Separating dark physics from physical darkness: Minimalist modified gravity versus dark energy

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The acceleration of the cosmic expansion may be due to a new component of physical energy density or a modification of physics itself. Mapping the expansion of cosmic scales and the growth of large scale structure in tandem can provide insights to distinguish between the two origins. Using Minimal Modified Gravity (MMG)—a single parameter gravitational growth index formalism to parametrize modified gravity theories—we examine the constraints that cosmological data can place on the nature of the new physics. For next generation measurements combining weak lensing, supernovae distances, and the cosmic microwave background we can extend the reach of physics to allow for fitting gravity simultaneously with the expansion equation of state, diluting the equation of state estimation by less than 25% relative to when general relativity is assumed, and determining the growth index to 8%. For weak lensing we examine the level of understanding needed of quasi- and nonlinear structure formation in modified gravity theories, and the trade off between stronger precision but greater susceptibility to bias as progressively more nonlinear information is used.

I. INTRODUCTION

Whether the acceleration of the cosmic expansion is due to a new physical component or a modification of gravitation, the answer will involve groundbreaking new physics beyond the current standard models for high energy physics and cosmology. To obtain the clearest, unbiased picture of the fundamental physics we need to allow for the possibility of gravitation beyond Einstein and see where the data lead. This article presents simultaneous fitting of the cosmic expansion and the theory of gravity.

Direct measurements of the expansion history can be interpreted generally as the equation of state of the universe; this may or may not correspond to a physical component. Measurements of the growth history of mass fluctuations combine information on the expansion and the theory of gravity. The two in complementarity thus allow separation of the possible origins for cosmic acceleration—a physical dark component, e.g. a new field in high energy physics, or dark (new) physics, e.g. a modification of Einstein gravity.

Within a specific theory of modified gravity one can attempt to calculate the cosmological observables and determine the goodness of fit with data. This model dependent approach must proceed on a case by case basis and moreover suffers from difficulties in computation of many quantities due to the complexity of the theories. Nor are the modifications necessarily well motivated or completely free from pathologies. The alternate approach taken here is phenomenological, using a model independent yet physically reasonable and broad parametrization of the gravity modification to gain insight into the effect of generalizing Einstein gravity. This is closely analogous to the widely successful equation of state approach to modifica-

tions of the expansion history, giving model independent constraints and understanding.

In Sec. II we discuss the development of the parametrized approach and its range of validity. Section III lays out the cosmological probes, fiducial models, and survey data characteristics used in our analysis. We examine in Sec. IV the ability of next generation cosmological probes to reveal new gravity simultaneous with fitting cosmological expansion, and the cosmological bias incurred if we neglect to allow the possibility of beyond Einstein gravity. Leverage and systematics from the nonlinear regime are discussed in Sec. V.

II. GRAVITATION AND GROWTH

Attempting to invent a general prescription for taking into account the effects of modified gravity is like seeking a general treatment of non-Gaussianity: there are so many ways in which a theory can be "not", in which a symmetry can be broken, that it seems a hopeless task. However, we do not seek an all-encompassing description of modified gravity in all its aspects, but rather its gross effects on cosmological observables beyond the expansion rate.

As a beginning step, we take a fairly conservative approach we call Minimal Modified Gravity (MMG). The modifications we consider are small, since general relativity gives a predominantly successful description of the universe and its large scale structure, and homogeneous, i.e. not dependent on environment à la chameleon scenarios [1]. We assume that structure formation continues to be described adequately by growth from Gaussian density perturbations. For weak gravitational lensing, we do not have to get too deeply involved in the nonlinear density

regime and can concentrate on the growth law; we assume changes to the gravitational deflection law are negligible.

This approach seeks to build understanding of modified gravity by taking a modest step away from general relativity. By examining an alteration in the linear perturbation growth, preserving the standard mapping from linear to nonlinear fluctuations, we obtain a clear picture of a specific effect of modified gravity (see Sec. V for relaxation of the assumption of standard mapping and discussion of scale dependence). This serves as a proxy for presumably more complicated and model dependent effects.

The form of the growth equation can be written so as to directly show the influence of the cosmic expansion and the additional effect of the gravity theory. For a matter density perturbation $\delta \equiv \delta\rho/\rho$, the linear growth factor $g = \delta/a$ (scaling out the matter dominated universe behavior $\delta \sim a$) evolves in general relativity as

$$g'' + \left[5 + \frac{1}{2}\frac{d\ln H^2}{d\ln a} \right] a^{-1} g'$$
$$+ \left[3 + \frac{1}{2}\frac{d\ln H^2}{d\ln a} - \frac{3}{2}\Omega_M(a) \right] a^{-2} g = 0, \quad (1)$$

where a prime denotes derivative with respect to the scale factor a, $H = \dot{a}/a$ is the Hubble parameter, and $\Omega_M(a)$ is the dimensionless matter density. Since the global cosmology parameters H and $\Omega_M(a)$ are, essentially, the expansion history, we see that the cosmic expansion determines the structure growth.

To make the relation between growth probes and expansion probes such as the luminosity distance-redshift relationship $r_l(z)$ even more explicit, we can start from the growth equation as

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi\rho\delta = 0, \quad (2)$$

where a dot denotes time derivative, and use

$$r_l = a^{-1} \int dt/a = a^{-1} \int da/(a^2 H) \quad (3)$$

(for a flat universe to keep the notation simple) to write

$$\frac{d^2\delta}{dr_l^2}(a^{-2} - Hr_l)^2 - \frac{d\delta}{dr_l}(Ha^{-2} + \ddot{a}/a) - 4\pi\rho\delta = 0. \quad (4)$$

Thus in general relativity the growth clearly contains the same cosmological information as the distance relation.

