Primordial Nongaussianity and Large-scale Structure

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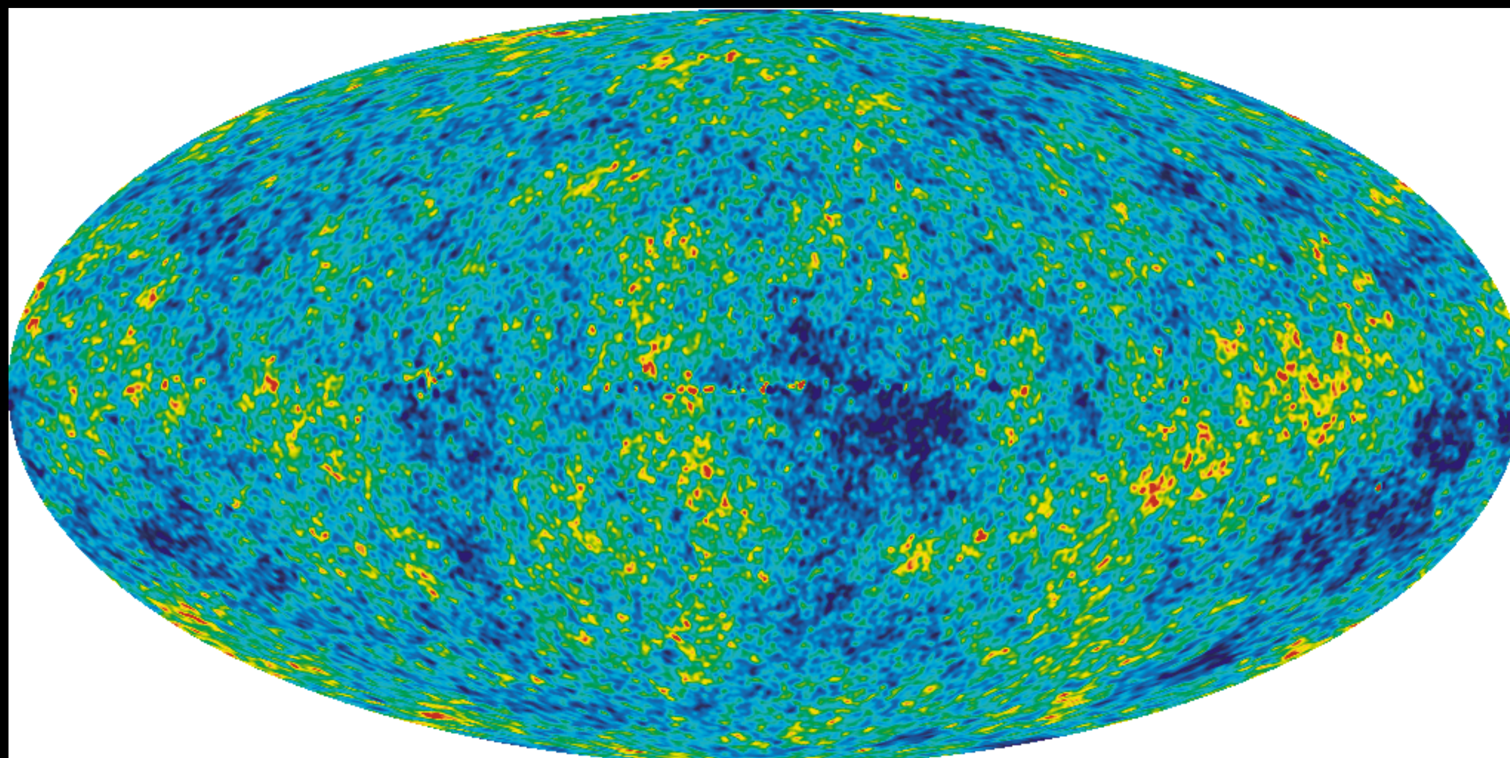
Some slides courtesy of O. Doré

background + work based on
arXiv:0710.4560 (PRD 2008)

Initial conditions in our universe

$$\frac{\delta T}{T}(\theta,\phi) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\theta,\phi)$$



Generic inflationary predictions:

- Nearly scale-invariant spectrum of density perturbations
- Background of gravity waves
- (Very nearly) gaussian initial conditions:

Isotropy:

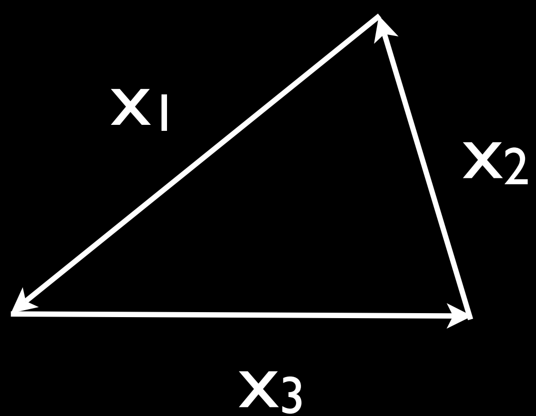
$$\langle a_{\ell m}\, a_{\ell' m'} \rangle = C_{\ell\ell' m m'} = C_\ell \delta_{\ell\ell'} \delta_{mm'}$$

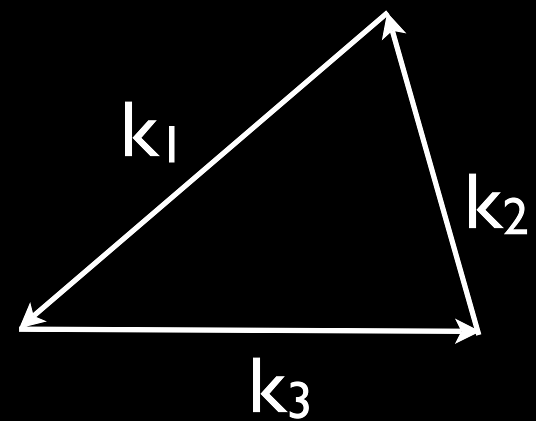
Gaussianity:

$$\langle a_{\ell m}\, a_{\ell' m'}\, a_{\ell'' m''} \rangle = 0 \quad \text{etc.}$$

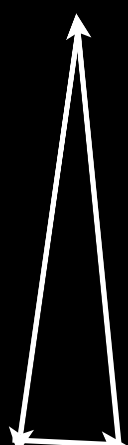
3-pt function as a measure of cosmological NonGaussianity (NG)

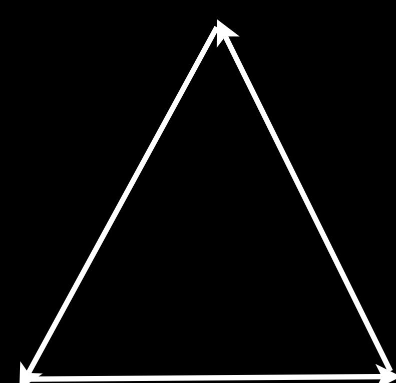
- Principal measure of NG: three-pt correlation function





"local"





"equilateral"

e.g. Luo & Schramm 1993

Inflation generically predicts (very nearly) gaussian random fluctuations

- Nongaussianity is proportional to slow-roll parameters, V′/V and V′′/V
- Reasonable and commonly used approximation $\Phi = \Phi_{\rm G} + f_{\rm NL}\left(\Phi_{\rm G}^2 - \langle\Phi_{\rm G}^2\rangle\right)$
- Inflation predicts $f_{\rm NL}$=O(0.1), which is basically extremely small
- More exotic inflationary models can produce observable NG, however

Salopek & Bond 1990; Verde et al 2000; Komatsu & Spergel 2001; Maldacena 2003

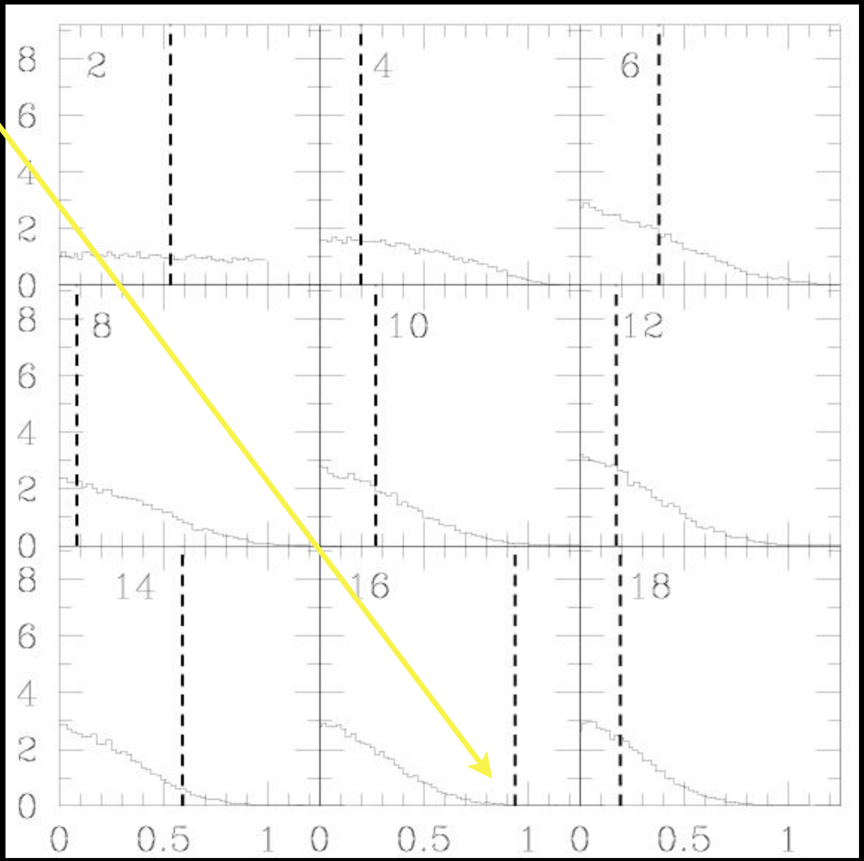
Brief history of NG measurements: 1990's

Early 1990s; COBE: Gaussian CMB sky (Kogut et al 1996)

1998; COBE: claim of NG at l=16 equilateral bispectrum
(Ferreira, Magueijo & Gorski 1998)

but explained by a known systematic effect!
(Banday, Zaroubi & Gorski 1999)

(and anyway isn't unexpected given all
bispectrum configurations you can measure;
Komatsu 2002)

