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# Importance of supernovae at $z>1.5$ to probe dark energy

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The accelerating expansion of the universe suggests that an unknown component with strongly negative pressure, called dark energy, currently dominates the dynamics of the universe. Such a component makes up $\sim 70\%$ of the energy density of the universe yet has not been predicted by the standard model of particle physics. The best method for exploring the nature of this dark energy is to map the recent expansion history, at which type Ia supernovae have proved adept. We examine here the depth of survey necessary to provide a precise and qualitatively complete description of dark energy. A realistic analysis of parameter degeneracies, allowance for natural time variation of the dark energy equation of state, and systematic errors in astrophysical observations all demonstrate the importance of a survey covering the full range $0<z\lesssim 2$ for revealing the nature of dark energy.

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The discovery of the acceleration of the expansion of the universe through the type Ia supernova distance-redshift relation is a major development in cosmology [1,2]. Exploring the expansion history of the universe is a key aim of cosmology, producing literally a textbook picture of the universe. Furthermore, such a map provides key clues to the underlying physics, independent of whether this is dark energy, higher dimensions, or an altered theory of gravitation [3].

In its interpretation as arising from a universal vacuum, or dark, energy, such a component would comprise some 70% of the critical density, be unclustered on subhorizon scales, and possess a substantially negative equation of state (EOS) $w=p/\rho\lesssim -0.6$ [4]. While these properties are unexpected from the standard model of particle physics, it has been suggested that they can be motivated by a number of fundamental theories [5,6]. Dark energy thus poses a crucial mystery to unravel for the fields of high energy physics, cosmology, and gravitation.

Supernovae studies, which first provided the evidence for the acceleration, are well suited for elucidating the nature of the dark energy [7,8]. One experiment being designed specifically to probe the accelerating universe using supernovae is the Supernova/Acceleration Probe (SNAP [9]). At an initial theoretical glance, the redshift range over which this exploration is most easily done seems simple to understand: the energy density dominance and dynamical influence (accelerating power) of dark energy enters at redshifts $z\lesssim 0.7$ (see Fig. 1). Moreover, an idealized perturbative, or Fisher matrix, calculation shows that the "sweet spot" of sensitivity to the equation of state $w$ lies at $z\approx 0.3$ [5,8,10]. So why are observations at $z>1$ necessary for characterizing the dark energy?

The answer lies in the breakdown of the ideal case: cosmological degeneracies; dark energy model degeneracies; systematic errors.

The required survey depth depends on the rigor of our scientific investigation, how much we are willing to assume about the other parameters entering into the determination of the dark energy equation of state. One could estimate a false precision without knowing how accurate, i.e. biased, the result is. We label this blind trust by three heresies, and here aim to demonstrate their insidious effects through simple illustrations rather than mathematical arguments.

Acceleration of the expansion must give way as we look further into the past to a normal, matter dominated decelerating phase so that structure could have formed. Observation of the turnover in the distance-redshift relation due to this transition provides both a critical check on our understanding and a discriminator from (generically monotonic) systematic effects; this requires redshifts $z>1$. While Fig. 1 shows the acceleration or deceleration transition occurs at lower $z$, the inertia caused by the integral nature of the distance relation prevents the turnover in the magnitude-redshift Hubble diagram from appearing until higher redshift [11,12]. The turnover occurs when the EOS of the total energy density $w_T=-1/3$. Distinguishing between dark energy models based on their distance-redshift behavior depends on the difference between their $w_T(z)$, but the models can cross in $w_T-z$ plane. Therefore, Hubble diagram curves of models may diverge only slowly with redshift. These effects preserve the importance of dark energy at higher redshifts. Figure 2 illustrates the falsity of the naïve assumption that dark energy is only important at low redshift: dark energy has an influence, significant on the precision scales SNAP can achieve, out beyond $z=1.5$. A survey extending this deep can clearly map out the transition from the accelerating to decelerating phase, basically seeing the onset of a present day inflation [3,13]. Moreover, mapping the redshift history of the universe beyond $z=1.5$ is critical for detecting unexpected late-time behavior of dark energy, such as the phase transition in the equation of state at $z\sim 2$ which may be favored by the current data [14].

