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Hiding dark energy transitions at low redshift

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We show that it is both observationally allowable and theoretically possible to have large fluctuations in the dark energy equation of state as long as they occur at ultralow redshifts $z \lesssim 0.02$. These fluctuations would masquerade as a local transition in the Hubble rate of a few percent or less and escape even future, high precision, high redshift measurements of the expansion history and structure. Scalar field models that exhibit this behavior have a sharp feature in the potential that the field traverses within a fraction of an e-fold of the present. The equation of state parameter can become arbitrarily large if a sharp dip or bump in the potential causes the kinetic and potential energy of the field to both be large and have opposite sign. While canonical scalar field models can decrease the expansion rate at low redshift, increasing the local expansion rate requires a noncanonical kinetic term for the scalar field.

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I. INTRODUCTION

With the ever-tightening constraints on the acceleration of cosmic expansion (e.g. [1]), it is interesting to ask whether the measurements are still compatible with any substantial deviations from a cosmological constant. While it is well known that at high redshifts ($z \gg 1$) order unity deviations may still exist in the dark energy equation of state, this is during an epoch where dark energy has very little impact on the expansion history.

Interestingly, the other place where an order unity transition in the dark energy equation of state can be hidden from data is in the local universe, at ultralow redshifts. Indeed there have been hints from Type Ia supernova (SN) data that there may be a discontinuous break in the Hubble diagram (a "Hubble bubble") of $\sim$5% at a redshift of $z \sim 0.023$ [2,3]. In the ΛCDM paradigm, a discontinuity could be explained by a local void, but its amplitude would have to be atypically large [4]. Moreover, these findings have been brought into question with the advent of new SN data and expanded studies of SN systematics, particularly color corrections [6–9].

In this *Brief Report*, we consider the local measurements as an upper limit on recent variations in the dark energy density. In Sec. II, we show how such sudden transitions are hidden from current and future high redshift measurements, determine the implied requirements on dark energy, and construct explicit scalar field models of the transition. We discuss these results in Sec. III.

II. HIDING DARK ENERGY TRANSITIONS

A. Low redshift transitions

Large transitions in the dark energy density can evade current observations if they only affect low redshifts $z_t \ll 0.1$. Galaxy surveys and the growth of structure are largely insensitive to such transitions simply because the enclosed volume and number of e-folds of the expansion are too small.

Distance measures are affected at all redshifts, but only have measurable changes at very low redshifts. For example, cosmic microwave background (CMB) and baryon acoustic oscillation (BAO) distances out to higher redshifts are largely unaffected since the shift is a small fraction of the total distance, $\delta d_L \sim -(z_t/H_0)\delta H_0/H_0$.

Given CMB and BAO absolute distance measures, one might expect their relationship to SN distance measures at $z \gg z_t$ to be affected by a dark energy transition. SN data measure the relative luminosity distance d_L between supernovae in the sample, $d_L(z)/d_L(z_{\rm min})$, where $z_{\rm min}$ is the minimum SN redshift in the survey. Ordinarily, one would assume the Hubble law

$$\lim_{z_{\rm min}\to 0} d_L(z_{\rm min}) = \frac{z_{\rm min}}{H_0} \tag{1}$$

and call $H_0 d_L(z)$ the observable SN distance. This measure would seem to be sensitive to local variations in the expansion rate when combined with $d_L(z)$ from the CMB and BAO. However, if the dark energy density undergoes a transition at $z_t < z_{\rm min}$, the expansion rate is no longer constant in z at $z < z_{\rm min}$, leading to departures from a pure Hubble law.

In other words, apparent SN magnitudes

$$m(z) = 5\log[H_0 d_L(z)] + (M - 5\log H_0 + 25) \tag{2}$$

are unaffected by local dark energy transitions. The quantity in parentheses is an unknown constant involving the absolute SN magnitude M. Since $d_L(z \gg z_t)$ is essentially unchanged if H_0 jumps in value locally, the observable $m(z)$ remains unchanged. In fact, the most precise measurement of H_0 to date uses a maser-Cepheid calibration of absolute SN distances and M *above* $z_{\rm min} = 0.023$ [9]. Only distance measurements below $z_t \lesssim 0.02$ would be sensitive to such a jump and current data limit its amplitude to be $\lesssim 5\%$.

