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INTENSIFICATION OF MAGNETIC FIELDS BY CONVERSION OF POTENTIAL ENERGY

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ABSTRACT

A strong superequipartition magnetic field strength on the order of 10 T (10^5 G) has been inferred at the bottom of the solar convection zone. We show that the "explosion" of weak magnetic flux tubes, which is caused by a sudden loss of pressure equilibrium in the flux loop rising through the superadiabatically stratified convection zone, provides a mechanism that leads to a strong field: the flow of high-entropy material out of the exploded loop leads to a significant intensification of the magnetic field in the underlying flux sheet at the bottom. In contrast to the amplification by differential rotation, this process converts the potential energy of the stratification into magnetic energy and thus is not dynamically limited by the back-reaction on the flow field via the Lorentz force.

Subject headings: MHD — Sun: interior — Sun: magnetic fields

1. INTRODUCTION

Studies of magnetic flux tubes in the solar convection zone indicate a field strength of about 10 T (10^5 G) at the base of the convection zone. This value was first suggested by van Ballegooijen (1982), who considered hydrostatic flux tubes extending across the whole depth of the convection zone. A field strength on the same order was found by Ferriz-Mas & Schüssler (1993, 1995) as a threshold value for the undulatory (Parker) instability (Spruit & van Ballegooijen 1982) of thin flux tubes in the solar overshoot region. The rise of flux tubes through the convection zone has been simulated in the thin flux-tube approximation by Choudhuri & Gilman (1987), Moreno-Insertis (1992), D'Silva & Choudhuri (1993), Fan, Fisher, & DeLuca (1993), Schüssler et al. (1994), and Caligari, Moreno-Insertis, & Schüssler (1995, 1998). They found that the simulations match basic sunspot properties, like the emergence at low latitudes and the variation of the tilt angle of bipolar groups with latitude, for an initial field strength of about 10 T at the base of the convection zone. The results of two- and three-dimensional simulations of rising flux tubes (Longcope, Fisher, & Arendt 1996; Moreno-Insertis & Emonet 1996; Fan, Zweibel, & Lantz 1998; Fan 2001; Dorch & Nordlund 1998) are in general accordance with the thin flux-tube simulations. It is doubtful whether the shear stress of the radial differential rotation at the base of the convection zone is capable of generating such a strong toroidal field. The magnetic energy density of a 10 T field exceeds the kinetic energy density of the differential rotation by a factor of 10–100, so that the back-reaction of the magnetic field on the differential rotation via the Lorentz force becomes vigorous.

A possible intensification mechanism, not relying on mechanical stress, was proposed by Moreno-Insertis, Caligari, & Schüssler (1995). In simulations of the undulatory (Parker) instability of thin flux tubes in the solar convection zone, they found that magnetic flux tubes with a low initial field strength do not reach the upper part of the convection zone. The reason is a sudden weakening of the field strength at the apex of the rising flux loop, which occurs if approximate hydrostatic equilibrium along the field lines holds during the rise. Since the radiative exchange of heat is negligible on the timescale of the development of the undulatory instability, the plasma remains nearly isentropic within the rising flux loop, whereas the entropy decreases with height in the surrounding superadiabatic convection zone. As a consequence of the higher entropy within the magnetic flux tube, the external pressure decreases faster with height than the internal pressure and thus eventually becomes equal to the internal gas pressure at a certain "explosion height" above the bottom of the convection zone. In the case of a constant superadiabaticity, $\delta = \nabla - \nabla_{\rm ad}$, the explosion height is given by $z_{\rm ex} \approx H_p\,[2/(\beta\delta)]^{1/2}$ (where H_p denotes the pressure scale height and $\beta = 2\mu_0 p_{\rm gas}/B^2$). When the loop apex approaches the explosion height, it expands drastically, since the pressure balance forces the magnetic field to approach zero, so that it is no longer dynamically relevant at the apex of the loop. The development after such an explosion is our primary interest since the buoyancy of the high-entropy material in the remaining "stumps" of the loop can drive an outflow that amplifies the magnetic field by reducing the gas pressure in the nonexploded part of the flux tube. Since the approximation of a thin magnetic flux tube breaks down during an explosion, this process can only be studied through full MHD simulations.

2. NUMERICAL MODEL

As a first step beyond the thin flux-tube approximation, we have performed simulations of exploding magnetic flux sheets in two-dimensional Cartesian geometry. We do not yet aim at a realistic simulation under solar conditions but rather consider a model that allows us to study the explosion process and its suitability as a field amplification mechanism in (artificial) isolation.