A leading candidate for the physics behind the accelerating universe is a dynamical scalar field acting as vacuum energy. But high energy field theories generically predict that the equation of state of such a dark energy—other than the

 

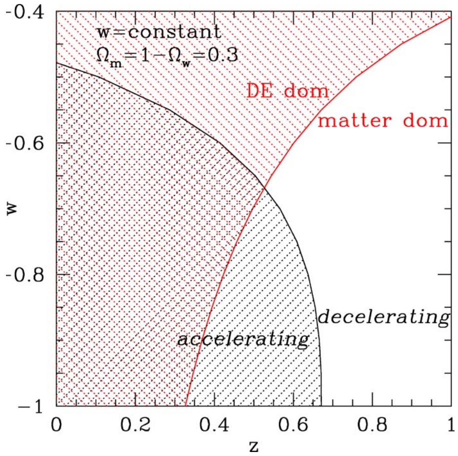


FIG. 1. The epochs of equality between the dark energy density and matter and of transition from acceleration to deceleration are plotted vs dark energy equation of state. The positively slanted hatching denotes the accelerating phase; the negatively slanted hatching shows when the dark energy density dominates over the matter density. Despite these both occurring below redshift $z \approx 0.7$, dark energy can be probed to much higher redshift.

cosmological constant—should vary with time. So consideration of only constant $w$ models severely prejudices the parameter space of theories. Conventionally one enlarges the classes of fundamental physics probed by including time variation to first order: $w(z)=w_0+w'z$ [15]. The parameter

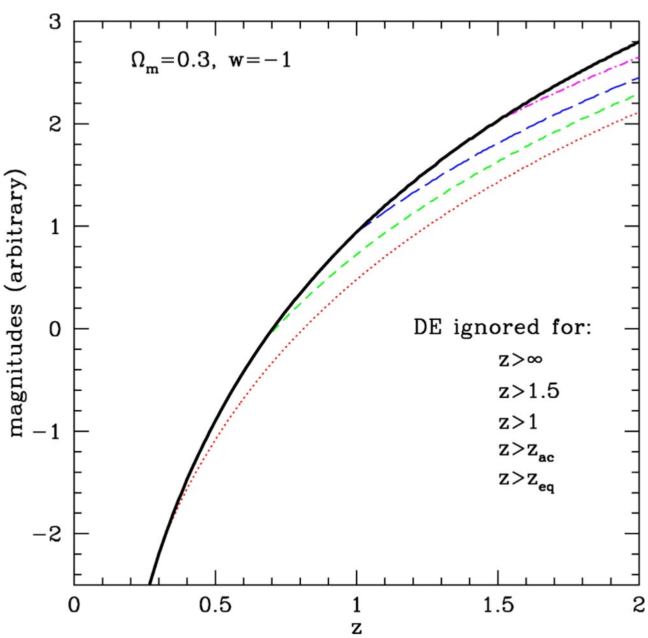


FIG. 2. Dynamical influence of dark energy persists substantially beyond the redshifts of equality $z_{eq}$ or the acceleration-deceleration transition $z_{ac}$. The curves show how the magnitude-redshift relation is distorted when the dark energy is ignored (i.e. treated as ordinary matter) above different redshifts (labeled from top down). The thickness of the solid black curve that fully incorporates dark energy represents 0.02 magnitudes—SNAP's projected sensitivity—so dark energy influence remains quite detectable even at $5z_{eq}$.

$w'$ is directly related to the scale length of the field potential $V'/V \equiv d\ln V/d\phi$.