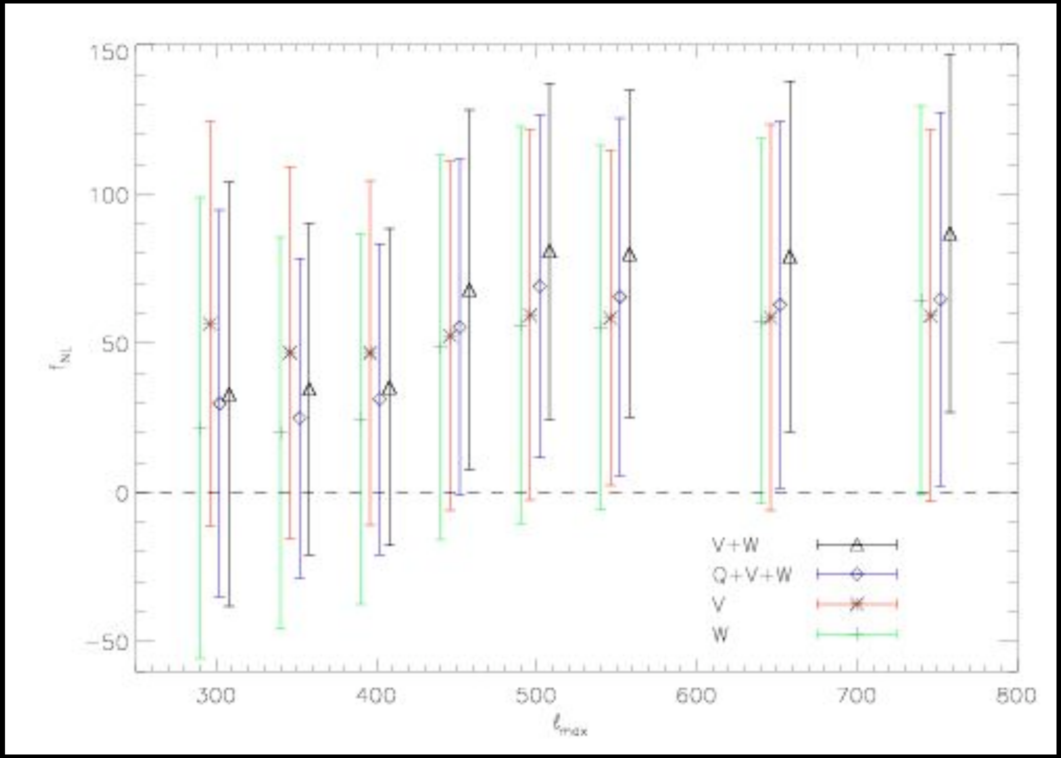


Brief history of NG measurements: 2000's

Pre-WMAP CMB: all is gaussian (e.g. MAXIMA; Wu et al 2001)

WMAP pre-2008: all is gaussian
(Komatsu et al. 2003; Creminelli, Senatore, Zaldarriaga & Tegmark 2007)

$-36 < f_{NL} < 100$ (95% CL)



Dec 2007, claim of NG in WMAP
(Yadav & Wandelt arXiv:0712.1148)

$27 < f_{NL} < 147$ (95% CL)

Future: much better constraints, f_{NL}<O(10) with Planck

TABLE 6

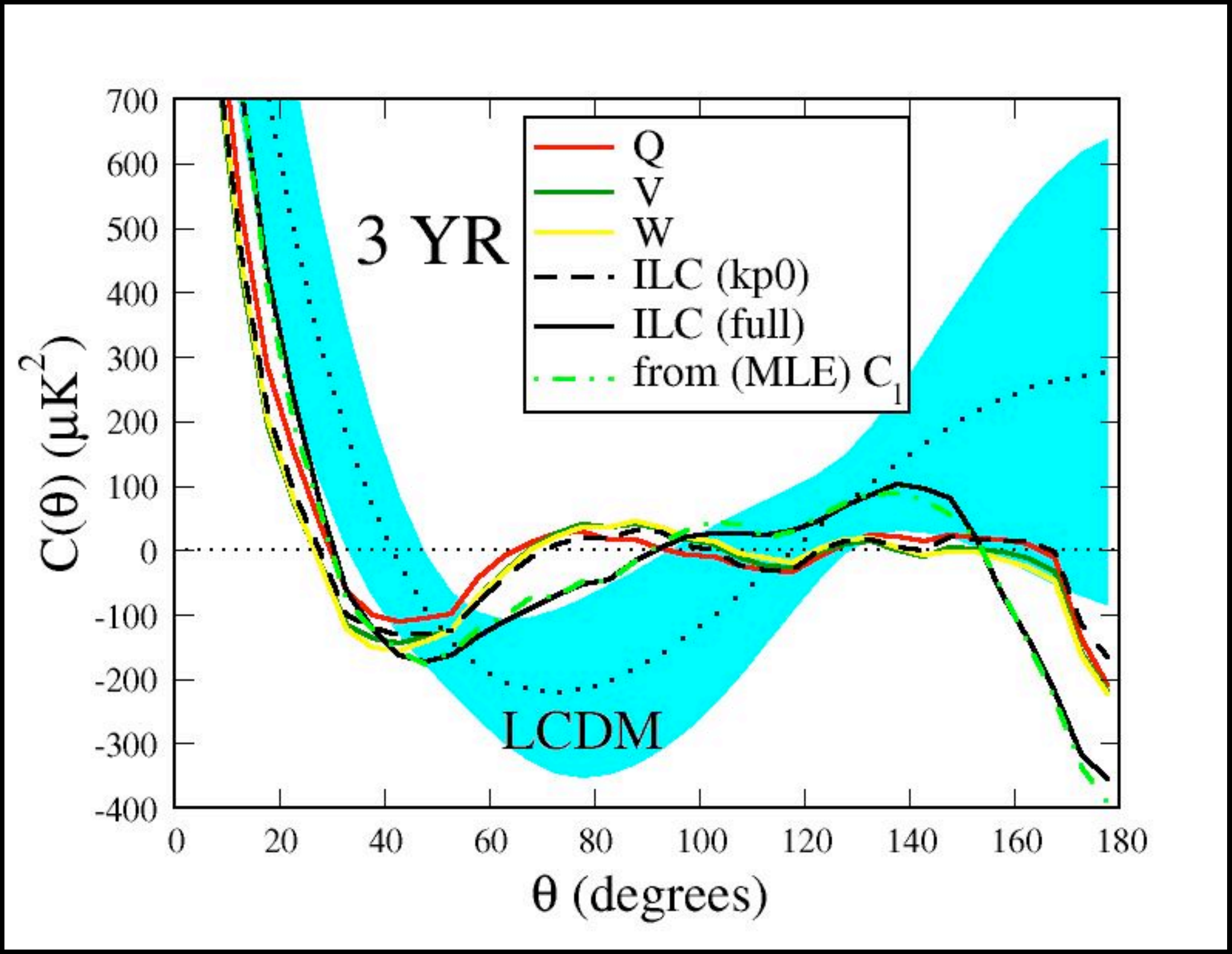
Null tests, frequency dependence, and raw-map estimates of the local form of primordial non-Gaussianity, $f_{NL}^{\rm local}$, for $l_{\rm max} = 500$

Band	Foreground	Mask	$f_{NL}^{\rm local}$
Q−W	Raw	*KQ75*	-0.53 ± 0.22
V−W	Raw	*KQ75*	-0.31 ± 0.23
Q−W	Clean	*KQ75*	0.10 ± 0.22
V−W	Clean	*KQ75*	0.06 ± 0.23
Q	Raw	*KQ75p1*[a]	-42 ± 45
V	Raw	*KQ75p1*	38 ± 34
W	Raw	*KQ75p1*	43 ± 33
Q	Raw	*KQ75*	-42 ± 48
V	Raw	*KQ75*	41 ± 35
W	Raw	*KQ75*	46 ± 35
Q	Clean	*KQ75p1*	9 ± 45
V	Clean	*KQ75p1*	47 ± 34
W	Clean	*KQ75p1*	60 ± 33
Q	Clean	*KQ75*	10 ± 48
V	Clean	*KQ75*	50 ± 35
W	Clean	*KQ75*	62 ± 35
V+W	Raw	*KQ85*	9 ± 26
V+W	Raw	*Kp0*	48 ± 26
V+W	Raw	*KQ75p1*	41 ± 28
V+W	Raw	*KQ75*	43 ± 30

[a]This mask replaces the point-source mask in *KQ75* with the one that does not mask the sources identified in the *WMAP* K-band data

Komatsu et al. 2008

… and also "large-scale anomalies"

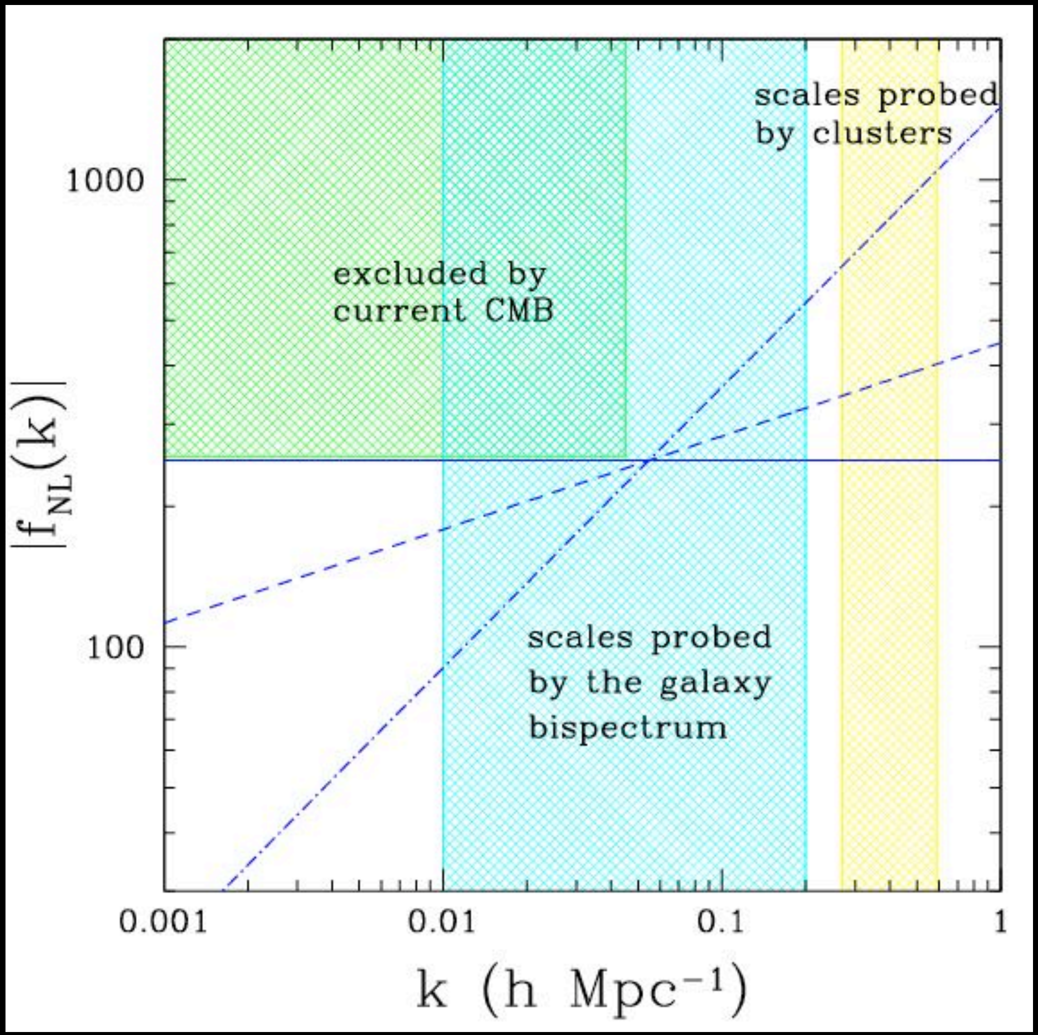


lack of power at >60 deg; significant at 99.97%

stronger evidence

Hinshaw et al. 1996 (COBE)
Spergel et al. 2003 (WMAP 1)
Copi, Huterer, Schwarz & Starkman 2007, 08 (WMAP 3, 5)

