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COSMOLOGY FROM SUPERNOVA MAGNIFICATION MAPS

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ABSTRACT

High-z Type Ia supernovae are expected to be gravitationally lensed by the foreground distribution of large-scale structure. The resulting magnification of supernovae is statistically measurable, and the angular correlation of the magnification pattern directly probes the integrated mass density along the line of sight. Measurements of the cosmic magnification of supernovae therefore complement measurements of galaxy shear in providing a direct measure of the clustering of the dark matter. As the surface density of supernovae is typically much smaller than that of sheared galaxies, the two-point correlation function of lensed Type Ia supernovae suffers from significantly increased shot noise. Nevertheless, we find that the magnification map of a large sample of supernovae provides an important cosmological tool. For example, a search over 20 deg^2 over 5 years leading to a sample of ~10,000 supernovae would measure the angular power spectrum of cosmic magnification with a cumulative signal-to-noise ratio of ~20. This detection can be further improved once the supernova distance measurements are cross-correlated with measurements of the foreground galaxy distribution. The magnification maps made using supernovae can be used for important cross-checks with traditional lensing shear statistics obtained in the same fields and can help to control systematics.

Subject headings: cosmology: observations — cosmology: theory — galaxies: fundamental parameters — gravitational lensing

Online material: color figures

1. INTRODUCTION

Type Ia supernovae (SNe Ia) are by now firmly established as powerful probes of the expansion history of the universe (Barris et al. 2004; Knop et al. 2003; Riess et al. 2004). In particular, the luminosity distance measurements from SNe provide a direct probe of dark energy in the universe and its temporal behavior (e.g., Huterer & Cooray 2005 and references therein). Numerous current and future SN Ia surveys are being planned or performed, and this community-wide effort is expected to reach its apex with the NASA/DOE Joint Dark Energy Mission (JDEM).

While SNe are very good (~10%–15% errors in flux) standard candles, the inferred luminosity of a given supernova is affected by cosmic magnification due to gravitational lensing from the mass distribution of the large-scale structure along the line of sight between the supernova and the observer (Frieman 1995; Holz & Wald 1998). This is a fundamental limitation to the utility of standard candles, and SNe at high redshift ($z > 1$) are especially prone to fluctuations of their flux due to lensing. There was a recent claim of evidence for weak lensing of SNe from the Riess et al. (2004) sample (Wang 2005), although this claim remains unconfirmed (Ménard & Dalal 2005).

Weak lensing biases the luminosity measurement from each SN and thus introduces a systematic error in the extraction of cosmological parameters. With a large number of SNe in each redshift interval at high z, this systematic can be essentially averaged out (Dalal et al. 2003; Holz & Linder 2005), although the full lensing covariance must be taken into account for accurate cosmological parameter estimates (Cooray et al. 2005). While lensing has mostly been considered as a nuisance, planned large-area SN surveys provide an opportunity to treat the lensing magnification of SNe as a signal. In practice, by comparing the SN Hubble diagram averaged over all directions with individual SN luminosity distance measurements, one can map out anisotropy in the SN Hubble diagram. This anisotropy will trace the cosmic magnification; in the weak gravitational lensing limit, it will be linearly proportional to the convergence and hence to the foreground dark matter distribution.

Previous studies have considered potential applications of cosmic convergence as a probe of the large-scale dark matter distribution (Jain 2002). Cosmic magnification has already been detected via cross-correlation between fluctuations in background source counts and low-redshift foreground galaxies (see Bartelmann & Schneider 2001 for a review). Past detections have often been affected by systematic uncertainties, but the Sloan Digital Sky Survey (SDSS) has recently made the first reliable detection of cosmic magnification (Scranton et al. 2005). Nevertheless, even this measurement using several hundred thousand quasars and upwards of 10 million foreground galaxies was at a relatively modest 8 σ level, illustrating the intrinsic difficulty in extracting the cosmic magnification. Some proposals for future detections involve the use of 21 cm background anisotropies of the neutral hydrogen distribution (Zhang & Pen 2005), but these studies are experimentally challenging and are affected by large theoretical uncertainties in the amplitude of the expected signal and its modification due to lensing (Cooray 2004).