As discussed in [2], we can alter the growth equation in modified gravity by changing the matter source term (proportional to $\Omega_M(a)$ in Eq. (1) or $\rho\delta$ in Eq. (2)) or adding a new source through a nonzero right hand side. Green function solutions for such modifications are given in [2]. The matter source term can be written as $Q\delta$, and $Q = \nabla^2\Phi/\delta$ arises from the equivalent of the Poisson equation relating the metric potential Φ to the matter perturbation δ. One possibility is to make a phenomenological modification to the Poisson equation and investigate its effects on structure growth; this has been investigated by [3–6] and we revisit it in Sec. V. An intriguing approach of general quadrature relations between the matter perturbations and metric potentials is discussed by [7].

A key aspect to note is that much of the growth *is* determined by the expansion history, even in modified gravity, so we should not throw away that knowledge. By following the effects, treating the expansion in terms of the well-developed equation of state formalism (whether arising from a physical dark energy or a modified Friedmann equation), i.e.

$$H^2(z)/H_0^2 = \Omega_M(1 + z)^3 + \delta H^2(z) \quad (5)$$

$$w(z) = -1 + \frac{1}{3}\frac{d\ln\delta H^2}{d\ln(1 + z)}, \quad (6)$$

and adding a new parameter to incorporate the effects of modified gravity specifically on the growth source term, we render the physics appropriately. This was the motivation behind the gravitational growth index formalism of [2], which we follow, calling the ansatz MMG. An alternate approach is to define wholly separate growth variables (see, e.g., [8,9]).

The gravitational growth index serves as a proxy for the full modified gravity theory. The linear growth factor is approximated by

$$g(a) = e^{\int_0^a d\ln a [\Omega_M(a)^\gamma - 1]}, \quad (7)$$

where γ is the growth index. This was shown to be accurate to 0.2% compared to the exact solution within general relativity for a wide variety of physical dark energy equation of state ratios. We verify explicitly that for dynamical dark energy where the equation of state ratio is parametrized as $w(a) = w_0 + w_a(1 - a)$, the formulas for $\gamma(w)$ given in [2] recover $g(a)$ to better than 0.3% when $w_0 + w_a < -0.1$. That is, the growth index parametrization of linear growth is extremely robust as long as the early matter dominated epoch is not upset.

The gravitational growth index formalism has also been tested and found accurate to 0.2% for a single modified gravity scenario [2], DGP [10,11] braneworld gravity, giving $\gamma = 0.68$. We conjecture that it may work for modified gravity theories with monotonic, smooth (Hubble timescale) evolution in the source term, so long as the matter dominated epoch is not disrupted. For example, in DGP the source term receives a smooth correction $1 - (1/3)(1 - \Omega_M^2(a))/(1 + \Omega_M^2(a))$ [11–14]. Preliminary results in scalar-tensor theory also indicate successful approximation [15]. Future work includes testing this for other specific models; here we use the growth index as an indicator of possible effects of modified gravity, with the advantage of knowing at least it is robust for many cases beyond ΛCDM.

III. FIDUCIAL MODEL AND COSMOLOGICAL DATA

To assess the leverage of cosmological observations to reveal dark physics vs. physical darkness, we simultaneously fit nine cosmological parameters: A, the normalization of the primordial power spectrum at $k_{\rm fid} = 0.05h$ Mpc^{-1}; physical matter and baryon densities $\Omega_M h^2$ and $\Omega_B h^2$, spectral index n, sum of the neutrino masses m_ν, matter energy density today relative to critical Ω_M, and parameters describing the effective dark energy equation of state w_0 and w_a, where $w(a) = w_0 + (1 - a)w_a$. The mass power spectrum $\Delta^2(k, a) \equiv k^3 P(k, a)/(2\pi^2)$ is written as

$$\Delta^2(k, a) = \frac{4A}{25\Omega_M^2}\left(\frac{k}{k_{\rm fid}}\right)^{n-1}\left(\frac{k}{H_0}\right)^4 g^2(a)T^2(k)T_{\rm nl}(k, a) \quad (8)$$

where $T(k)$ is the transfer function, and $T_{\rm nl}(k, a)$ is the prescription for the nonlinear power. Modified gravity enters through the ninth parameter, the gravitational growth index γ in the linear growth function $g(a)$ from Eq. (7), with the growth index $\gamma = 0.55$ for the fiducial cosmology corresponding to a flat universe with Einstein gravity and a cosmological constant. The other fiducial values adopted correspond roughly to the current concordance cosmology, with $\Omega_M = 0.3$, $w_0 = -1$, $w_a = 0$, $\Omega_B h^2 = 0.023$, $\Omega_M h^2 = 0.14$, $n = 0.97$, $m_\nu = 0.2$ eV (one massive species), and $A = 2 \times 10^{-9}$ (corresponding to $\sigma_8 \simeq 0.9$). While the exact values of some of these parameters, especially σ_8, are still a subject of much debate, we do not expect that different values allowed by current data will change any of our conclusions on the detectability of nonstandard growth.