B. Dark energy requirements

Let us determine what is required of dark energy to achieve such a Hubble transition. Consider a scenario in which the true Hubble constant is given by

$$H_0 \approx (1+\delta)\tilde{H}_0, \tag{3}$$

while the high redshift expansion rate is left unchanged. Here and throughout tildes denote values in a flat ΛCDM reference model.

To achieve this let us take a dark energy density of the form

$$\rho_{\rm DE}(z) = [1 + f(z)]\tilde{\rho}_\Lambda \tag{4}$$

and demand that

$$f(z) = \begin{cases} 0 & z \gg z_t, \\ 2\delta/\tilde{\Omega}_\Lambda & z = 0, \end{cases} \tag{5}$$

where z_t is the transition redshift. Since we have not altered the physical matter density, $\Omega_m h^2 = \tilde{\Omega}_m \tilde{h}^2$ and

$$\Omega_m = \frac{\tilde{\Omega}_m}{1+f(0)} = \frac{1-\tilde{\Omega}_\Lambda}{1+2\delta/\tilde{\Omega}_\Lambda} = 1 - \Omega_{\rm DE}. \tag{6}$$

Furthermore, CMB constraints on the matter density at recombination are automatically satisfied.

Given these requirements, three general features of a dark energy Hubble transition remain to be specified. The first two, the transition redshift z_t and the duration of the transition Δz, stringently constrain the equation of state

$$1 + w = \frac{1}{3}\frac{(1+z)f'}{1+f}, \tag{7}$$

where $f' \equiv df/dz$, given our requirements that $z_t \ll 1$ and $\Delta z < z_t$. The average equation of state at low redshifts is $1 + w \sim -\delta/\Delta z$. Thus a transition at $z_t \lesssim 0.02$ with an amplitude greater than a few percent requires an exotic equation of state which deviates from a cosmological constant by more than order unity. Moreover, transitions with $0 < \delta \ll 1$ require phantom equations of state with $w < -1$.

The remaining freedom is somewhat more subtle. Although the dark energy density, Hubble parameter, and distance-redshift relation are specified by our description so far, the dark energy equation of state *after* the transition (i.e. at $z \ll z_t$) is not. Given the small fraction of an e-fold of expansion between the transition and the present, any post-transition $1 + w$ that is order unity or less would give the same cosmological observables. This freedom in $w(z \ll z_t)$ allows one to build many models that produce a given Hubble transition.

C. Dark energy models

Let us construct scalar field models that satisfy the Hubble transition requirements. For a canonical kinetic term, we can take the dark energy density and equation of state from Eqs. (4) and (7) and reconstruct the scalar field potential [10–13]. Since the potential energy is $V = (\rho_{\rm DE} - p_{\rm DE})/2$, we have

$$V(z) = \tfrac{1}{2}(1 - w(z))\rho_{\rm DE}(z) = [(1+f) - (1+z)f'/6]\tilde{\rho}_\Lambda. \tag{8}$$

The kinetic energy of the field is $\dot{\phi}^2/2 = (\rho_{\rm DE} + p_{\rm DE})/2$, so

$$\begin{aligned} \phi(z) &= \int_0^z |(1 + w(z'))\rho_{\rm DE}(z')|^{1/2}\frac{dz'}{(1+z')H(z')} \\ &= \sqrt{\frac{\tilde{\rho}_\Lambda}{3}}\int_0^z \left(\frac{|f'|}{1+z'}\right)^{1/2}\frac{dz'}{H(z')}, \end{aligned} \tag{9}$$

where, without loss of generality, we have taken the sign of the field to be positive and set its present value to zero. From the two equations above one can implicitly get $V(\phi)$. During the transition, the field in units of the reduced Planck mass $M_{\rm Pl} \equiv (8\pi G)^{-1/2}$ rolls a distance $\Delta\phi/M_{\rm Pl} \sim |\delta\Delta z|^{1/2}$ which for typical values gives 10^{-2}.

