The Astrophysical Journal, 578:L95–L98, 2002 October 20

WEAK LENSING AS A CALIBRATOR OF THE CLUSTER MASS-TEMPERATURE RELATION

Dragan Huterer[1] and Martin White[2]
Received 2002 June 17; accepted 2002 September 12; published 2002 September 27

ABSTRACT

The abundance of clusters at the present epoch and weak gravitational lensing shear both constrain roughly the same combination of the power spectrum normalization σ_8 and matter energy density Ω_M. The cluster constraint further depends on the normalization of the mass-temperature relation. Therefore, combining the weak-lensing and cluster abundance data can be used to accurately calibrate the mass-temperature relation. We discuss this approach and illustrate it using data from recent surveys.

Subject headings: cosmology: theory — large-scale structure of universe

1. INTRODUCTION

The number density of galaxy clusters as a function of their mass, the mass function, and its evolution can provide a powerful probe of models of large-scale structure. Historically, the most important constraint coming from the present-day abundance of rich clusters has been the normalization of the *linear theory* power spectrum of mass density perturbations (e.g., Evrard 1989; Frenk et al. 1990; Bond & Myers 1991; Henry & Arnaud 1991; Lilje 1992; Oukbir & Blanchard 1992; Bahcall & Cen 1993; White, Efstathiou, & Frenk 1993; Viana & Liddle 1996, 1999; Henry 2000). The normalization is typically quoted in terms of σ_8, the rms density contrast on scales of 8 h^{-1} Mpc, with the abundance constraint forcing models to a thin region in the Ω_M-σ_8 plane.

Since the mass, suitably defined, of a cluster is not directly observable, one typically measures the abundance of clusters as a function of some other parameter that is used as a proxy for mass. Several options exist, but much attention has been focused recently on the X-ray temperature. Cosmological *N*-body simulations and observations suggest that X-ray temperature and mass are strongly correlated with little scatter (Evrard, Metzler, & Navarro 1996; Bryan & Norman 1998; Eke, Navarro, & Frenk 1998; Horner, Mushotsky, & Scharf 1999; Nevalainen, Markevitch, & Forman 2000). How well simulations agree with observational results is far from clear, and several issues need to be resolved. On the simulation side there are the usual issues of numerical resolution and difficulties with including all of the relevant physics. On the observational side, instrumental effects can be important (especially for the older generation of X-ray facilities) in addition to the worrying lack of a method for estimating "the mass." In this respect it is worth noting that there are numerous differing definitions of which M and T are to be related in the M-T relation (White 2001)!

With current samples, the *dominant* uncertainty in the normalization in fact comes from the normalization of the M-T relation (Eke, Cole, & Frenk 1996; Viana & Liddle 1996; Donahue & Voit 1999; Henry 2000; Pierpaoli, Scott, & White 2001; Seljak 2002). Or phrased another way, the cluster abundance is a sensitive probe of the normalization of the M-T relation.

The abundance of clusters is, of course, not the only way to constrain the cosmological parameters. In this regard it is interesting to note that weak gravitational lensing provides a constraint on a very similar combination of Ω_M and σ_8. Therefore, the two constraints can be combined to check for consistency of our cosmological model, to provide a normalization for the M-T relation, to probe systematics in either method, and/or to measure other parameters not as yet included in the standard treatments.

While the cluster constraint comes primarily from scales of about $R = 10\ h^{-1}$ Mpc, current weak-lensing surveys constrain somewhat smaller scales. These surveys probe scales between roughly 1′ and 10′, which for source galaxies located at $z \simeq 1$ in a Λ cold dark matter (ΛCDM) cosmology corresponds to $0.7\ h^{-1}\ \mathrm{Mpc} < R < 7\ h^{-1}$ Mpc. Therefore, weak lensing probes slightly smaller scales than clusters. As lensing surveys push to larger scales, the overlap will become even better.

