 (.0016)	0.000 (.0002)	0.000 (.0004)	0.000 (.0000)		-0.298* (.0788)	0.000 (.0000)	-0.002 (.0018)	-0.002 (.0018)
ESP	-0.001 (.0009)	0.008* (.0038)	0.000 (.0003)	0.000 (.0000)	-0.152* (.0411)	0.008* (.0037)	-0.001* (.0004)	-0.002 (.0011)	0.000 (.0000)	-0.149* (.0394)		0.000 (.0001)	0.008 (.0043)	0.008 (.0043)
SWE	-0.001* (.0005)	-0.001* (.0004)	-0.099* (.0258)	-0.086* (.0196)	-0.001* (.0004)	0.014 (.0073)	0.000 (.0000)	-0.001* (.0005)	-0.086* (.0196)	0.000 (.0000)	0.000 (.0000)		-0.001* (.0004)	0.000 (.0001)
CHE	-0.093* (.0225)	0.009* (.0039)	0.004* (.0017)	0.000 (.0000)	-0.095* (.0238)	-0.080* (.0161)	0.000 (.0002)	0.003 (.0010)	0.000 (.0000)	-0.001 (.0006)	0.006* (.0029)	-0.001* (.0004)		0.005* (.0021)
GBR	-0.001 (.0007)	-0.064* (.0142)	-0.001 (.0005)	0.000 (.0000)	-0.071* (.0180)	0.015 (.0073)	-0.074* (.0189)	-0.070* (.0173)	0.000 (.0000)	-0.001* (.0004)	0.004 (.0021)	0.000 (.0001)	0.003 (.0016)	

Notes: The off-diagonal elements of the table report the effect of a one-unit increase in the column country's labor-market-training expenditures on its European counterparts. These numbers are calculated using the spatial multiplier matrix $(\mathbf{I}-\rho\mathbf{W})^{-1}$ and thus reflect all feedback effects. Parentheses contain standard errors calculated by the delta method.

Table 3. Steady-State Spatial Effects of labor Market Training Expenditures in Europe (Binary Contiguity Weights Matrix)