Allowing for $w'$ has a dramatic effect on the physical content of the results. Consider the analogy of the now classic confidence contours in the dark energy (cosmological constant) density vs. matter density, or $\Omega_\Lambda-\Omega_M$, plane. Finding a precise value of, say, $\Omega_M=0.45$, $\Omega_\Lambda=1$—purely hypothetical but consistent with current supernova data—would contradict cosmic microwave background (CMB) results on flatness. Should we interpret this as evidence for a radical reworking of cosmology? Not necessarily, for the simpler explanation is that we unnecessarily limited the dark energy parameter space by forcing $w=-1$, a cosmological constant. Such a hypothetical result could be equally well fit (over a redshift range $z \lesssim 1$) by a consistent flat model with $\Omega_M=0.3$, $w=-1.15$. Analogously, confining ourselves to constant $w$ can skew the results from the true model containing a natural $w'$ term—with a very different underlying physics. That is, a restricted phase space is subject to bias because of ignoring other parameters.[1]

The mere possibility of time variation also carries important implications for error estimation. An *a priori* assumption of constant behavior not only biases the conclusions on cosmology and dark energy, but gives strongly deviant estimations of the associated errors, illustrated in Fig. 3. That is, one gets inaccurate results extremely precisely. The error $\sigma(w)$—assuming a constant equation of state—disagrees with $\sigma(w_0)$—merely *allowing* for the possibility of time variation—by a factor 3 for a survey observing 2000 (plus 300 low $z$) SNe out to $z_{max}=0.5$. Another virtue of a deep survey to $z>1.5$ is that this disagreement is only 25% at $z_{max}=1.7$. This is shown by the dotted arrows.

The necessity for a long baseline survey is even more evident in Fig. 4, which shows the uncertainty $\sigma(w')$. The error sensitivity curve steepens dramatically as the depth decreases below $z_{max}=1.5$, rapidly worsening to uselessness.

Along with the uncertainty in dark energy properties is that in our cosmological knowledge. So rather than fixing the dimensionless matter density $\Omega_M$, we take as a realistic case a Gaussian prior $\sigma(\Omega_M)=0.03$, i.e. $\Omega_M=0.3\pm0.03$.

Uncertainties in source, propagation, or detector impose a floor on our ability to reduce errors merely by gathering large numbers of supernovae. While the great advantages of supernovae as a probe are the long history of supernova studies, the rich data stream and crosschecks they provide in their light curves and spectra, and their underlying physical simplicity, we still cannot ignore the impact of astrophysics on our attempts to measure cosmology.

In Fig. 3 we see the huge discrepancy between the precision claimed in the ideal situation (actually with a prior $\sigma(\Omega_M)=0.01$, not fixed $\Omega_M$) and in the presence of systematics (see solid arrows). The systematic error essentially represents imperfect knowledge of all the astrophysics lying be-

[1]Rather than calling these families of models degenerate, it is more evocative to call them congeneric: resembling in nature or action. This has the connotation in chemistry of a molecule that acts analogously but yields a very different taste.

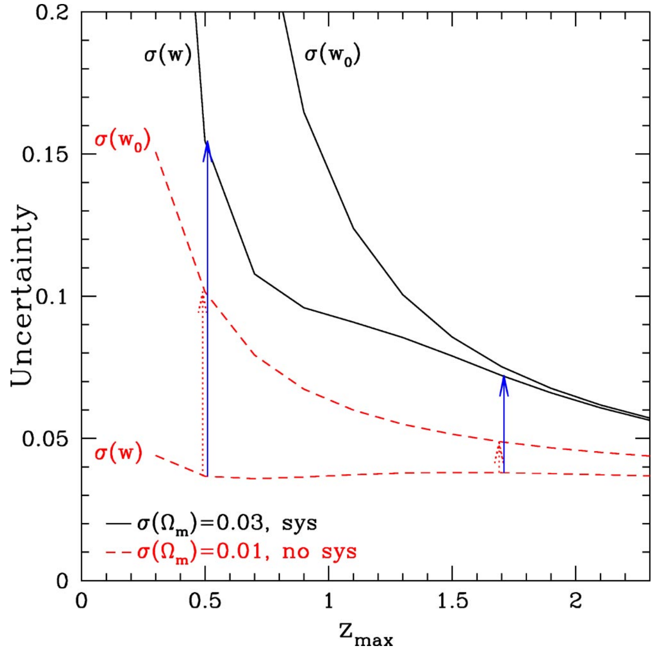


FIG. 3. Uncertainty in determination of the dark energy equation of state today as a function of survey depth $z_{max}$; $w$ denotes assuming *a priori* that there is no time variation while $w_0$ allows the possibility. The dotted arrows denote the difference; ignoring the possibility that $w$ varies with time grossly underestimates the error, especially for shallow surveys. The solid arrows show the effect of ignoring systematic errors. Precisely (and accurately) determining the equation of state requires supernovae at $z>1.5$.