Implicit in this construction is the requirement that the kinetic energy $\rho_{\rm DE} + p_{\rm DE}$ remain a positive quantity, which implies that f must monotonically increase with z. Note that if $\rho_{\rm DE}$ can switch signs, this differs from the requirement that $1 + w > 0$. A change of sign in both the dark energy density and $1 + w$ can occur if $f < -1$ but requires such a large change in $H(z)$ that $\delta H_0^2/\tilde{H}_0^2 \lesssim -\tilde{\Omega}_\Lambda$, i.e. almost to the point that the expansion becomes a contraction at low redshift. For all models with canonical kinetic terms, the requirement that $f' > 0$ combined with the restrictions of Eq. (5) implies that $\delta < 0$.

To construct $\delta > 0$ models we require a noncanonical kinetic term. The simplest possibility is to just reverse the sign of the kinetic term. The expressions in Eq. (8) and (9) are then identical, but the scalar field now rolls up the potential.

Even given these requirements, there are many scalar field potentials that can reproduce a given δ. Let us start with the assumption that the scalar field on the low redshift side of the transition becomes potential energy dominated directly after the transition. This implies that $f(z)$ strictly approaches a constant for $z \ll z_t$. For example, we can take

$$f(z) = \frac{2\delta}{\tilde{\Omega}_\Lambda}\frac{S(z)}{S(0)}, \qquad S(z) = \frac{1}{2}\left[1 - \tanh\left(\frac{z - z_t}{\Delta z}\right)\right]. \tag{10}$$

The scalar field potentials reconstructed from this assumption are shown in Fig. 1. Note that to achieve the $|1 + w| > 2$ equation of state for a large amplitude, rapid transition, we require *negative* potentials where a large positive ki-