We consider a rectangular computational box with the vertical coordinate z representing depth and the horizontal coordinate x corresponding to the azimuthal direction on the Sun. We impose periodic side boundaries and closed top and bottom boundaries; the magnetic field (B_x, B_z) is taken to be vertical at the top boundary and symmetric at the bottom boundary. The simulations have been carried out with the Versatile Advection Code[1] (Tóth 1996) using a Riemann-solver–based total variation diminishing scheme. We solve the ideal MHD equations without taking into consideration the very small values of the physical viscosity and magnetic diffusivity. Since we have to resolve very thin magnetic structures, numerical grid sizes between 384 × 384 and 768 × 512 points are required

[1] Available at http://www.phys.uu.nl/~toth.

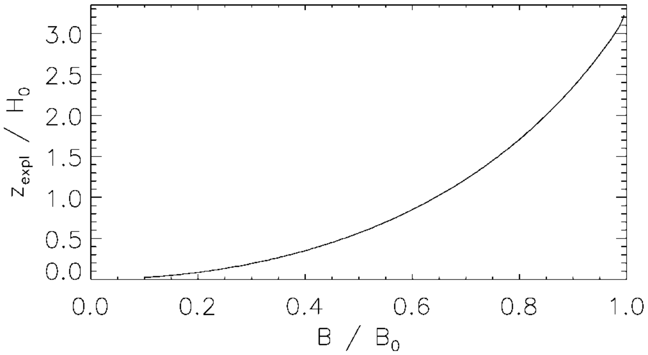

FIG. 1.—Explosion height (in units of the pressure scale height at $z = 0$) as a function of field strength at $z = 0$ (normalized to the value for which the explosion takes place at the top of the integration domain).

in order to keep the influence of the numerical diffusivity sufficiently small. By varying the grid resolution, we have convinced ourselves that the evolution of the magnetic field strength in time (see Fig. 4 below) converged.

The initial magnetic configuration consists of a thin magnetic flux sheet located near the bottom of the box. The stratification, covering about 5 pressure scale heights, is isentropic (adiabatic, $\delta = 0$) above the flux sheet and strongly subadiabatic ($\delta = -0.2$) below, so that the global buoyancy instability of the sheet is suppressed. The specific entropy of the gas in the flux sheet is larger than that of the medium above by a value $\Delta s > 0$, providing the necessary condition for the explosion. Figure 1 gives the corresponding explosion height, $z_{\rm ex} \approx H_p c_p/\beta\Delta s$, as a function of field strength B.

In order to trigger the explosion in the simulation, the flux sheet is deformed into a Gaussian loop, keeping hydrostatic equilibrium along the field lines. In our setup of the simulation, only a slight deformation of about $(0.1\text{–}0.3)H_0$ (see Fig. 2) is required to bring the apex close to the explosion height. As the loop then rises owing to its buoyancy, its apex crosses the explosion height.

Although differing from the conditions in the real Sun, this configuration nevertheless captures the essential properties relevant for the intensification of exploding flux tubes in the lower convection zone of the Sun, i.e., a rising flux loop near hydrostatic equilibrium with a positive entropy contrast, $\Delta s > 0$, and an "anchored" horizontal part. Realistic solar values of β in the range of 10^5–10^7 cannot be used in numerical simulations based on the full set of ideal MHD equations. The typical timescale for the dynamics is the Alfvénic timescale, whereas the time step of the code is limited by the sound speed that exceeds the Alfvén speed by a factor of 300–3000. Therefore, we performed our simulations with values of β in the range of 20–800, which are still large compared to unity. We show below that the results can be extrapolated to realistic solar values of β by an appropriate adaptation of the timescale.

