Reconstructing quintessence

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astro-ph/9808133
+ work in progress

Quintessence - a dynamical scalar field

Origin: particle physics (yet unknown)

History: Starting in the late 1980's, shows up in literature as 'Rolling Scalar field', 'Dynamical Lambda', 'Quintessence'.

Features:

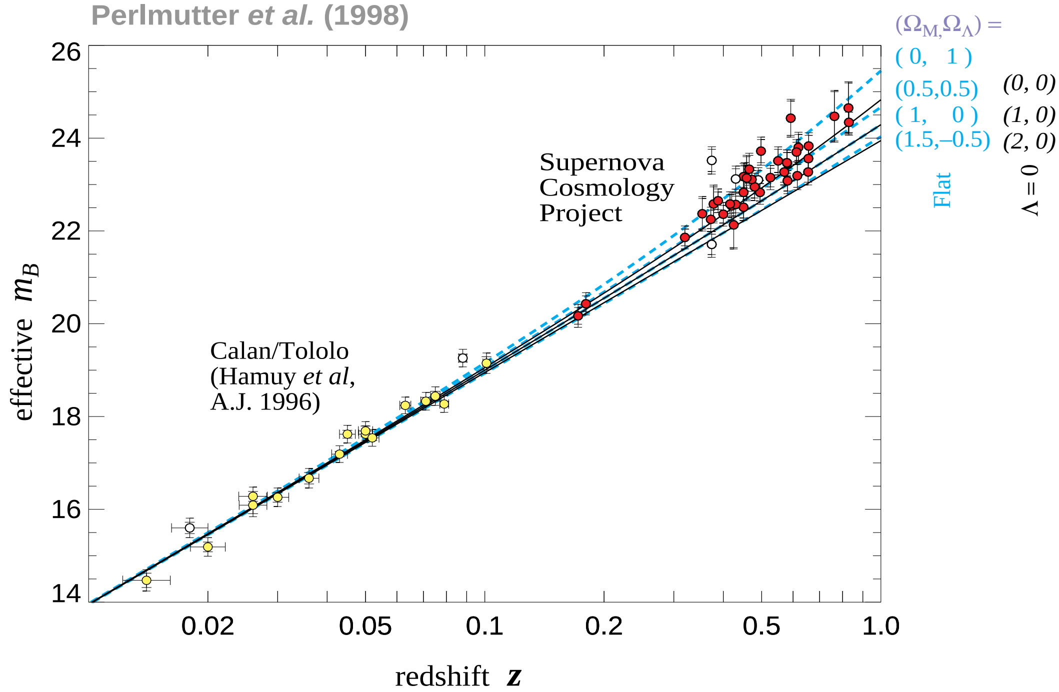
- rolls down its (effective) potential
- provides significant energy density Ω_Q (missing energy?).
- has negative equation of state today

$$w_Q = \frac{\dot{\phi}^2/2 - V(\phi)}{\dot{\phi}^2/2 + V(\phi)} < 0 \qquad (-1 \lesssim w_Q \lesssim -0.5)$$

- in addition, quintessence may have other nice properties. . .

Supernova Ia Search

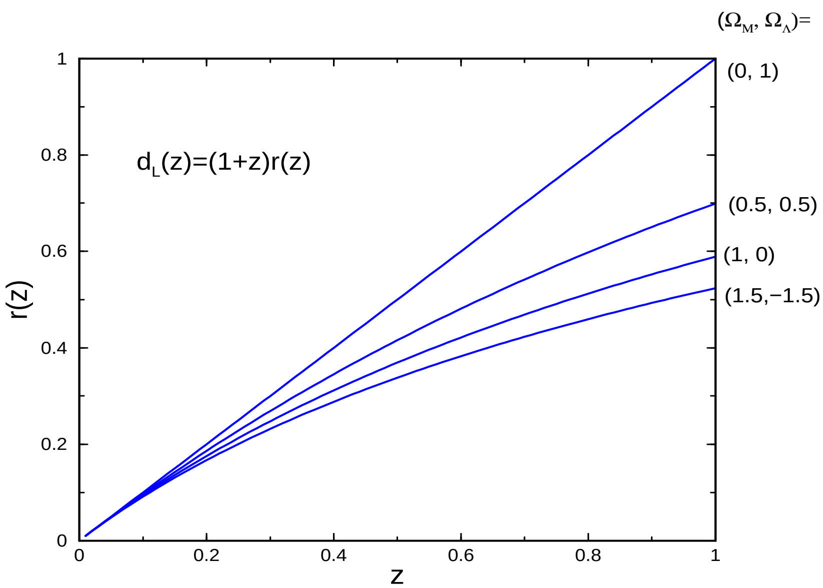
Supernova Cosmology Project
Perlmutter *et al.* (1998)



