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SUPERNOVAE, THE LENSED COSMIC MICROWAVE BACKGROUND, AND DARK ENERGY

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ABSTRACT

Supernova distance and primary cosmic microwave background (CMB) anisotropy measurements provide us with powerful probes of the dark energy evolution in a flat universe, but they degrade substantially once curvature is marginalized. We show that lensed CMB polarization power spectrum measurements, accessible to next generation ground-based surveys such as SPTpol or QUIET, can remove the curvature degeneracy at a level sufficient for the *SNAP* and *Planck* surveys and allow a measurement of $\sigma(w_p) = 0.03$, $\sigma(w_a) = 0.3$ jointly with $\sigma(\Omega_K) = 0.0035$. This expectation assumes that the sum of neutrino masses is independently known to better than 0.1 eV. This assumption is valid if the lightest neutrino has negligible mass in a normal neutrino mass hierarchy and is potentially testable with upcoming direct laboratory measurements.

Subject headings: gravitational lensing — large-scale structure of universe

Online material: color figures

1. INTRODUCTION

Currently, observations of the expansion history of the universe are remarkably consistent with cosmic acceleration driven by a cosmological constant in a spatially flat universe. When testing this hypothesis, one typically looks for evidence of spatial curvature in the absence of dark energy evolution *or* evolution in the absence of spatial curvature. It is of course possible that spatial curvature and dark energy evolution conspire to mimic a cosmological constant in a flat universe. Nonetheless, while the data remain consistent with the simpler hypothesis, this approach is justified.

More worrying is the possibility that as measurements improve, we find evidence for nonstandard dark energy in a flat universe—a dark energy equation of state $w \neq -1$. Should we then believe that the universe is flat and dark energy varying in time, or that it has a small curvature and the dark energy is simply the cosmological constant? While the standard inflationary theory predicts that the curvature of our Hubble volume is below measurable limits ($\Omega_K \lesssim 10^{-4}$), models with larger values do exist and are arguably on sounder footing than dynamical dark energy models.

Ideally, of course, we would like to measure both Ω_K and $w(z)$, but this is difficult because of degeneracies. Moderately good constraints are obtained once Type Ia supernovae (SNe) data and cosmic microwave background (CMB) data are combined with high-precision Hubble constant measurements (Hu 2005; Linder 2005), weak gravitational lensing (Knox 2006; Bernstein 2006), baryon oscillations (Knox et al. 2006; Ichikawa et al. 2006), or cluster abundances (Albrecht et al. 2006). However, these techniques are subject to systematic uncertainties that have to be accounted for carefully.

Only two methods—SNe Ia and CMB—have proven so far to be both powerful and robust probes of cosmology. Here we show that the information required to break the degeneracy between curvature and dark energy to a level sufficient for future SN missions such as *SNAP* (Aldering et al. 2004) lies within the reach of next-generation ground-based CMB polarization power spectrum measurements. This information comes from weak gravitational lensing of the CMB in the linear regime at redshifts $z \sim 1$–3 (see Lewis & Challinor [2006] for a recent review). Furthermore, this information is contained in the lensed power spectra and is not subject to systematic errors from non-Gaussian lensing reconstruction. Consequently, if the primary science goal of limiting gravitational wave power is achieved, then this information will also be recovered.

We employ a recently developed framework for CMB lensing power spectra observables that includes the non-Gaussian nature of the lensing signal (Smith et al. 2006). This method is ideally suited for investigating the complementarity between different cosmological probes in a wide range of dark energy models.

2. METHODOLOGY

To describe the information content of the various cosmological probes, we model the observables and employ the usual Fisher approach. For SNe Ia, we model the magnitudes m_i of the SNe as

$$m_i = 5 \log H_0 d_L(z_i) + \mathcal{M} + \epsilon_i, \tag{1}$$

where i runs through the observed SNe. Here the luminosity distance is given by

$$d_L(z) = (1+z)\frac{1}{\sqrt{\Omega_K H_0^2}} \sinh\left(\sqrt{\Omega_K H_0^2} D\right), \tag{2}$$

where Ω_K is the curvature in units of the critical energy density, H_0 is the Hubble constant, $D(z) = \int dz/H(z)$ is the comoving radial distance, $\mathcal{M}$ is a nuisance parameter involving intrinsic SN luminosity, and ϵ_i represents the statistical and systematic errors.

