Problems with Small Area Surveys: Lensing Covariance of Supernova Distance Measurements

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While luminosity distances from type Ia supernovae (SNe) are a powerful probe of cosmology, the accuracy with which these distances can be measured is limited by cosmic magnification due to gravitational lensing by the intervening large-scale structure. Spatial clustering of foreground mass leads to correlated errors in SNe distances. By including the full covariance matrix of SNe, we show that future wide-field surveys will remain largely unaffected by lensing correlations. However, ''pencil beam'' surveys, and those with narrow (but possibly long) fields of view, can be strongly affected. For a survey with 30 arcmin mean separation between SNe, lensing covariance leads to a $\sim$45% increase in the expected errors in dark energy parameters.

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Introduction.—Type Ia supernovae (SNe) have proven to be powerful probes of the expansion history of the universe [1], contributing to the discovery that this expansion is accelerating. A mysterious dark energy component is presumed to be responsible for this acceleration; however, its properties and provenance remain a complete mystery. As the nature of dark energy has profound implications for both cosmology and particle physics, the elucidation of its properties is one of the foremost observational and theoretical challenges. It is hoped that more accurate cosmological measurements will eventually shed light on the underlying physical mechanism [2]. Several ongoing programs, including the Supernova Legacy Survey (http://www.cfht.hawaii.edu/SNLS/), Carnegie Supernova Project (http://csp1.lco.cl/~cspuser1/CSP.html), Essence (http://www.ctio.noao.edu/~wsne/), Sloan Supernova Survey, and Supernova Factory (http://snfactory.lbl.gov), are underway to observe large samples of low, intermediate, and high redshift SNe and thereby obtain $\sim$10% constraints on the equation of state parameter of dark energy. Future attempts to measure crucial properties of dark energy, such as its time evolution, include a dedicated space-based instrument as part of the NASA/DOE Joint Dark Energy Mission.

The slight modification of the observed SN flux due to lensing by the intervening large-scale structure limits the accuracy with which the true luminosity distance can be determined for an individual SN [3]. In fact, the total error budget for SNe at redshifts higher than $z \sim 1$ will have statistical errors due to lensing that exceed the intrinsic luminosity distance dispersion [4]. These lensing effects may have already been detected in the current SN sample [5], although the evidence is still inconclusive [6]. Assuming that lensing contributes to the variance of the observed SN luminosity distribution (i.e., affects each SN observation individually) and using the expected distribution function for the cosmic magnification [7], it has been suggested that the intrinsic power of SNe type Ia observation can be restored in the presence of lensing provided the SN sample is increased by a factor of 2–3 [4].

In addition to the increased variance of SN distance measurements due to lensing, spatial fluctuations in the foreground mass structures will lead to correlation of distance estimates of SNe. Even SNe that are widely separated in the radial direction will be lensed by common (sufficiently large-scale) modes of the foreground mass distribution.

In principle, one can use fluctuations of the mean intrinsic luminosity to measure magnification statistics [8]. While such measurements are useful in the context of weak lensing studies, lensing correlations provide a significant challenge for precision measurement of dark energy properties. The additional covariance due to lensing can lead to significant degradation of cosmological parameter estimates for future small-field SN searches.

Calculational method.—Lensing modifies the true SN flux by a magnification μ, so that the observed flux is given by $f^{\rm obs}(\hat{\mathbf{n}}, z) = \mu(\hat{\mathbf{n}}, z) f^{\rm true}(z)$, where $\hat{\mathbf{n}}$ represents the direction of the SN on the sky. In the weak lensing limit, this magnification can be related to other well-known quantities through [9]

$$\mu = [(1-\kappa)^2 - |\gamma|^2]^{-1} \approx 1 + 2\kappa + 3\kappa^2 + |\gamma|^2 + \cdots, \quad (1)$$

where $\kappa(\ll 1)$ is the lensing convergence and $|\gamma| = \sqrt{\gamma_1^2 + \gamma_2^2}$ is the total lensing shear. Since $f^{\rm obs} \propto d_L^{-2}(z)$, where $d_L(z)$ is the luminosity distance to a source at a redshift of z, fluctuations in μ lead to fluctuations in inferred distance so that $\delta d_L/\bar{d}_L = -\delta\mu/2$. Ignoring higher-order terms (which are suppressed by an order of