In this Letter we argue that a natural application of combining the cluster abundance and weak-lensing constraints is to calibrate the M-T relation for galaxy clusters (see also Hu & Kravtsov 2002). In § 2 we define the M-T relation and derive how cluster abundance constraints depend on Ω_M and σ_8. In § 3 we illustrate how combining the two constraints can fix the normalization of the M-T relation using two recently obtained data sets. Finally, in § 4 we discuss this approach further.

2. THE MASS-TEMPERATURE RELATION

Throughout, we are interested in the abundance of massive clusters at low redshifts, so we parameterize the M-T relation as

$$\frac{M(T,z)}{M_{15}} = \left(\frac{T}{T_*}\right)^{3/2} (\Delta_c E^2)^{-1/2} \left[1 - 2\,\frac{\Omega_\Lambda(z)}{\Delta_c}\right]^{-3/2}, \qquad (1)$$

where $M_{15} = 10^{15}\ h^{-1}\ M_\odot$, Δ_c is the mean overdensity inside the virial radius in units of the critical density, which we compute using the spherical top-hat collapse model, and $E^2 = \Omega_M(1+z)^3 + \Omega_\Lambda + \Omega_k(1+z)^2$. T_* is the normalization coefficient that we seek to constrain; it roughly corresponds to the temperature of an $M = 7.5 \times 10^{13}\ h^{-1}\ M_\odot$ cluster. If measured in units of keV, the value of T_* is precisely equivalent to β from Pierpaoli et al. (2001) and is $1.34 f_T$ of Bryan & Norman (1998).

Let us explore the sensitivity of cluster abundance on Ω_M and σ_8. The Press-Schechter formula gives the number of collapsed objects dn per mass interval $d \ln M$ (Press & Schechter 1974); we define $N(M,z) = dn/(d \ln M)$. Further defining $\nu \equiv \delta_c/\sigma(M,z)$, where $\sigma(M,z)$ is the rms density fluctuation

[1] Department of Physics, Case Western Reserve University, 10900 Euclid Avenue, Cleveland, OH 44106; dxh72@po.cwru.edu.

[2] Departments of Astronomy, 601 Campbell Hall, and Department of Physics, 366 Lefonte Hall, University of California at Berkeley, Berkeley, CA 94720.

on a mass scale M evaluated at redshift z using linear theory and $\delta_c \approx 1.686$ is the linear threshold overdensity for collapse, we have

$$N(M, z) = \sqrt{\frac{2}{\pi}} \frac{\rho_M}{M} \frac{d \ln \sigma(M, z)}{d \ln \nu} \nu \exp\left(-\frac{\nu^2}{2},\right) \quad (2)$$

where ρ_M is the present-day matter density. Assuming we are dealing with the current cluster abundance, $z \simeq 0$. Following Pen (1998), for the mass scales of interest we can approximate $\sigma(M) \propto M^{-\alpha}$, where $\alpha \simeq 0.27$ for the currently popular ΛCDM cosmology.

Let us examine the dependence of N on Ω_M, σ_8, and M. Ignoring the term $d \ln \sigma/ (d \ln \nu)$ (which slowly varies), one obtains

$$\frac{\delta N}{N} = \frac{\delta\Omega_M}{\Omega_M}(1 - \alpha + \nu^2\alpha) + \frac{\delta\sigma_8}{\sigma_8}(\nu^2 - 1) - \frac{\delta M}{M}(1 - \alpha + \nu^2\alpha). \quad (3)$$

Setting the left-hand side to zero and using the fact that $\delta M/M = -(3/2)\,\delta T_*/T_*$, for our fiducial cosmology and massive clusters ($M \sim 10^{15}\ h^{-1}\ M_\odot$, or $\nu \simeq 2$) we have[3]

$$T_* \propto (\sigma_8 \Omega_M^{0.6})^{-1.1}. \quad (4)$$

Therefore, measurements of the cluster abundance at the present epoch constrain a degenerate combination of T_* and $\sigma_8\Omega_M^{0.6}$. One of them cannot be determined without knowing the other. Thankfully, weak lensing happens to measure roughly this combination of Ω_M and σ_8 accurately, and the orthogonal combination much less accurately (e.g., Bernardeau, van Waerbeke, & Mellier 1997). Consequently, weak lensing in conjunction with cluster abundance can be used to constrain T_* quite strongly.