We assume a survey similar to the planned *SNAP* mission (Albert et al. 2005) with 2800 SNe distributed in redshift out to $z = 1.7$ given by Aldering et al. (2004) (middle curve of their Fig. 9, reproduced here in the top panel of Fig. 1). We combine the high-z data set with 300 local SNe uniformly distributed in the $z = 0.03$–0.08 range.

Following Albert et al. (2005) we model the error as a sum of the statistical error and an irreducible, but unbiased, systematic error. The latter imposes a floor on the errors at a given redshift that is uncorrelated across broad redshift ranges. Given

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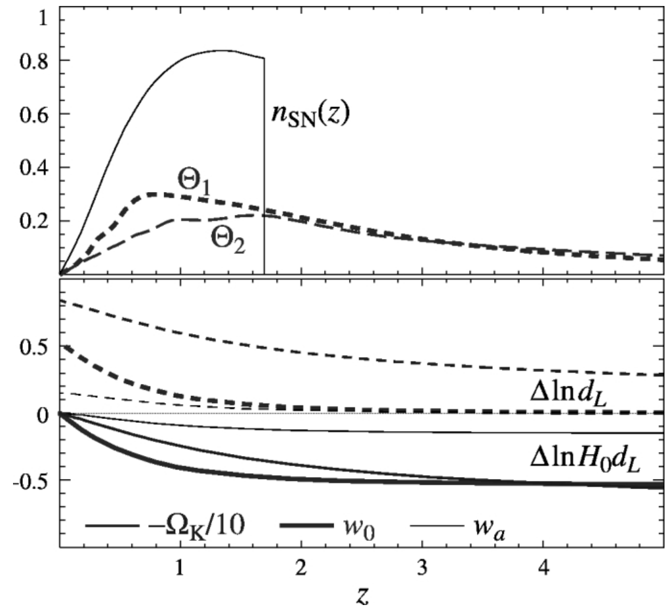

FIG. 1.—*Top*: Redshift distribution of the high-*z* SNe n_{SN} and weights of the lensing observables $\{\Theta_1, \Theta_2\}$ normalized to integrate to unity. *Bottom*: Derivatives of the luminosity distance d_L and relative luminosity distance $H_0 d_L$ with respect to curvature Ω_K and the dark energy parameters w_0 and w_a. Here Ω_{DE} is adjusted to keep the angular diameter distance to recombination fixed in each case. [*See the electronic edition of the Journal for a color version of this figure.*]

a binning of SNe into some arbitrary bins in z denoted as Δz_I, we assume that

$$\sum_{i\in I, j\in J} \frac{\langle (m_i - \bar{m}_i)(m_j - \bar{m}_j)\rangle}{N_I N_J} = \delta_{IJ}\left(\frac{\sigma_m^2}{N_I} + \sigma_{\rm sys}^2\right), \qquad (3)$$

where N_I is the number of SNe in Δz_I. Following Tegmark et al. (1998) we can replace the sum over discrete SNe with an integral over the redshift distribution, $N_I = n_{SN}(z)\Delta z_I$ and construct the Fisher matrix for a parameter set p_μ as

$$F_{\mu\nu}^{\rm SN} = \int dz n_{\rm SN}(z) \frac{1}{\sigma_\epsilon^2(z)} \frac{\partial \bar{m}(z)}{\partial p_\mu} \frac{\partial \bar{m}(z)}{\partial p_\nu}, \qquad (4)$$

where $\sigma_\epsilon^2 = \sigma_m^2 + \sigma_{\rm sys}^2 n_{\rm SN}(z)\Delta z$. We take $\sigma_m = 0.15$ and $\sigma_{\rm sys} = 0.02\ (1+z)/2.7$, $\Delta z = 0.1$. When constructing the Fisher matrix in cosmological parameters, we marginalize over $\mathcal{M}$.