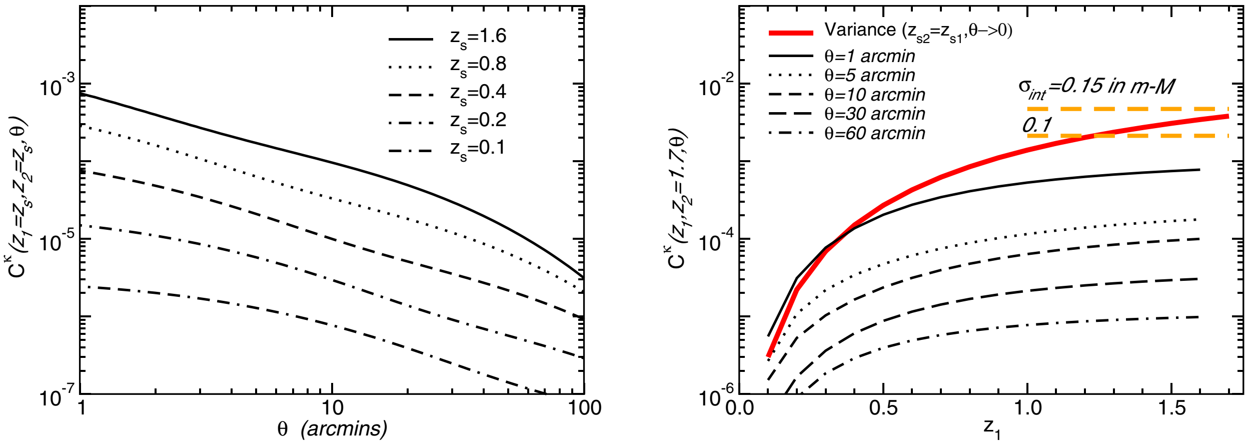

FIG. 1 (color online). Covariance of weak lensing convergence, $C^\kappa(z_1, z_2, \theta)$, as a function of two source redshifts, z_1 and z_2, and their projected angular separation θ. Left panel: Covariance as a function of θ for several values of $z_s \equiv z_1 = z_2$. Right panel: Covariance as a function of source redshift with the other source fixed at $z = 1.7$ and for several illustrative values of θ. For comparison we also show the lensing variance as a function of redshift. The two horizontal lines represent an intrinsic SN measurement error of 0.10 and 0.15 mag (or 0.046 and 0.069 in $\delta d_L/\bar{d}_L$), respectively. Note that the lensing variance becomes comparable to intrinsic dispersion at $z \gtrsim 1.2$ for $\sigma_{\rm int} = 0.1$ mag, and $z \gtrsim 1.7$ for $\sigma_{\rm int} = 0.15$ mag. The lensing variance at low redshift may become smaller than the covariance of closely separated SNe when one SN is at high redshift ($z = 1.7$ in this case). The correlation coefficient is always less than unity, but can be more than 0.5 if the SNe are separated by 10 arcmin or less.

magnitude [8,10]), one can take $\mu \approx 1 + 2\kappa$ and relate SN distance fluctuations due to lensing to the convergence along the line of sight. Thus, the full covariance matrix of fractional distance estimates for a sample of SNe is

$$\mathrm{Cov}_{ij} \approx \sigma_{\rm int}^2 \delta_{ij} + C^\kappa(z_i, z_j, \theta_{ij}), \tag{2}$$

where $\sigma_{\rm int}$ is the intrinsic error that affects each SN distance measurement. Note that higher-order terms ignored above that relate magnification to convergence are likely to become important at nonlinear scales. The higher-order terms correspond to the strong lensing regime. Only a small percentage of events will suffer large magnifications, and these will be easily identifiable by their large excursions on the Hubble diagram [11].