In this Letter we propose mapping cosmic magnification with a sample of Type Ia SNe. We compute the predicted angular power spectrum of lensing magnification and estimate how accurately it can be measured, as well as how this measurement can be improved through cross-correlating supernova distances with the foreground distribution of galaxies. We discuss several important applications of this technique. We adopt a flat cosmological model with a Hubble constant of $h = 0.7$, matter

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density $\Omega_m = 0.3$, and normalization of the matter power spectrum $\sigma_8 = 0.85$.

2. WEAK LENSING OF SUPERNOVAE

We begin by summarizing the effect of the lensing magnification of SNe. Note that our study applies to any standard candle (e.g., Holz & Hughes 2005). Luminosity of a given supernova at a redshift z and located in the direction $\hat{\boldsymbol{n}}$, $L(z, \hat{\boldsymbol{n}})$, is affected by weak-lensing magnification so that $L(z, \hat{\boldsymbol{n}}) = \mu(z, \hat{\boldsymbol{n}})L$, where $\mu(z, \hat{\boldsymbol{n}})$ is the weak-lensing–induced magnification in the direction $\hat{\boldsymbol{n}}$ and at redshift z, and L is the true luminosity of the supernova. Note that μ can take values between the empty-beam value and infinity; the probability distribution function of μ, $P(\mu)$, has been extensively studied (e.g., Holz 1998; Wang et al. 2002). Since $\langle\mu\rangle = 1$, one can average over large samples to determine the mean luminosity L (Wang 2000; Holz & Linder 2005). One can now consider spatial fluctuations in the luminosity, $\delta_L(z, \hat{\boldsymbol{n}}) = [L(z, \hat{\boldsymbol{n}}) - L]/L$, which trace fluctuations in the cosmic magnification, μ. In the weak-lensing limit ($\mu, \kappa \ll 1$), we have

$$\mu = [(1-\kappa)^2 - |\gamma|^2]^{-1} \approx 1 + 2\kappa + 3\kappa^2 + |\gamma|^2 + \dots, \quad (1)$$

where κ is the lensing convergence and $|\gamma| = (\gamma_1^2 + \gamma_2^2)^{1/2}$ is the total shear. To first order in the convergence, $\delta_L(z, \hat{\boldsymbol{n}})$ traces spatial fluctuations of 2κ, although higher order corrections may be important (Ménard et al. 2003). Traditional weak lensing involves measurement of the statistics of the shear, γ_i, as this leads to a distortion of background galaxy shapes (Bartelmann & Schneider 2001). Magnification, on the other hand, changes images' sizes but suffers from the problem that the true size of cosmological objects is highly uncertain. Fluctuations in the luminosity of standard candles provide us with a reliable way to probe the cosmic magnification.

Assuming statistical isotropy, the angular power spectrum of magnification fluctuations is $\langle\mu^\star_{\ell m}\mu_{\ell' m'}\rangle = C_\ell^{\mu\text{-}\mu}\delta_{\ell\ell'}\delta_{mm'}$, where $\mu_{\ell m}$ are the multipole moments of the magnification. Using the Limber approximation, the angular power spectrum can be written as (Kaiser 1998; Cooray et al. 2000)

$$C_\ell^{\mu\text{-}\mu} = \int dr\, \frac{W^2(r)}{d_A^2} P_{\rm dm}\left(k = \frac{\ell}{d_A}, r\right),$$
$$W(r) = 3\int dr'\, n(r')\Omega_m \frac{H_0^2}{c^2 a(r)} \frac{d_A(r) d_A(r'-r)}{d_A(r')}, \quad (2)$$

where r is the comoving distance, d_A is the angular diameter distance, and $n(r)$ is the radial distribution of SNe normalized so that $\int dr\, n(r) = 1$. $P_{\rm dm}(k, r)$ is the three-dimensional power spectrum of dark matter evaluated at the distance r; we calculate it using the halo model of the large-scale structure mass distribution (Cooray & Sheth 2002). The next order correction term, $\langle\kappa^2_{\ell m}\kappa_{\ell' m'}\rangle$, is easily related to the convergence bispectrum (Cooray et al. 2000).