For the CMB, we quantify the information from *Planck* in the absence of lensing with the Fisher matrix $F_{\mu\nu}^{Planck}$ of the unlensed power spectra out to multipole $\ell = 2000$ (e.g., Zaldarriaga et al. 1997). We assume for *Planck* 80% usable sky and three usable channels for cosmology: FHWM 5.′0 with temperature noise $\Delta_T = 51\ \mu$K′ and polarization noise $\Delta_P = 135\ \mu$K′; 7.′1 with $\Delta_T = 43\ \mu$K′, $\Delta_P = 78\ \mu$K′; 9.′2 with $\Delta_T = 51\ \mu$K′, $\Delta_P = \infty$.

For the additional information from lensing, we use the lensing observables framework (see Smith et al. 2006 for details). Here there are two lensing observables Θ_1 and Θ_2 that represent the principal components of the convergence power spectrum obtained from temperature/*E*-polarization and *B*-polarization, respectively. They reflect fractional changes in the amplitude of the convergence power spectrum around $\ell_1 \sim 100$ and $\ell_2 \sim 500$. The redshift sensitivities of Θ_1 and Θ_2 are plotted in Figure 1 (*top*); they extend to $z \gg 1$, which is why CMB lensing has higher sensitivity to curvature than dark energy. The CMB lensing Fisher matrix is given by

$$F_{\mu\nu}^{\rm CMBlens} = \sum_{i=1,2} \frac{\partial \Theta_i}{\partial p_\mu} \frac{1}{\sigma_{\Theta_i}^2} \frac{\partial \Theta_i}{\partial p_\nu}. \qquad (5)$$

For the errors on the observables, we assume a deep CMB survey that is comparable to the proposed SPTpol survey. Specifically, we take a deep temperature survey on 4000 deg^2 and $\Delta_T = 11.5\ \mu$K′ and a deep polarization survey on 625 deg^2 with $\Delta_P = \sqrt{2}\Delta_T = 4\ \mu$K′. We take a FWHM beam of 1′. With these specifications combined with sensitivity to Θ_1 from *Planck*, the two observables can be measured with an accuracy of $\sigma_{\Theta_1} = 0.041$ and $\sigma_{\Theta_2} = 0.032$. For reference, the latter represents a ∼3% measurement of the overall power in lensing *B*-modes and dominates the overall constraints. Moreover, the deep temperature survey provides little weight in the Θ_1 constraint itself and would mainly serve as an internal cross-check for foregrounds, systematics, and other secondaries. Likewise, other planned surveys such as QUIET will have comparable precision in Θ_2 with very different frequency bands.

Finally, we sum the Fisher matrices as usual

$$F_{\mu\nu} = F_{\mu\nu}^{\rm SN} + F_{\mu\nu}^{Planck} + F_{\mu\nu}^{\rm CMBlens} \qquad (6)$$

and approximate the joint parameter covariance matrix as $C_{\mu\nu} = (\mathbf{F})^{-1}_{\mu\nu}$.

3. FORECASTS WITH CURVATURE

It is well known that CMB information from recombination allows SNe to determine the dark energy equation of state, parameterized by $w(a) = w_0 + (1-a)w_a$, in a flat universe. In the three-dimensional space $\{\Omega_{\rm DE}(=0.76),\ w_0(=-1),\ w_a(=0)\}$, *Planck* CMB measurements limit the allowed region to a two-dimensional surface or plane in the Fisher approximation (see Fig. 2). Values in parentheses represent those of the fiducial model. Here we have marginalized over the baryon density $\Omega_b h^2(=0.022)$, cold dark matter density $\Omega_c h^2(=0.106)$, tilt $n_s(=0.958)$, initial amplitude of curvature fluctuations $\delta_\zeta(=4.52\times10^{-5})$ at $k = 0.05$ Mpc^{-1}, and reionization optical depth $\tau(=0.092)$. These eight parameters represent the minimal cosmological set in the Fisher matrix analysis to which we add below the curvature $\Omega_K(=0)$ and sum of the neutrino masses $m_\nu(=0.06$ eV).

SN measurements constrain a flat tube in this space that is nearly orthogonal to the *Planck* surface. Note that the premarginalization of any one of the three parameters before combining does not bring out this complementarity.

