# MODELING AND INTERPRETING INTERACTIVE HYPOTHESES IN REGRESSION ANALYSIS

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**ABSTRACT**

Despite occasional constructive pedagogical treatises on the topic in the past (e.g., Friedrich 1982), a common methodology for employing and interpreting interaction terms in regression analysis continues to elude the field; and, partly as a consequence, their misinterpretation remains sadly rampant. This paper aims to redress these problems.

We first document the widespread and expanding use of interaction terms in political science. We then elaborate and support our claim that, despite the increased use of interaction terms in regression analyses, many inherently interactive arguments are not being modeled as such. Finally, we note and review several inconsistencies and much confusion in the literature regarding the substantive interpretation of interaction terms and the statistical inferences from the coefficient and standard-error estimates surrounding them.

After this review, we discuss three pedagogical themes. First, we offer a generic consideration of the process of writing empirical models that embody interactive hypotheses. Our second theme focuses more specifically on a set of statistical practices and *rules of thumb* when utilizing interaction models suggested by previous methodological treatments of the topic and frequently employed in current political science literature. We demonstrate that many of these can be misleading. Finally, we discuss the presentation of interaction effects. We show that, while the existing practice of reporting standard errors for individual coefficients remains useful, that practice is decidedly insufficient regarding coefficients on interactive terms. Assessment of interaction effects virtually requires graphical or tabular presentation of results, neither of which is currently commonly published. In this context, we strongly suggest the use of *effect-line* graphs or *conditional-coefficient* tables, complete with standard errors, hypothesis tests, and/or confidence intervals of those *effects* or *conditional coefficients*. We show how to construct these graphs and tables, giving detailed spreadsheet formulae for doing so in addition to the standard mathematical formulae for their elements.

# THE PREVALENCE OF INTERACTION TERMS

*A Generic Interactive Hypothesis:*

The effects of some independent variable(s), X, on some dependent variable(s), Y, depend upon a third (set of) independent variable(s), Z. The statistical tool most often employed to capture this theoretical claim is the interactive, or multiplicative term.

*Current Usage:*

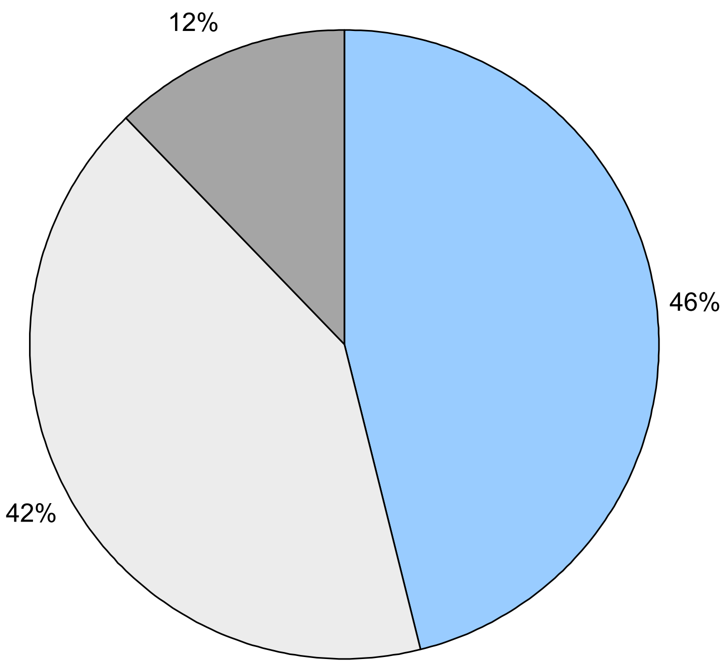
**TYPES OF ARTICLES IN MAJOR POLITICAL SCIENCE JOURNALS, 1996-1997**

| **JOURNALS (1996-1997)** | **Total Articles** | **Articles Employing Statistical Analysis** | | **Articles Employing Interaction Terms** | | |
|---|---|---|---|---|---|---|
| | | *Count* | *% of Total* | *Count* | *% of Total* | *% of Statistical* |
| *American Journal of Political Science* | 122 | 96 | 79% | 27 | 22% | 28% |
| *American Political Science Review* | 95 | 42 | 44% | 12 | 13% | 29% |
| *Comparative Politics* | 47 | 1 | 2% | 0 | 0% | 0% |
| *Comparative Political Studies* | 48 | 25 | 52% | 4 | 8% | 16% |
| *International Organization* | 46 | 11 | 24% | 0 | 0% | 0% |
| *International Studies Quarterly* | 54 | 19 | 35% | 2 | 4% | 11% |
| *Journal of Politics* | 94 | 79 | 84% | 19 | 20% | 24% |
| *Legislative Studies Quarterly* | 49 | 38 | 78% | 8 | 16% | 21% |
| *World Politics* | 38 | 8 | 21% | 0 | 0% | 0% |
| ***TOTAL*** | **593** | **319** | **54%** | **72** | **12%** | **23%** |