The linear power spectrum uses the fitting formulae of [16]. We always use the linear growth function from Eq. (7) and account for its dependence on cosmological parameters Ω_M, w_0, w_a and γ when taking the derivatives for the Fisher matrix. To complete the calculation of the full nonlinear power spectrum we use the halo model fitting formulae of [17].

For the cosmological probes, we assume future weak lensing (WL) and Type Ia supernova (SN) data as provided by the SNAP experiment [18] as well as cosmic microwave background anisotropy (CMB) data provided by the upcoming Planck satellite [19].

In this work, for weak lensing we only consider the two point correlation function. The weak lensing shear power spectrum measures cosmology through both the mass power spectrum and distance factors,

$$P_{ij}^\kappa(\ell) = \int_0^\infty \frac{dz}{r(z)^2 H(z)} W_i(z)W_j(z)\Delta^2\left(\frac{\ell}{r(z)}, z\right), \quad (9)$$

where $r(z)$ is the comoving distance and $H(z)$ is the Hubble parameter. The weights W_i are given by $W_i(\chi) = (3/2)\Omega_M H_0^2 f_i(\chi)(1 + z)$ where $f_i(\chi) = r(\chi) \times$

$\int_\chi^\infty d\chi_s n_i(\chi_s)r(\chi_s - \chi)/r(\chi_s)$, χ is the radial coordinate and n_i is the comoving density of galaxies if χ_s falls in the distance range bounded by the ith redshift bin and zero otherwise. We employ the redshift distribution of galaxies of the form $n(z) \propto z^2 \exp(-z/z_0)$ that peaks at $2z_0 = 1.0$. The observed convergence power spectrum is [20]

$$C_{ij}^\kappa(\ell) = P_{ij}^\kappa(\ell) + \delta_{ij}\frac{\langle\gamma_{\rm int}^2\rangle}{\bar{n}_i}, \quad (10)$$

where $\langle\gamma_{\rm int}^2\rangle^{1/2}$ is the rms intrinsic shear in each component, taken to be 0.22, and $\bar{n}_i$ is the average number of galaxies in the ith redshift bin per steradian. The cosmological constraints can then be computed from the Fisher matrix

$$F_{ij}^{\rm WL} = \sum_\ell \frac{\partial \mathbf{C}}{\partial p_i}\mathbf{Cov}^{-1}\frac{\partial \mathbf{C}}{\partial p_j}, \quad (11)$$

where p_i are the cosmological parameters and $\mathbf{Cov}^{-1}$ is the inverse of the covariance matrix between the observed power spectra whose elements are given by

$$\text{Cov}[C_{ij}^\kappa(\ell), C_{kl}^\kappa(\ell')] = \frac{\delta_{\ell\ell'}}{(2\ell + 1)f_{\rm sky}\Delta\ell}[C_{ik}^\kappa(\ell)C_{jl}^\kappa(\ell) + C_{il}^\kappa(\ell)C_{jk}^\kappa(\ell)]. \quad (12)$$

The fiducial WL survey assumes 1000 square degrees with tomographic measurements in 10 uniformly wide redshift bins extending out to $z = 3$. The effective source galaxy density is 100 per square arcminute.

We will also sometimes consider a South Pole Telescope (SPT [21]) type cluster survey with sky coverage of 5000 deg^2 and a total of about 25000 clusters (for the assumed cosmology with $\sigma_8 = 0.9$), giving a number-redshift test involving the geometric volume and the number density from growth of structure. For simplicity, we neither consider the additional information provided by masses of the clusters, nor degradation of constraints due to the imperfectly calibrated mass-observable relation (here the observable is the Sunyaev-Zel'dovich flux). Our previous tests have shown that these two effects roughly cancel out in the final cosmological constraints [22]. Then the cluster Fisher matrix is

$$F_{ij}^{\rm clus} = \sum_k \frac{1}{N(z_k)}\frac{\partial N(z_k)}{\partial p_i}\frac{\partial N(z_k)}{\partial p_j} \quad (13)$$

where $N(z_k)$ is number of clusters in kth redshift bin.

The SN survey provides a luminosity distance-redshift test, with 2800 SNe distributed in redshift out to $z = 1.7$ as given by [18], and combined with 300 local supernovae uniformly distributed in the $z = 0.03$–0.08 range. We add systematic errors in quadrature with intrinsic random Gaussian errors of 0.15 mag per SNe. The systematic errors create an effective error floor of $0.02(1 + z_i)/2.7$ mag per bin of $\Delta z = 0.1$ centered at redshift z_i.

For the CMB we use the full Fisher matrix predicted for the Planck experiment with polarization information (W. Hu, private communication). Note that most, though not all, information about dark energy is captured in the distance to the last scattering surface from the acoustic peaks of the power spectrum (e.g. [23]). The effective precision of the angular diameter distance to $z = 1089$ from Planck is 0.4% with temperature and polarization information [24].

IV. FITTING GRAVITY

The combination of probes of the expansion history of the universe and the growth history of large scale structure tests the nature of the acceleration physics. The expansion history is described by the effective equation of state parameters w_0 and w_a, and the deviation of the growth history from that given by Einstein gravity under that expansion is measured by the growth index γ. Almost all cosmological analyses to date, however, have assumed Einstein gravity or worked within a specific alternate theory of gravity, rather than fitting for gravity.