Constraints from future LSS surveys



Sefusatti, Vale, Kadota & Frieman, 2006

LoVerde, Miller, Shandera & Verde, arXiv:0711.4126

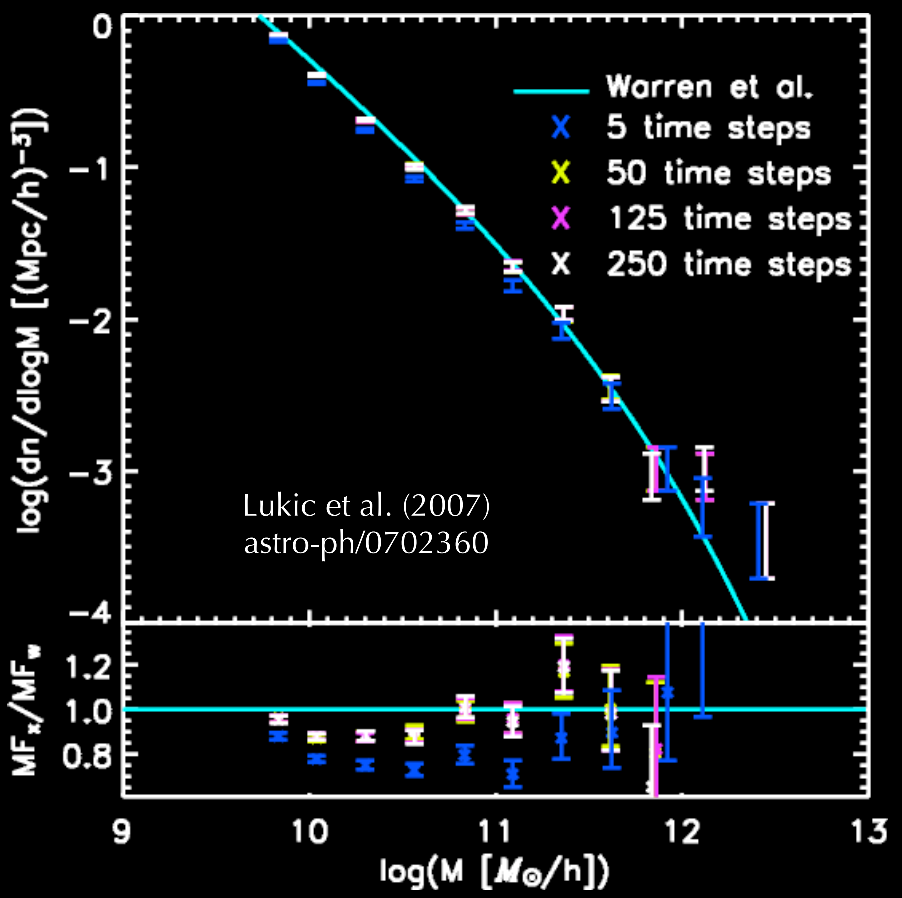
Abundance of halos: the mass function

Lots of interest in using halo counts as a cosmological probe.

- Mass function can be computed precisely (~5%) and robustly for standard cosmology (Jenkins et al. 01, Warren et al. 03)

- dN/dM appears universal — i.e. f(σ) — for standard cosmologies

$$\sigma^2(M,z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k) W^2(k,M)\, dk$$



Mass function, usual analytic approach

Press & Schechter 1974:

$$\frac{dn}{dM}dM = \frac{\rho_M}{M}\left|\frac{dF}{dM}\right|dM \qquad F(>M) = 2\int_{\delta_c/\sigma(M)}^{\infty} P_G(\nu)d\nu$$

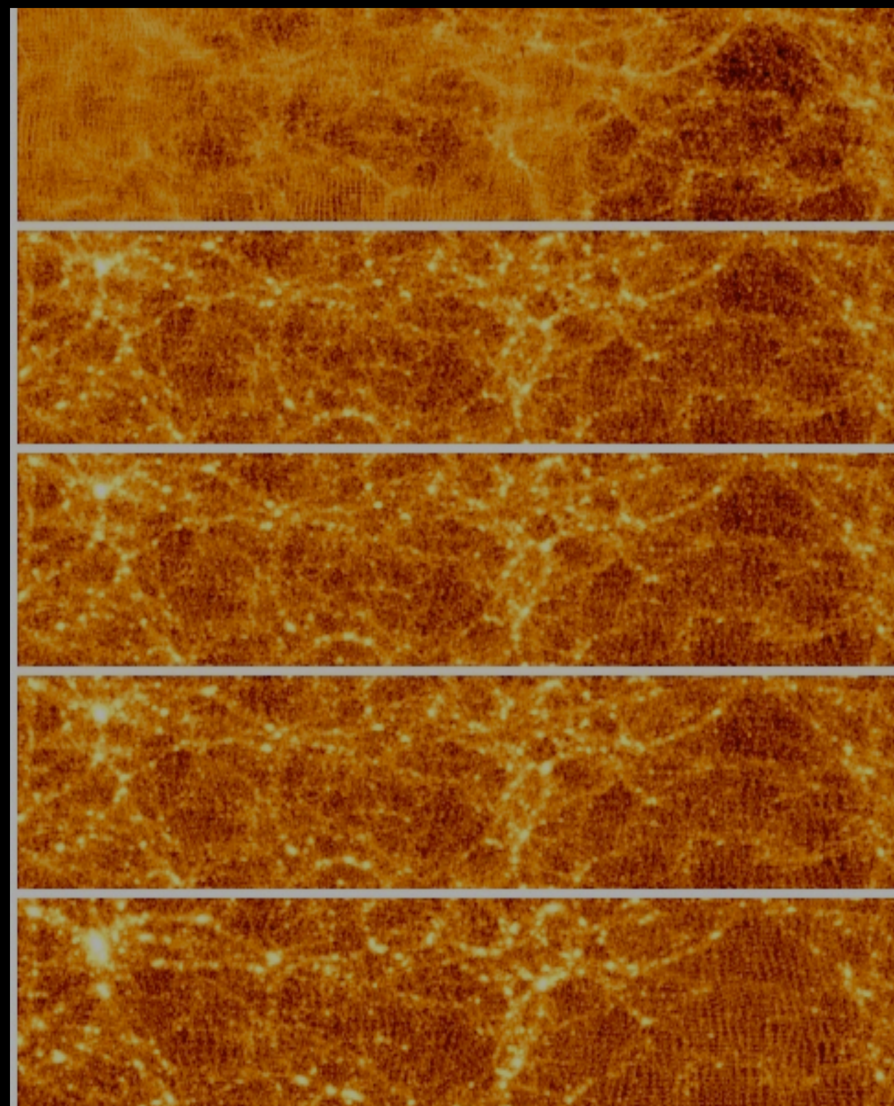
therefore $$\left(\frac{dn}{d\ln M}\right)_{\mathrm{PS}} = 2\,\frac{\rho_M}{M}\frac{\delta_c}{\sigma}\left|\frac{d\ln\sigma}{d\ln M}\right| P_{\mathrm{G}}(\delta/\sigma)$$

"Extended Press-Schechter" (EPS): $P_G(\nu) \rightarrow P_{NG}(\nu)$

Matterese, Verde & Jimenez (2000; MVJ):
follow EPS, then expand P_{NG} in terms of skewness, do the integral

However, no convincing reason why either should work!
Need to check these formulae with simulations

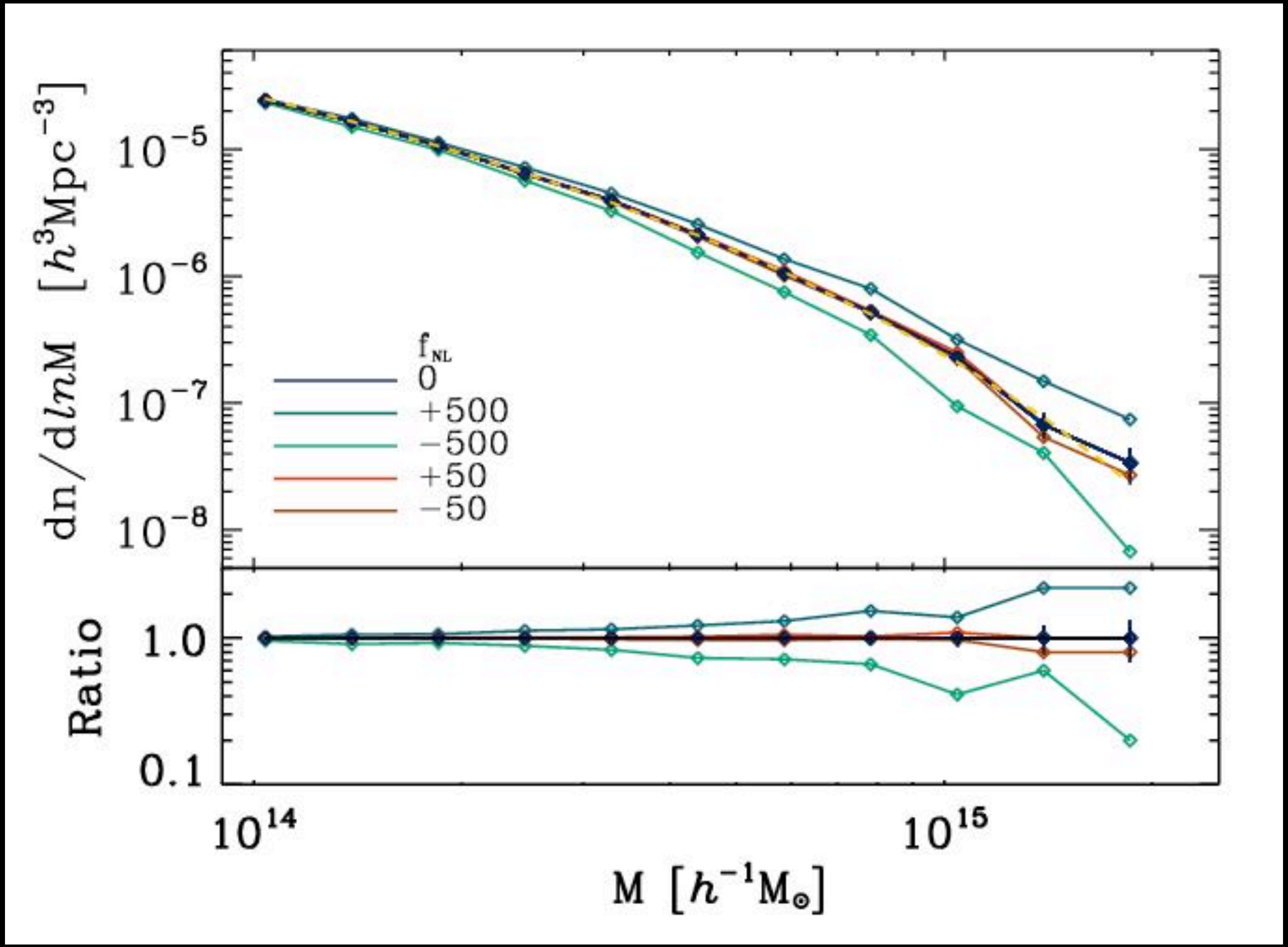
Simulations with nongaussianity (f_{NL})



- Under-dense region evolution decrease with f_{NL}
- Over-dense region evolution increase with f_{NL}

- Same initial conditions, different f_{NL}
- Slice through a box in a simulation $N_{part}=512^3$, L=800 Mpc/h