**TYPES OF ARTICLES IN MAJOR POLITICAL SCIENCE JOURNALS, 1996-1997**

| **Journals** | **1996** | | | | | | **1997** | | | | | | **% Change in Articles Using Interactions, 1996 to 1997** | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Total Articles* | *Employing Statistical Analysis* | | *Employing Interaction Terms* | | | *Total Articles* | *Employing Statistical Analysis* | | *Employing Interaction Terms* | | | | | |
| | | Stat Count | Stat % of Tot | Stat Count | Inter % of Tot | Inter % of Stat | | Stat Count | Stat % of Tot | Stat Count | Inter % of Tot | Inter % of Stat | Stat % of Tot | Inter % Tot | Inter % Stat |
| *American Journal of Political Science* | 57 | 45 | 79% | 14 | 25% | 31% | 65 | 51 | 78% | 13 | 20% | 25% | -1% | -5% | -6% |
| *American Political Science Review* | 45 | 17 | 38% | 5 | 11% | 29% | 50 | 25 | 50% | 7 | 14% | 28% | +12% | +3% | -1% |
| *Comparative Politics* | 20 | 1 | 5% | 0 | 0% | 0% | 27 | 0 | 0% | 0 | 0% | 0% | -5% | 0% | 0% |
| *Comparative Political Studies* | 24 | 12 | 50% | 1 | 4% | 8% | 24 | 13 | 54% | 3 | 13% | 23% | +4% | +9% | +15% |
| *International Organization* | 23 | 7 | 30% | 0 | 0% | 0% | 23 | 4 | 17% | 0 | 0% | 0% | -13% | 0% | 0% |
| *International Studies Quarterly* | 24 | 6 | 25% | 0 | 0% | 0% | 30 | 13 | 43% | 2 | 7% | 15% | +18% | +7% | +15% |
| *Journal of Politics* | 46 | 41 | 89% | 6 | 13% | 15% | 48 | 38 | 79% | 13 | 27% | 34% | -10% | +14% | +19% |
| *Legislative Studies Quarterly* | 24 | 18 | 75% | 4 | 17% | 22% | 25 | 20 | 80% | 4 | 16% | 20% | +5% | -1% | -2% |
| *World Politics* | 15 | 2 | 13% | 0 | 0% | 0% | 23 | 6 | 26% | 0 | 0% | 0% | +13% | 0% | 0% |
| ***TOTAL*** | **278** | **149** | **54%** | **30** | **11%** | **20%** | **315** | **170** | **54%** | **42** | **13%** | **25%** | **0%** | **+2%** | **+5%** |

**TYPES OF ARTICLES IN MAJOR POLITICAL SCIENCE JOURNALS, 1996-1997**




Total number of articles: 593
Journals: *American Journal of Political Science, American Political Science Review, Comparative Politics, Comparative Political Studies, International Organization, International Studies Quarterly, Journal of Politics, Legislative Studies Quarterly, and World Politics.* Issues from 1996 & 1997.

## <u>CURRENT PRACTICE</u>

1. Despite the prevalence of interaction terms in statistical models, many scholars have proposed interactive hypotheses but have failed to model them as such.

a) **MISMATCH BETWEEN THEORETICAL AND EMPIRICAL MODELS:** Authors propose arguments that involve, explicitly or implicitly, an interactive hypothesis (the effect of X on Y depends on Z), but estimate a strictly additive model, thus misstating the hypothesis actually proposed.

b) **DISAGGREGATING THE SAMPLE:** Authors argue that the effect of some set of X's on Y varies across sub-samples; e.g., they may be interested in the different effects of X on Y among women and men or in different time-periods. They split samples and model Y as a separate function of X in each sub-sample and then discuss the relative magnitude and signs of the coefficients in the sub-samples. Unfortunately, "eyeballing" the coefficients cannot provide adequate information about how and how certainly different these coefficients are. Without analyzing the sub-samples together, (one way is to use an interaction term), no statistical comparison of the resulting coefficients is possible.

c) **EMPLOYING ANOVA INSTEAD OF INTERPRETING REGRESSION RESULTS:** We have no technical qualms about using ANOVA instead of interactive terms within regressions to assess the *presence* of interactive effects, but we argue that *substantive interpretation* of interactive effects within regressions is more straightforward. ANOVA does not facilitate discussion of magnitudes or directions of the effects of X on Y. Moreover, as a variance accounting method; ANOVA allocates shares of variation in a dependent variable, Y, to individual variables, X, among a set of explanatory variables, **X**. This is logically problematic if some elements of **X** are deterministic functions of each other, such as $X_3=X_1*X_2$.