Using the angular cross power spectrum of convergence between two different redshifts, computed under the Limber approximation [12]

$$C_\ell^{\kappa\kappa}(z_i, z_j) = \int_0^{\min(r_i, r_j)} dr \frac{W(r, r_i) W(r, r_j)}{d_A^2} \times P_{\rm dm}\left(k = \frac{l}{d_A(r)}, r\right)$$

$$W(r, r_s) = \frac{3}{2}\Omega_m \frac{H_0^2}{c^2 a(r)} \frac{d_A(r) d_A(r_s - r)}{d_A(r_s)}, \tag{3}$$

the lensing contribution to the covariance is

$$C^\kappa(z_i, z_j, \theta_{ij}) = \int \frac{d^2\boldsymbol{l}}{(2\pi)^2} C_\ell^{\kappa\kappa}(z_i, z_j) J_0(l\theta_{ij}). \tag{4}$$

Here J_0 is the 0th order Bessel function of the first kind. In Eq. (3), r_i and r_j are comoving distances corresponding to SNe at redshifts z_i and z_j, respectively, d_A is the angular diameter distance, and $P_{\rm dm}(k, r)$ is the three-dimensional power spectrum of dark matter evaluated at the distance r; we calculate it using the halo model of the large-scale structure mass distribution [13].

Equation (2) defines the full covariance matrix due to lensing for SNe at redshifts z_i and z_j with projected angular separation of θ_{ij} on the sky. For reference, the previously considered excess variance due to lensing corresponds to diagonal elements of Cov_{ij} with $z_i = z_j$ and $\theta_{ij} = 0$. In this limit, $J_0(l\theta_{ij}) \rightarrow 1$ in Eq. (4), and one recovers the variance, $\sigma^2(z) = \int ldlC_\ell^{\kappa\kappa}(z)/2\pi$.

In the left panel of Fig. 1 we show the covariance $C^\kappa(z_i, z_j, \theta)$ as a function of $\theta \equiv \theta_{ij}$ (which is assumed fixed for the moment) for several values of $z_i = z_j$, while in the right panel we show the covariance as a function of z_1 with the other redshift fixed at $z_2 = 1.7$. For reference we also plot the variance as a function of redshift z and compare it to the intrinsic SN magnitude errors of 0.10 and 0.15 mag, roughly spanning the error expected in upcoming surveys. To estimate the resulting effect on cosmological parameter estimates, we compute the Fisher information matrix

$$\mathbf{F}_{\alpha\beta} = \sum_{ij} \frac{\partial d_L(z_i)}{\partial p_\alpha} (\mathrm{Cov}^{-1})_{ij} \frac{\partial d_L(z_j)}{\partial p_\beta}. \tag{5}$$

If the variance of SN distance measurements alone is considered, the Fisher matrix reduces to the familiar form, with the inverse covariance terms of $N(z_i)/(\sigma_{\rm int}^2 + \sigma_{\rm lens}^2)$; here $N(z_i)$ is the number of SNe in the redshift bin centered at z_i and $\sigma_{\rm lens}^2$ is the variance due to lensing. With