Ignoring the possibility of modified gravity creates the risk of the biasing our cosmological conclusions. This holds not only for "gravitational" parameters but all information. Neglecting possible modification is equivalent to fixing the gravitational growth index γ to its Einstein value (e.g. $\gamma = 0.55$ for general relativity and a cosmological constant model); however this will bias the other parameters due to their covariances with γ (this "gravity's bias" was illustrated for the linear growth factor alone, rather than the weak lensing shear power spectrum, in Fig. 5 of [25]).

Suppose the true value of the growth index differs by $\Delta\gamma$ from its assumed general relativity value. This propagates through into the weak lensing shear cross power spectrum $C_\alpha^\kappa(\ell)$ at multipole ℓ for the pair of redshift bins $\alpha \equiv \{i, j\}$, changing it from the assumed general relativity value of $\bar{C}_\alpha^\kappa(\ell)$. Using the Fisher matrix formalism, the bias on any of the P cosmological parameters is

$$\delta p_i = \tilde{F}_{ij}^{-1} \sum_\ell [C_\alpha^\kappa(\ell) - \bar{C}_\alpha^\kappa(\ell)]$$

$$\times \text{Cov}^{-1}[\bar{C}_\alpha^\kappa(\ell), \bar{C}_\beta^\kappa(\ell)] \frac{\partial \bar{C}_\beta^\kappa(\ell)}{\partial p_j} \quad (14)$$

$$\approx (\Delta\gamma) \tilde{F}_{ij}^{-1} F_{jg} \quad (15)$$

where the last line follows if the finite difference is replaced by a derivative. Here $\tilde{F}$ is the $(P - 1) \times (P - 1)$ Fisher matrix that specifically does not include the growth index γ, F is the full $P \times P$ Fisher matrix, summations over j and the redshift bin indices α, β are implied, and g is the index corresponding to the γ parameter in the matrix F.

Bias in the cosmology from neglecting the possibility of modified gravity can be significant. The red (open) contours in Fig. 1 demonstrate that assuming Einstein gravity in a universe with γ actually higher by 0.1 can shift the expansion characteristics (effective equation of state parameters) by $\sim 2\sigma$. Recall that the DGP braneworld model has $\Delta\gamma = 0.13$ with respect to the standard cosmological constant case. Note too that within this simple treatment of modified gravity a shift in γ moves the w_0-w_a contour along the degeneracy direction, else the bias would be even larger. Given the price of closing our eyes to the issue of possible gravitational modifications, we must attempt to fit for beyond Einstein deviations.

Including the gravitational growth index as an additional parameter, and marginalizing over it to estimate the effective equation of state parameters describing the expansion, removes the bias but necessarily degrades parameter determination. (This would of course become more severe if we required more growth variables than just γ.)

The degradation on the weak lensing shear constraints from marginalizing over γ causes a factor ~ 2 increase in the contour area. While adding CMB or supernovae data does not directly constrain the growth, they prove valuable in breaking degeneracies between parameters. Adding CMB to WL improves constraints by 30–35%, but still suffers the factor of 2 weakening relative to fixing γ. The inclusion of SN is potent in reducing uncertainties. WL + SN + CMB allows for fitting modified gravity, improving parameter estimation by 5–7 times and the area constraint by 40 times over WL alone. Conversely, it only dilutes estimation of w_0 by 23%, w_a by 14%, and the contour area by 35% relative to the "fixed to Einstein" case. This seems
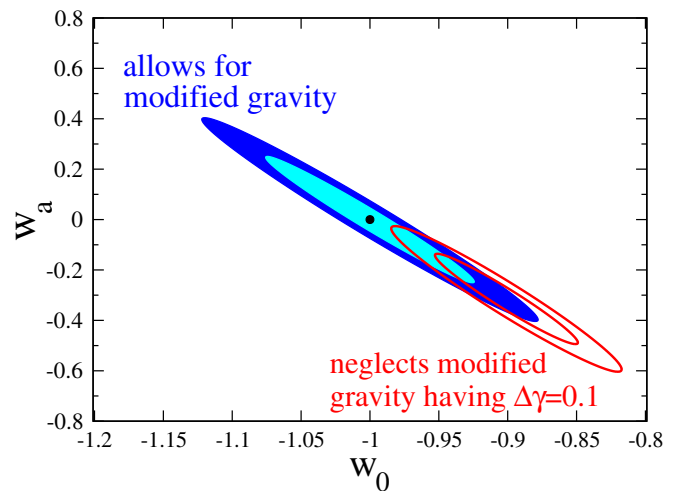

FIG. 1 (color online). 68% and 95% CL constraints on the expansion history equation of state parameters w_0 and w_a, marginalized over all other parameters. The two blue (filled) contours give constraints from combining weak lensing, supernovae, cluster, and CMB data, while simultaneously fitting for beyond Einstein gravity. The black dot shows the fiducial model. The two red (empty) contours show the biased constraints if the modified gravity growth rate, here with $\Delta\gamma = 0.1$, is ignored (i.e. fixing γ to the general relativity value).

TABLE I. Fiducial constraints on the gravitational growth index γ (last column), as well as the two expansion history parameters w_0 and w_a. Starting with the weak lensing survey, we then add the supernova, CMB, and cluster information consecutively. Note that both WL and cluster surveys are partially sensitive to nonlinear physics; constraints that only rely on the linear regime are discussed in Sec. V.

Probe	$\sigma(w_0)$	$\sigma(w_a)$	$\sigma(\gamma)$
Weak lensing	0.33	1.16	0.23
+SNe Ia	0.06	0.28	0.10
+Planck	0.06	0.21	0.044
+Clusters	0.05	0.16	0.037

a fairly modest price to pay for extending the physics reach to beyond Einstein gravity.