The marginalization of spatial curvature can be visualized as the superposition of independent shifts in the *Planck* plane and SN tube. Given that even the unlensed CMB has distance-independent, albeit weak, curvature information from both the integrated Sachs-Wolfe effect and the acoustic peaks (see Hu & White 1996, Fig. 11), the *Planck* plane only widens marginally (see Fig. 2). On the other hand, the SN tube widens substantially. The net effect on the joint constraints in the w_0-w_a plane marginalized over $\{\Omega_{\rm DE},\ \Omega_K\}$ is shown in Figure 3. It represents a factor of 4.8 increase in the 68% CL area (Huterer & Turner 2001) as measured by $A_w = \sigma(w_p)\sigma(w_a)$. Here w_p is the equation

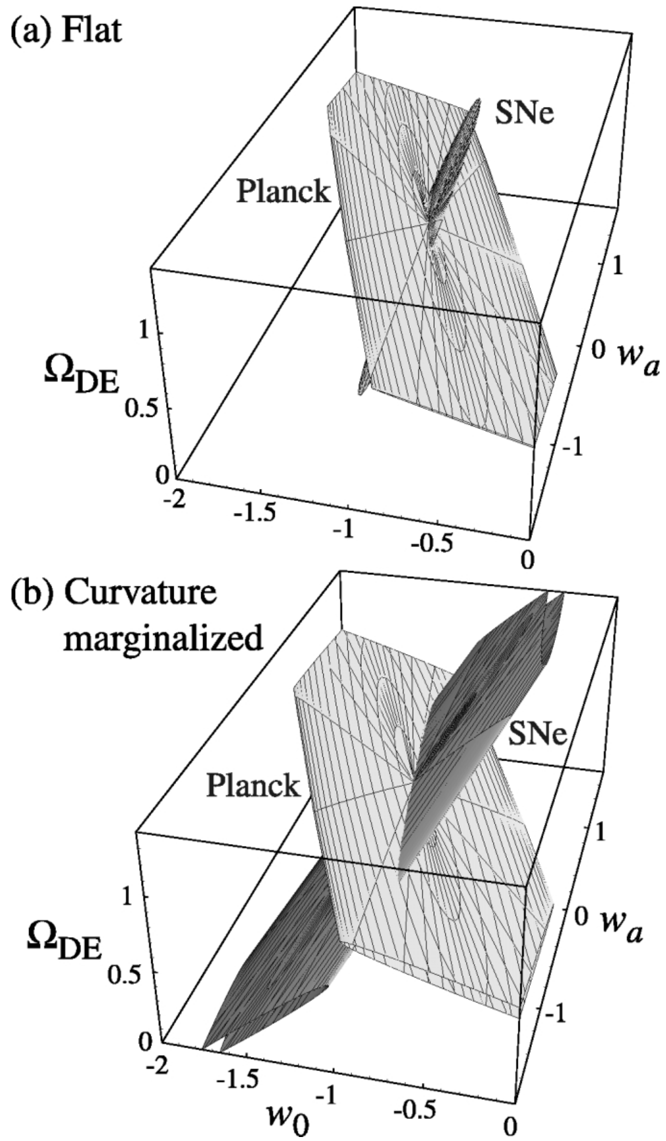

FIG. 2.—Constraints on dark energy parameters Ω_{DE}, w_0, and w_a, shown for SNe and unlensed CMB (*Planck*) separately: (*a*) constraints assuming a flat universe; (*b*) weakened constraints from curvature marginalization. [*See the electronic edition of the Journal for a color version of this figure.*]