The measured halo mass function

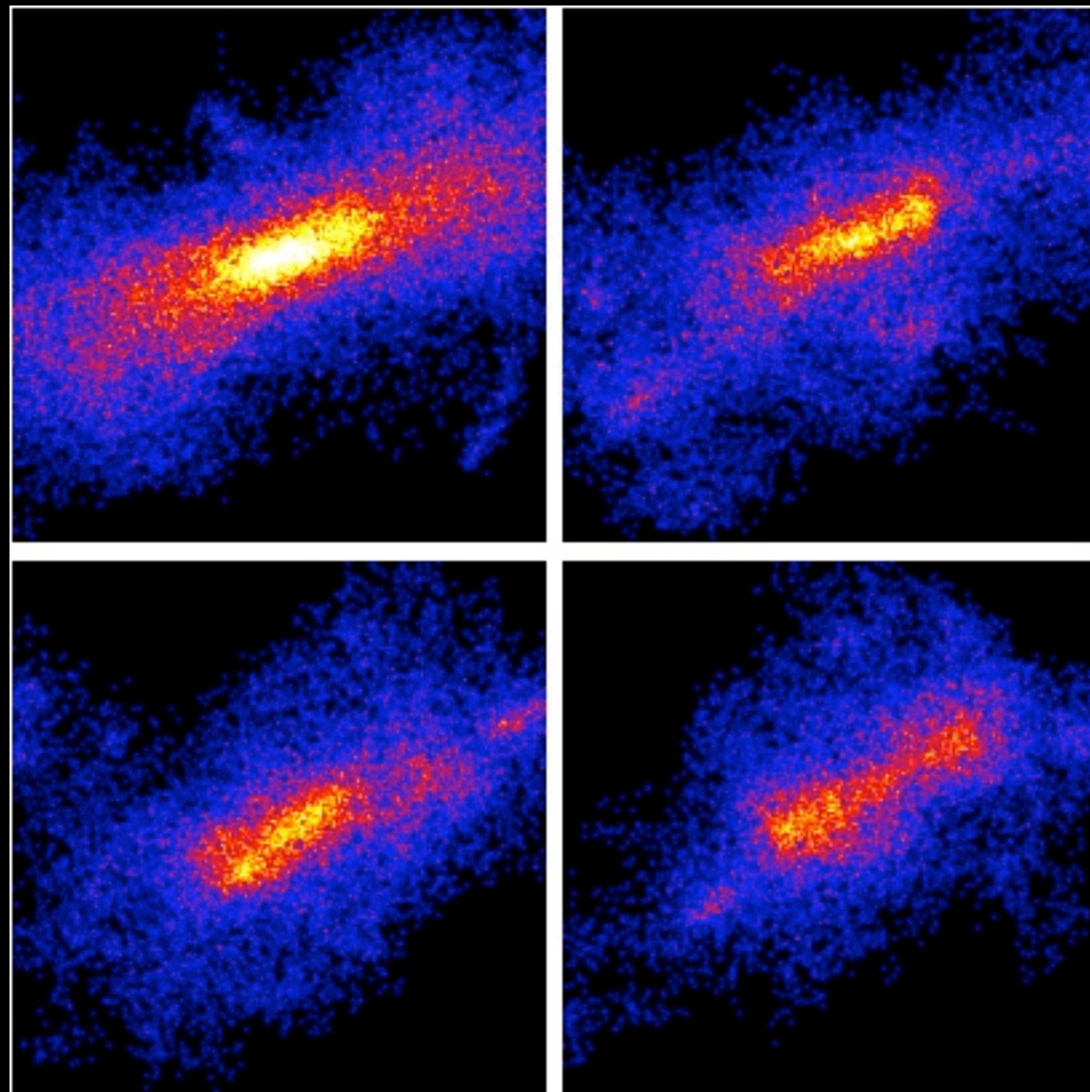


- 512^3 (1024^3) particle simulations with box size 800 (1600) Mpc/h
- Gracos code (www.gracos.com); add quadratic Phi term in real space; apply transfer function in Fourier space

Looking at one individual cluster

f_{NL}=+5000
M=1.2 10^{16} $M_\odot$

f_{NL}=+500
M=5.9 10^{15} $M_\odot$

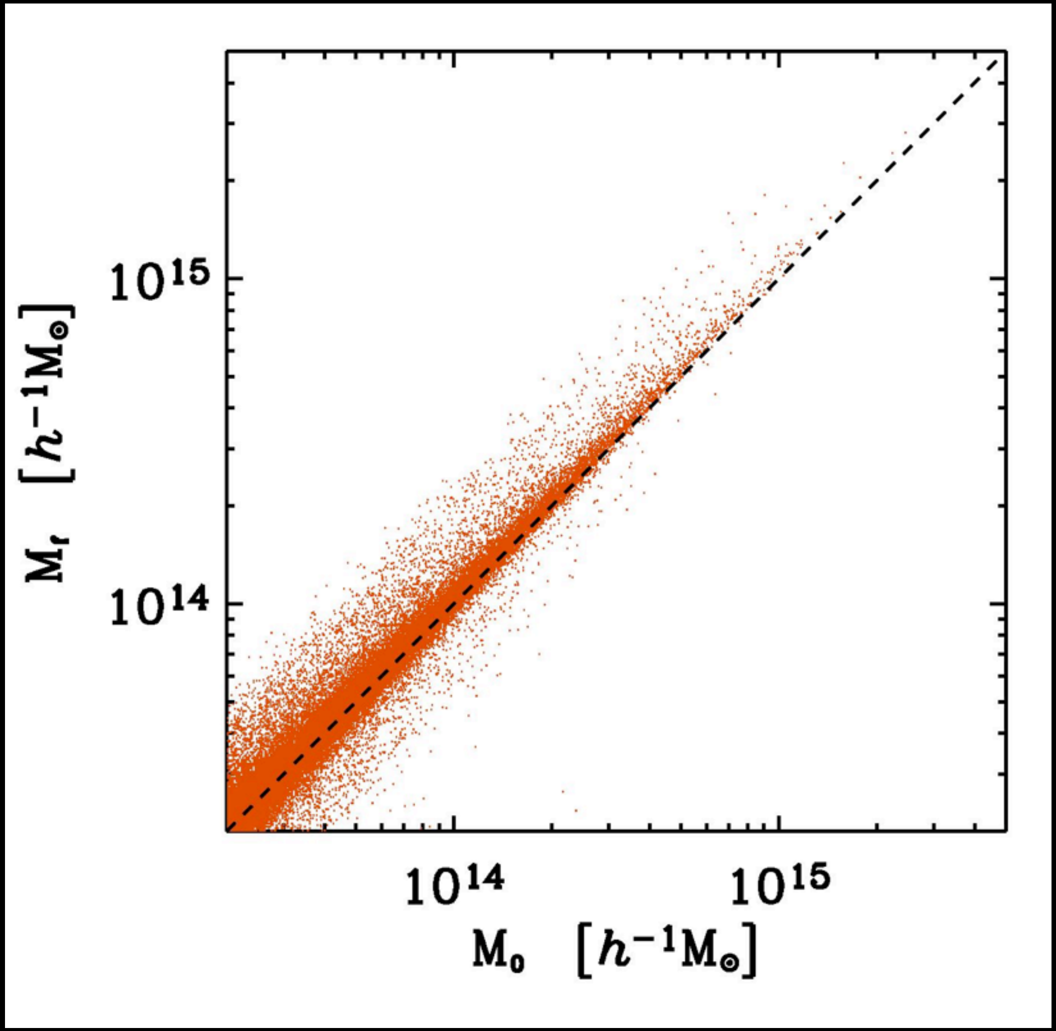


f_{NL}=0
M=5.1 10^{15} $M_\odot$

f_{NL}=-500
M=4.3 10^{15} $M_\odot$

- Most massive cluster in our simulation
- For small enough f_{NL}, same peaks arise, with different heights (implying different masses)
- Can we extend to any cluster?

Building the $P(M_f|M_0)$ distribution



$f_{NL} = 500$

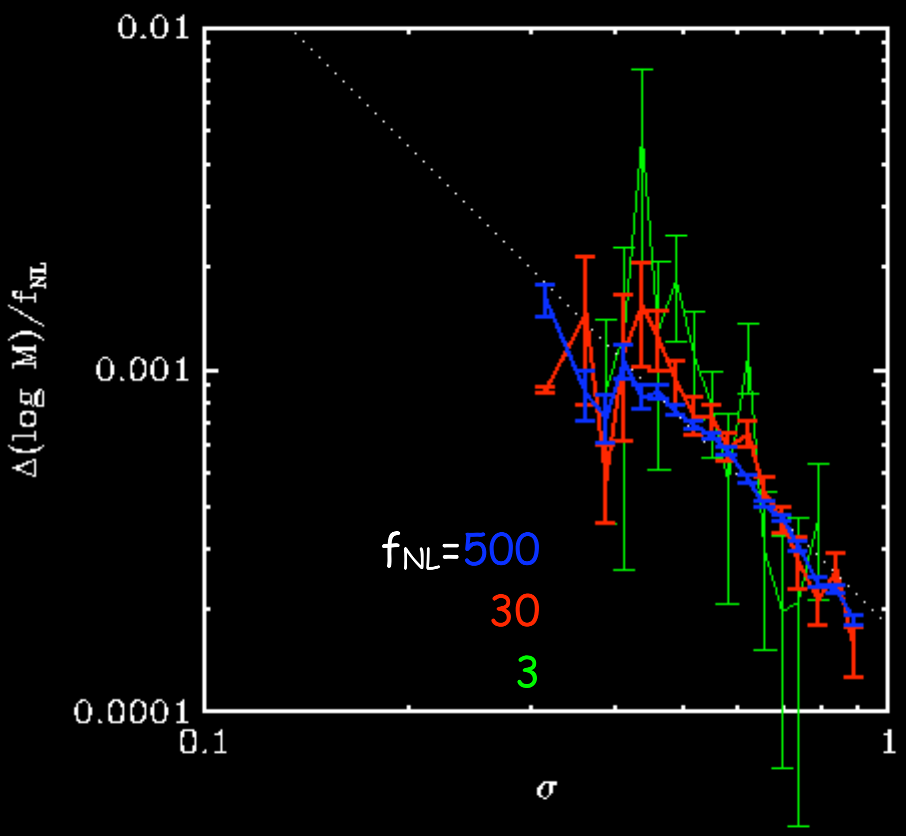
- Idea: identify the *same* cluster for different f_{NL}, keep track how its mass changed!
- Significantly saves computational expenses

Towards a fitting function

- If the mapping $M_0 \rightarrow M_f$ is described by a PDF $dP/dM_f(M_0)$, then the non-gaussian mass function is a convolution over the (known) gaussian mass function

$$\frac{dN}{dM} = \int \frac{dP(M_f|M_0)}{dM_f} \frac{dN}{dM_0} dM_0$$

(e.g. Jenkins)