In flat universe: Ω_M = 0.28 [± 0.085 statistical] [± 0.05 systematic]

Prob. of fit to $\Lambda = 0$ universe: 1%

astro-ph/9812133



Reconstruction Equations: $r(z) \rightarrow V(\phi)$

Assume a Universe where $\Omega_M + \Omega_Q = 1$. Then, from the Friedmann equations:

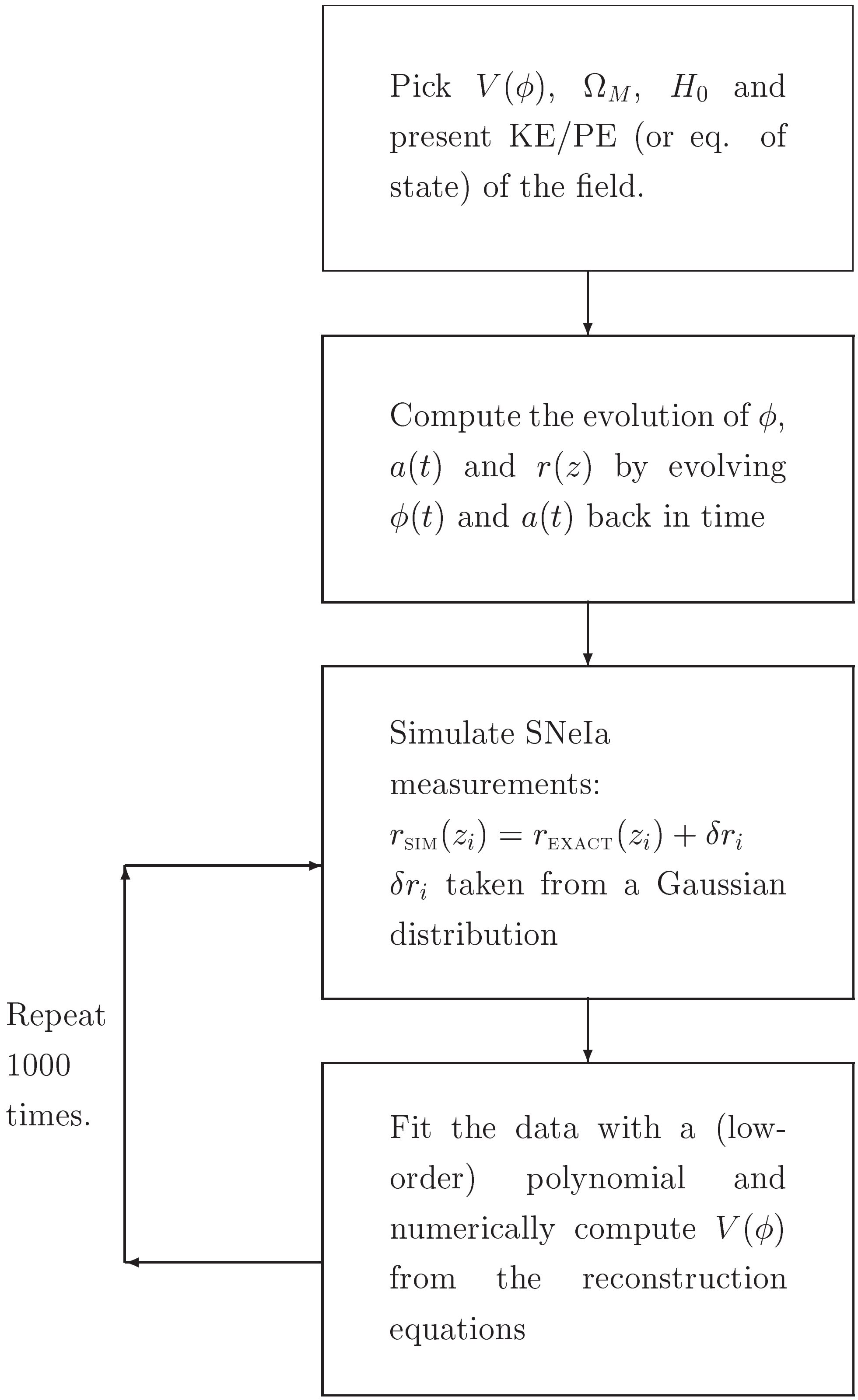
$$V[r(z)] = \frac{1}{8\pi G}\left[\frac{3}{(dr/dz)^2} + (1+z)\frac{d^2r/dz^2}{(dr/dz)^3}\right] - \frac{3\Omega_M H_0^2(1+z)^3}{16\pi G}$$

$$\frac{d\phi}{dz} = \mp\frac{dr/dz}{1+z}\left[-\frac{1}{4\pi G}\frac{(1+z)d^2r/dz^2}{(dr/dz)^3} - \frac{3\Omega_M H_0^2(1+z)^3}{8\pi G}\right]^{1/2}$$

- Only need to know Ω_M
- $r(z)$ comes in only as dr/dz and d^2r/dz^2

To demonstrate the feasibility of this approach, we use Monte Carlo simulation.