2. Among articles that do employ interactive terms, we observe the following problems:

a) **DISJUNCTURE BETWEEN THE VERBAL ARGUMENT AND THE EMPIRICAL SPECIFICATION:**

E.g., Linear-Additive Models when Linear-Interactive Models Apply: (see above).

E.g., Simplicity First: Scholars occasionally include higher-order interactions without examining lower-order ones first, failing to apply *Occam's Razor* (see below).

E.g., Get Complexity Right: An argument that "the effect of X on Y depends on the combination of V and W present" might imply Y = f (X , X*V*W ), but scholars often model only Y = f ( X , X*V , X*W ) in such cases.

b) **CONFUSION ABOUT INTERPRETATION**: Scholars occasionally commit the following errors in interpreting the results of their interactive estimations:

i) Attributing "the effect of X" to the coefficient and t-statistic on X alone, but "the effect of X" involves the coefficients on both X and X*Z and the value of the variable Z.

E.g., to ascertain whether X has non-zero effect on Y, scholars must consider the joint-hypothesis test that the coefficients on both X and X*Z are both zero.

E.g., [a pseudo-quote]: "If the nonsignificant interaction term with [Z] is dropped, then the statistical significance of the coefficient on [X] improves to p<.09[RM & CK1]." The authors take this to imply that the *effect* of X was insignificant in the interactive regression

ii) Referring to the coefficient on X as "the effect of X 'considered independently' of Z" or as "the main effect of X," but the coefficient on X refers to the effect of X *when Z (and therefore X*Z) equals zero*. This is neither "independent of Z" nor necessarily "main" (it may not even be logically possible or occur in the sample).

iii) These stem primarily from confusing *coefficients* for *effects*.

c) **OBSESSION WITH MULTICOLLINEARITY**: Scholars sometimes use multicollinearity to excuse statistically insignificant coefficients, to justify rescaling or reformulating variables, *etc.*, all of which suggest some underlying misunderstanding about how to interpret these coefficients.

E.g., [pseudo-quote] "Very high multicollinearity prevents the coefficients in this [interactive] model from achieving statistical significance." [RM & CK2] The author then replaces an interval-measured variable with an indicator truncating its information content to 0-1.

E.g., [pseudo-quote] "...interactive terms tend to correlate highly with parent variables...while not biasing [these] coefficients, multicollinearity inflates standard errors, perhaps making variables look statistically insignificant when they are actually significant." This is not technically correct; the coefficients in question are in fact statistically indistinguishable from zero. Neither coefficients nor standard errors are biased by multicollinearity, so tests are not biased. Crook and Hibbing, apsr 91(4)

d) **LACK OF MEASURES OF UNCERTAINTY (STANDARD ERRORS):** When scholars do report predicted values of the dependent variable at different values of X and Z, or effects of X or Z at different values of the other, they often provide no standard errors of these effect estimates.

With interaction terms, standard errors of the *estimated effects* of some variable cannot be read simply from the standard errors on the *estimated coefficients*. The (estimated) standard errors of the (estimated) effects of X depend on the values of Z just like the effects themselves do.

## POTENTIALLY MISLEADING *RULES of THUMB*

1. **CENTERING / RESCALING:**

a) Originally suggested to alleviate the multicollinearity *problem* (Smith & Sasaki 1979, Cronbach 1987)

b) Examples of Centering or Rescaling in the Literature: [pseudo-quotes]

> E.g.: "Interaction terms in multiple regression…are often correlate highly with their component parts, producing big standard errors and low statistical significance. The standard remedy for this is to *center* the components about their means before constructing the interaction…This reduces the standard errors but does not affect the magnitude of the interaction coefficient or of the calculated slopes for the components [RM & CK3]."
>
> "Centering reduces the possibility of collinearity among the variables and their interactions and allows a simpler interpretation of the estimated baselines [RM & CK4]."

c) Advice from Previous Pedagogical Treatments:

> "[T]he major threat of multicollinearity in interactive models is not substantive…but rather practical. Multicollinearity does not affect the properties of OLS estimates… High correlations between predictors, however, can cause computational errors on standard computer programs, given the algorithms that are typically used for regression analysis. Cronbach (1987) suggests centering the $X_1$ and $X_2$ variables (prior to transforming the multiplicative term) as a means of addressing this problem. Such a transformation will tend to yield low correlations between the product term and the component parts of the term. We also recommend the transformation suggested by Cronbach" (Jaccard et al., p. 31).

d) **Centering substantively changes in neither the effects *nor their standard errors*:** Aside from this technical concern about computer math in binary numbers, centering (or any linear rescaling of variables) changes *none* of the answers to *any* of the substantive questions one might ask about interactions. At the same substantive levels of X & Z, the estimated effects of X & Z *and the standard errors of those effects* are the same with or without centering (rescaling).

e) **Centering does not substantively change statistical significance:** Centering variables does tend to give smaller standard errors (and often more significant t-test results) on the *coefficients* for the separate terms, X and Z. The standard errors and coefficients for the interaction term are identical. However, this is only because the substantive level of Z (or X) to which these standard errors (and t-tests) refer has changed; Z=0 is different from centered-Z=0 and X=0 is different from centered-X=0.