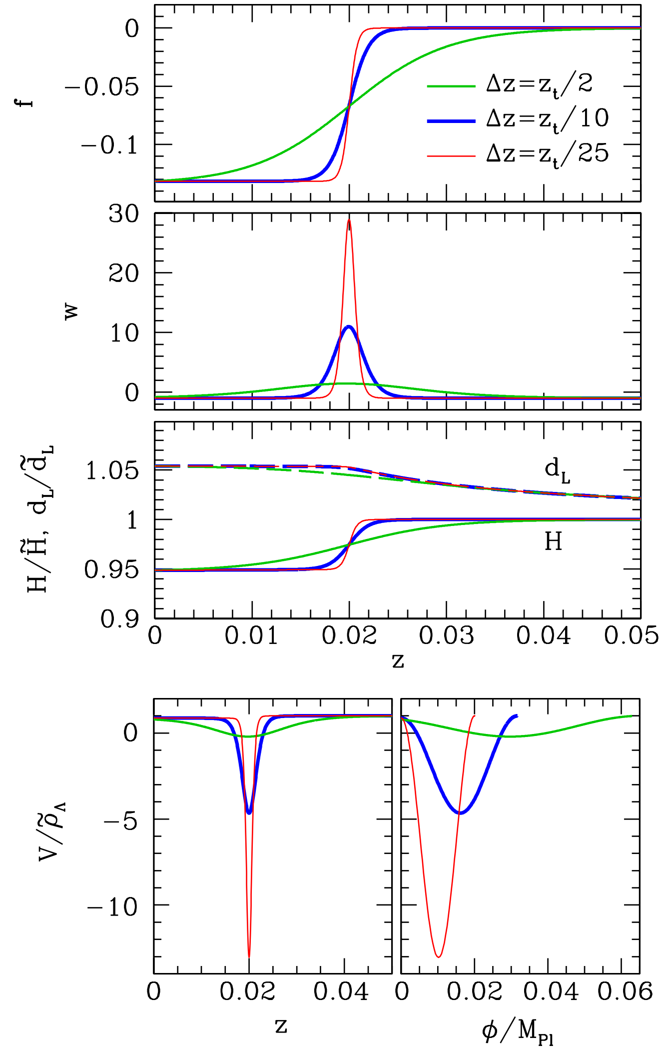

FIG. 1 (color online). Hubble transitions with different redshift widths corresponding to the potential-dominated model of Eqs. (8) and (10): $\Delta z = z_t/2$ (medium thickness, green), $z_t/10$ (thick, blue), and $z_t/25$ (thin, red). All models have a transition at $z_t = 0.02$ with amplitude $\delta = -0.05$ and tildes denote the ΛCDM reference model with $\tilde{\Omega}_m = 0.24$, $\tilde{\Omega}_\Lambda = 0.76$, and $\tilde{h} = 0.73$. The upper panels show redshift evolution of various observables. The lower panels show scalar field model potential.

netic energy and large negative potential energy cancel to leave a small total energy (see Fig. 1). In fact, the potential can become arbitrarily negative and $|1 + w|$ arbitrarily large without measurably changing the main observable $d_L(z)$. This is because no matter how sharp the transition in the expansion rate $H(z)$, distances are always a smooth function of redshift.

Models that leave the field with more kinetic energy after the transition are also possible. For example, consider the class of potentials defined by

$$V(z)/\tilde{\rho}_\Lambda - 1 = A\frac{S(z)}{S(0)} - B\frac{(1+z)}{6}\frac{S'(z)}{S(0)}. \qquad (11)$$

Our potential-dominated model in Eq. (10) corresponds to $A = 2\delta/\tilde{\Omega}_\Lambda$, $B = A$. After the transition, the A term dominates and therefore sets the level of $V(z = 0)$ (see Fig. 2). The corresponding model for $f(z)$, obtained by inverting Eq. (8), is

$$f(z) = 6\int_z^\infty dz' \frac{(1+z)^6}{(1+z')^7}[V(z')/\tilde{\rho}_\Lambda - 1]$$
$$\approx B\frac{S(z)}{S(0)} + 3(A - B)\ln[1 + e^{2(z_t - z)/\Delta z}]\Delta z, \qquad (12)$$

which implicitly defines δ through $f(0)$. The approximation assumes that $z_t \ll 1$.

Note that a sharp change in $V(z)$ is not sufficient to induce a sharp change in $f(z)$. For example, a step function