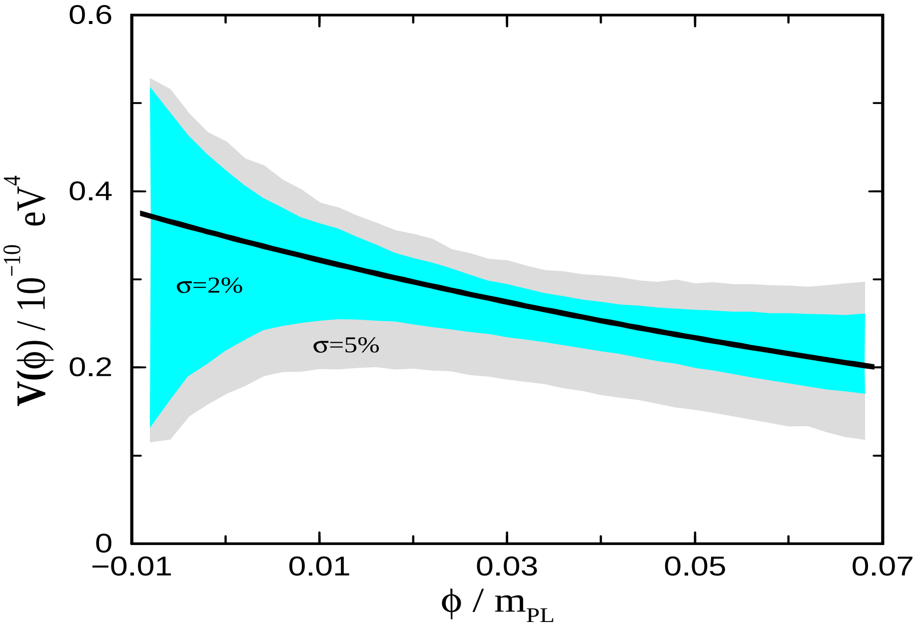
Monte Carlo demonstration of the potential reconstruction



Examples of reconstruction

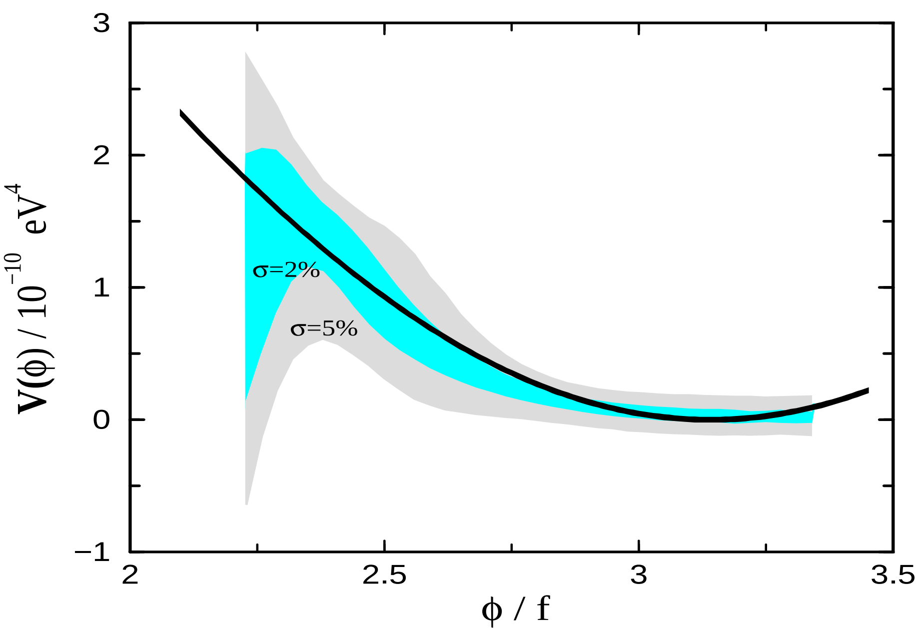
$V(\phi) = V_0 \exp(-\beta\phi/m_{\rm Pl})$ $\quad V_0 = (2.43 \times 10^{-3}\,{\rm eV})^4$ $\quad \beta = 8$