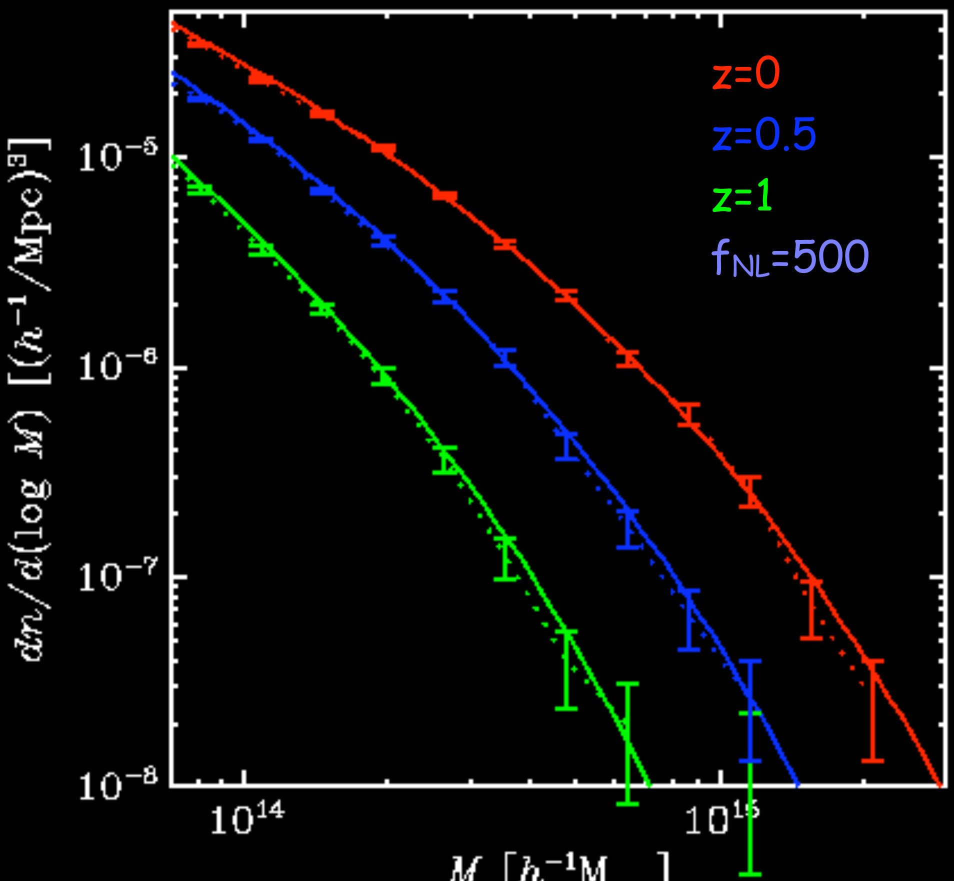


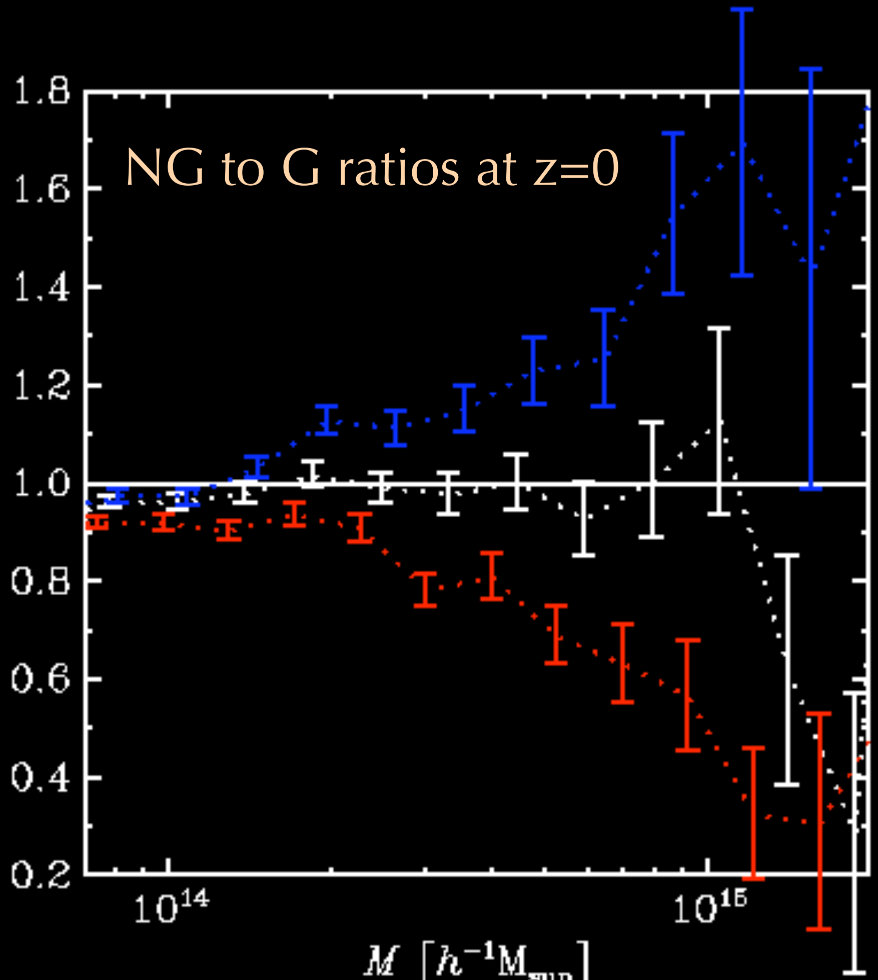
- We thus aim at fitting the mean and rms of $\Delta(\log M)(z)$
- The simplest thing to do is to consider a… Gaussian…
- We'd expect the mean of the PDF to be shifted by $\Delta(\log M) \propto f_{NL}$
- We find that a good fit is given by

$$\left[\frac{\bar{M}_f}{M_0}\right] - 1 = 6.\ 10^{-5} f_{NL} \sigma_8\ \sigma(M_0, z)^{-2}$$

$$\sigma\left(\left[\frac{\bar{M}_f}{M_0}\right] - 1\right) = 0.012\ (f_{NL}\sigma_8)^{0.4}\ \sigma(M_0, z)^{-0.5}$$

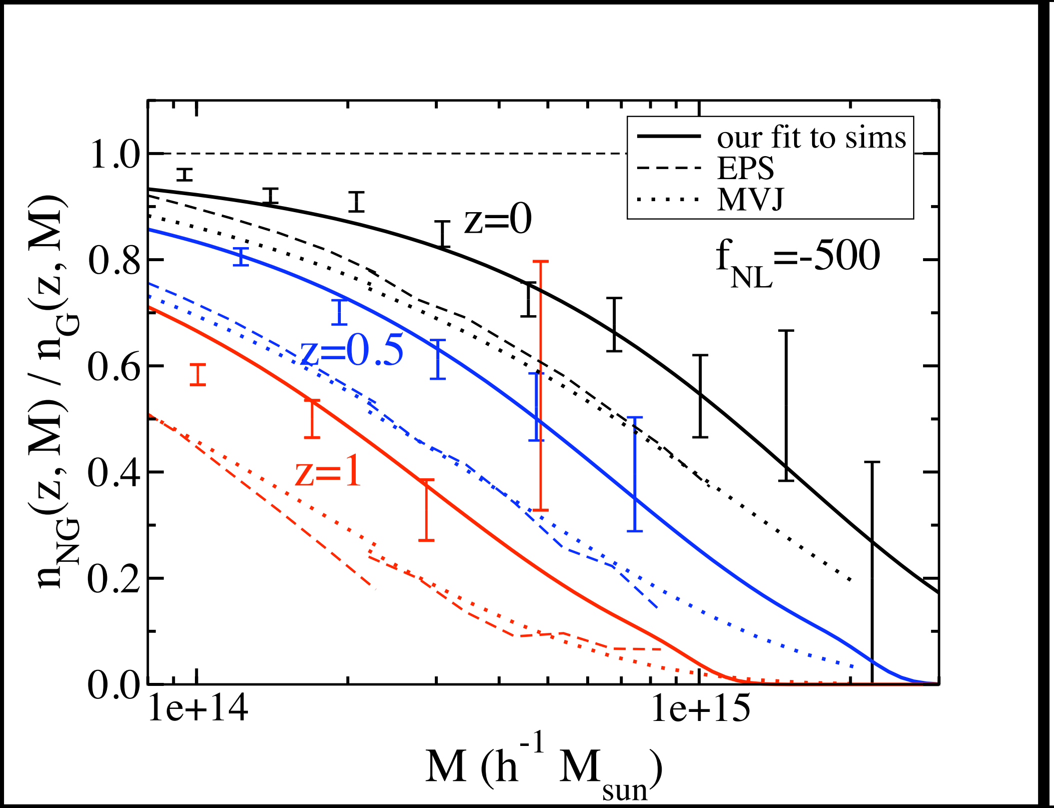
Mass function from N-body simulation and our fitting formula