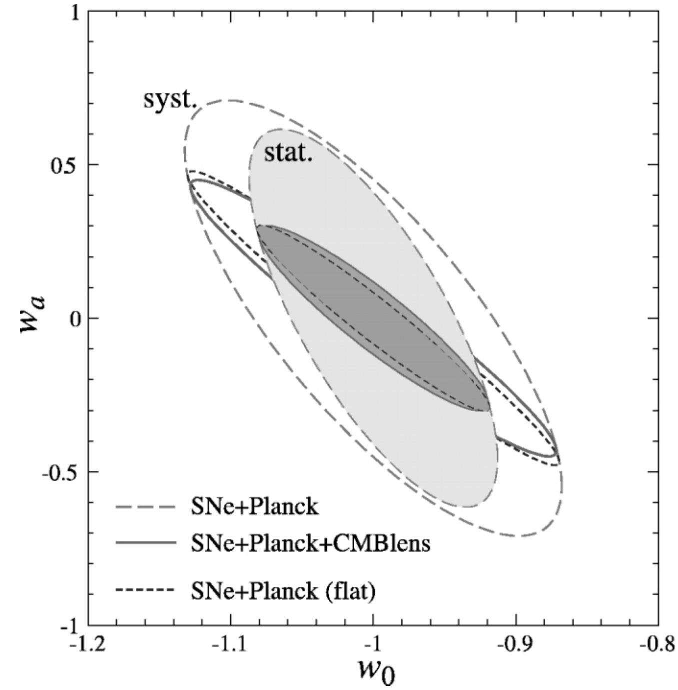

FIG. 3.—68% CL region in the w_0-w_a plane with CMB lensing (CMBlens, *solid curves*) information. *Dashed ellipses*: *Planck*+SN errors alone with curvature marginalized (*long dashed line*) and in a flat universe (*short dashed line*). *Filled ellipses*: SN statistical errors. *Open ellipses*: SN systematic and statistical errors. [*See the electronic edition of the Journal for a color version of this figure.*]

of state at the best constrained or pivot redshift, and its errors are equal to those of w_0 at fixed w_a (Hu & Jain 2004).

This degeneracy is also illustrated in Figure 1 (*bottom*). Here the fractional deviations in the SNe observable $H_0 d_L$ from the fiducial model are shown as parameter derivatives at a fixed distance to recombination. Without spatial curvature, w_0 and w_a make distinguishable changes in the relative distance at $z < 2$. With spatial curvature, the effects become largely degenerate.

The effect of spatial curvature on observables persists to high redshift $z \gg 1$ whereas that of the dark energy parameters flatten and depend only on H_0, the difference between relative ($H_0 d_L$), and absolute distances d_L. This degeneracy may therefore be broken either by high-precision Hubble constant (Hu 2005; Linder 2005) or high-z distance measurements (Knox 2006; Bernstein 2006).

CMB lensing supplies the latter kind of information. Transforming the sensitivity to distances d_L and the matter power spectrum at the redshift range shown in Figure 1 into the cosmological parameters used in equation (5) yields

$$\Delta\Theta_1 \approx -1.01\Omega_{\rm DE} - 0.399\Delta w_0 - 0.146\Delta w_a - 5.17\Delta\Omega_K + 12.3\Delta\Omega_c h^2 + 2\Delta \ln \delta_\zeta - 0.33\frac{\Delta m_\nu}{1\ \rm eV},$$

$$\Delta\Theta_2 \approx -1.27\Omega_{\rm DE} - 0.446\Delta w_0 - 0.154\Delta w_a - 5.30\Delta\Omega_K + 18.8\Delta\Omega_c h^2 + 2.09\Delta \ln \delta_\zeta - 0.45\frac{\Delta m_\nu}{1\ \rm eV}. \quad (7)$$

Here we have used the fact that other parameters in the latter class such as n_s are sufficiently well determined by *Planck*. The sum of the neutrino masses m_ν, however, is not well determined and changes both the shape and growth rate of the matter power spectrum.

First let us consider the impact of CMB lensing constraints assuming that the sum of the neutrino masses m_ν is fixed. This is a good assumption if the lightest neutrino has a mass <0.01 eV and a normal mass hierarchy due to the measurement of the solar and atmospheric neutrino mass squared differences. The same assumption in an inverted hierarchy would also be sufficient in that it only adds a second discrete possibility.

In this fixed neutrino case, the addition of the lensing constraint nearly fully restores the ability of the SN survey and *Planck* to measure the dark energy (see Fig. 3). It allows a measurement to $\sigma(w_a) = 0.30$ and $A_w^{-1} = 137$. This restoration of sensitivity occurs even if the SN survey is limited by only statistical errors such that $\sigma(w_a) = 0.19$ and $A_w^{-1} = 241$.