$N = 40$ points $\quad z_{\rm max} = 1.5$ $\quad \Omega_M = 0.4$



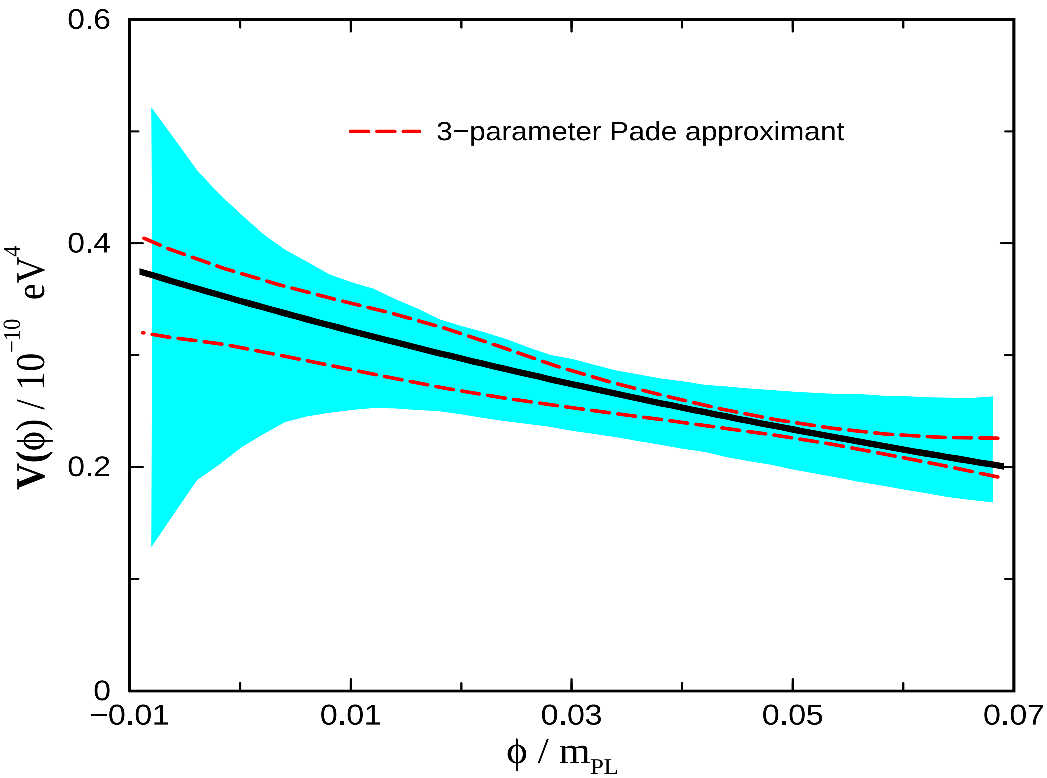
$V(\phi) = V_0[1 + \cos(\phi/f)]$ $\quad V_0 = (4.65 \times 10^{-3}\,{\rm eV})^4$ $\quad f/m_{\rm Pl} = 0.154$

$N = 40$ points $\quad z_{\rm max} = 1.0$ $\quad \Omega_M = 0.3$



Padé Approximants:

