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Three-Dimensional Instability of a Magnetized Wake Flow Embedded in the Current Sheet

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Introduction

A magnetohydrodynamic equilibrium configuration with a planar wake flow embedded into the three-dimensional sheared magnetic field of a constant magnitude is widely used to model physical systems which have both magnetic and velocity shears, such as heliospheric current sheet embedded in the solar wind, coronal streamers, etc. In this paper, we revisit the stability properties for a compressible wake – current sheet model of streamer belt dynamics, and extend earlier two-dimensional nonlinear simulations [1, 2] to three-dimensional MHD computations [3]. It is the main result of this paper to show that (1) strongly supersonic magnetized wakes, characterized by a plasma beta of order unity, have dominant 3D sinuous instabilities which (2) can lead to fully 3D structured fast magnetosonic shock fronts in their nonlinear evolution.

To model the wake-current sheet system we used the set of one-fluid compressible resistive MHD equations. The magnetized wake flow co-spatial with the current sheet was idealized as a force-free equilibrium configuration:

$$\begin{aligned} V_x = 1 - \cosh^{-1} y,\ V_y = 0,\ V_z = 0 \\ B_x = M_a^{-1} \tanh \frac{y}{w},\ B_y = 0,\ B_z = M_a^{-1} \cosh^{-1} \frac{y}{w} \\ \rho = 1.0,\ p = (\gamma M_s^2)^{-1}, \end{aligned} \qquad (1)$$

where M_s and M_a are the sonic and Alfvénic Mach numbers for the fast flow streams, respectively, and w describes the thickness of the current sheet relative to the width of the wake flow. The latter, together with the density and velocity of the fast flow streams, has been used to define our unit system. The plasma β is then found from $\beta = (2M_a^2)/(\gamma M_s^2)$. If not mentioned otherwise, we set $w = 1$.

To analyze linear stability properties of the wake-current sheet configuration we used the LEDAFLOW code [4], which computes the complete MHD spectrum of all waves and instabilities that are eigenfrequencies of one-dimensionally varying equilibria containing background plasma flows. Nonlinear modeling was performed by the general finite-volume-based Versatile Advection Code (VAC) [5]. For all runs we used the full set of compressible resistive MHD equations with a constant resistivity $\eta = 10^{-4}$ value. The one-step total variation diminishing (TVD) method was employed, which is a second-order accurate shock-capturing scheme. The rectangular computational domain has periodic boundaries in the streamwise x-direction, and open boundaries in the cross-stream direction y. For 3D runs, the spanwise direction z is periodic as well. We used non-equidistant rectangular grids with a symmetric (around $y = 0$) accumulation near the core of the wake.

Linear stability analysis

Depending on the parameters of the one-dimensionally varying (but fully 3D structured) equilibrium configuration given by Eqs. (1), up to three different mode types (sinuous, varicose and resistive varicose) may be destabilized in the wake-current sheet configuration. Here we shall focus at the instability of ideal sinuous type, since in the range of Mach and Alfvén Mach numbers investigated, it always has the largest growth rate. The maximum sinuous mode growth rate is achieved for an incompressible, purely hydrodynamical case. Compressibility, as well as the increase of magnetic field acts to suppress the sinuous instability. This is also seen in Fig. 1, panel (A). The range of destabilized k_x decreases, and the streamwise wavelength of the fastest growing mode decreases. When M_a becomes smaller than some critical value M_a^{cr}, the system is stable. This critical value M_a^{cr} was found to be approximately equal to 2.5, and this value is rather universal in a wide range of studied cases ($0 < M_s < 10$). Generally, with an increase of the sonic Mach number M_s, the overall maximal growth rate of the sinuous instability goes down, as well as the diapason of destabilized wavenumbers. For an Alfvén Mach number $M_a = 3.5$, variations from Mach 2 till Mach 15 flows on the sinuous growth rates are shown in panel (B) from Fig. 1.

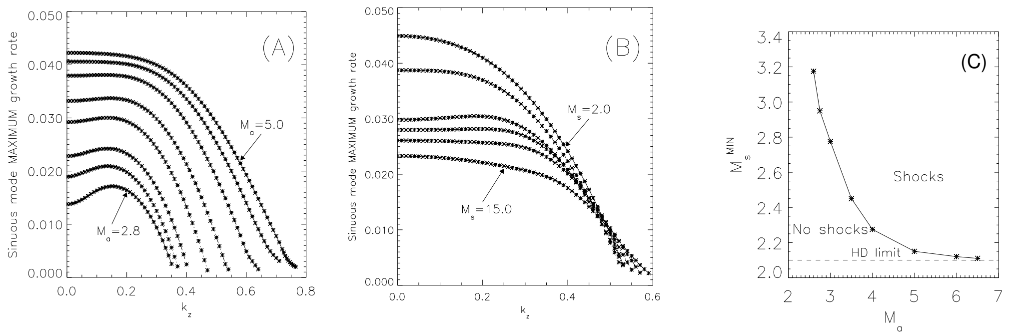

Figure 1: (A),(B): Maximum growth rates of the ideal sinuous mode in a wake – current sheet system versus spanwise wavenumber for different M_s and M_a. The streamwise wavenumber k_x varies from curve to curve. (A) At fixed $M_s = 3$, curves corresponding to $M_a = \{5.0, 4.5, 4.0, 3.5, 3.25, 3.0, 2.9, 2.8\}$ top to bottom, (B) At fixed $M_a = 3.5$, curves corresponding to $M_s = \{2.0, 2.5, 3.5, 4.0, 5.0, 15.0\}$ top to bottom. (C): Threshold value of sonic Mach number necessary for shock formation versus Alfvén Mach number.