2. **NESTED INTERACTIONS:**

> Including all the sub-elements of interactions is a matter of practical advice, and should be taken as such, rather than as a logical requirement grounded in statistical reasoning. Indeed, for any variable Z, several X and W exist such that X*W=Z. **The rule of thumb is just an application of Occam's Razor.** Omitting sub-elements forces (some) effect-lines' intercepts to zero, which is similar to leaving constants or fixed effects out. Generally, we suggest reporting non-interactive, full-interactive, and semi-interactive models if scholars are going to drop some sub-terms.

## OUR RECOMMENDATIONS

**MODEL BUILDING:** Examples of Arguments that Imply Interactions

1. **BEHAVIORAL STUDIES:**

    **a)** Between-group differences: e.g., black and white political participation may depend in different ways on group-consciousness

    **b)** Experimental conditions (replacing, or adding to, ANOVA): e.g., examination not only of whether but also of how the effects of experimental condition 1 on the subject's behavior, depend on condition 2

    **c)** Contextual analyses: e.g. ,the relationship between citizen sense of efficacy and attention to campaigns may be contingent upon the level of activism within the citizen's neighborhood

2. **INSTITUTIONAL STUDIES:**

    **a)** "Funneling": e.g., interests are funneled through institutions to affect policies; institutions moderate the impacts of various structures of interests on policies

    **b)** Principle-agent models/ Divided control of outcomes: e.g., principal would act according to f(X), and agent would act according to g(Z), each left unmolested. Some institutional conditions, I, (e.g., monitoring costs) determine degree to which P maintains control/enforces her will on A. Yields: Outcome= $h(I)*f(X) + (1-h(I))*g(Z)$

    **c)** Structural-change models: e.g. changes in institutions (Z) may influence how X affects Y; comparison across sub-samples

3. **STRATEGIC BEHAVIOR (MANY AREAS):** e.g., the behavior of one actor may be contingent on that actors' characteristics as well as another actor's behavior or characteristics

## MODELING AND TESTING INTERACTIVE HYPOTHESES

We argue for linking *(a) the theoretical hypothesis* with *(b) its mathematical expression,* and *(c) the corresponding statistical test*. Suppose, e.g., we were interested in the effect of X on Y. We think that the effect of X is contingent upon some other variable, Z. We also think that X can affect Y when Z is zero, and Z can affect Y when X is zero.

In a linear model, these contentions imply the following:

$$\mathbf{Y} = a + b_x\mathbf{X} + b_{xz}\mathbf{XZ} + b_z\mathbf{Z} + e$$

…which allows the effect of X to be non-zero when Z is zero, *via* $\beta_x$, the effect of Z to be non-zero when X is zero, *via* $\beta_z$, and the effect of X (or of Z) to depend on Z (on X), *via* $\beta_{xz}$.

Of such models, we typically ask the following types of theoretical questions:

1. ***Does Y depend on X (on Z)? Or, equivalently, is Y a function of X (of Z)?***
2. ***Is Y's dependence on X (on Z) contingent upon or moderated by Z (by X)? Or, equivalently, does the effect of X (of Z) on Y depend on Z (on X)?***
3. ***Does Y depend on X, Z, or X*Z at all? Or, equivalently, is Y a function of X, Z, and/or X*Z?***

## LINKING INTERACTIVE ARGUMENTS TO STATISTICAL TESTS

### *1. DOES Y DEPEND ON X?*

| *Hypothesis* | *Mathematical expression* | *statistical test* |
|---|---|---|
| "X affects Y", or "Y is a function (depends on) X" | Y=f(X) $H_0$: $\delta Y/\delta X = \hat{b}_x + \hat{b}_{xz} Z = 0$ | F- test: $\hat{b}_x = \hat{b}_{xz} = 0$ |
| "X increases Y" | $H_0$: $\delta Y/\delta X = \hat{b}_x + \hat{b}_{xz} Z > 0$ | Multiple t-tests: $\hat{b}_x + \hat{b}_{xz} Z > 0$ |
| "X decreasesY" | $H_0$: $\delta Y/\delta X = \hat{b}_x + \hat{b}_{xz} Z < 0$ | Multiple t- tests: $\hat{b}_x + \hat{b}_{xz} Z < 0$ |