The marginalized uncertainties on the key parameters describing the nature of the effective dark physics are shown in Table I. Since SN and CMB do not probe growth, we start with WL measurements and add other probes in sequence.

While marginalizing over the growth index allows us to account for modified gravity rather than ignoring it, we also want to measure the modification itself. That is, we want to extract information quantifying the deviation, to

FIG. 2 (color online). 68% CL constraints on the gravitational growth index γ and the matter density Ω_M, marginalizing over the effective equation of state (and all other parameters). To fit for beyond Einstein gravity as well as the expansion history requires a combination of probes. Other effects on growth, such as neutrino masses, should be taken into account as well; the inner contour of each pair shows the effect of holding this fixed, most severe for weak lensing in isolation.

guide us toward any "dark physics", not just say that there is some inconsistency with general relativity. Figure 2 show the constraints on the growth index from the cosmological probes, in the γ-Ω_M plane (marginalizing over the equation of state and other parameters).

We see an enormous difference in the leverage on modified gravity as we combine probes, again due to the breaking of degeneracies. The determination of γ reaches 0.044 for the combination WL + SN + CMB, representing 8% precision with respect to the Einstein value. If we do not include supernova data, then the uncertainty increases by a factor 2 (5 with only WL), and furthermore the overall contour area in the γ-Ω_M plane increases by 11 times (44 for WL only). So complementarity between WL and SN is quite important for answering the question of whether we are facing dark physics or physical darkness.

Complementarity of probes is important for separation of different physical effects on growth. While inclusion of neutrino mass, which can suppress growth (as does increasing γ), broadens the uncertainty area of WL by 74%, adding other methods immunizes against such a "theory systematic", with the effect on WL + SN + CMB limited to 10%.

V. STRUCTURE IN THE NONLINEAR REGIME

A. Beyond the linear regime

The gravitational growth index modifies the linear growth factor, which then propagates into the full nonlinear growth of structure. Here we examine some issues related to the nonlinear regime. In particular, clusters of galaxies involve nonlinear growth and we might wonder whether special sensitivity to modified gravity arises in this regime. The top panel of Fig. 3 shows cluster counts per $\Delta z = 0.1$ as a function of redshift for our fiducial survey (motivated by the South Pole Telescope), for two values of the growth index ($\gamma = 0.55$ and 0.65). Since we normalized the power spectrum at high redshift, the number density of clusters is independent of the growth index at high redshift. As $z \to 0$, the volume element, which is independent of the growth index by definition, dominates over the number density and makes the counts go to zero for either model. Therefore the biggest difference between the two models is around the peak of the redshift distribution at $z \sim 0.6$, as illustrated in the top panel of Fig. 3. Also shown are number count predictions when the equation of state and neutrino mass have been perturbed from their fiducial values by 0.05 and 0.3 eV, respectively. While the strong degeneracy of the growth index with other cosmological parameters is apparent, it is worth noting that clusters can in principle provide several other observables to break this degeneracy, chief among them being the mass information. However, in the nonlinear regime we are at increased risk of the simple MMG model breaking down, possibly requiring model dependent simulations for a specific theory of gravity.
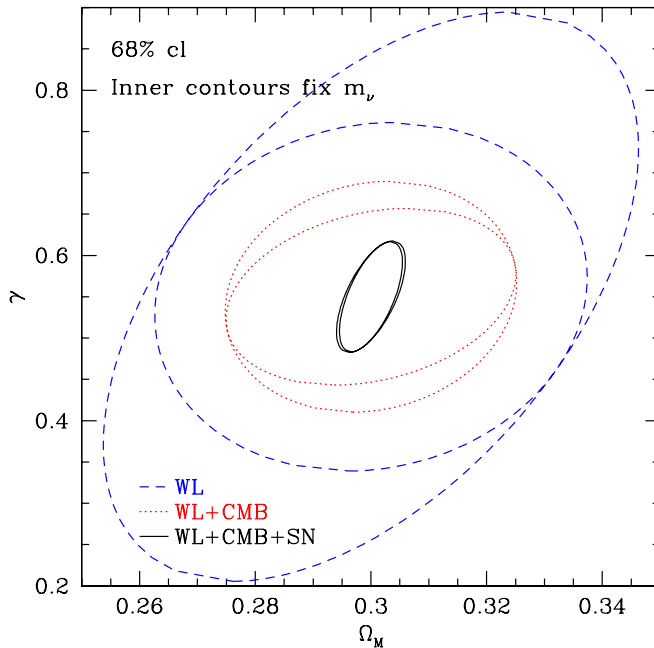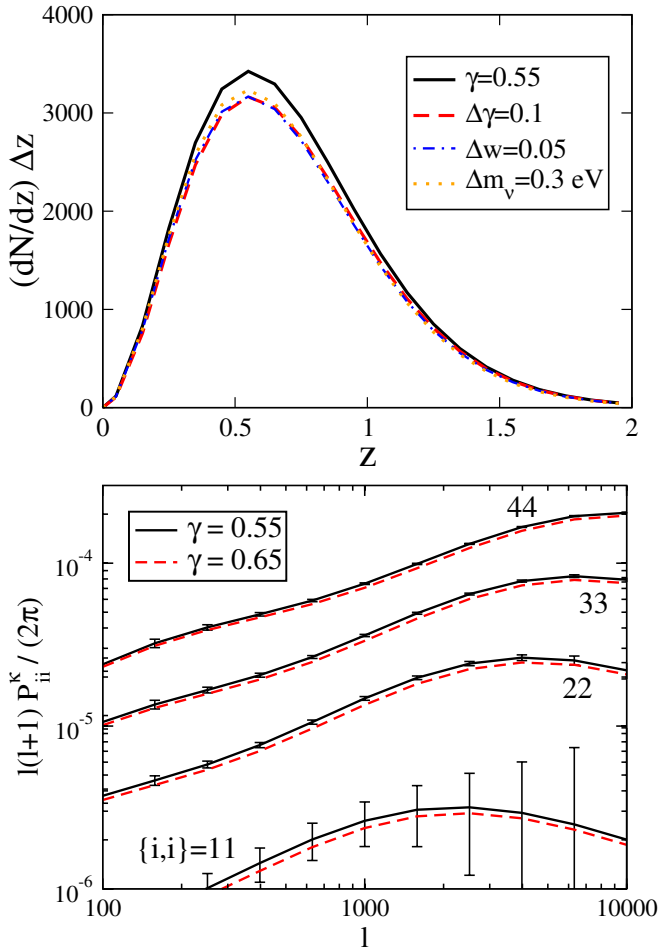
FIG. 3 (color online).   Top panel: Cluster counts per $\Delta z = 0.1$ as a function of redshift for our fiducial survey, for two values of the growth index ($\gamma = 0.55$ and 0.65). Also shown are cases when the equation of state and neutrino mass have been perturbed from their fiducial values by 0.05 and 0.3 eV, respectively. Bottom panel: Autocorrelation power spectra of 4-bin weak lensing tomography for the two values of the growth index, with statistical errors shown for the $\gamma = 0.55$ model.