In addition to a measurement of the projected angular power spectrum of cosmic magnification, one can also cross-correlate the magnification with the foreground galaxy distribution. The idea here is that the dark matter distribution that causes the magnification pattern $\delta\mu(z, \hat{\boldsymbol{n}})$ is traced by galaxies, and therefore δ_μ and the normalized galaxy overdensity, $\delta_{\rm gal}$, are correlated. The projected cross-correlation between the two fields is described by the angular power spectrum

$$C_\ell^{\mu\text{-gal}} = \int dr\, \frac{W(r) n_{\rm gal}(r)}{d_A^2} P_{\rm dm\text{-}gal}\left(k = \frac{\ell}{d_A}, r\right), \quad (3)$$

where $n_{\rm gal}(r)$ is the normalized radial distribution of foreground galaxies. When relating galaxy density fluctuations to dark matter fluctuations, we assume a bias factor of unity. Since $\delta_L = \delta_\mu$, the cross-correlation is independent of the power-law slope of the source number counts, unlike in the case of traditional galaxy-quasar cross-correlation measurements (Scranton et al. 2005).

To estimate how well these angular power spectra can be measured with upcoming surveys, we compute the cumulative signal-to-noise ratio for detection

$$\left(\frac{\rm S}{\rm N}\right)^2 = \sum_\ell \left(\frac{C_\ell^i}{\Delta C_\ell^i}\right)^2, \quad (4)$$

where the index i references either the magnification power spectrum or the magnification-galaxy cross power spectrum. The error in the magnification power spectrum is given by

$$\Delta C_\ell^{\mu\text{-}\mu} = \sqrt{\frac{2}{(2\ell+1) f_{\rm sky}\Delta\ell}}\left(C_\ell^{\mu\text{-}\mu} + \frac{\sigma_\mu^2}{N_{\rm SN}}\right), \quad (5)$$

where $N_{\rm SN}$ is the surface density of SNe (number per steradian), σ_μ is the uncertainty in the δ_μ measurement from each supernova, $f_{\rm sky}$ is the fraction of sky covered by the survey, and $\Delta\ell$ is the binning width in multipole space. For the SN luminosity-galaxy count cross-correlation, the error is

$$\Delta C_\ell^{\mu\text{-gal}} = \sqrt{\frac{1}{(2l+1) f_{\rm sky}\Delta\ell}} \times \left[(C_\ell^{\mu\text{-gal}})^2 + \left(C_\ell^{\mu\text{-}\mu} + \frac{\sigma_\mu^2}{N_{\rm SN}}\right)\left(C_\ell^{\rm gal\text{-}gal} + \frac{1}{N_{\rm gal}}\right)\right]^{1/2}, \quad (6)$$

where $C_\ell^{\rm gal\text{-}gal}$ is the angular clustering power spectrum of foreground galaxies and $N_{\rm gal}$ is their surface density.

For definitiveness we assume a magnification measurement error for each SN of $\sigma_\mu = 0.1$.[4] For simplicity we consider the SNe uniformly distributed in $0.1 \le z \le 1.7$, which roughly approximates the distribution expected from the *Supernova/Acceleration Probe* (*SNAP*; Aldering et al. 2004).