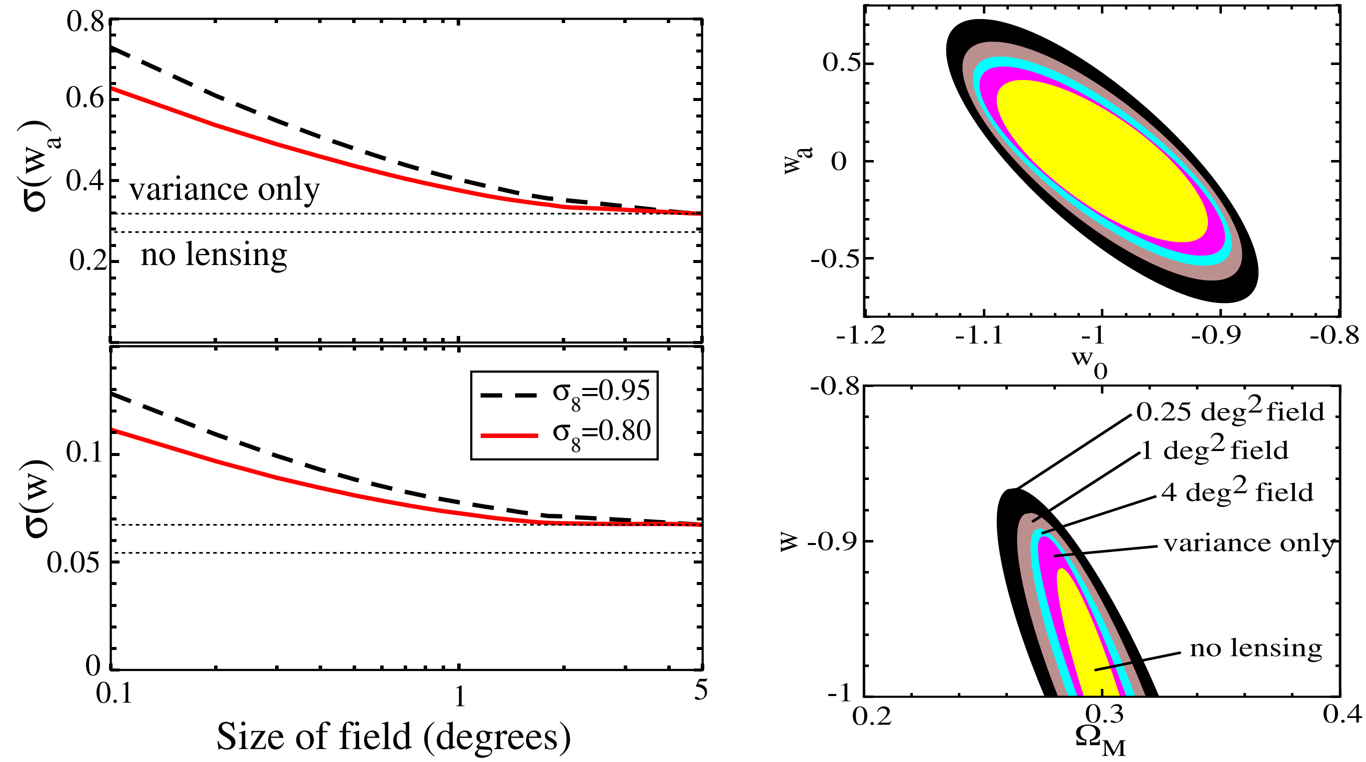

FIG. 2 (color online). Left panel: Expected errors on $w = \text{const}$ (bottom plot) or w_a (with a prior on Ω_M of 0.01, top plot) as a function of the side length of the observed field. The two horizontal dotted curves show errors in corresponding parameters when lensing is completely ignored, and when only the lensing variance is considered. We show results for two values of σ_8 that roughly span the currently favored values of the amplitude of mass fluctuations and hence the SN lensing covariance. Right panel: The full expected constraints projected into the $\Omega_M - w$ plane (bottom plot; assuming $w = \text{const}$) and $w_0 - w_a$ plane (top plot; with a prior on Ω_M of 0.01) when $\sigma_8 = 0.95$ and for the cases of no lensing, lensing variance only, and a few selected survey sizes. We have assumed a fixed total number of SNe ($N = 1700$) throughout, regardless of the parameter set and survey sky coverage.

the full covariance matrix considered, this simple form no longer holds. Moreover, a full $N_{\text{tot}} \times N_{\text{tot}}$ Fisher matrix (and not the redshift-binned smaller version) is now required in order to obtain the cosmological parameter accuracy estimates; however, this is not a novel problem since a correct treatment of SN calibration uncertainties similarly requires the full $N_{\text{tot}} \times N_{\text{tot}}$ (or even larger) covariance matrix [14]. Here we implicitly neglect information from the cosmological parameter dependence of the covariance matrix—with small covariance terms this information is expected to be small. There would be significant information in the covariance only if the off-diagonal terms were comparable to the diagonal ones.

Discussion.—To estimate cosmological parameter measurement errors, we assume a survey with 1700 SNe distributed uniformly in redshift out to $z = 1.7$ (roughly following Ref. [15]). To speed up the calculation of the 1700×1700 covariance matrix, we compute it in discrete redshift bins, stepping by 0.1 in both z_i and z_j. The covariance also depends on the angular separation of SNe, and we distribute the SNe randomly in a square field whose side (or total area) we are free to change. The histogram of the angular separations is a smooth bell curve that peaks at roughly half the field size.