The most important finding from Fig. 1 is that the most unstable sinuous mode may become oblique to the shear layer in a wake – current sheet system for sufficiently supersonic flows. We found that for a given sonic Mach number larger than $M_s \approx 2.6$, there exists a range in M_a where the maximum growth occurs for a mode with nonzero spanwise wavenumber k_z. That diapason of M_a for dominant three-dimensional instability is bounded from below and from above, and most wide near $M_s \approx 2.6$ where it is $2.9 < M_a < 4$. Its size decreases slowly with increasing M_s, so that even for highly supersonic wake flows the 3D instability domain is non-negligible, e.g. for $M_s = 10$ it is found in the range $2.65 < M_a < 3.025$.

Nonlinear evolution and shock formation

To start the sinuous instability in fully nonlinear time-dependent MHD simulations, the initial equilibrium configuration given by Eqs. (1) was perturbed by a symmetric cross-stream velocity perturbation. At the nonlinear stage of the instability, exactly at the point of maximal density contrast, shock waves may be formed. The shocks carry strong density contrasts, up to a factor of 2. Using the Rankine-Hugoniot relations we identified these shocks to be of fast magnetosonic type. The steepening of the sinuous wave fronts into shock waves is possible only above a threshold value of sonic Mach number. That threshold depends on the amplitude of the magnetic field (hence on M_a). In a pure hydrodynamical case ($M_a = \infty$), sonic shocks are formed for supersonic wake flows with $M_s \geq 2.10$. Shocks appeared for all studied hydrodynamic cases in the range $2.10 < M_s < 10.0$ and no upper limit of M_s for their formation was found. This hydro critical value is recovered in weak ($M_a > 5$) magnetized wakes. However, with an increase of the magnetic field magnitude, the minimum value M_s^{Min} of the sonic Mach number needed for shock formation, is increased. This is presented in Fig. 1 (C), where threshold sonic Mach numbers are plotted versus the Alfvén Mach numbers. Formation of the shock fronts was shown to be possible for any M_a where the sinuous mode is still unstable ($M_a > 2.5$). Hence, for any magnetic field strength, where the sinuous instability is not suppressed linearly, we were able to identify a shock-treshold value of Mach number M_s^{Min}. For all $M_s > M_s^{Min}(M_a)$ shock fronts form self-consistently through wave front steepening. 3D shock structures can even emerge from wakes with dominant 2D linear instabilities, but only under rather selective initial excitations, as it is shown in Fig. 2. There, the two-dimensional mode starts to grow from the noise level and obtains noticeable amplitude only at times when the 3D instability is close to saturation. The shock fronts have a truly 3D topology.

Conclusions

In this paper we performed a numerical study of the linear properties and nonlinear evolution of wake – current sheet configurations by means of compressible resistive MHD simulations. In contrast to previous studies, we have shown that there exist ranges of Mach number M_s and Alfvén Mach number M_a, where the most unstable sinuous mode is oblique to the shear layer. Such pure three-dimensional instabilities develop nonlinearly into fully 3D deformations of the wake system containing fast magnetosonic shock fronts. For dominant 3D instabilities, the Mach number should exceed a threshold value $M_s > 2.6$. With increase

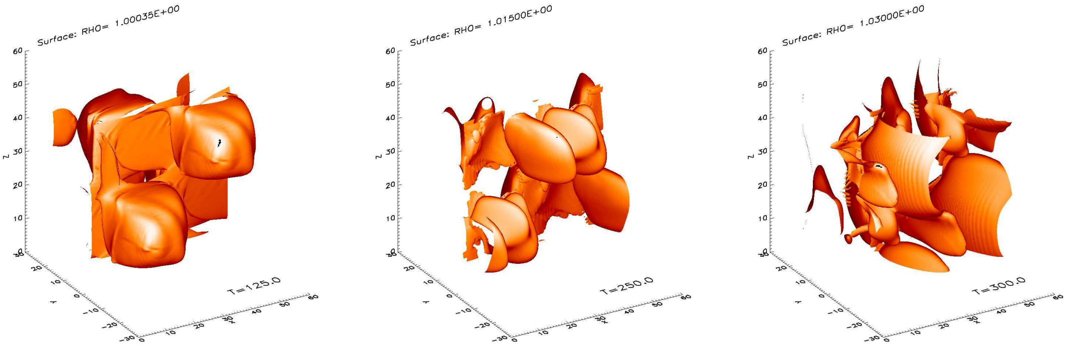

Figure 2: 3D MHD simulations of a wake-current sheet with $M_s = 3$, $M_a = 5$. 3D mode ($k_x = 0.35/3 = 0.11667$, $k_z = 0.125$) is excited, although the most unstable mode is 2D having ($k_x = 0.35$, $k_z = 0$). Note the 3D structuring of the wake due to mode-mode interplay.

of M_s, the range of M_a corresponding to 3D instability decreases, but even for highly supersonic cases, a dominant 3D instability still exists in some range of M_a. At the nonlinear stage of this sinuous instability in the supersonic regime, self-consistent formation of fast magnetosonic shocks is observed. They carry density jumps (by factors of 2) in cross-stream fast flow regions far away from the wake center. Shock fronts appear only for Mach numbers above some critical value, which depends on magnetic field strength, but is always larger than the hydro limit $M_s = 2.1$.

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