*N.b., the effect of X on Y will be negative for some values of Z and positive for others because the interaction is linear. This implies that the effect will statistically differ from zero at some values of Z and be statistically indistinguishable from zero at other values of Z. The latter two hypotheses must therefore be stated more precisely to refer to some values or range of values of Z. See our recommendations below.*

### *2. IS Y'S DEPENDENCE ON X CONTINGENT ON Z?*

| *Hypothesis* | *Mathematical expression* | *statistical test* |
|---|---|---|
| "The effect of X on Y depends on Z" | Y = f ( XZ , • ) $H_0$: $\delta Y/\delta X = \hat{b}_x + \hat{b}_{xz} Z = g(Z)$ $H_0$: $\delta(\delta Y/\delta X)/\delta Z = \hat{b}_{xz} = 0$ | t- test: $\hat{b}_{xz} = 0$ |
| "The effect of X on Y increases in Z" | $H_0$: $\delta(\delta Y/\delta X)/\delta Z = \hat{b}_{xz} > 0$ | t-test: $\hat{b}_{xz} > 0$ |
| "The effect of X on Y decreases in Z" | $H_0$: $\delta(\delta Y/\delta X)/\delta Z = \hat{b}_{xz} < 0$ | t-test: $\hat{b}_{xz} < 0$ |

### *3. DOES Y DEPEND ON X, Z, OR X*Z?*

| *Hypothesis* | *Mathematical expression* | *statistical test* |
|---|---|---|
| "Y is a function of (depends on) X, Z,and/or their interaction." | $H_0$: Y = f ( X , Z , XZ ) | F-test: $\hat{b}_x = \hat{b}_{xz} = \hat{b}_z = 0$ |

## INTERPRETING STATISTICAL RESULTS

Conditional effects:

They are logically symmetric. If the effect of X on Y is conditional on Z, then the effect of Z on Y must be conditional on X: $\delta Y/\delta X = f(Z) \Longleftrightarrow \delta Y/\delta Z = h(X)$.

$$\text{E.g., } Y = \alpha + \beta_x X + \beta_{xz} XZ + \beta_z Z + \varepsilon \Longrightarrow \delta Y/\delta X = \beta_x + \beta_{xz} Z \ \&\ \delta Y/\delta Z = \beta_z + \beta_{xz} X \text{ and}$$
$$\delta(\delta Y/\delta Z)/\delta X = \delta(\delta Y/\delta X)/\delta Z = \beta_{xz}$$

*The derivative (or difference) method:*

Calculate $E[(\delta Y/\delta X)|Z]$ with associated standard errors, t-statistics, confidence intervals, p-values.

$$E[(\delta Y/\delta X)|Z=\{low..high\}] = \hat{b}_x + \hat{b}_{xz} Z$$

$$Var[(\delta Y/\delta X)|Z=\{low..high\}] = Var(\hat{b}_x) + Var(\hat{b}_{xz})Z^2 + 2Cov(\hat{b}_x, \hat{b}_{xz})Z$$

Conditional expectations:

The predicted-values method: predictions with associated standard errors, t-statistics, confidence intervals, p-values.

$$E[Y|X=\{low..high\},Z=\{low..high\},else=fixed]$$

$$\Longrightarrow \hat{y} = X^0\hat{B} = \hat{a} + x^0\hat{b}_x + z^0\hat{b}_z + x^0z^0\hat{b}_{xz} \ \hat{y}^0$$

$$Var[\hat{y}^0] = X^{0\prime}[V(\hat{B})]X^0$$

confidence (or forecast) interval: $\hat{y}^0 \pm t_{\lambda/2} se(\hat{y}^0)$

## THE PRESENTATION OF INTERACTIVE HYPOTHESES

Suppose we were to estimate the following equation:[1]

Political Participation = $\alpha+\beta_1$Education+$\beta_2$Media Exposure+$\beta_3$Education*Media Exposure+$\varepsilon$

### REGRESSION RESULTS

| VARIABLE NAME | COEFFICIENT (STANDARD ERROR) | VARIANCE COVARIANCE MATRIX | | | |
|---|---|---|---|---|---|
| | | *Education* | *Education* Media Exposure* | *Media Exposure* | *Constant* |
| ***Education*** | .161 (.032) | .00103 | | | |
| ***Education* Media Exposure*** | .096 (.046) | -.00132 | .00208 | | |
| ***Media Exposure*** | .071 (.024) | .00058 | -.00094 | .00058 | |
| ***Constant*** | .090 (.016) | -.00045 | .00058 | -.00035 | .00026 |

n=2045, Source: 1992 National Election Study.