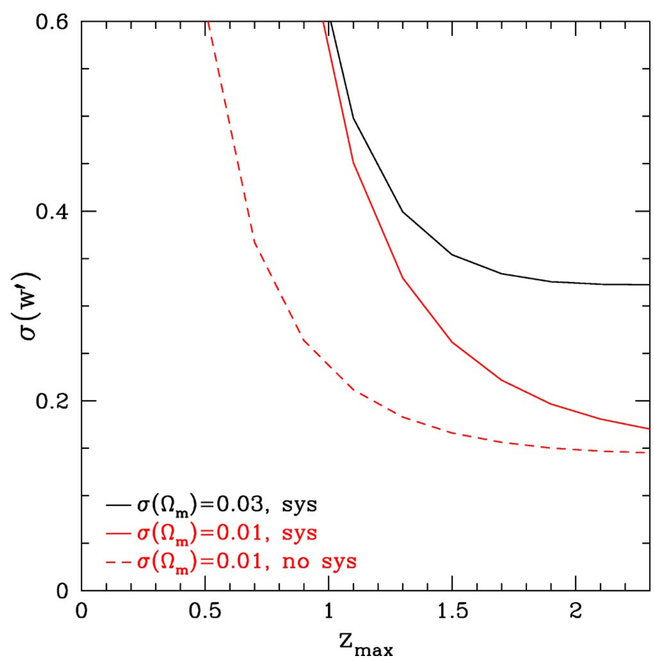


FIG. 4. Uncertainty in determination of the time variation of the dark energy equation of state as a function of survey depth $z_{max}$. Even in the idealized case of no systematic error the uncertainty rises steeply as $z_{max}$ decreases. One needs a survey extending to $z_{max}\gtrsim 1.5$ to detect this key discriminator of fundamental theories.

hind the observations, leaving a small residual error once we have carried out as good a fit as possible to the data. The systematic imposes an upper limit on the number of supernovae useful for reducing the statistical error in the magnitude through Poisson statistics. One example of such a systematic is nonstandard host galaxy dust extinction. To model the slow variation of astrophysical systematics we adopted a floor to the magnitude error within a bin of width $\Delta z=0.1$ of $dm=0.02\ (1.7/z_{max})(1+z)/2.7$. Despite the error growing with redshift, we see from Fig. 3 that the long baseline of a deep survey provides crucial leverage.

Indeed this conclusion might be made even stronger. Despite an increased magnitude error for short redshift baselines, our adopted systematic might be said to be overly generous to shallow surveys (e.g. it gives an error of 0.02 at $z=0.5$ for a survey reaching $z_{max}=0.9$), since the level of the residual systematic will depend on how elaborately the survey is designed. Without a long redshift baseline, broad wavelength coverage into the near infrared, spectral observations, a rapid observing cadence, small point spread function, etc. this number can be large. SNAP is specifically designed to achieve 0.02 mag. For a typical ground based survey, a more realistic estimate might be 0.05 mag.

For the time variation $w'$ in Fig. 4 the discrepancy due to ignoring systematics is also strong. For any reasonable prior on $\Omega_M$, systematics have an extreme effect for shallow surveys: a factor $\sim 5$ degradation of our estimate $\sigma(w')$ at $z_{max}=0.5$. Compare this to a mere 12% (40%) degradation for $z_{max}=1.7$ when the $\Omega_M$ prior is 0.03 (0.01); this clearly shows the vast utility of including supernovae at $z>1.5$.