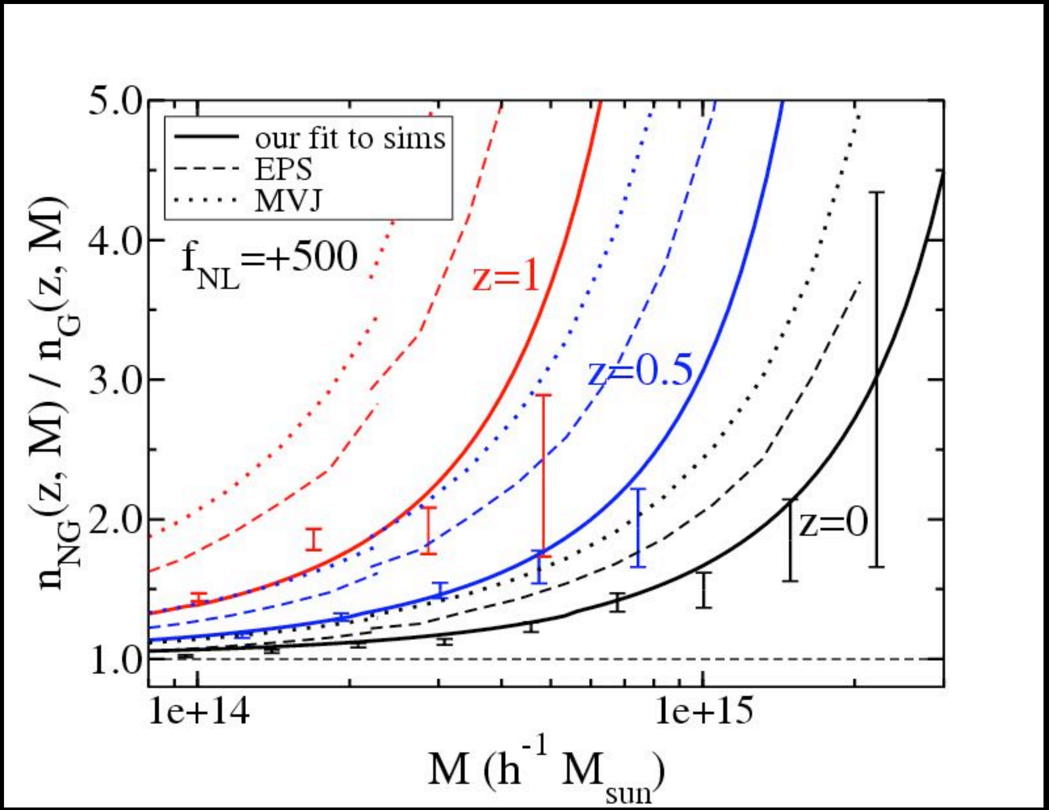




Dalal, Doré, Huterer & Shirokov, arXiv:0710.4560

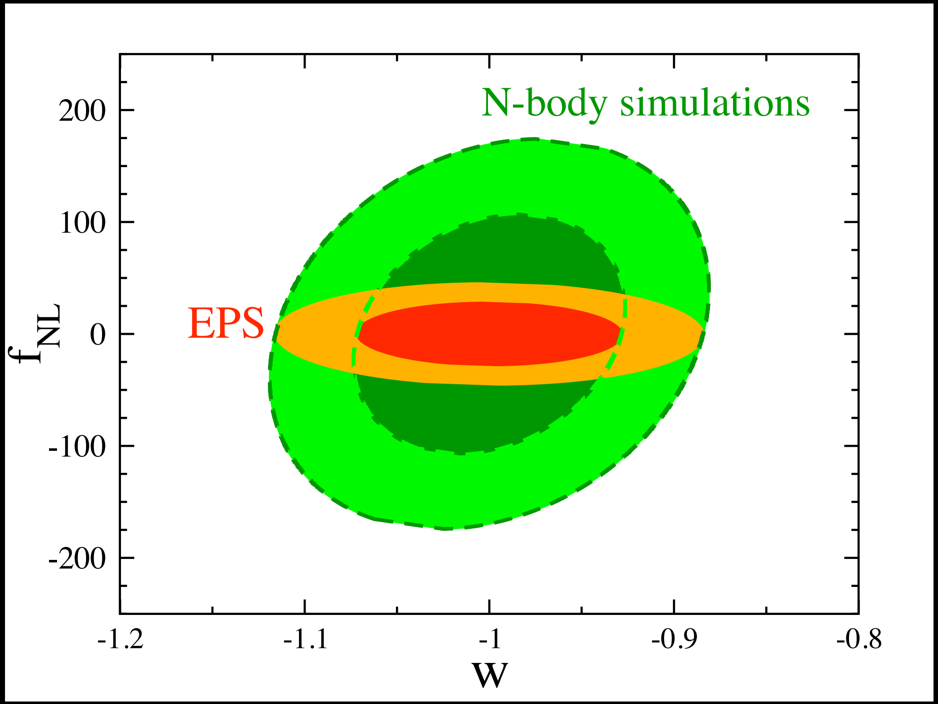
Old fitting functions are discrepant; off by O(100%) wrt truth

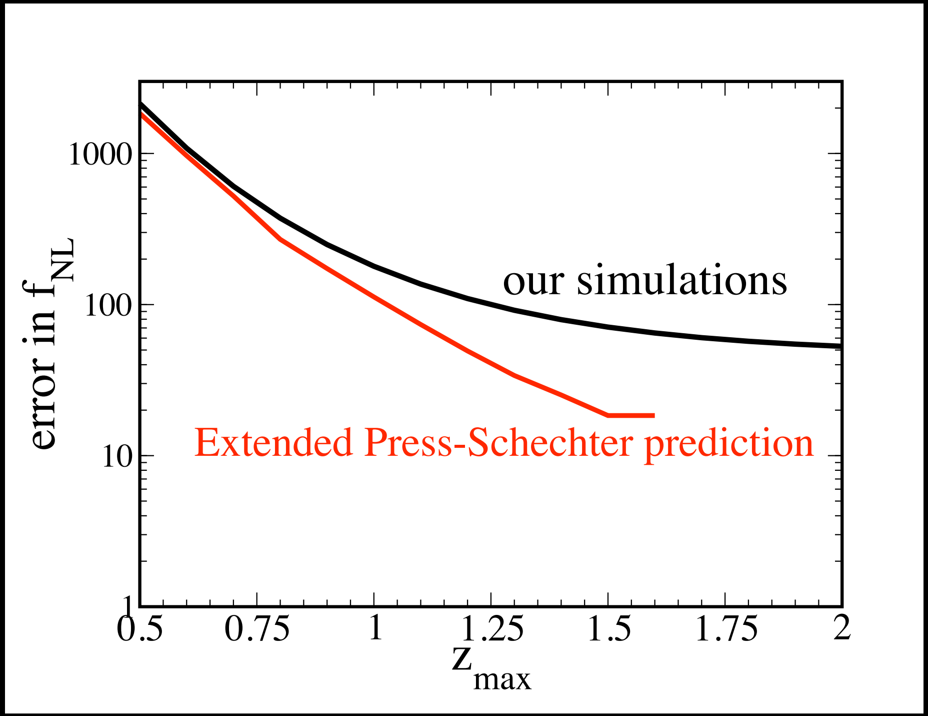




Moreover, it is not much harder to run a simulation than evaluate Extended Press-Schechter n(M)

Cosmological constraints - dark energy and NG





SPT-type survey, ~7,000 clusters, 4000 sq.deg., 0.1<z<1.5

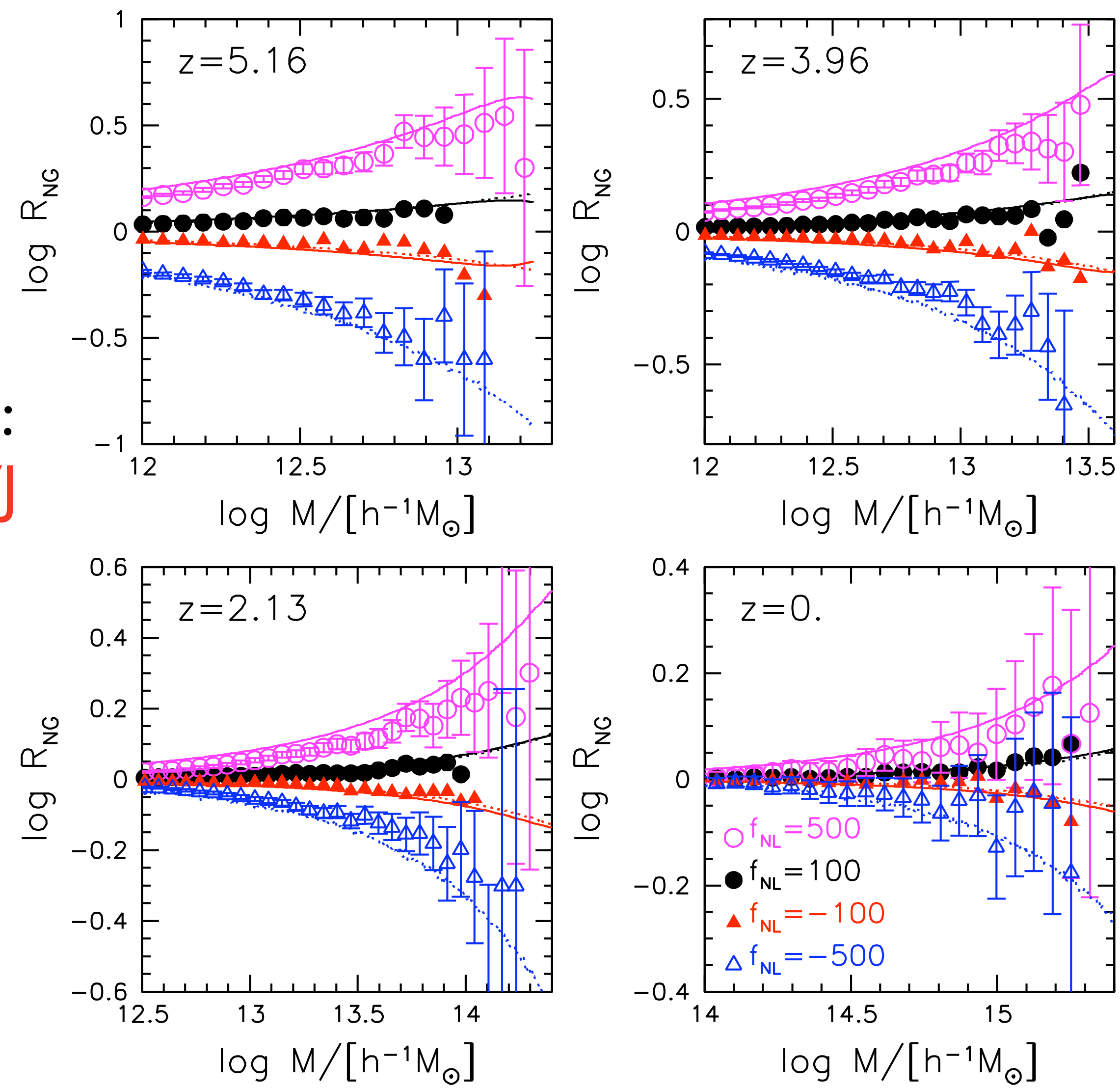
Planck prior

Recall, this is just from the cluster counts;
CMB provides stronger constraints

Comparison to other (numerical) work

1) Kang, Norberg & Silk (astro-ph/0701131):
claim much *bigger* discrepancy with MVJ,
but: their simulations are 128^3 (insufficient, as they note)

2) Grossi et al (arXiv:0707.2516):
claim perfect agreement with MVJ



We looked at the galaxy bias

cosmologists measure

$$\text{bias} \equiv \frac{\text{clustering of galaxies}}{\text{clustering of dark matter}} = \frac{\left(\frac{\delta\rho}{\rho}\right)_{\text{halos}}}{\left(\frac{\delta\rho}{\rho}\right)_{\text{DM}}}$$

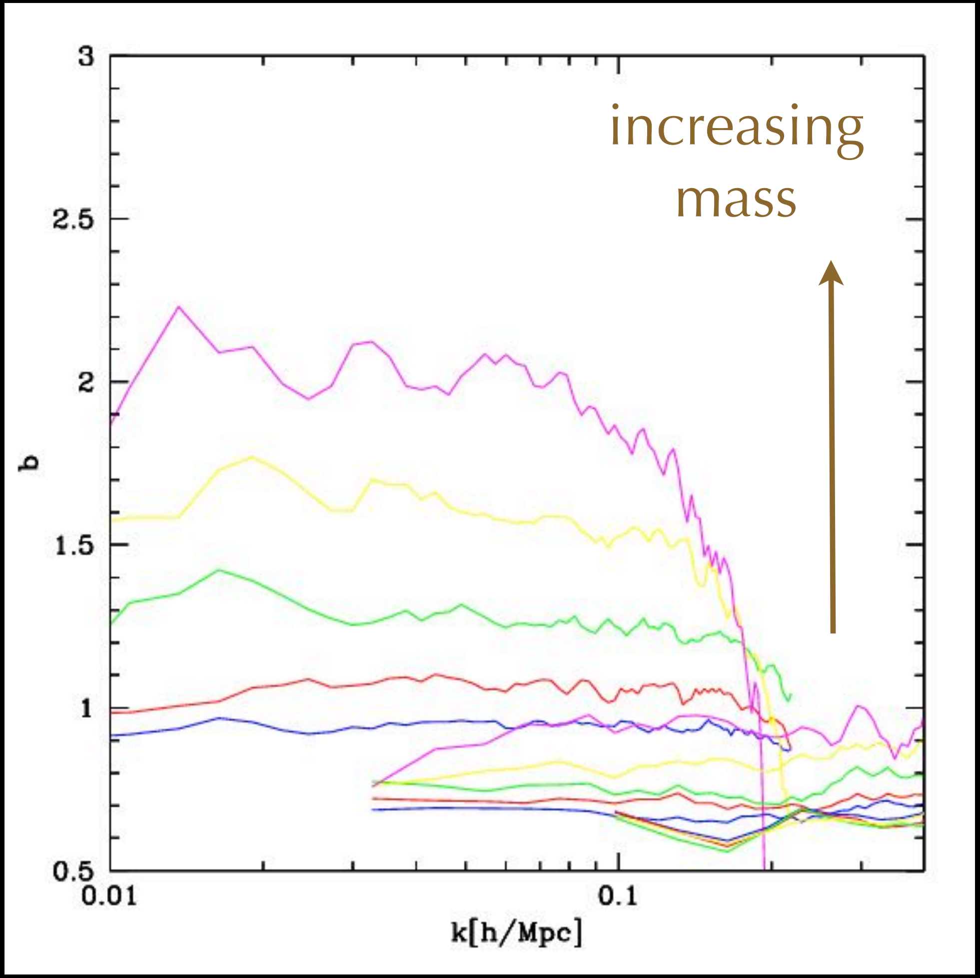
usually nuisance parameter(s)