### 1. TABLE OF CONDITIONAL EFFECTS OF EDUCATION, WITH STANDARD ERRORS

| ***Media Exposure (Sample or Logical Range)*** | | ***Conditional Effect*** $(\delta Y/\delta X\|Z)= \hat{b}_x+(\hat{b}_{xz})Z$ | ***Standard Error of Conditional Effect*** $Var(\delta Y/\delta X\|Z)= [Var(\hat{b}_x)+Var(\hat{b}_{xz})Z^2+2Cov(\hat{b}_x,\hat{b}_{xz})Z]^{.5}$ | ***Lower and Upper Bounds of 95% Confidence Interval*** $CI_L = (\delta Y/\delta X\|Z) - t_{.05/2}*s.e.(\delta Y/\delta X\|Z)$ | $CI_U = (\delta Y/\delta X\|Z) + t_{.05/2}*s.e.(\delta Y/\delta X\|Z)$ |
|---|---|---|---|---|---|
| **Col\Row** | **A** | **B** | **C** | **D** | **E** |
| 11 | 0 | 0.16 | 0.03 | 0.10 | 0.22 |
| 12 | 0.1 | 0.17 | 0.03 | 0.12 | 0.23 |
| 13 | 0.2 | 0.18 | 0.02 | 0.13 | 0.23 |
| 14 | 0.3 | 0.19 | 0.02 | 0.15 | 0.23 |
| 15 | 0.4 | 0.20 | 0.02 | 0.16 | 0.23 |
| 16 | 0.5 | 0.21 | 0.02 | 0.18 | 0.24 |
| 17 | 0.6 | 0.22 | 0.01 | 0.19 | 0.25 |
| 18 | 0.7 | 0.23 | 0.01 | 0.20 | 0.26 |
| 19 | 0.8 | 0.24 | 0.02 | 0.21 | 0.27 |
| 20 | 0.9 | 0.25 | 0.02 | 0.21 | 0.28 |
| 21 | 1 | 0.26 | 0.02 | 0.21 | 0.30 |

### Spreadsheet Formulae Entered To Generate Table

**Column A:** Enter “0” in row 11. In row 12 enter *+A11+.1* and copy it down to rest of column.

**Column B:** Place the estimated coefficient on Education in cell A9 and the coefficient on Education * Media Exposure in B9. Enter *+$A$9+$B$9*A11* in cell B11 and copy down.

[1] All variables are scaled from 0-1. Political Participation is an additive scale of participatory acts. Education has 7 categories. Media Exposure measures how often respondent watches the TV news and reads the newspaper. This is a pedagogical example that is not to be taken too seriously.

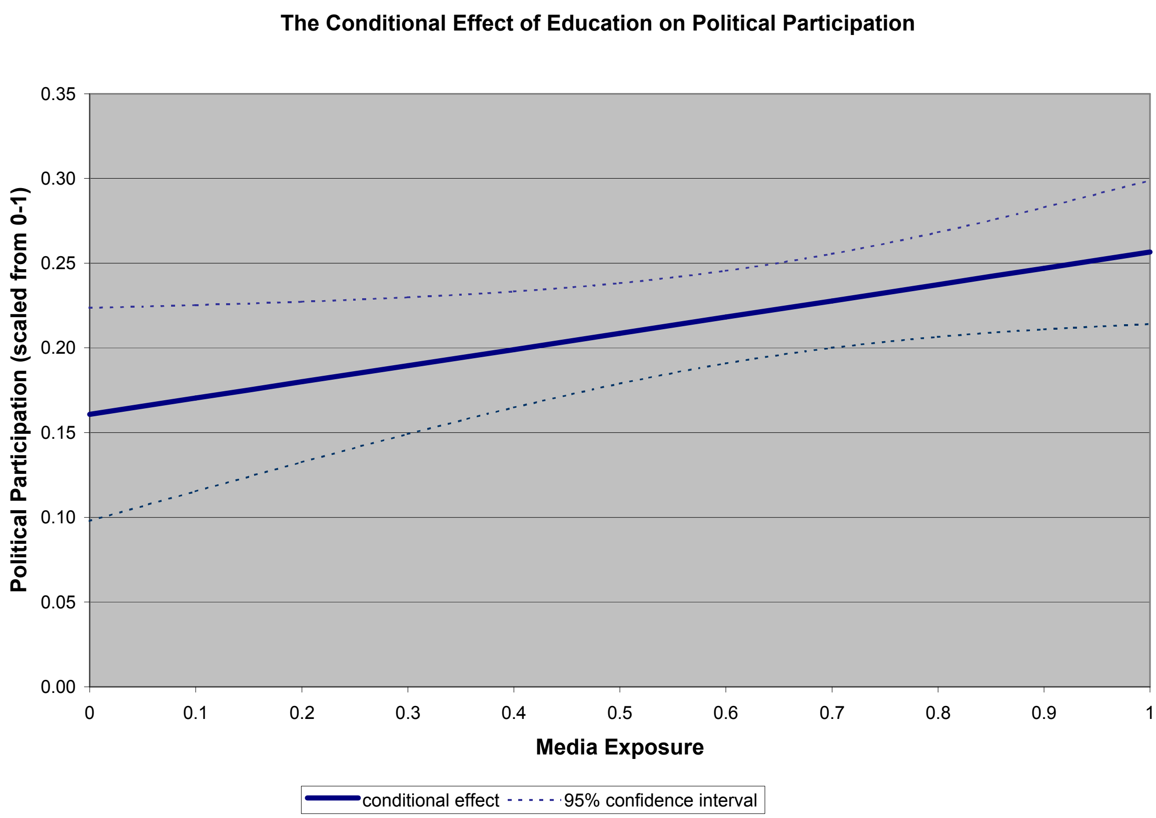
**Column C:** Place the estimated variance-covariance matrix of the coefficients on Education and Education * Media Exposure in cells C8…D9. Enter *($C$8+$D$9*(A11^2)+2*$C$9*A11)^.5* in cell C11 and copy down.