Weak lensing also involves scales in the quasilinear and nonlinear regimes. The sensitivity of a weak lensing survey is shown in the bottom panel of Fig. 3. For simplicity, we consider the same fiducial survey as before but with 4-bin (instead of 10-bin) tomography, with divisions $z = [0, 0.5], [0.5, 1], [1, 1.5], [1.5, 3]$. We show the four autocorrelation power spectra with corresponding statistical errors for $\gamma = 0.55$, and the same but without errors for $\gamma = 0.65$. The raw signal-to-noise for distinguishing the two values of the growth index increases with redshift, giving an advantage to a deeper survey. The angular scale at which the two values are best distinguished decreases with redshift, so the multipole increases, going from $\ell \sim 500$ for the first bin to $\ell \sim 5000$ for the last bin, making advantageous a higher resolution survey.

We have also checked that the sensitivity to the growth index increases as the density of resolved source galaxies

n_g and their mean distance, parametrized by the mean of the redshift distribution, are increased. All these factors indicate that a space-based weak lensing survey has certain definite advantages for testing beyond Einstein gravity. At this time, however, we have not further pursued these sensitivity tests nor tried to devise an optimal strategy to determine the growth index. The reason is that by far the dominant uncertainty is a theory uncertainty—our ability to predict the nonlinear clustering statistics and associated observables—in a given modified gravity theory.

B. Uncertainty and bias

So far we have included nonlinear scales when obtaining constraints from measurements involving the growth of structure. Indeed, none of the cosmological growth probes is solely a linear theory probe, with the possible exception of the Integrated Sachs-Wolfe effect (which, however, very directly depends on the metric potentials which themselves are likely to be altered in the modified gravity theory). By including nonlinear scales we increase the statistical discrimination power with respect to growth but may bias the results as a result of employing an improper nonlinear prescription for the modified gravity growth. We now consider the trade off between these two trends.

Since weak lensing probes a range of scales, we can consider limiting the weak lensing information to scales $k \leq k_{cut}$ and investigate how the information on the growth index is degraded. This "k-cutting" can remove those physical scales where we lack dependable estimates of modified gravity effects. The most straightforward way to implement the cutting is to use weak lensing power spectrum nulling tomography (see Sec. 5 of [26]) where, for a given multipole ℓ, only the lens planes with distances $r(z) > \ell/k_{cut}$ are allowed to contribute information. Higher ℓ then contribute increasingly less information, and exhaust all information below k_{cut}. For a fixed k_{cut}, we compute the ratio of the error in the growth index relative to the error with $k_{cut} = \infty$. The results are shown with the solid line in Fig. 4. Restricting the information to purely linear scales ($k_{cut} < 0.2h$ Mpc^{-1}) leads to degradations in the marginalized error in γ of more than a factor of 10 relative to the full nonlinear fiducial case. However, when the quasilinear scales are used ($k_{cut} < 1h$ Mpc^{-1}), the degradation is kept to a factor of a few, and the resulting constraints on the growth index are still interesting. Therefore, even if the nonlinear prediction out to $k \approx 10h$ Mpc^{-1} ($\ell \approx 10000$) in modified gravity theories is unfeasible, efforts to understand predictions on quasilinear scales are well worthwhile.