3. WEAK LENSING PLUS CLUSTERS: AN EXAMPLE

As a more concrete example of these ideas, let us examine what value of T_* is required to bring current cluster and weak lensing results into agreement. This analysis is necessarily illustrative but is already quite enlightening.

3.1. *The Cluster Data*

We compute σ_8 using a Monte Carlo method following the steps outlined in Pierpaoli et al. (2001). Since some of the details have changed, we sketch the procedure here.

We use HIFLUGCS of Reiprich & Böhringer (1999), restricted to clusters with $0.03 < z < 0.10$. For simplicity we do not include "additional" clusters of lower flux/temperature, which could scatter into the sample. The cosmic microwave background (CMB) frame redshifts from Struble & Rood (1999) were used when available and so were the two-component temperatures published in Ikebe et al. (2002). For each Ω_m, we sample from a distribution of cosmological parameters including h, n, and T_* (the normalization of the M-T relation). For each such realization we generate 50 mass functions, where the temperature is chosen from a Gaussian with the mean and variance appropriate to the observational value and errors and a scatter of 15% in mass at fixed T is assumed for the M-T relation. Using the mean values of the M-T relation and the L-T relation from Ikebe et al. (2002),

$$L_X = 1.38 \times 10^{35} \left(\frac{kT}{1\ \mathrm{keV}}^{2.5}\ h^{-2}\ \mathrm{W},\right) \quad (5)$$

we compute the volume to which clusters of mass M could be seen above the flux limit $f_{\rm lim} = 1.99 \times 10^{-14}$ ergs s^{-1} cm^{-2} of the survey. For each realization of the mass function we compute the best-fitting σ_8 by maximizing the Poisson likelihood of obtaining that set of masses from the theory with all parameters except σ_8 fixed. The mass function can be computed using the Press-Schechter (1974), Sheth-Tormen (1999), or Jenkins et al. (2001) formulae. We have used the Sheth-Tormen prescription throughout, with the mass variance $\sigma^2(M)$ computed using the transfer function fits of Eisenstein & Hu (1999) and masses converted from $M_{180\,\Omega}$ to M_{Δ_c} assuming a Navarro, Frenk, & White (1997) profile with $c = 5$. The best-fitting σ_8 is corrected from $\bar{z}$ to $z = 0$. The mean of the 50 $z = 0$ normalizations is then taken as the fit for that set of cosmological parameters (since the error from Poisson sampling is completely subdominant to the error in the M-T normalization we do not keep track of it here). When quoting a best fit for a given triplet of (Ω_m, σ_8, T_*), we marginalize (average) over the other cosmological parameters h and n.

3.2. *The Weak-lensing Data*

As an example of weak-lensing measurements, we use shear measurements obtained using Keck and William Herschel telescopes (Bacon et al. 2002). These joint measurements used two independent telescopes covering 0.6 and 1 deg^2, respectively, and enabled careful assessment of instrument-specific systematics. The authors compute the shear correlation function and compare with the theoretical prediction. Assuming the shape parameter $\Gamma = 0.21$, the results are well fitted by

$$\sigma_8 \left(\frac{\Omega_M}{0.3}^{0.68} = 0.97 \pm 0.13,\right) \quad (6)$$

which captures the total 68% CL error: statistical and redshift uncertainty and uncertainty in the ellipticity-shear conversion factor. These results are consistent with other recent measurements of cosmic shear (van Waerbeke et al. 2002; Refregier, Rhodes, & Groth 2002; Hoekstra, Yee, & Gladders 2002).

3.3. *Calibrating the M-T Relation*

Figure 1 shows the constraints in the Ω_M-σ_8 plane. The cluster constraint has been marginalized over h and n as explained above and plotted for three different values of T_*. We have checked that the allowed ranges for h and n are wide enough so that essentially all of the likelihood is contained within those ranges. The weak-lensing constraints assume the shape parameter $\Gamma = 0.21$. Note that the constraint regions from the two methods are indeed parallel, with very similar degeneracy directions. This enables an accurate determination of the normalization T_*.