3. RESULTS AND DISCUSSION

Figure 1 shows the angular power spectra of the cosmic magnification of SNe, the cross power spectrum between the magnification anisotropies and the foreground galaxy distribution, and the galaxy angular power spectrum. Here we are particularly interested in the μ-μ and μ-gal spectra, for which we also show error bars. We have assumed the same foreground galaxy sample as in Scranton et al. (2005), with a redshift distribution of the form $dn/dz \sim z^{1.3} \exp[-(z/0.26)^{2.17}]$ and a number density of usable galaxies of 3 arcmin^{-2}. To avoid overlap of SNe with the galaxy sample, in the case of cross-

[4] The magnification error is equal to the relative error in measuring luminosity and is roughly equal to twice the relative luminosity distance error.

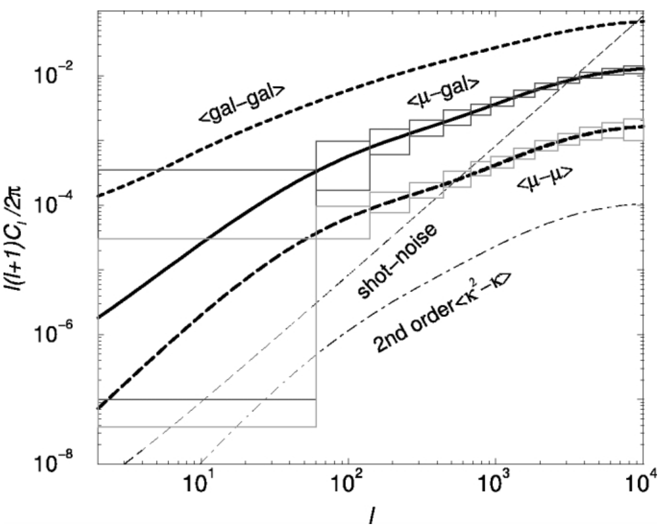

FIG. 1.—Angular power spectrum of cosmic magnification (*heavy long-dashed curve*), cross-correlation between magnification and foreground galaxies (*heavy solid curve*), and foreground galaxy clustering (*heavy short-dashed curve*). The fractional error in the luminosity of each SN has been assumed to be 0.1, and the error boxes account for both the sample (cosmic) variance due to the limited survey area and the presence of shot noise due to the finite number of SNe. We assume 10,000 SNe obtained over an area of 10 deg^2 with a uniform distribution in redshift between 0.1 and 1.7. The thin dashed line shows the shot noise in the magnification power spectrum. The cross-correlation makes use of the sample of SNe at $z > 0.7$, while the foreground galaxy sample is from Scranton et al. (2005). The thin dot-dashed line shows a second-order correction to magnification. [*See the electronic edition of the Journal for a color version of this figure.*]

correlation, we only consider an SN subsample with redshifts greater than 0.7. In Figure 1 we also show the second-order correction to magnification corresponding to the κ^2 term in equation (1).

In Figure 2 we show the total signal-to-noise ratio for the detection of the angular power spectrum of SN magnifications. The left panel shows the S/N as a function of the smallest scale (maximum multipole) probed by the survey. We show cases of 3000 and 10,000 SNe observed over 20 deg^2 (i.e., $f_{\rm sky} \approx 0.0005$). Note that, while the signal-to-noise ratio for the magnification power spectrum is usually around 20 or below, the cross power spectrum can be detected with considerably better significance due to the much smaller shot noise in the foreground galaxy population. The right panel of Figure 2 shows the signal-to-noise ratio, except now as a function of the sky coverage of the survey, $f_{\rm sky}$, as we hold the total number of observed SNe fixed. This allows us to optimize the magnification measurement for a given amount of telescope time. Very small $f_{\rm sky}$ leads to large cosmic variance, while large $f_{\rm sky}$ decreases the surface density of SNe (since both their number and the observation time are held fixed) and therefore increases the shot noise. In addition, the upper limit on the rate of SNe translates into a minimum $f_{\rm sky}$. Measurements of the actual rate of SNe (Pain et al. 2002) combined with theoretical estimates (Oda & Totani 2005) suggest that a year-long survey should find up to $\sim 10^3$ SNe deg^{-2}, and this limit, assuming a 5 yr survey, is shown as a lower limit on $f_{\rm sky}$ in the right panel of Figure 2.