Since we desire to retain values of k beyond the linear regime, for their leverage, it is instructive to make at least an approximate estimate of the bias that might be induced by keeping such data. Within the halo model, the linear regime corresponds to the 2-halo term of clustering between dark matter halos, while the nonlinear regime in-
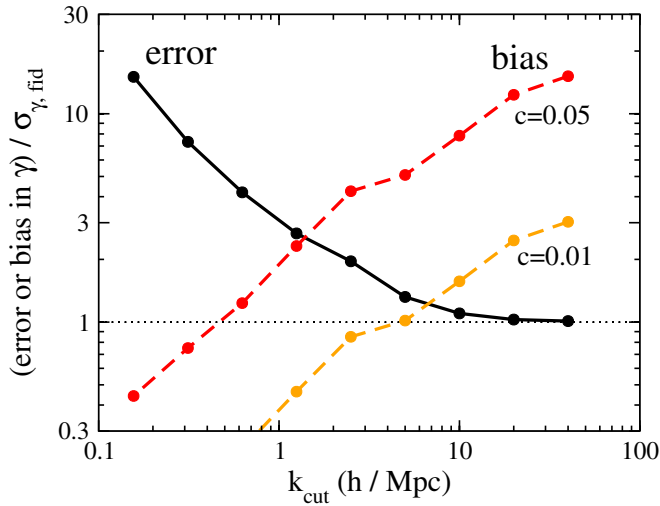
FIG. 4 (color online). The solid (black) line shows the degradation in constraints on the growth index γ from a SNAP-type weak lensing survey (plus SN + CMB) as small-scale weak lensing information is increasingly dropped. For each value of k_{cut}, we drop all information from $k > k_{cut}$ following [26] and compute the ratio of the constraint on γ relative to the case when all information is used, that is, when $k_{cut} = \infty$. The dashed curves (dark (red) and light (orange)) lines show the bias in the growth index divided by the $k_{cut} = \infty$ statistical error for the bias model given in Eq. (17) with $c = 0.05$ or 0.01. The optimum choice of k_{cut}, with the least risk, occurs near the intersection of the error and bias curves.

volves the 1-halo term of the profile and concentration within a halo. For $k \approx 0.2 - 1h$ Mpc^{-1}, the 1-halo contribution may be approximated by a white noise term in the power spectrum [27], due to Poisson fluctuations in the number of halos. In other words,

$$\Delta^2(k, z) \simeq \Delta_{lin}^2(k, z) + \left(\frac{k}{k_*(z)}\right)^3. \qquad (16)$$

We find a good fit to the full power spectrum in the quasilinear regime for $k_*(z) = (5/3)k_{nl}(z)$, where the nonlinear scale $k_{nl}(z)$ is defined via $\Delta^2(k_{nl}(z), z) = 1$.

Since neither the presence of a 1-halo/2-halo split nor the Poisson fluctuations in number should rely on the specific gravity theory, we adopt this approach and consider what happens if we improperly estimate the 1-halo effects. This would shift the white noise term to a different value (equivalent to changing the scale at which the 2-halo and 1-halo terms are comparable), i.e. biasing the power spectrum by

$$\delta(\Delta^2(k, z)) = c\left(\frac{k}{k_*(z)}\right)^3 \qquad (17)$$

where c is a dimensionless constant which represents the fractional error in the 1-halo term. We then use the Fisher formalism to propagate this offset into biases on cosmological parameters. (The expansion parameters w_0 and w_a are not appreciably affected by nulling, so we focus on γ.)

The less of the nonlinear regime we use, the less effect the misestimated 1-halo, or c, term has. The results are shown in Fig. 4 where the dashed curves give the bias in the growth index divided by the fiducial statistical error (i.e. the statistical error for $k_{cut} = \infty$). It is clear that, as we perform increasingly more drastic nulling, decreasing the value of k_{cut}, the bias/error ratio decreases significantly. For $c = 0.05$ (0.01), cutting information beyond $k_{cut} = 1(6)h$ Mpc^{-1} gives bias in γ that is below the statistical error, while increasing the error by a factor of 3 (25%). One might expect the bias model of Eq. (17) to be cut off in the stable clustering regime, causing bias in γ to level off or decline at k greater than a few times k_{nl}, e.g. $k > 1h$ Mpc^{-1}. This would allow use of more of the nonlinear regime.

C. Scale dependent growth

MMG, with the gravitational growth index formalism, has been adopted as the simplest reasonable method of accounting for the effects of beyond Einstein gravity on cosmological probes involving the growth of structure. One part of its simplicity is that γ acts in a scale independent fashion; this should reproduce global effects such as time varying gravitational coupling. However many modifications to gravity will introduce scale dependence in the growth. For example, in the DGP braneworld we might expect changes to the growth equation beyond the varying coupling on both small and large scales, due to the Vainshtein radius (where the scalar degrees of freedom become relevant) and relativistic effects, respectively, (R. Scoccimarro, private communication).

Scale dependence on small scales affects the nonlinear regime, and this is just what we looked at with the bias calculations above. On large scales, we may treat the modification of the source term mode by mode in the linear growth equation. From a harmonic analysis of metric perturbations, [28,29] found long wavelength corrections to the Poisson equation; recall from Sec. II this alters the factor Q in the source term $Q\delta$. Such JLW corrections would multiply $\Omega_M(a)$ by $Q \sim 1 + \alpha e^{-(k/k_{JLW})^2}$. We can then either solve the modified differential equation for the linear growth, or we can retain the growth index approach, but similarly multiply $\Omega_M(a)$ in Eq. (7) by that same factor (this can also be viewed as making the growth index γ a function of k). The characteristic scale is the horizon scale, $k_{JLW} \sim H$, and the amplitude $\alpha \sim O(1)$. We find that effects on the growth are then negligible for $k > 10^{-3}h$ Mpc^{-1}.