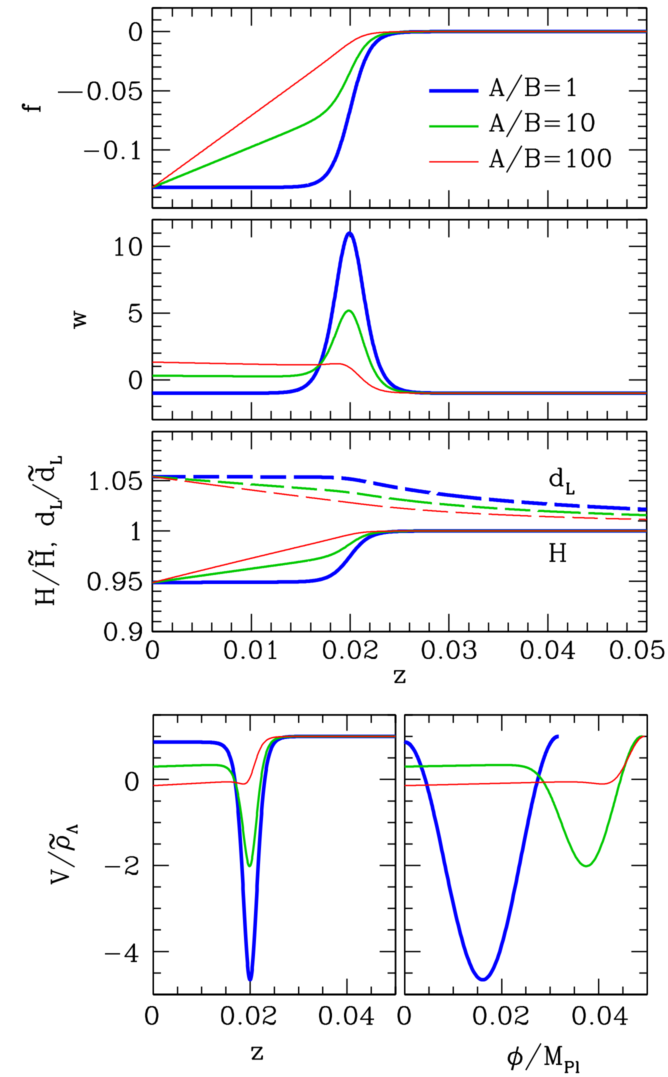

FIG. 2 (color online). Hubble transitions with different ratios of potential to kinetic energy at $z = 0$, using the generalized model of Eqs. (11) and (12) with $A/B = 1$ (thick, blue), 10 (medium thickness, green), and 100 (thin, red). The $A/B = 1$ model corresponds to zero kinetic energy after the transition as in Fig. 1, and $A/B \to \infty$ to maximal kinetic energy. All models have $z_t = 0.02$ of width $\Delta z = z_t/10$ and amplitude $\delta = -0.05$.

potential is included in the class of Eq. (11) with ($B = 0$, $\Delta z \rightarrow 0$) and has a smooth transition in f that is linear in z out to z_t (see Fig. 2).

Finally, the $\delta > 0$ phantom models have identical behavior except for a change in the sign of $(V - \tilde{\rho}_\Lambda)$ and so we do not illustrate them separately.

III. DISCUSSION

We have shown that it is both observationally allowable and theoretically possible to have arbitrarily large fluctuations in the equation of state of dark energy as long as they occur at ultralow redshifts. The possibility of such fluctuations that are hidden from data creates degeneracies that are important to understand in model-independent analyses of the dark energy constraints [14].

These fluctuations in $w(z)$ would appear as a local transition in the Hubble rate. So long as this change is of order a few percent or less at $z \lesssim 0.02$ it would escape current observational constraints. Moreover, as long as the transition is from a constant high redshift dark energy density, it would be practically indistinguishable from a cosmological constant for even future high precision distance and growth of structure measurements at high redshift. On the other hand, future percent-level Hubble constant measurements could place stronger limits on such transitions but will require accurate modeling of peculiar velocities (e.g. [15]).

Although theoretically possible with scalar field dark energy, a Hubble transition of this sort requires some unusual properties. First, to make even a percent-level change in the expansion rate over the low redshifts in question the average equation of state *must* deviate by order unity from a cosmological constant. Moreover, models with very rapid transitions require $|1 + w| \gg 1$. This can be achieved in scalar field models where the potential and kinetic energy are of opposite sign and nearly cancel. Scalar field potentials that realize these properties have a sharp feature that must coincidentally be traversed within a fraction of an e-fold of the present epoch.

In fact, $|1 + w|$ can be made arbitrarily large during the transition without a readily observable effect since the transition in the distance-redshift relation remains smooth. Nonetheless, the absence of order unity changes in the expansion rate in the data rules out a transition that is large enough to switch the sign of the dark energy density and make $w(z)$ diverge before crossing $w = -1$. Finally, to obtain *enhancements* of the low-redshift expansion rate (such as those suggested by some recent supernova data), the scalar field must in addition have a noncanonical kinetic term so that $1 + w < 0$.

The utility of studying the low redshift end of the Hubble diagram for dark energy extends beyond the extreme context taken here of hiding order unity transitions from current observations. More generally, while intermediate redshift measurements from BAO can largely take the place of local H_0 measurements for dark energy models that evolve smoothly near the present [16,17], precision measurements of the low redshift end of the Hubble diagram not only help to constrain such smooth models (e.g. [18]), but also offer the only empirical way to test whether dark energy has undergone recent variations in its equation of state.

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