The surface density of galaxies, estimated to be around 10^9 sr^{-1} down to 27 mag (Smail et al. 1995), is far larger than the surface density of SNe (which is of order 10^6 sr^{-1} over a year-long integration). Nevertheless, cosmological methods that use weak lensing of galaxies to probe the formation of structure in the universe are subject to systematic errors that range from theoretical uncertainties to a variety of measurement systematics (see Huterer et al. 2005 and references therein). Supernova measurements of the magnification can be extremely valuable in helping us control these systematics and break certain degeneracies.

For example, in order to measure the masses of shear-selected galaxy clusters, one can reconstruct the convergence $\kappa(\hat{\boldsymbol{n}})$ from the measured shear $\gamma_{1,2}(\hat{\boldsymbol{n}})$. While this can be done using well-known techniques (e.g., Kaiser & Squires 1993), the reconstruction is insensitive to a multiplication of $[1 - \kappa(\hat{\boldsymbol{n}})]$ and $\gamma_{1,2}(\hat{\boldsymbol{n}})$ with a constant; this operation preserves the reduced shear and thus also the measured ellipticities. Since SNe are

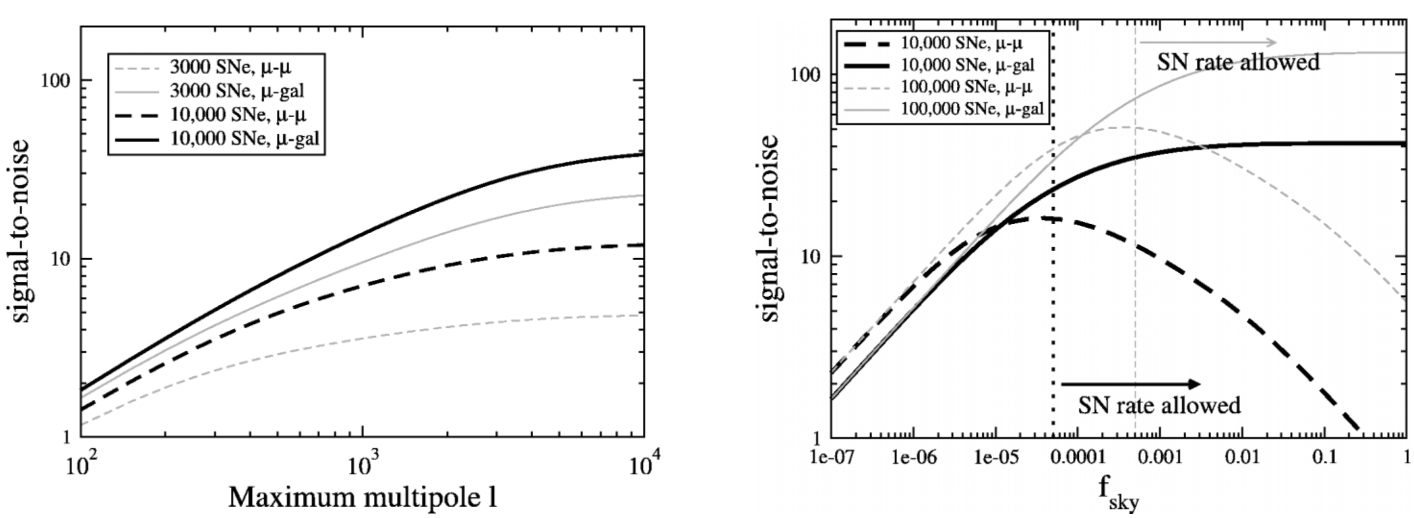

FIG. 2.—*Left panel:* Signal-to-noise ratio for the detection of the magnification power spectrum (*dashed curves*) and the magnification-galaxy cross power spectrum (*solid curves*) as a function of the smallest scale (maximum multipole) probed by the survey. We show cases of 3000 SNe (*light curves*) and 10,000 SNe (*heavy curves*) collected over 20 deg^2 ($f_{\rm sky} \approx 0.0005$). *Right panel:* Signal-to-noise ratio of the magnification power spectrum (*dashed curves*) and the galaxy-magnification cross-power (*solid curves*) as a function of the fraction of the sky covered, $f_{\rm sky}$, and assuming 10,000 SNe (*heavy curves*) and 100,000 SNe (*light curves*). The vertical lines show the minimal $f_{\rm sky}$ for a given number of SNe, which is given by the SN rate over the survey area and assuming an observing time of 5 years. We assume an error in measuring the magnification of each SN of $\sigma_\mu = 0.1$; for $\sigma_\mu = 0.15$, the signal-to-noise ratio decreases by $\sim$30%. [*See the electronic edition of the Journal for a color version of this figure.*]