In the example above, we see that a relatively low T_* is preferred ($T_* \lesssim 1.7$ keV) in order for cluster results to agree with the weak-lensing results. While systematics in both methods could still be important, it is interesting to note that this result is in line with most earlier estimates (Evrard et al. 1996; Eke et al. 1998; Bryan & Norman 1998; Yoshikawa, Jing, & Suto 2000),

[3] Note that the dependence of M (or T_*) on σ_8 is stronger for more massive clusters; a more detailed analysis gives $T_* \propto \sigma^{-5/3}$ for the most massive clusters (Evrard et al. 2002).

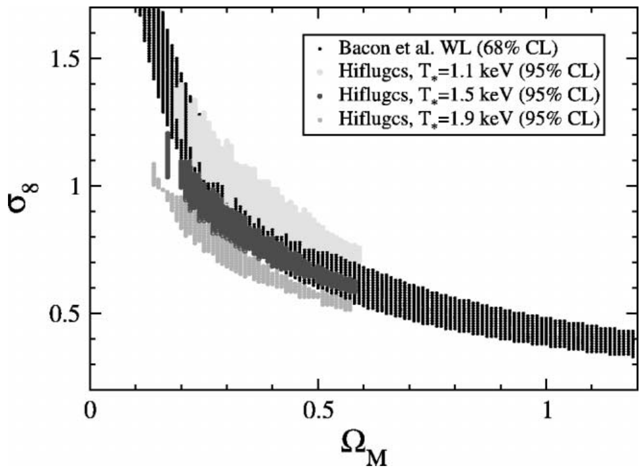

FIG. 1.—The 68% CL uncertainty contours in the Ω_M-σ_8 plane, for a weak-lensing survey (Bacon et al. 2002) and 95% CL uncertainties for a cluster survey (Reiprich & Böhringer 1999). Cluster results are shown for three different values of the mass-temperature normalization parameter T_* and are marginalized over n and h. The degeneracy regions for the two methods are very similar, which in principle enables an accurate determination of T_*.

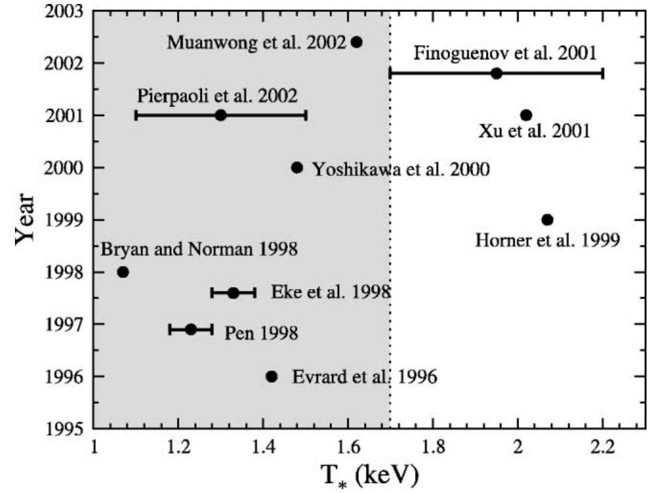

FIG. 2.—Estimates of the M-T normalization T_* collected from the literature. The three points on the right are estimates from simulations, while the seven points on the left are from the observations. Points with no error bars had none quoted. Shaded region is roughly our favored range of values of T_*.

while it disagrees with values adopted more recently (e.g., Seljak 2002).

The fact that cluster abundance and weak lensing probe different scales opens a possibility that one might be able to secure the agreement between the two methods by varying the shape of the power spectrum or the spectral index n rather than the M-T normalization. Unfortunately, the constraints we have combined above have individually been marginalized over h and n. Ideally, one would combine the cluster and weak-lensing likelihood functions and then marginalize over the relevant parameters to get the probability distribution of T_*:

$$P(T_*) = \int L_{\rm clus}(T_*, \Omega_M, \sigma_8, n, h) \times L_{\rm WL}(\Omega_M, \sigma_8, n, h)\, d\Omega_M\, d\sigma_8\, dn\, dh. \quad (7)$$

Then the results would be manifestly independent of the power spectrum parameters. We do not have the ability to perform such an analysis here.