Thus horizon size scale dependence, as should hold for the DGP case and scalar-tensor theories as well, will have essentially no effect on the weak lensing probe, as the WL power spectrum errors on the near-horizon scales are very large due to sample variance; see e.g. Fig. (10) in [30]. Weak lensing can be reasonably treated by the scale independent γ factor over $10^{-3} \lesssim k/h$ Mpc$^{-1} \lesssim 10$. Such

scale dependence may however need to be taken into account for attempts to use the ISW effect to probe cosmology, as mentioned previously. Only if the gravitation theory possesses a substantially smaller scale, approaching the nonlinear scale, as in the phenomenological alterations of the Poisson equation in [3–6], are we forced to more elaborate parametrizations than γ.

The specific optimum of the trade off between leverage on cosmological parameter constraints from the added information of smaller scales and increasing risk of bias from gaps in our understanding will depend on the specific gravity theory. Since this is what we are trying to obtain insight into, a rational, model independent approach might be to carry out both a wide area survey to squeeze the most statistical power out of the relatively weakly discriminating low ℓ (linear) regime and a deep, high resolution and high number density survey to probe the richer high ℓ (quasi- and nonlinear) regime. This indicates possible strong complementarity between a ground-based weak lensing survey such as LSST [31] and a space-based survey like SNAP.

VI. DISCUSSION

This article presents an approach for simultaneous fitting of the cosmic expansion and the theory of gravity. We have advocated a ''minimalist'' strategy of distinguishing modified gravity from dark energy, which consists in measuring a single parameter, the growth index γ. In addition to reproducing the linear growth function for essentially all standard gravity, dark energy models (parametrized with w_0 and w_a), the growth index also fits the linear growth of a single known modified gravity theory, the DGP braneworld scenario. Therefore, it is reasonable to expect that the growth index can be used to measure deviations from standard gravity: given the measurements of the background expansion rate (parametrized, say, by Ω_M, w_0 and w_a), standard gravity predicts the value of γ, and a statistically significant deviation from this value can in principle be interpreted as evidence for—and characterization of— beyond Einstein gravity.

This is a first step, hence our emphasis in calling it Minimal Modified Gravity. One could examine more complex schemes but these may be more model dependent or employ more parameters and are therefore likely to give weaker results. The MMG approach therefore has more statistical power, being more suitable to near-future data, while we think the loss in generality is minimal. We also emphasize the advantage in retaining the maximal physics information by treating the expansion effects on growth through the effective equation of state, giving clear separation from deviations in the gravity theory.

How accurately such a program can reveal beyond Einstein gravity—dark physics vs physical darkness—is the main topic of this paper. We have shown that measurements from weak gravitational lensing, Type Ia supernovae, and the CMB combined can measure the growth index to about 8%, or to ± 0.04 around its ΛCDM value (and galaxy cluster data could potentially reduce this even further). At the same time, the constraints on other cosmological parameters are not appreciably degraded, essentially because the surveys probe a range of scales and thus their complementarity breaks the degeneracies between parameters.

One particular concern in the program of distinguishing general relativity from modified gravity is the nonlinear density regime of structure formation. Even for the limited number of well-defined modified gravity theories, details of nonlinear clustering are currently unknown. While in principle the nonlinear structure formation is calculable from N-body simulations of modified gravity, creating these simulations in practice is extremely difficult except for some very simple cases. The structure and evolution of galaxy clusters, which are nonlinear objects, is fairly strongly dependent on the nonlinear physics, and is consequently problematic.

Weak lensing, on the other hand, probes a range of scales, and we studied how our results behave if we drop small-scale (that is, nonlinear) information. Using the nulling tomography approach and a reasonably well motivated toy model for bias due to uncertainty in nonlinear structure, we found that cutting out the small-scale information ($k \gtrsim 1h \text{ Mpc}^{-1}$) can lead to significant decrease in the resulting bias in the growth index, at the expense of increasing the statistical error in it by a factor of a few.

Other cosmological observables exist with sensitivity to the growth of fluctuations, and hence can be used to constrain MMG, but we have not discussed them in any detail. For example, the bispectrum of weak gravitational lensing is a potentially powerful probe [32]; however, it has proved to be a tough task to calibrate the bispectrum even in standard general relativity. The same holds for Lyman-alpha forest observations. The Integrated Sachs-Wolfe effect is a potentially strong discriminant of modified gravity models (see, in particular, recent predictions of the ISW in DGP models [14]); however, the metric potentials are particularly sensitive to the structure of the modified gravity theory. Neither we nor anyone else has yet succeeded in finding a generic parametrization of the deviations from general relativity for the ISW effect.

Our work outlines a first step in treating modified gravity models. At the time of this writing, there is hardly a single well-defined modified gravity theory that does not look like standard general relativity in terms of observables, but is not already ruled out or disfavored by data. Considerable effort is underway to construct such theories (e.g. [33–47]; for a review see [48–50]) and test them experimentally.

It is heartening that the strong complementarity in cosmological probes such as the combination of weak lensing, supernovae, and the CMB does provide important information on the question of dark physics vs physical dark-

ness. Such data from next generation experiments, here calculated specifically for SNAP and Planck, does furnish a real test not just of individual models but of the physics framework beyond Einstein. We will be able to measure the effective equation of state describing the cosmic expansion and simultaneously reveal the theory of gravity. By continuing forward with advances in measurements, theory, and computation we can lift the darkness on the new physics.

ACKNOWLEDGMENTS

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