standard candles, this "mass-sheet degeneracy" (e.g., Falco et al. 1985; Bradac et al. 2004) can be broken with exact convergence measurements directly via magnification. Updating the calculations in Kolatt & Bartelmann (1998), we find that up to 2% of the SNe are magnified by foreground clusters at a factor greater than 1.3 (a 3 σ or better detection). A survey covering ∼20 deg with $\sim 10^4$ SNe, combined with the shear information, can provide mass measurements (enclosed out to the impact radius of the background supernova) of ∼100 clusters to better than 10%.

This approach can also test the consistency between shear measurements from galaxy shapes and convergence from SNe luminosity anisotropies. One can construct *E*- and *B*-modes of shear, and, in the weak-lensing limit, $C_\ell^\kappa = C_\ell^E$ and $C_\ell^B = 0$. Departures from these relations are expected from both physical and theoretical systematic uncertainties. For example, intrinsic correlations between galaxies may produce an additional but unequal contribution to the *E*- and *B*-modes (Heavens et al. 2000). Moreover, there will exist contributions from higher order effects due to slight departures from the weak-lensing limit (see eq. [1]; Ménard et al. 2003), higher order corrections to lensing (Cooray & Hu 2002), and a gravitational wave background (Dodelson et al. 2003). The power of this consistency test is limited by the size of the higher order corrections and requires taking into account higher order contributions of shear and convergence to the magnification.

To quantify the detectability of the difference between the shear and convergence power spectra, we assume for a moment that this difference is given by the second-order term $C_\ell^{\kappa^2-\kappa}$ plotted as the dot-dashed line in Figure 1. We calculate the signal-to-noise ratio in measuring the quantity $C_\ell^\Delta = |C_\ell^\kappa - C_\ell^\gamma|$ following the procedure similar to that in equation (4). Since future weak-lensing shear surveys will have much smaller shot noise ($\gamma_{\rm rms}^2/\bar{n} \sim 10^{-11}$) than the corresponding magnification power measurements ($\sim 10^{-9}$), the former source of noise can be ignored in the calculation. Using an SN surface density of 10^3 deg^{-2} yr^{-1}, we find that the difference between the power spectra can be detected with a signal-to-noise ratio of $\sim 10(20\ {\rm deg}^2/A)^{-1/2}$, where A is the total survey area. Alternatively, if we assume that the fiducial difference between the power spectra has a shot-noise power spectrum (i.e., flat in ℓ) with $C_\ell^\Delta = \Delta$, the minimum detectable amplitude (with a signal-to-noise ratio of unity) is $\Delta \approx 5 \times 10^{-7}(20\ {\rm deg}^2/A)^{1/2}$. Consequently, corrections to the shear signal that are due to intrinsic correlations may be detectable (Jing 2002).