theory predicts

Simulations and theory both say:
large-scale bias is scale-independent

Bias of dark matter halos - Gaussian case

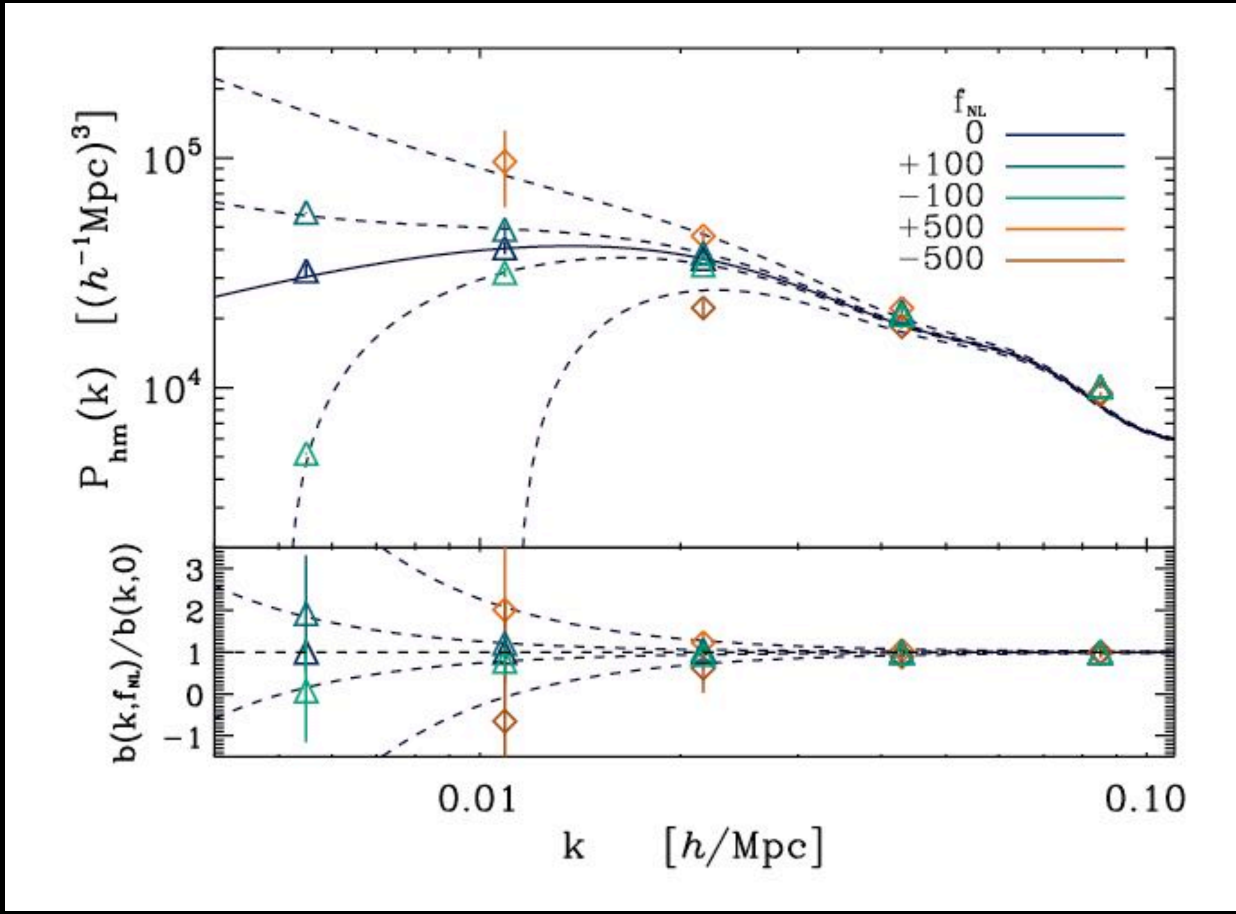
$$b \equiv \delta_h / \delta_{\rm DM}$$



Seljak & Warren 2006

Simulations and theory both say: large-scale bias is scale-independent
(theorem if halo abundance is function of local density)

Scale dependence of NG halo bias!



- Strong scale dependence of bias - i.e. b(k) - even deep in linear regime
- 512^3 (1024^3) particle simulations with box size 800 (1600) Mpc/h

Halo clustering with NG: Analytic confirmation

$$\Phi_{\rm NG} = \phi + f_{\rm NL}(\phi^2 - \langle\phi^2\rangle)$$

Then

$$\nabla^2\Phi_{\rm NG} = \nabla^2\phi + 2f_{\rm NL}\left(\phi\nabla^2\phi + |\nabla\phi|^2\right)$$

We know the statistics of all terms, so we can compute anything, e.g.

Skewness $$S_3 = \frac{\langle\delta^3_{\rm NG}\rangle}{\langle\delta^2_{\rm NG}\rangle^2} = 6f_{\rm NL}\frac{\langle\phi\delta\rangle}{\sigma^2_\delta}$$

And in particular $$\delta_{\rm NG} = \delta(1 + 2f_{\rm NL}\phi)$$

Halo clustering with NG: Analytic confirmation

Definition of bias: $$\delta_h = b_L\,\delta$$

With NG, for peaks: $$\delta \rightarrow \delta + 2 f_{\rm NL}\phi_p\delta_c$$

Assuming $$\delta_h \rightarrow (b_L + \Delta b(k))\delta$$

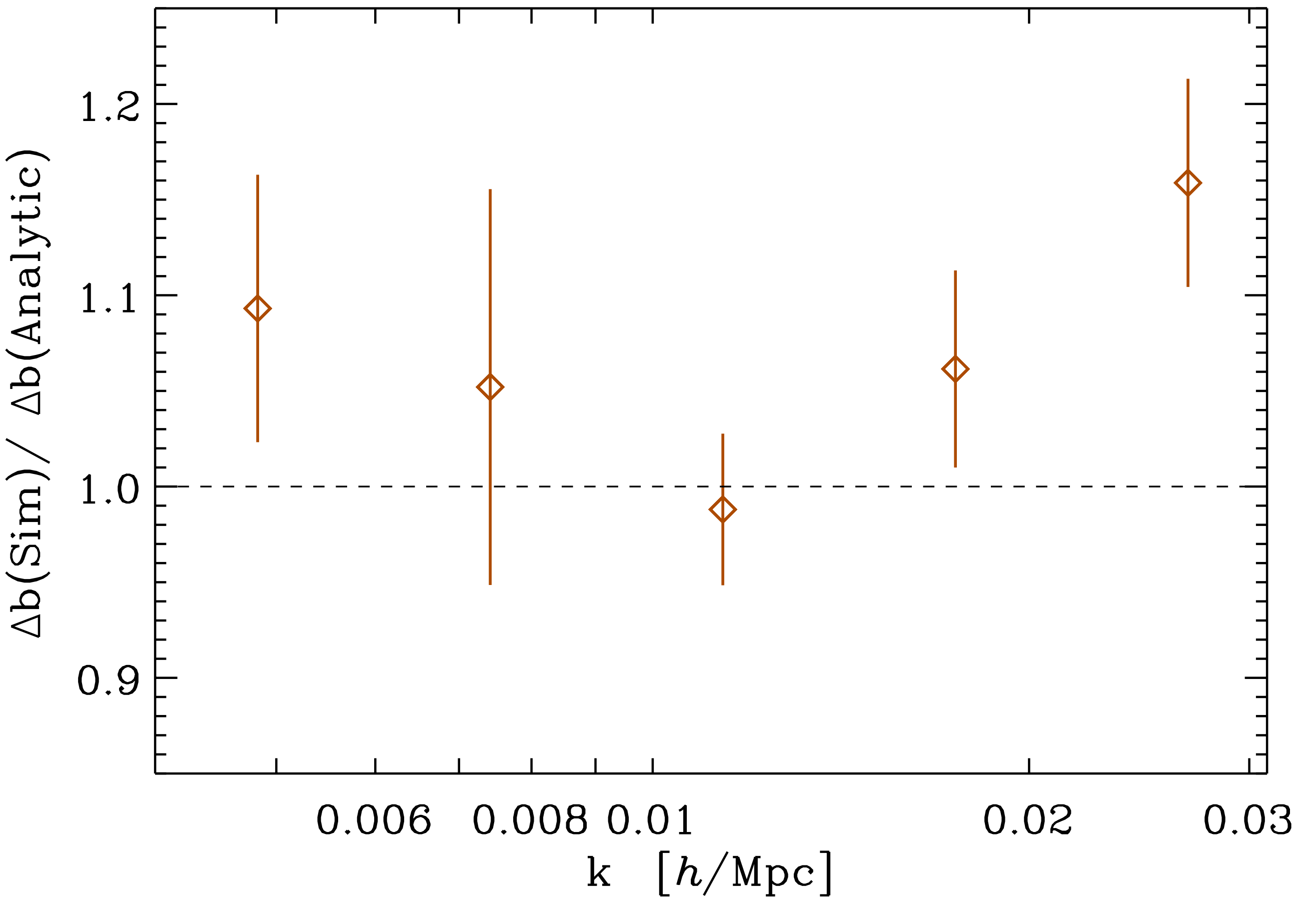
and using Poisson equation it follows that

$$\Delta b(k) = 2 b_L f_{\rm NL}\delta_{\rm crit}\frac{3\Omega_M}{2 a r_H^2 k^2}$$

Dalal, Doré, Huterer & Shirokov, arXiv:0710.4560

see also Matarrese & Verde 2008; Slosar et al. 2008; Afshordi & Tolley, 2008; McDonald 2008