Figure 4 shows how A_w^{-1} depends on prior knowledge of

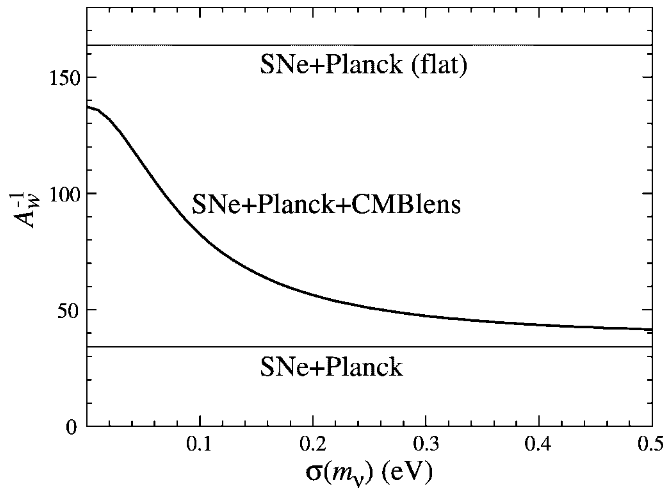

FIG. 4.—Improvement in the area statistic A_w in the w_0-w_a plane of as a function of the prior on the sum of the neutrino masses. The top line represents the SN+*Planck* constraint alone in a flat universe, the bottom represents the degradation once curvature is marginalized. CMB lensing can recover much of this information if the sum of the neutrino masses is known to $\sigma(m_\nu) <$ 0.1 eV. SN systematic errors are included here but the relative effect for statistical errors only is similar. [*See the electronic edition of the Journal for a color version of this figure.*]

the sum of the neutrino masses in the case that the lightest neutrino does not have negligible mass. In this case all three neutrinos could have degenerate masses. External constraints on the sum of neutrino masses begin to help at the 0.2 eV level and would be fully sufficient at a few $\times\ 10^{-2}$ eV. For example, the KATRIN experiment is expected to reach $\sigma(m^2_{\nu_e}) = (0.16\ \mathrm{eV})^2$ from tritium β decay (Aalseth et al. 2004). Such a measurement would test the degenerate mass scenario.

As an aside, it is interesting to note that even with spatial curvature and dark energy marginalized, the combination of data sets would allow a measurement of $\sigma(m_\nu) = 0.24$ eV. With curvature fixed, $\sigma(m_\nu) = 0.14$ eV. Hubble constant measurements with 1%–7% precision would provide neutrino measurements that interpolate between these two limits by fixing the spatial curvature.

4. DISCUSSION

Constraints on the temporal evolution of dark energy benefit particularly strongly from the addition of CMB lensing information to that of SNe and the primary CMB at recombination. The three methods probe very different epochs: SNe are sensitive to distances at $z \lesssim 1$, and the primary CMB to $z \sim 1089$, whereas CMB lensing probes $1 \lesssim z \lesssim 3$. Given that spatial curvature affects distances and growth out to high redshift, CMB lensing is ideally suited to breaking the degeneracy between curvature and the dark energy. It has the additional advantage of being nearly entirely in the linear regime and a lensing test of curvature where the source distance can be considered fixed.

Furthermore, this degeneracy breaking requires only already planned ground-based CMB polarization power spectrum measurements. We have demonstrated that even if the SNe and *Planck* surveys are limited only by statistical errors, a ground-based survey like SPTpol will be sufficient to extract the full information: $\sigma(w_p) = 0.02$, $\sigma(w_a) = 0.2$ and $\sigma(\Omega_K) = 0.0034$; with some accounting for SN systematic errors these degrade to 0.025, 0.3, and 0.0035.

There are two critical assumptions that make this possible. First, the ground-based CMB survey will be able to remove foregrounds and systematics at a level sufficient to enable few percent level measurements of the lensing *B*-mode polarization power. Second, we assume that the neutrino masses are fixed by oscillation measurements and a theoretical assumption about the neutrino mass hierarchy. This assumption will be tested by next-generation laboratory experiments. In the more general context, the sum of the neutrino masses must be externally determined to 0.1 eV or better.

The lensing observables approach we have taken here can be easily extended to consider different combinations of probes. Furthermore, we have only considered the simplest description of the time-dependent dark energy density, and a more detailed parameterizatoin may be even further assisted by CMB lensing.

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