	AUT	BEL	DEN	FIN	FRA	DEU	IRE	NTH	NOR	PRT	ESP	SWE	CHE	GBR
AUT		0.027	0.052	0.002	0.159	-0.530*	0.005	0.050	0.002	0.006	-0.021	-0.016	-0.557*	-0.033
		(.0136)	(.0296)	(.0024)	(.0914)	(.1588)	(.0050)	(.0292)	(.0024)	(.0067)	(.0189)	(.0134)	(.1663)	(.0277)
BEL	0.013		0.023	0.001	-0.254*	-0.238*	0.033	-0.236*	0.001	-0.009	0.033	-0.007	0.047	-0.237*
	(.0068)		(.0125)	(.0010)	(.0723)	(.0636)	(.0167)	(.0581)	(.0010)	(.0079)	(.0204)	(.0058)	(.0237)	(.0636)
DEN	0.052	0.047		0.094	0.051	-0.640*	0.003	0.056	0.094	0.002	-0.007	-0.648*	0.039*	-0.025
	(.0407)	(.0346)		(.0505)	(.0305)	(.2218)	(.0038)	(.0335)	(.0505)	(.0022)	(.0062)	(.2271)	(.0183)	(.0211)
FIN	0.002	0.002	0.094		0.002	-0.028	0.000	0.002	-0.520*	0.000	0.000	-0.493*	0.002	-0.001
	(.0785)	(.0640)	(.0739)		(.0570)	(.2043)	(.0096)	(.0621)	(.0076)	(.0057)	(.0149)	(.0403)	(.0266)	(.0489)
FRA	0.064*	-0.203*	0.020	0.001		-0.207*	0.034	0.082*	0.001	0.080*	-0.286*	-0.006	-0.240*	-0.247*
	(.0031)	(.0027)	(.0505)	(.2121)		(.0228)	(.0002)	(.0026)	(.1477)	(.0001)	(.0004)	(.1402)	(.0016)	(.0014)
DEU	-0.177*	-0.158	-0.213*	-0.009	-0.173		-0.012	-0.191*	-0.009	-0.006	0.023	0.065	-0.133	0.083
	(.0502)	(.0864)	(.0122)	(.0010)	(.2112)		(.0209)	(.0466)	(.0010)	(.0514)	(.1164)	(.0054)	(.0808)	(.0910)
IRE	0.009	0.132	0.007	0.000	0.172	-0.069		0.163	0.000	0.006	-0.023	-0.002	-0.015	-1.257*
	(.0131)	(.0928)	(.0075)	(.0005)	(.1045)	(.0576)		(.0940)	(.0005)	(.0074)	(.0212)	(.0029)	(.0127)	(.4242)
NTH	0.033	-0.314*	0.038	0.002	0.137	-0.382*	0.054		0.002	0.005	-0.018	-0.011	0.011	-0.390*
	(.0267)	(.1186)	(.0223)	(.0018)	(.0777)	(.1241)	(.0313)		(.0018)	(.0057)	(.0161)	(.0099)	(.0007)	(.1319)
NOR	0.002	0.002	0.094	-0.520*	0.002	-0.028	0.000	0.002		0.000	0.000	-0.493*	0.002	-0.001
	(.0031)	(.0027)	(.0505)	(.1477)	(.0024)	(.0228)	(.0002)	(.0026)		(.0001)	(.0004)	(.1402)	(.0016)	(.0014)
PRT	0.012	-0.037	0.004	0.000	0.398	-0.038	0.006	0.015	0.000		-1.345*	-0.001	-0.044	-0.045
	(.0175)	(.0424)	(.0044)	(.0003)	(.2572)	(.0341)	(.0074)	(.0171)	(.0003)		(.5073)	(.0016)	(.0398)	(.0425)
ESP	-0.021	0.067	-0.007	0.000	-0.714*	0.068	-0.011	-0.027	0.000	-0.672*		0.002	0.079	0.082
	(.0252)	(.0559)	(.0062)	(.0004)	(.2910)	(.0448)	(.0106)	(.0242)	(.0004)	(.2537)		(.0025)	(.0524)	(.0566)
SWE	-0.011	-0.009	-0.432*	-0.329*	-0.010	0.129	-0.001	-0.011	-0.329*	0.000	0.001		-0.008	0.005
	(.0119)	(.0103)	(.1514)	(.0935)	(.0090)	(.0805)	(.0010)	(.0099)	(.0935)	(.0005)	(.0016)		(.0059)	(.0056)
CHE	-0.371*	0.062	0.026	0.001	-0.400*	-0.266*	-0.005	0.011	0.001	-0.015	0.053	-0.008		0.037
	(.1644)	(.0439)	(.0122)	(.0011)	(.1346)	(.0532)	(.0042)	(.0007)	(.0011)	(.0133)	(.0350)	(.0059)		(.0217)
GBR	-0.017	-0.237*	-0.012	-0.001	-0.309*	0.125	-0.314*	-0.293*	-0.001	-0.011	0.041	0.004	0.028	
	(.0185)	(.0959)	(.0105)	(.0007)	(.1137)	(.0734)	(.1061)	(.0989)	(.0007)	(.0106)	(.0283)	(.0042)	(.0163)	

Notes: The off-diagonal elements of the table report the effect of a one-unit increase in the column country's labor-market-training expenditures on its European counterparts. These numbers are calculated using the long-run spatio-temporal-multiplier matrix $\left[\mathbf{I}-(\mathbf{I}-\rho\mathbf{W})^{-1}\phi\right]^{-1}[\mathbf{I}-\rho\mathbf{W}]^{-1}$. Parentheses contain standard errors calculated by the delta method.

Figure 1. Short-run Spatial Effects of a Positive One-unit Shock to German LMT Expenditures

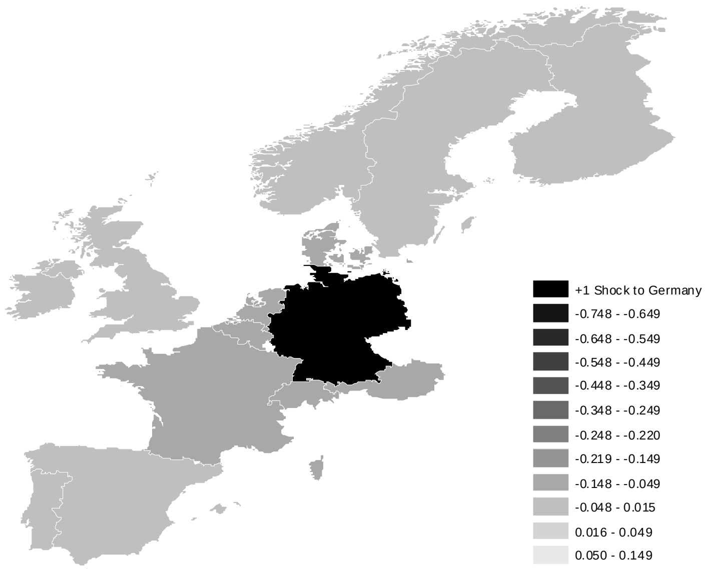

Figure 2. Steady-state Spatial Effects of a Positive One-unit Shock to German LMT Expenditures