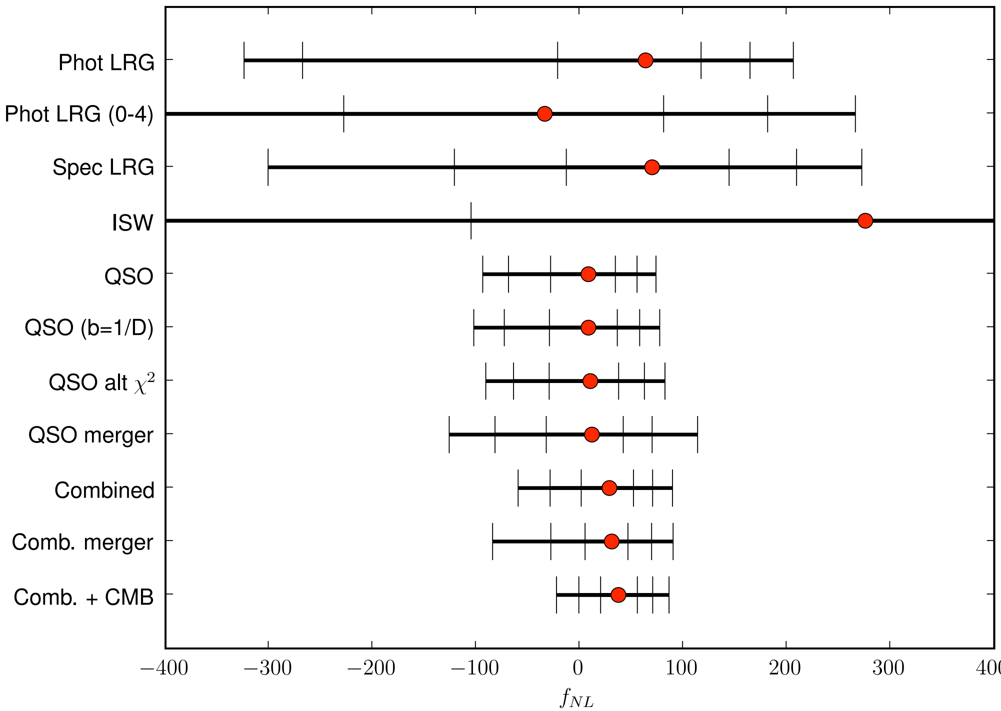
Analytic and numerical results agree

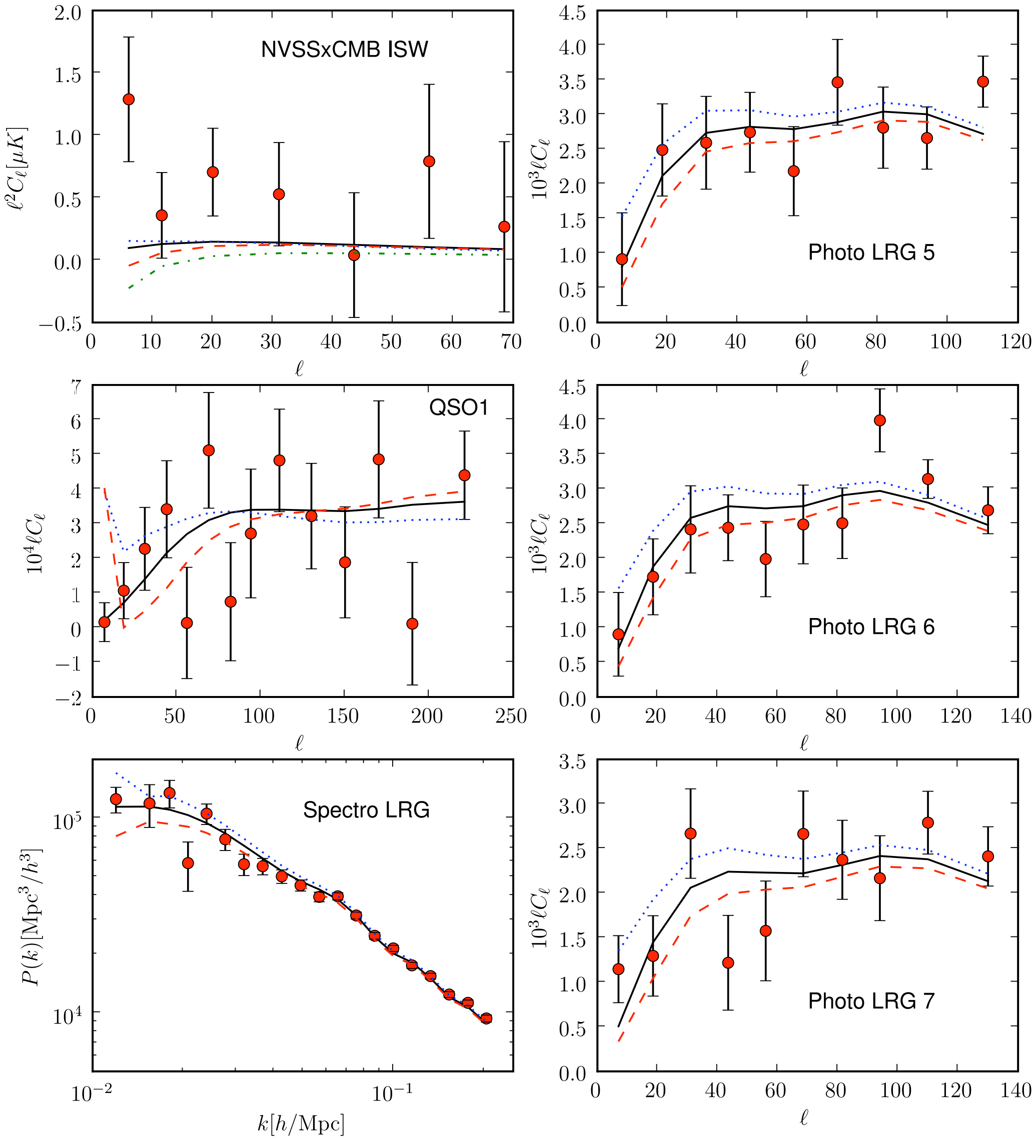


$$\Delta b(k) = 2 b_L f_{\rm NL} \delta_{\rm crit} \frac{3\Omega_M}{2 a r_H^2 k^2}$$

Very recent, exciting developments...

Constraints from *current* data - west coast team



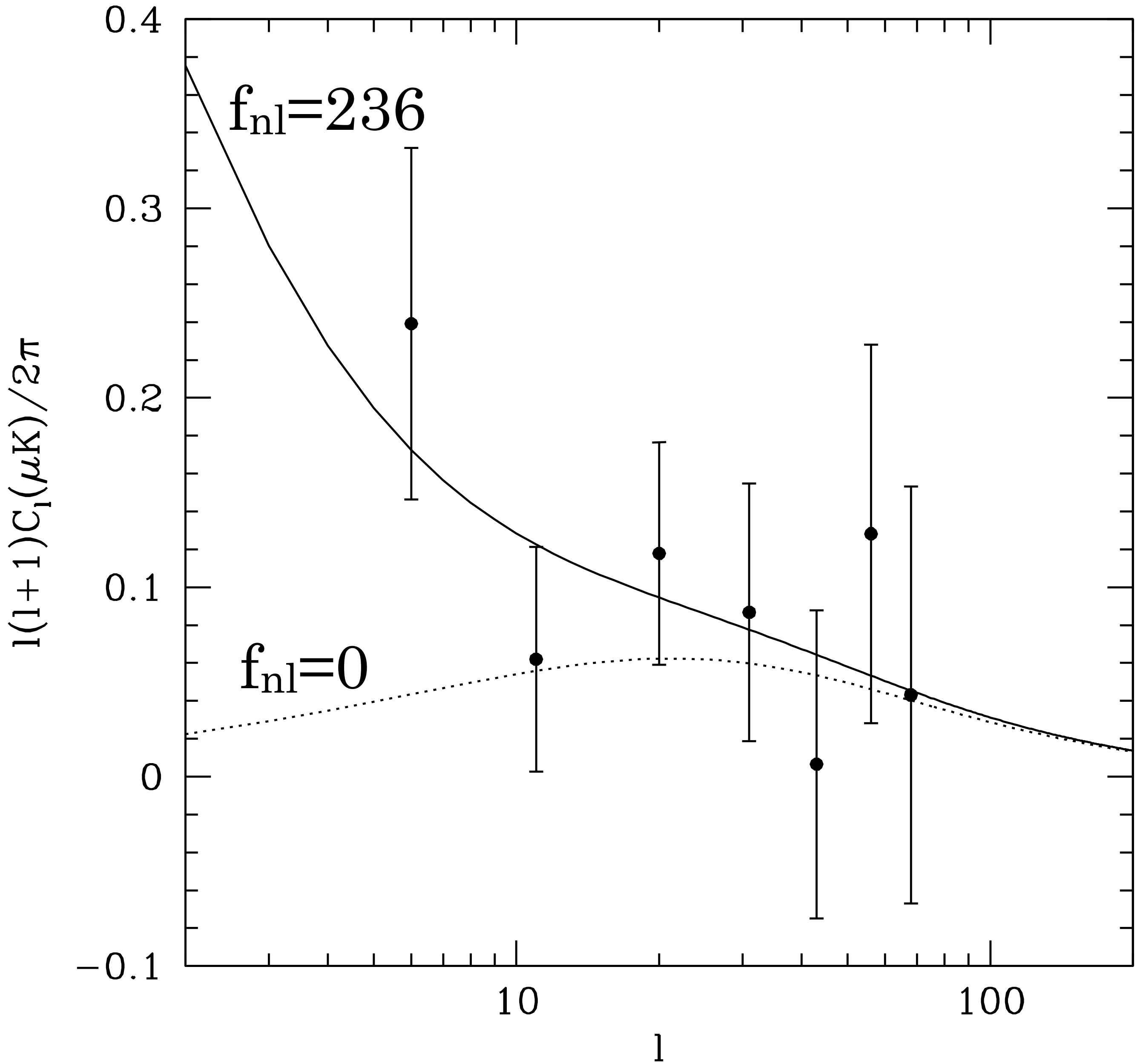


$f_{nl} = 8$ +/- 30 (68%, QSO)

$f_{nl} = 23$ +/- 23 (68%, all)

Slosar, Hirata, Seljak, Ho & Padmanabhan 2008

Constraints from *current* data - Canada team



f_{nl} = 236 +/- 127 (68%)

Afshordi & Tolley 2008

Future NG from measurements of b(k)

- Numerous cosmological probes, such as the baryon acoustic oscillations (BAO) or probes of Integrated Sachs-Wolfe effect (galaxy-CMB cross-corr) can be used to measure b(k)

- The effect (going as k^{-2}) provides a fairly unique signature and a clear target

- Expect accuracy of order sigma(f_{NL})<10 or even ~1 in the future

TABLE 1
GALAXY SURVEYS CONSIDERED

survey	z range	sq deg	mean galaxy density $(h/Mpc)^3$	$\Delta f_{\rm NL}/q'$ LSS
SDSS LRG's	$0.16 < z < 0.47$	7.6×10^3	1.36×10^{-4}	40
BOSS	$0 < z < 0.7$	10^4	2.66×10^{-4}	18
WFMOS low z	$0.5 < z < 1.3$	2×10^3	4.88×10^{-4}	15
WFMOS high z	$2.3 < z < 3.3$	3×10^2	4.55×10^{-4}	17
ADEPT	$1 < z < 2$	2.8×10^4	9.37×10^{-4}	1.5
EUCLID	$0 < z < 2$	2×10^4	1.56×10^{-3}	1.7
DES	$0.2 < z < 1.3$	5×10^3	1.85×10^{-3}	8
PanSTARRS	$0 < z < 1.2$	3×10^4	1.72×10^{-3}	3.5
LSST	$0.3 < z < 3.6$	3×10^4	2.77×10^{-3}	0.7

!

Carbone, Verde & Matarrese 2008; Afshordi & Tolley 2008

Conclusions

- Searching for primordial nongaussianity is one of the most fundamental tests of cosmology

- CMB bispectrum traditionally most promising tool; current results favor $f_{NL}>0$ but only at 1-2 sigma

- Cluster counts are in principle sensitive to NG, but not competitive with the CMB, especially if you trust the numerical results from Dalal et al.

- Cosmological models with (local) primordial NG lead to significant scale dependence of halo bias; theory and simulations appear to be in remarkable agreement on this

- Therefore, LSS probes (baryon oscillations, galaxy-CMB cross-correlations, etc) are likely to lead to constraints on NG an order of magnitude stronger than previously thought

- Fisher matrix calculations show sigma(f_{NL})~1 expected from future LSS surveys (DES, LSST, JDEM etc)