ECON 211A/240, Problem Set 3.

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Question 1: Intergenerational Investments in Human Capital.

The economy lasts two periods. In period 1, an individual (parent) works for wage w, consumes c, saves s, decides how much education e to purchase on behalf of his/her offspring, and then dies at the end of the period. Utility of household i is given as $U_i = \ln c_i + \ln \hat{c}_i$ in which $\hat{c}_i$ is the consumption of the offspring. There is heterogeneity among children, so the cost of education, $\theta_i e_i$, varies across i. In the second period, individuals receive a wage $w(e)$, where $w' > 0$ and $w'' < 0$ as usual.

1. Consider the case in which credit markets are perfect: households can borrow and lend at the same interest rate r. Characterize the household's decision problem. Show that the choice of education is independent of the form of the utility function.

2. Now assume a credit-market friction: households can lend at r, but cannot borrow going from period 1 to period 2. Write down the household's decision problem, including this new constraint. Show how the education and consumption decisions are no longer separable.

3. Now consider the following hypothetical situation: two households are identical, except that household A has a higher income and household B values offspring consumption more (e.g., U_B might be $\frac{1}{2}\ln c_B + \ln \hat{c}_B$.). How will their investments differ if credit markets are perfect? How will their investments differ if the households face a borrowing constraint as in (1.2)?

4. Describe how the equilibrium would change if the investment in education were "lumpy": e.g., $e = 0$ or $e = \bar{e} > 0$.

Question 2: Illiquidity and Panics.

(Based on Diamond/Dybvig.) Consider the following economy. There are three periods ($t = 0, 1, 2$) and N identical agents. Each agent is initially endowed with one unit of consumption good. All agents have access to a common investment technology that has a gross return of R in $t = 2$. Investment must be made at date $t = 0$. If α units are withdrawn at date 1, the output is $R(1-\alpha)$. There is a also a storage technology with a constant gross return of 1.

Agents are either type 1 or type 2, and each agent's type is revealed to him or her (but not publicly) at time $t = 1$. Type-1 agents have utility $u(c_1)$ and type-2 have utility $u(c_2)$ where $u()$ is increasing and strictly concave. In other words, agents of type i only get utility from consumption at date i. Assume that the coefficient of relative risk aversion is everywhere greater than 1. Exactly $M < N$ agents are of type 1. All agents have an independent and equal chance of being either type.

1. Assume that, in each period, there is a competitive market in claims on future goods (but no banks). What are the prices of future consumption?

2. Now assume that, when types are revealed in period 1, this information is publicly observable. Describe an optimal contract that a social planner can offer. Which type of agent has higher consumption? Why isn't this contract available in the private-information case? Explain why the agents *strictly* prefer the outcome in the public-information case.

3. For the remainder of the problem, assume that the agent's type is private information. Suppose a planner can offer a contract contingent only on an individual's announcement of his or her type at $t = 1$ (and not conditional upon the announcements of others). At $t = 1$, the planner meets each agent once, with the meeting order randomly determined. Write out a contract that the planner can offer agents. Can the allocation from part 2 be achieved? Explain (briefly) how the planner can be interpreted as a bank. What is a "bank run" in this setting?

4. Suppose that contracts can also depend on the announcement of previously encountered agents. Can the planner offer a contract that eliminates the possibility of bank runs? Suggest a modification to the model so that it may be optimal to refuse to pay out some agents of type 1 in period 1.

5. Assume that a bank run as described in your answer to part (2.3) occurs with probability p. For what values of p is it nevertheless optimal to have the bank. Describe the optimal solution otherwise.

Question 3: Equity versus Debt.

Assume that the economy is the same as in question 2. Now, consider a financial firm that offers shares with the following arrangement. A share pays d in period 1 and then pays the remainder of the firm's value at time 2. (Normalize the number of shares to one and allow the share to be infinitely divisible.) After the realization of types at $t = 1$ and the payment of dividends, shares can be traded at a price p (to be determined).

1. Find c_1 and c_2 in terms of d, p, and R.

2. Now use the supply and demand for shares at $t = 1$ to find p. What value for d will investors choose at $t = 0$? Explain.

3. Which crucial assumption of the Diamond/Dybvig model does this question seem to violate? How and why?

Question 4: Managing Currency Risk.

Consider the financing decision of the following firm. It has production function $f(k)$. There are two periods, $t = 0$ and $t = 1$. There is no discounting. A fraction α of its output is tradeable and $(1 - \alpha)$ is nontradeable. The market interest rate on tradeable debt is r^* and on nontradeable debt is r. For some reason, $r^* \geq r$. The exchange rate e denotes the price of tradeables in units of nontradeables (i.e., how much nontradeables are required to buy one unit of tradeables). Suppose further that $E_0[e] = 1$, and that e has a p.d.f. $h(e)$.

Capital is rented at $t = 0$ and returned (undepreciated) in $t = 1$. The firm has $w_0 > 0$ of net worth in $t = 0$. It borrows $L = K - w_0$ in period $t = 0$. A fraction β of this loan is denominated in units of tradeable goods and the rest in nontradeables. At $t = 1$, production occurs and loans are repaid. The firm keeps the residual, w_1, and has preferences $u(w_1)$ such that $u'() > 0$ and $u''() < 0$.

1. Justify the assumption that $u''(w_1) < 0$ as a reduced form for a credit friction. Why might this be a poor approximation?

2. Write down w_1 in units of nontradeables.

3. Describe the firm's decision problem.

4. Fix L. Write down the FOC for the optimal choice of β.

5. Call ($\beta = \alpha$) "perfect" matching or hedging. Under what circumstances is perfect hedging optimal?

Consider a two-point distribution for $h(e)$: a depreciation to e_d with probability q or an appreciation to e_a with probability $(1 - q)$. Remember that, in these units, $e_d > 1 > e_a$.

5. Solve for the optimal L and β.

6. Fix L again. Maintain through the assumption that $E_0[e] = 1$. Take as a stylized contrast between a fixed and flexible exchange rate that the fixed regime has a higher (more devalued) e_d, but a lower q. Should we expect more currency mismatch in the fixed or flexible exchange-rate regime? How does endogenizing L affect your conclusion?

7. Suppose that the wedge between r^* and r arises due to some distortion. Would the socially optimal equilibrium have more or less debt than in (4.5)? Explain.