We have seen that low redshift sensitivity to the form of the dark energy depends on idealized conditions: (1) reduction of the parameter space by fixing the cosmological model (i.e. the matter density $\Omega_M$), (2) reduction of the parameter space by restricting the dark energy model (i.e. *ad hoc* adoption of constant $w$, ignoring $w'$), (3) reducing errors by increasing statistics without limit (i.e. no systematics floor from unknown uncertainties). This perfect knowledge of cosmology, physics, and astrophysics is unrealistic and misleading.

Compounding approximations takes us further from reality. Here we take the three oversimplifications two at a time to show the distortions they cause. The conclusion in each case will be that realistic analysis of probing dark energy leads inexorably to the necessity for the observations to extend beyond $z>1.5$.

For clarity and conciseness, we demonstrate this in simple illustrations. Figure 5 shows the effects of correcting the first two oversimplifications. When both $\Omega_M$ and the dark energy model (e.g. constant $w$) are not overassumed, then degeneracies can lead to complete inability to discriminate very different cases using only data from a survey out to $z\leqslant 1$. A deep survey gains both by the divergence of the curves and the longer redshift observation baseline. The curves in Fig. 5 would be distinguishable by SNAP, which will attain a precision, including systematics, below 0.02 mag.

The effect of the second and third heresies is to mistake the uppermost, more realistic curve on Fig. 3 for the lowest one. Ignoring both time variation and systematics would mis-

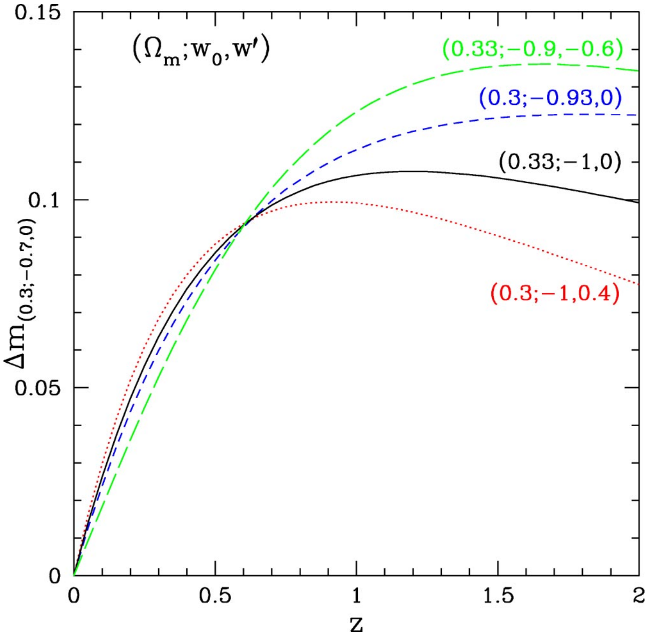


FIG. 5. Degeneracies due to the dark energy model, e.g. equation of state value $w_0$ or evolution $w'$, and to the cosmological model, e.g. value of $\Omega_m$, cannot be resolved at low redshifts. In this differential magnitude-redshift diagram the three parameters to be determined are varied two at a time. Only at $z\approx1.7$ do these very different physics models exceed 0.02 mag discrimination; SNAP will be able to distinguish them.