Note, however, that the scales probed by lensing and clusters are quite close, separated by an order of magnitude at most. For example, it would require a spectral tilt of $n \sim 1.2$ to make the recently obtained "low" normalization from cluster abundance ($\sigma_8 \sim 0.6$) agree with the "high" normalization from weak lensing ($\sigma_8 \sim 0.9$), and such a high value of n is already disfavored by recent CMB experiments (Balbi et al. 2000; Netterfield et al. 2002; Pryke et al. 2002; Sievers et al. 2002).

4. CONCLUSIONS

There has been a lot of discussion recently regarding the value of cluster normalization σ_8. While the "old" results favor $\sigma_8 \sim 1$ (Viana & Liddle 1999; Pierpaoli et al. 2001 and references therein), several new cluster abundance analyses favor a significantly lower normalization (Reiprich & Böhringer 1999; Borgani et al. 2001; Viana, Nichol, & Liddle 2002; Seljak 2002; Ikebe et al. 2002; Bahcall et al. 2002). The lower normalization is also favored by the combined analysis of Two-Degree Field Galaxy Redshift Survey and CMB data (Lahav et al. 2002). On the other hand, recent weak-lensing results (van Waerbeke et al. 2002; Bacon et al. 2002; Refregier et al. 2002; Hoekstra et al. 2002) tend to favor a higher value of σ_8. The cause is of this discrepancy between various measurements has not been identified yet; one candidate is larger than anticipated systematic errors in one or both methods. Another possibility is the bias in the relation between the mass and the observable quantity—temperature or luminosity—used to construct the abundance of clusters.

The cluster abundance constraint on σ_8 crucially depends on the M-T normalization T_*. Figure 2 summarizes the current status of our knowledge of T_*. It shows seven determinations from N-body simulations and three from direct observations, as compiled in Pierpaoli et al. (2001) and Muanwong et al. (2002). The shaded region is roughly our favored range of values of T_*. Points without error bars had none quoted, and the three observed values of T_* assumed the isothermal β model. The measurement due to Muanwong et al. corresponds to their "radiative" and "preheating" cases that are cooling-flow corrected, while the value due to Pierpaoli, Scott, & White is an average over the simulations. The large discrepancy between the different measurements is apparent, and it also appears that the observed values are systematically higher than the ones obtained from simulations (see Muanwong et al. 2002 for further discussion).

We argue here that the cluster abundance–weak lensing complementarity can be used to cross check the M-T relation. By combining recent weak-lensing constraints from Bacon et al. and HIFLUGCS of Reiprich & Böhringer, we have demonstrated the utility of this method. While potential systematic errors in both data sets are still a concern, the example we use prefers relatively low values of the M-T normalization ($T_* \lesssim 1.7$ keV). We conclude that future weak-lensing surveys (the Visible and Infrared Survey Telescope for Astronomy, the Large-aperture Synoptic Survey Telescope, and the *Supernova/Accleration Probe*) combined with new cluster data from *Chandra* and *XMM-Newton* observations will provide a strong probe of the M-T relation.

D. H. is supported by the DOE grant to Case Western Reserve University. M. W. is supported by NASA and by the Sloan Foundation.