Figure 2 summarizes the effect of lensing covariance on dark energy measurements from the assumed future SN survey. We model the evolution of the dark energy equation of state with redshift as $w(a) = w_0 + (1 - a)w_a$ where a is the scale factor, and consider measurements of four parameters: the matter energy density relative to critical, Ω_M, w_0, w_a, and the nuisance parameter $\mathcal{M}$ that combines the Hubble constant and absolute SN magnitude information. Our fiducial model is standard ΛCDM with $\Omega_M = 0.3$, $w_0 = -1$, and $w_a = 0$. Figure 2 (left panel) shows the expected errors on $w = \text{const}$ (bottom plot) or w_a (with a prior on Ω_M of 0.01; top plot) as a function of the size of the observed field. The two horizontal dotted curves show errors in corresponding parameters when lensing is completely ignored, and when solely the lensing variance is considered. It is apparent that the lensing covariance contributes to the error budget appreciably when the size of the field is $\lesssim 1$ deg. Furthermore, the effects of lensing covariance depend on the fiducial convergence power $C^{\kappa}(z_i, z_j, \theta_{ij})$, which in turn is sensitive to the amplitude of mass fluctuations σ_8 (and, to a lesser extent, other cosmological parameters). Since σ_8 is somewhat uncertain at present, we show results for two values, $\sigma_8 = 0.8$ and $\sigma_8 = 0.95$, that roughly span the currently favored values of the amplitude of mass fluctuations in the universe.

Figure 2 (right panel) shows the full expected constraints projected into the $\Omega_M - w$ plane (bottom plot; assuming $w = \text{const}$) and $w_0 - w_a$ plane (top plot; with a prior on Ω_M of 0.01) with $\sigma_8 = 0.95$ and for the cases of no lensing, lensing variance only, and a few selected survey sizes. Again we see that surveys of less than about one square degree will suffer from considerable error due to lensing covariance. As the mean separation between SNe is increased, off-diagonal terms in the covariance matrix decrease, and the resulting effect on cosmological parameters is reduced.

Our results can be understood simply in the limit of equal off-diagonal covariance terms. In this case, the Fisher matrix estimate of error in parameter p_α is increased by a factor $\sqrt{1 + (N-1)r^2}$ relative to the case with no off-diagonal terms, where $r = C^\kappa(z_i, z_j, \theta)/\sqrt{C^\kappa(z_i, z_i, 0)C^\kappa(z_j, z_j, 0)}$ and N is the total number of SNe in the sample. With $N \sim 2000$ or more in upcoming searches, parameter errors will increase by a factor of $\sqrt{2}$ when $r \sim 1/\sqrt{N} \sim 0.02$, and Fig. 1 reveals that at $z_1 \gtrsim 1$ correlations are at the percent level when SNe are separated by $\theta \sim 10$ arcmin. Note that in order to accurately estimate the errors on dark energy parameters one will need to allow for the dependence of the covariance matrix on imprecisely known cosmological parameters that determine the weak lensing convergence power spectrum. While galaxy shear maps are not effective in correcting the individual lensing-modified SN flux [16], such maps may be useful for statistical studies of correlations between SNe.

Conclusions.—We have discussed gravitational lensing covariance as an additional source of error for cosmological surveys utilizing standard candles. Future SN surveys that plan ~10–20 deg^2 coverage with ~2000 SNe will be largely unaffected by lensing covariance. Lensing variance remains an issue, but is reduced through increased numbers of SNe (about 50 SNe per redshift bin of width 0.1 are necessary to reduce the lensing variance so that it is negligible compared to the systematic floor [4]). Cosmological parameter accuracies for a survey with a rectangular field that is wide in one direction and narrow in another may be compromised, since the histogram of the angular distribution of SNe now has a peak at an angle of order the narrow side of the survey (albeit with a very pronounced tail). The consideration of lensing covariance thus argues against pencil beams and in favor of wider-field surveys. While our discussion generally applies to any standard candle, gravitational wave standard sirens [17] are expected to be less affected as these observations involve detectors that are intrinsically sensitive to the whole sky.

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