3. RESULTS

Figure 2 illustrates the evolution of a magnetic sheet with $\beta = 400$ through and after the explosion. Owing to the buoyancy of its high-entropy gas, the elevated loop rises and crosses the explosion height. As a consequence of the ensuing explosion, the flux loop forms two vertical stumps, which are connected by a "cloud" of weak magnetic field (Fig. 2, *middle panel*). After the formation of the stumps, the buoyancy-driven outflow continues and drains material from the lower horizontal part of the magnetic sheet (Fig. 2, *bottom panel*), so that the gas pressure decreases, and the magnetic field is intensified. At the interface between magnetized and unmagnetized plasma at $z \approx 2H_0$, Kelvin-Helmholtz vortices form because the flow velocity exceeds the Alfvén speed. The high-entropy gas rises to the upper half of the box and remains there owing to its buoyancy (Fig. 3). The development of the field strength in the horizontal part of the magnetic sheet is shown in Figure 4. The initial evolution ($t \lesssim 0.4\tau_{\rm A}$) corresponds to the formation of the stumps. Once the outflow has started, the magnetic field strength increases approximately linearly in time ($0.4\tau_{\rm A} \lesssim t \lesssim 0.8\tau_{\rm A}$) until it saturates. Changing the initial field strength while keeping the same entropy contrast leads to nearly the same final field strength. Depending on the initial field strength, the amplification factor of the magnetic field varies between 2.5 and 4 in the case shown here, which corresponds to factors of 6.3–16 in the magnetic energy density.

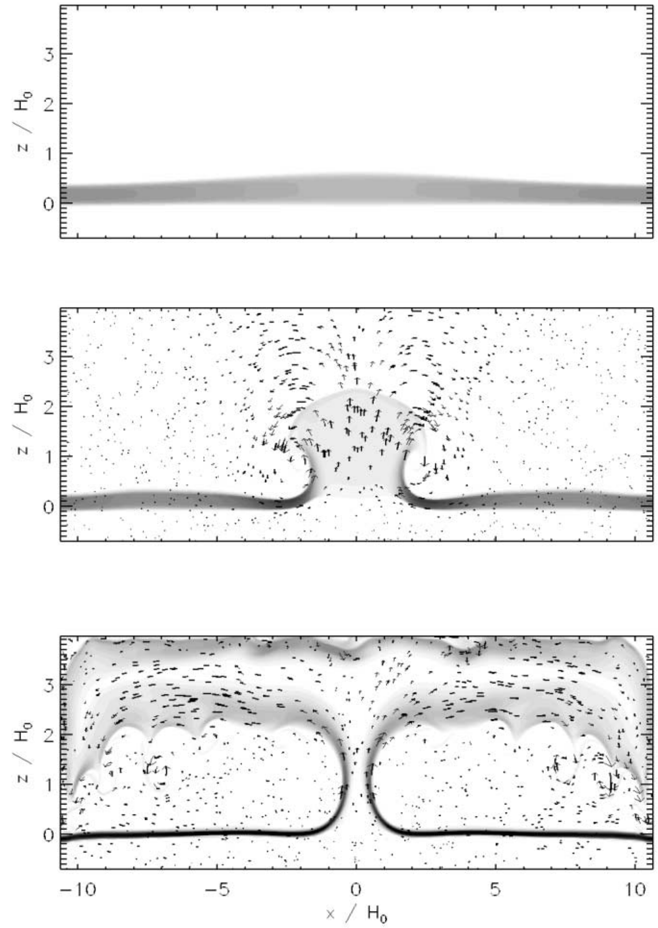

FIG. 2.—Evolution of magnetic field strength (*gray scale*; the dark gray scale denotes a strong field) and velocity field (*arrows*) during the explosion. The magnetic field vectors are within the *x-z* plane. Lengths are given in units of H_0, and the pressure scale height at $z = 0$. The sequence shows the initial state (*top panel*), the formation of two vertical stumps of the magnetic field (*middle panel*) after $0.4\tau_{\rm A}$, and the buoyancy-driven outflow of plasma out of the stumps and the formation of a cloud of weak field (*bottom panel*) after $0.8\tau_{\rm A}$. The stumps reach up to $z \approx 2H_0$, which corresponds to the explosion height of the intensified field (amplified by a factor of 3) in the horizontal part of the sheet; $\tau_{\rm A}$ is the traveling time of an Alfvén wave over the horizontal extension of the box.

The final field strength is mainly determined by the initial entropy contrast Δs. With increasing field strength, the effective

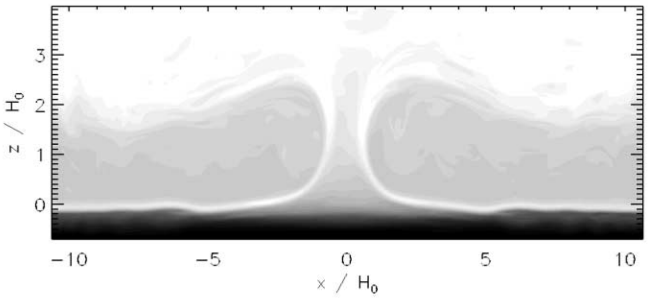

FIG. 3.—Entropy distribution (*gray scale*; the dark gray scale denotes low entropy) after the explosion ($t = 2\tau_A$). Owing to its buoyancy, the high-entropy plasma that was initially within the flux sheet remains in the upper half of the domain.