Magnification statistics from SNe also provide us with information on the cosmological parameters in a similar fashion to the conventional weak lensing of galaxies (Hu & Tegmark 1999; Huterer 2002). With the magnification power spectrum detected at a signal-to-noise ratio of 10 (100), one linear combination of parameters, typically with large weights in the Ω_m and σ_8 directions, can be constrained to 10% (1%). While the conventional weak lensing of galaxies can provide us with a more accurate overall determination of cosmological parameters, the strength of the proposed method is that it combines lensing shear and magnification information in the same field, thereby providing a number of cross-checks on systematics.

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REFERENCES

Aldering, G., et al. 2004, PASP, submitted (astro-ph/0405232)
Barris, B. J., et al. 2004, ApJ, 602, 571
Bartelmann, M., & Schneider, P. 2001, Phys. Rep., 340, 291
Bradac, M., Lombardi, M., & Schneider, P. 2004, A&A, 424, 13
Cooray, A. 2004, NewA, 9, 173
Cooray, A., & Hu, W. 2002, ApJ, 574, 19
Cooray, A., Hu, W., & Miralda-Escude, J. 2000, ApJ, 535, L9
Cooray, A., Huterer, D., & Holz, D. E. 2005, Phys. Rev. Lett., in press (astro-ph/0509581)
Cooray, A., & Sheth, R. 2002, Phys. Rep., 372, 1
Dalal, N., Holz, D. E., Chen, X. L., & Frieman, J. A. 2003, ApJ, 585, L11
Dodelson, S., Rozo, E., & Stebbins, A. 2003, Phys. Rev. Lett., 91, 021301
Falco, E. E., Gorenstein, M. V., & Shapiro, I. I. 1985, ApJ, 289, L1
Frieman, J. A. 1995, Comments Astrophys., 18, 323
Heavens, A., Refregier, A., & Heymans, C. 2000, MNRAS, 319, 649
Holz, D. E. 1998, ApJ, 506, L1
Holz, D. E., & Hughes, S. A. 2005, ApJ, 629, 15
Holz, D. E., & Linder, E. V. 2005, ApJ, 631, 678
Holz, D. E., & Wald, R. M. 1998, Phys. Rev. D, 58, 063501
Hu, W., & Tegmark, M. 1999, ApJ, 514, L65
Huterer, D. 2002, Phys. Rev. D, 65, 063001
Huterer, D., & Cooray, A. 2005, Phys. Rev. D, 71, 023506
Huterer, D., Takada, M., Bernstein, G., & Jain, B. 2005, MNRAS, in press (astro-ph/0506030)
Jain, B. 2002, ApJ, 580, L3
Jing, Y.-P. 2002, MNRAS, 335, L89
Kaiser, N. 1998, ApJ, 498, 26
Kaiser, N., & Squires, G. 1993, ApJ, 404, 441
Knop, R. A., et al. 2003, ApJ, 598, 102
Kolatt, T. S., & Bartelmann, M. 1998, MNRAS, 296, 763
Ménard, B., & Dalal, N. 2005, MNRAS, 358, 101
Ménard, B., Hamana, T., Bartelmann, M., & Yoshida, N. 2003, A&A, 403, 817
Oda, T., & Totani, T. 2005, ApJ, 630, 59
Pain, R., et al. 2002, ApJ, 577, 120
Riess, A. G., et al. 2004, ApJ, 607, 665
Scranton, R., et al. 2005, ApJ, 633, 589
Smail, I., Hogg, D. W., Yan, L., & Cohen, J. G. 1995, ApJ, 449, L105
Wang, Y. 2000, ApJ, 536, 531
———. 2005, J. Cosmol. Astropart. Phys., 0503, 005
Wang, Y., Holz, D. E., & Munshi, D. 2002, ApJ, 572, L15
Zhang, P., & Pen, U.-L. 2005, Phys. Rev. Lett., 95, 241302