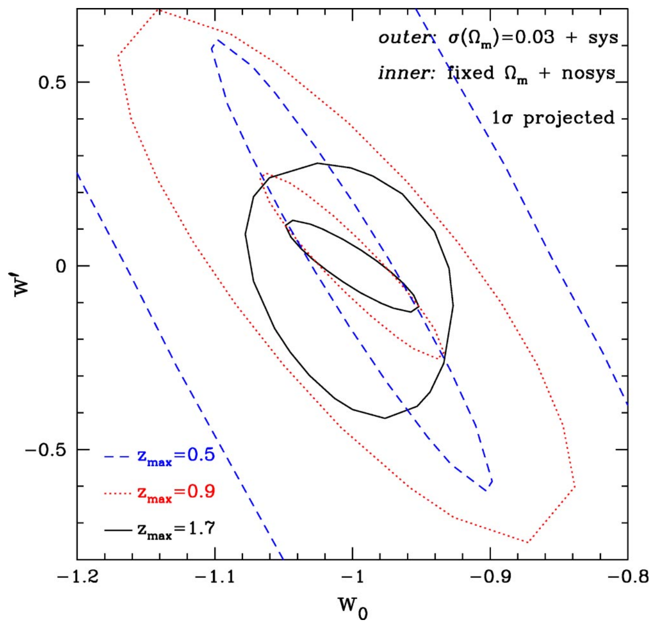


FIG. 6. The effect of breaking oversimplifying assumptions on cosmological parameter determination as a function of survey depth $z_{max}$. Uncertainties in $\Omega_m$ and the presence of systematics drastically weaken constraints from shallow surveys but the long baseline and depth $z_{max}>1.5$ immunize against systematics. The outer contours of each of the three pairs represent realistic estimates for the cosmological parameters as a function of survey depth (see [16]). Contours here enclose 39% of the probability so the $1\sigma$ errors can be read off by projection onto the axes.

estimate the errors by a factor 12.5 at $z_{max}=0.5$ but only 2 at $z_{max}=1.7$.

Finally, consider the first and third together: the idealized case vs. realistic knowledge of the cosmology in the form of flatness, a prior on $\Omega_M$ of 0.03, and systematic error. Figure 6 illustrates several important properties:

(1) $w'$: A shallow survey is incapable of appreciably limiting $w'$, even for perfect assumptions; a medium survey fails under any realistic conditions.

(2) Depth: While there appears to be only moderate difference between the results of a $z_{max}=0.9$ and 1.7 survey under the ideal case, for the realistic case the $1\sigma$ constraints on $w_0$, $w'$ degrade by a full sigma. Depth plus long redshift baselines immunize against the effect of systematics. The main remaining influence is the degeneracy from an uncertain $\Omega_M$, which can be dealt with by complementary cosmological information (see the next section).[2]

(3) Like to like: Experiments should be compared under the appropriate assumptions. An idealized $z=0.9$ survey might unfairly claim limits on $w_0$, $w'$ *better* than the realistic $z=1.7$ one, in noted contrast to the above like to like comparison.

As a final wrap up, consider Fig. 7. This illustrates the comparison between surveys to $z_{max}=0.7$ and 1.7, roughly corresponding to the depths for completeness and precision from ground based and space based supernova surveys in the next decade. Each includes 2000 supernovae plus an additional 300 at $z<0.1$, and makes realistic assumptions about cosmological and astrophysical knowledge. The deep survey is seen to represent a huge advancement in determination of the dark energy model.

Complementary probes of cosmology such as the cosmic microwave background (CMB), weak gravitational lensing, galaxy counts, etc. play an important role in elucidating dark energy. In particular, they are crucial for constraining flatness and the matter density $\Omega_M$. They will also impact, together with supernovae and perhaps independently, the determination of a redshift averaged form of the equation of state $\langle w\rangle$. But these probes possess very little sensitivity to the physically decisive time variation $w'$, and even any prior constraint provided on $\langle w\rangle$ contributes minimally to finding $w'$. Furthermore, except for the CMB (which does not see time variation since it measures the distance to a single redshift), they are first generation experiments, with their own systematic effects (over the 2/3 of the age of the universe stretching back to $z\approx1.5$) at best partially accounted for.

Several supernova cosmology surveys will go forward over the next several years. For example, the "$w$ Project"

[2]Note also that uncertainty in $\Omega_M$ tends to fatten contours in one direction. Especially for the shallow survey cases the limits on $w_0$, $w'$ change relatively little with increasing uncertainty on $\Omega_M$, but the area of the error contours increases by up to a factor three. So one must be cautious at low redshift of simple quotes such as "this experiment determines $w_0$ to $\pm0.1$."