REFERENCES

Bacon, D. J., Massey, R. J., Refregier, A. R., & Ellis, R. S. 2002, MNRAS, submitted (astro-ph/0203134)
Bahcall, N., & Cen, R. 1993, ApJ, 407, L49
Bahcall, N., et al. 2002, ApJ, submitted (astro-ph/0205490)
Balbi, A., et al. 2000, ApJ, 545, L1
Bernardeau, F., van Waerbeke, L., & Mellier, Y. 1997, A&A, 322, 1
Bond, J. R., & Myers, S. T. 1991, in Trends in Astroparticle Physics, ed. D. Cline & R. Peccei (Singapore: World Scientific), 262
Borgani, S., et al. 2001, ApJ, 561, 13
Bryan, G. L., & Norman, M. L. 1998, ApJ, 495, 80
Donahue, M., & Voit, G. M. 1999, ApJ, 523, L137
Eisenstein, D., & Hu, W. 1999, ApJ, 511, 5
Eke, V., Cole, S., & Frenk, C. S. 1996, MNRAS, 282, 263
Eke, V., Navarro, J. F., & Frenk, C. S. 1998, ApJ, 503, 569
Evrard, A. E. 1989, ApJ, 341, L71
Evrard, A. E., Metzler, C., & Navarro, J. F. 1996, ApJ, 469, 494
Evrard, A. E., et al. 2002, ApJ, 573, 7
Frenk, C. S., White, S. D. M., Efstathiou, G., & Davis, M. 1990, ApJ, 351, 10
Henry, J. P. 2000, ApJ, 534, 565
Henry, J. P., & Arnaud, K. A. 1991, ApJ, 372, 410
Hoekstra, H., Yee, H. K. C., & Gladders, M. D. 2002, ApJ, submitted (astro-ph/0204295)
Horner, D. J., Mushotsky, R. F., & Scharf, C. A. 1999, ApJ, 520, 78
Hu, W., & Kravtsov, A. 2002, ApJ, submitted (astro-ph/0203169)
Ikebe, Y., Reiprich, T. H., Böhringer, H., Tanaka, Y., & Kitayama, T. 2002, A&A, 383, 773
Jenkins, A., Frenk, C. S., White, S. D. M., Colberg, J. M., Cole, S., Evrard, A. E., & Yoshida, N. 2001, MNRAS, 321, 372
Lahav, O., et al. 2002, MNRAS, 333, 961
Lilje, P. B. 1992, ApJ, 386, L33
Muanwong, O., Thomas, P. A., Kay, S. T., & Pearce, F. R. 2002, MNRAS, in press (astro-ph/0205137)
Navarro, J. F., Frenk, C. S., & White, S. D. M. 1997, ApJ, 490, 493
Netterfield, C. B., et al. 2002, ApJ, 571, 604
Nevalainen, J., Markevitch, M., & Forman, W. 2000, ApJ, 532, 694
Oukbir, J., & Blanchard, A. 1992, A&A, 262, L21
Pen, U.-L. 1998, ApJ, 498, 60
Pierpaoli, E., Scott, D., & White, M. 2001, MNRAS, 325, 77
Press, W. H., & Schechter, P. 1974, ApJ, 187, 425
Pryke, C., et al. 2002, ApJ, 568, 46
Refregier, A., Rhodes, J., & Groth, E. J. 2002, ApJ, 572, L131
Reiprich, T. H., & Böhringer, H. 2002, ApJ, 567, 716
Seljak, U. 2002, MNRAS, submitted (astro-ph/0111362)
Sheth, R. K., & Tormen, G. 1999, MNRAS, 308, 119
Sievers, J. L., et al. 2002, ApJ, submitted (astro-ph/0205387)
Struble, M. F., & Rood, H. 1999, ApJS, 125, 35
van Waerbeke, L., Mellier, Y., Pelló, R., Pen, U.-L., McCracken, H. J., & Jain, B. 2002, A&A, submitted (astro-ph/0202503)
Viana, P. T. P., & Liddle, A. 1996, MNRAS, 281, 323
———. 1999, MNRAS, 303, 535
Viana, P. T. P., Nichol, R. C., & Liddle, A. R. 2002, ApJ, 569, L75
White, M. 2001, A&A, 367, 27
White, S. D. M., Efstathiou, G., & Frenk, C. S. 1993, MNRAS, 262, 1023
Xu, H., Jin, G., & Wu, X.-P. 2001, ApJ, 553, 78
Yoshikawa, K., Jing, Y. P., & Suto, Y. 2000, ApJ, 535, 593