**Column D:** Enter *+$B11-[TVALUE]*$C11* in cell D11 and copy it down to E21. (The requisite t-value can also be calculated by spreadsheet, but formulae differ from Excel to 1-2-3 to Quattro.)

**Column E:** In cell E11, edit the formula just copied from D11, replacing the – sign with a + sign. Copy that down.

*Note: Calculating t-statistics and associated p-levels to test whether the conditional effect significantly differs from zero at each level of media exposure is straightforward. Divide Column B by Column C for t-statistics. Use spreadsheet formula for t-distributions to calculate the associated p-levels.*

## GRAPHS OF CONDITIONAL EFFECTS, WITH MEASURES OF UNCERTAINTY




We recommend that scholars plot E[d(Y)/d(X)|Z] (column B above) over some logically possible range of Z, its sample range, its inter-quartile range, or some other substantively meaningful but large range, with confidence intervals. Such graphs provide the effect of a one-unit increase in, e.g., education on political participation. Since education in our example is scaled from 0-1, the lines illustrate the "maximum effect" of education on political participation, conditional on varying levels of media exposure. To plot effects of smaller changes, say 0.1 increases, multiply columns B and C by 0.1 (e.g., edit cell B11 to *+.1*($A$9+$B$9*A11)* and copy down. Similarly edit cell C11 to *0.1*(($C$8+$D$9*(A11^2)+2*$C$9*A11)^.5)* and copy down. Columns D and E will adjust automatically.

**TABLE OF CONDITIONAL EXPECTATIONS OF PARTICIPATION, WITH STANDARD ERRORS**

*Conditional Expectations:*

$$\hat{y}^0 = .090 + .161(\text{Education}) + .096(\text{Education*Media Exposure}) + .071(\text{Media Exposure})$$

$$\text{Var}[\hat{y}^0] = X^{0\prime}[\text{Variance-Covariance Matrix of } \hat{b}]X^0$$

where $X^0$ is the vector of values at which the variables are fixed.