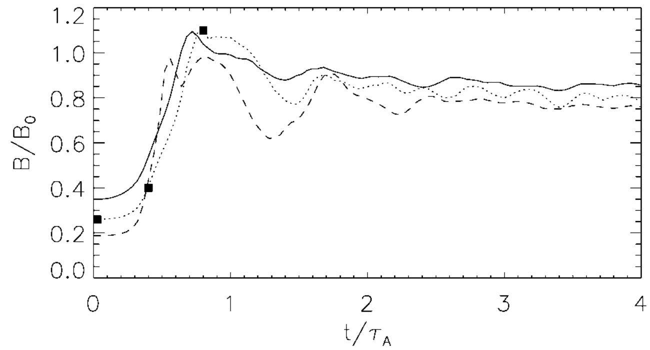

FIG. 4.—Time evolution of the magnetic field strength in the horizontal bottom part of the flux sheet. The field strength is normalized to the field strength for which the explosion would take place at the top of the domain. The time is given in units of the horizontal Alfvén travel time for a field strength of $0.3B_0$ at the level $z = 0$. The three lines correspond to initial states with different field strength but equal magnetic flux. The time steps shown in Fig. 2 are indicated as squares on the dotted line.

explosion height is shifted upward until it reaches the height range where the material with high entropy has been deposited. Since the entropy contrast vanishes there, the material flowing out of the stumps of the sheet no longer has an effective buoyancy and the outflow stops. The field strength in the horizontal (anchored) part of the sheet is now sufficiently high for hydrostatic equilibrium along the field lines without forcing B to approach zero below the level $z \approx 2H_0$ above which the high-entropy plasma has been deposited. From to Figure 1, we see that this requires a field strength of about $0.8B_0$, in accordance with the final field strength reached in the simulation (cf. Fig. 4).

Simulations with different values of β show basically the same pattern of evolution, only the timescale varies. The amplification time is roughly given by the travel time of an Alfvén wave along the nonexploded part of the flux tube. The outflow of plasma at the apex of the flux sheet causes a disturbance that travels roughly with Alfvén speed along the flux sheet. The typical outflow velocities are of the same order, so that the amplification time can be written as

$$\tau \approx \frac{l}{c_s}\left(\frac{\beta\gamma}{2}\right)^{1/2}, \qquad (1)$$

where l is the length of the nonexploded part of the flux sheet and c_s is the sound speed. This timescale corresponds to the phase of the linear increase of the field strength in Figure 4. The significantly steeper increase of the field strength for the lowest initial field is due to the fact that in this case, a second loop develops whose ensuing explosion accelerates the draining of mass from the flux sheet.

During the intensification process, material with higher entropy is transported upward in the box, so that the energy source is the potential energy of the system. This process is not dynamically limited by the Lorentz force since all relevant motions are parallel to the magnetic field. The final field strength is limited by the initial entropy contrast in the model considered here. In the solar case, this is determined by the superadiabaticity (entropy gradient) of the stratification. Since $\delta > 0$ is maintained by the energy flux through the convection zone, the necessary potential energy source for the amplification is always available.

4. RELEVANCE FOR THE SUN

The main deviations of our numerical model from the real Sun are the two-dimensional geometry, the low values of β, and the adiabatic background stratification without convective motion. Nevertheless, the results indicate that the explosion of flux tubes may be relevant for field intensification at the bottom of the solar convection zone.

The entropy difference between the magnetic loop and the surrounding plasma is crucial for the dynamics of the explosion process. The variation of the entropy contrast along the elongated loop determines both the explosion height and the buoyancy force that drives the outflow from the stumps. Our assumption of an adiabatic background stratification leads to a constant entropy difference, whereas the entropy difference in the solar case grows with height. This primarily has an influence on the explosion height as a function of field strength but not on the dynamics of the explosion process itself.

According to Moreno-Insertis et al. (1995), vertical convective motions of about 10 m s^{-1} would be sufficient to lift flux tubes with an equipartition field out of the overshoot region and thus initiate the explosion process as an alternative to the undulatory instability. For the explosion itself, the influence of the convective motions can be neglected since the relevant dynamical timescale of the explosion is much shorter than the convective timescale. The timescale of the outflow and intensification of the magnetic field is comparable to the convective timescale, but there is no obvious reason why large-scale convective motions should significantly affect the buoyancy-driven outflow of the high-entropy plasma. The main effect of convection would be to transport the outflowing material away from the flux tube and possibly turbulent mixing with the background plasma. Since this would diminish the entropy inversion of the background stratification visible in Figure 3, it can be expected that the effect of convection increases the saturation field strength of the amplification process.