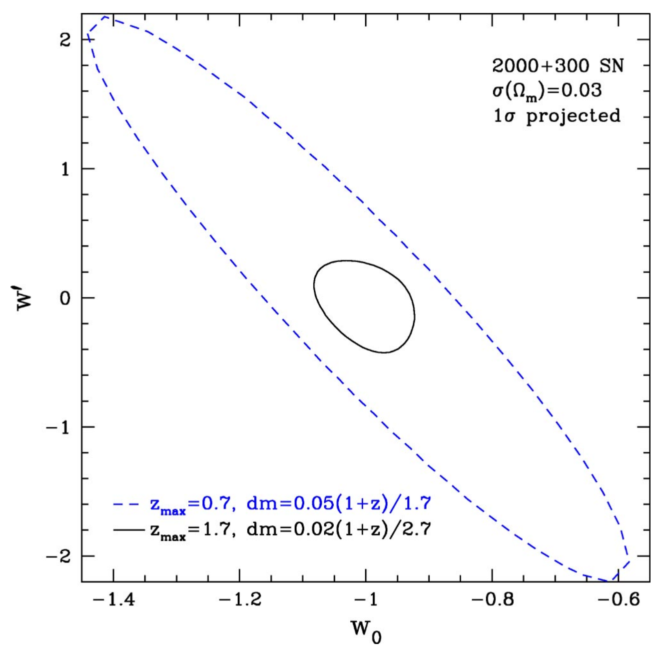


FIG. 7. Realistic assessment of cosmological parameters from complete and precise surveys in the next decade from the ground ($z_{max}=0.7$) and space ($z_{max}=1.7$) [16]. Contours here enclose 39% of the probability so the $1\sigma$ errors can be read off by projection onto the axes.

[17] at CTIO should obtain 200 SN at redshifts $z=0.15-0.75$ over the course of five years. With a quoted systematic [18] of $dm=0.03(z/0.5)$, and using a prior of $\sigma(\Omega_M)=0.04$ and the crucial low redshift data of the Nearby Supernova Factory [19], this should determine $w$ to $+0.10$, $-0.12$. Suppose $\sigma(\langle w\rangle)=0.1$, where $\langle w\rangle$ is interpreted as an average value of the EOS over the redshift range. This would of course be quite interesting in itself, but for the further important parameter $w'$ such middle redshift experiments provide no useful prior. In fact, such a prior on $\langle w\rangle$ would improve SNAP's constraint on $w'$ by less than 3%. In this sense SNAP is very much a next generation experiment.

One promising method of adding value to SNAP is the information the Planck Surveyor experiment [20] provides via the cosmic microwave background anisotropies. This constrains a combination of the matter density and the dark energy parameters; the result of this complementarity is not only to strengthen the advantage of a high redshift supernova survey, but to greatly improve its precision [21]. For example, adding the information expected from Planck would improve SNAP's determination of $w'$ by roughly a factor of two. In fact, using a new, well behaved parametrization of the function $w(z)$, Linder [3] shows that one could attain $\sigma(dw/d\ln(1+z)|_{z=1})\approx 0.1$ for a model such as supergravity inspired dark energy. For the particular SUGRA model [22] this would represent a 99% confidence level detection of time variation in the EOS.

The discussions and illustrations presented here show that expectations based on oversimplified cosmology, physics, and astrophysics prove insufficient and misleading for understanding how to probe dark energy. Could we detect dark energy with measurements at $z<1$? Assuredly—we already have through the supernova method. Could we reliably distinguish its equation of state from that of a cosmological constant? Possibly—wide field ground based surveys, possibly together with higher redshift Hubble Space Telescope observations, could well give indications of this, though not necessarily definitive ones. Could we see the critical evidence of time variation in the equation of state that sets us on the path of a fundamental theory? No. For that we required detailed observations out to $z\approx 1.5$–2 and control of systematics.

In the realistic view, one clearly appreciates the need for a precision survey reaching out to $z_{max}\approx 1.5$–2. More rigorous Monte Carlo simulations [16] implementing a variety of systematic error, cosmology, and dark energy models bear out this conclusion.

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