*Confidence intervals:*

$$\hat{y}^0 \pm t_{.05/2}*\text{se}(\hat{y}^0)$$

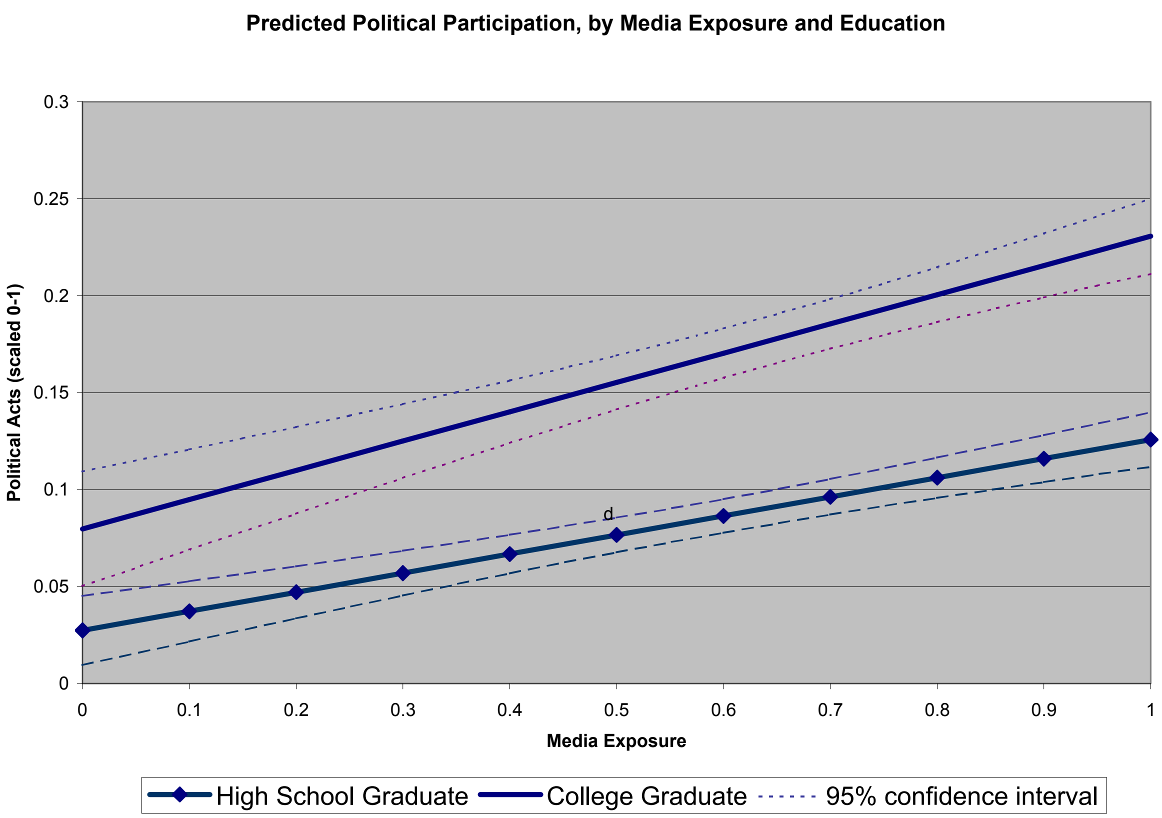
**Predicted Value of Participatory Acts, Given Level of Education and Amount of Media Exposure**

| exposure | High School (HS) Graduate | lower and upper bounds of confidence interval for HS | | College Graduate (CG) | lower and upper bounds of confidence interval for CG | |
|---|---|---|---|---|---|---|
| 0 | 0.03 | 0.03 | 0.03 | 0.08 | 0.05 | 0.11 |
| 0.1 | 0.04 | 0.04 | 0.04 | 0.09 | 0.07 | 0.12 |
| 0.2 | 0.05 | 0.05 | 0.05 | 0.11 | 0.09 | 0.13 |
| 0.3 | 0.06 | 0.06 | 0.06 | 0.13 | 0.11 | 0.14 |
| 0.4 | 0.07 | 0.07 | 0.07 | 0.14 | 0.12 | 0.16 |
| 0.5 | 0.08 | 0.08 | 0.08 | 0.16 | 0.14 | 0.17 |
| 0.6 | 0.09 | 0.09 | 0.09 | 0.17 | 0.16 | 0.18 |
| 0.7 | 0.10 | 0.10 | 0.10 | 0.19 | 0.17 | 0.20 |
| 0.8 | 0.11 | 0.11 | 0.11 | 0.20 | 0.19 | 0.21 |
| 0.9 | 0.12 | 0.12 | 0.12 | 0.22 | 0.20 | 0.23 |
| 1 | 0.13 | 0.13 | 0.13 | 0.23 | 0.21 | 0.25 |

Note: The spreadsheet formulae from the previous table provide a useful template for contructing this one. However, matrix manipulation can be cumbersome in spreadsheets. We are exploring simpler methods, possibly combining spreadsheet and statistical software.

### Graphs of Conditional Expectations (or Predicted Values), with Measures of Uncertainty

These tables and graphs present much the same information in slightly different format. Whereas the conditional-effect tables and graphs document the estimated effects on participation of increasing education by on unit (from any startin level) as a function of the level of media exposure, the conditional-expectations tables and graphs document the estimated amount of participation as a function of media exposure for some given levels of education.




Conditional expectation graphs and tables are more useful for comparing whether predicted levels of, e.g., participation vary by education at different levels of media exposure. Conditional effect graphs and tables are more useful for comparing whether the impact of, e.g., education on participation depends substantively and statistically importantly on media exposure.

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[RM & CK1](Gelpi, APSR 91(2))

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[RM & CK2]Gelpi: "Very high levels of multicollinearity, however, prevent the coefficients in that model from achieving statistical significance." That's why he converted some of the categorical variables into dummies. Technically, statistical significance should depend on the values of the variables…

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this quote is from Gilens, 1996. APSR.
Page: 5
this quote is from Box-Steffensmeier et al., 1997. APSR.