Obviously, the dynamics of a two-dimensional flux sheet is different from the dynamics of a three-dimensional flux tube since the flux sheet divides the integration domain so that plasma cannot flow around the magnetic field as it can in the three-dimensional flux-tube case, so that the horizontal speed of matter away from the rising apex of the sheet is exaggerated. This has a marked influence on the wake of the rising loop. The initial field configuration that we use would be unstable in three-dimensional geometry and would form rising flux tubes. However, our basic result that the outflow of plasma and the intensification of the magnetic field continue until a new hydrostatic equilibrium is reached is not affected by this since there is no difference between the hydrostatics along field lines in two and three dimensions. Since three-dimensional geometry offers more space for the explosion and the outflowing high-entropy material (in the case of a single exploding flux tube), this would probably inhibit the formation of an entropy inversion and thus support the intensification process.

The values of β used here are much smaller than the solar ones; i.e., the field strength assumed is larger. However, we still have $\beta \gg 1$, so that the magnetic pressure is a small perturbation of the gas pressure in the background stratification. Consequently, our simulations are in the same physical regime as the lower solar convection zone. To test this, we have varied the value of β while keeping the quantity $\beta\Delta s$ constant (in order to maintain similar conditions for the explosion) and found no significant effect of β on the explosion process, except for the timescale: lower β (larger field strength) leads to a more rapid evolution. Inserting typical solar values ($l = 10^9$ m, $c_s = 2 \times 10^5$ m s^{-1}) in equation (1) leads to the following scaling of the amplification time with β:

$$\tau \approx \sqrt{\frac{\beta}{5 \times 10^7}}\ \text{yr}, \tag{2}$$

so that $\tau \lesssim 0.5$ yr for $\beta \lesssim 10^7$. Consequently, field intensification by explosion takes place on a timescale that is short compared with the solar cycle length.

We have found that the field strength is on the order of the value for which the explosion height is close to the top of the stratification. Moreno-Insertis et al. (1995) show that for the solar convection zone model of Skaley & Stix (1991), the explosion height is near the solar surface if the field strength at the base of the solar convection zone has a value of 10 T. Consequently, the inferred field strength at the bottom of the solar convection zone may be reached through intensification by the outflow of gas after an explosion without recourse to mechanical stress exerted by differential rotation.

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REFERENCES

Caligari, P., Moreno-Insertis, F., & Schüssler, M. 1995, ApJ, 441, 886

———. 1998, ApJ, 502, 481

Choudhuri, A. R., & Gilman, P. A. 1987, ApJ, 316, 788

Dorch, S. B. F., & Nordlund, Å. 1998, A&A, 338, 329

D'Silva, S., & Choudhuri, A. R. 1993, A&A, 272, 621

Fan, Y. 2001, ApJ, 546, 509

Fan, Y., Fisher, G. H., & DeLuca, E. E. 1993, ApJ, 405, 390

Fan, Y., Zweibel, E. G., & Lantz, S. R. 1998, ApJ, 493, 480

Ferriz-Mas, A., & Schüssler, M. 1993, Geophys. Astrophys. Fluid Dyn., 72, 209

———. 1995, Geophys. Astrophys. Fluid Dyn., 81, 233

Longcope, D. W., Fisher, G. H., & Arendt, S. 1996, ApJ, 464, 999

Moreno-Insertis, F. 1992, in Sunspots: Theory and Observations, ed. J. H. Thomas & N. O. Weiss (NATO ASI Ser. C, 375; Dordrecht: Kluwer), 385

Moreno-Insertis, F., Caligari, P., & Schüssler, M. 1995, ApJ, 452, 894

Moreno-Insertis, F., & Emonet, T. 1996, ApJ, 472, L53

Schüssler, M., Caligari, P., Ferriz-Mas, A., & Moreno-Insertis, F. 1994, A&A, 281, L69

Skaley, D., & Stix, M. 1991, A&A, 241, 227

Spruit, H. C., & van Ballegooijen, A. A. 1982, A&A, 106, 58

Tóth, G. 1996, Astrophys. Lett., 34, 245

van Ballegooijen, A. A. 1982, A&A, 106, 43