- Fit the (simulated) data with $$r(z) = \frac{z(1+az)}{1+bz+cz^2}$$



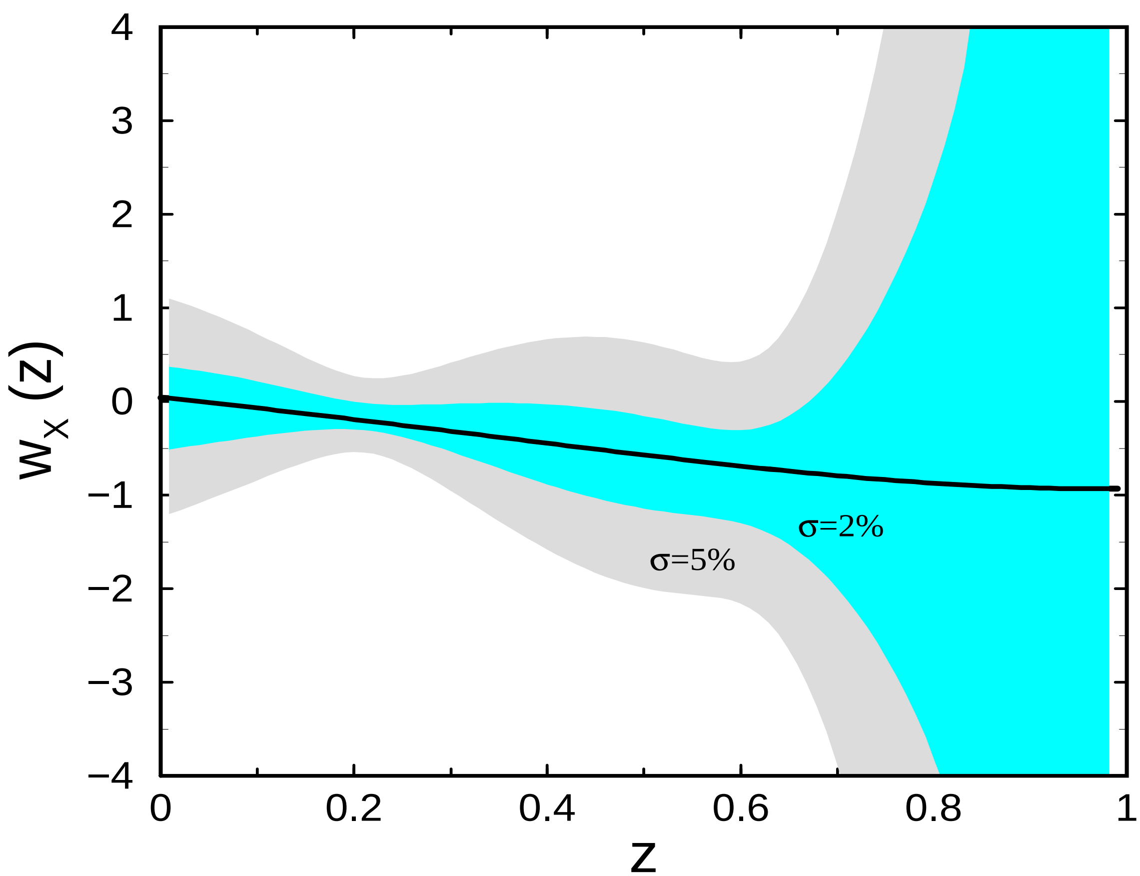
Summary of potential reconstruction

- Need to know only Ω_M and $\Omega_Q = 1 - \Omega_M$.
- The uncertainty in the reconstruction will decrease as more supernovae are discovered (roughly as $1/\sqrt{N}$).
- Inferring d^2r/dz^2 from the data is required for reconstruction.

Reconstructing the equation of state

- No need to assume that quintessence is the missing energy!

- $1 + w_X(z) = \frac{1+z}{3} \, \frac{3H_0^2\Omega_M(1+z)^2 + 2(d^2r/dz^2)/(dr/dz)^3}{H_0^2\Omega_M(1+z)^3 - (dr/dz)^{-2}}$



- This gives evidence that beyond $z \sim 0.8$ it is difficult to get information about the missing component.

Optimal supernova search strategies

Q: What is the ideal distribution of supernovae in redshift?

Minimize $A \propto [\det(F)]^{-1/2}$

$$m_n = 5\log[H_0 d_L(z_n, \Omega_M, \Omega_\Lambda)] + m_0 + \epsilon_n,$$

$$\begin{aligned}
F_{ij} &= -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_{\mathbf{x}} \\
&= \frac{1}{\Delta m^2} \sum_{n=1}^{N} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_i} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_j} \\
&= \frac{1}{\Delta m^2} \sum_{n=1}^{N} \omega_i(z_n)\, \omega_j(z_n) \qquad \text{(Tegmark et al., astro-ph/9804168)}
\end{aligned}$$

If we represent the measurements as a sum of delta-functions

$$g(z) = \sum_{i=1}^{BINS} g_i\, \delta(z - z_i),$$

then

$$F_{ij} = \frac{N}{(\Delta m)^2} \int_0^\infty g(z)\, \omega_i(z)\, \omega_j(z)\, dz,$$

With two parameters:

$$\det(F) = \int_0^\infty \int_0^\infty g(z_1)\, g(z_2)\, \omega^2(z_1, z_2)\, dz_1\, dz_2$$

$$= \sum_{i,j=1}^{BINS} g_i\, g_j\; \omega^2(z_i, z_j)$$

with

$$\sum_{i=1}^{BINS} g_i = 1 \quad \text{and} \quad g_i > 0$$

The result is, for $\Omega_M - \Omega_\Lambda$ case

$$\mathbf{g(z) = 0.50\,\delta(z - 0.44) + 0.50\,\delta(z - 1.00)},$$

and for the $\Omega_M - w_Q$ case

$$\mathbf{g(z) = 0.50\,\delta(z - 0.36) + 0.50\,\delta(z - 1.00)}.$$

Simulating and fitting the data