Multipole Vectors and the CMB Sky

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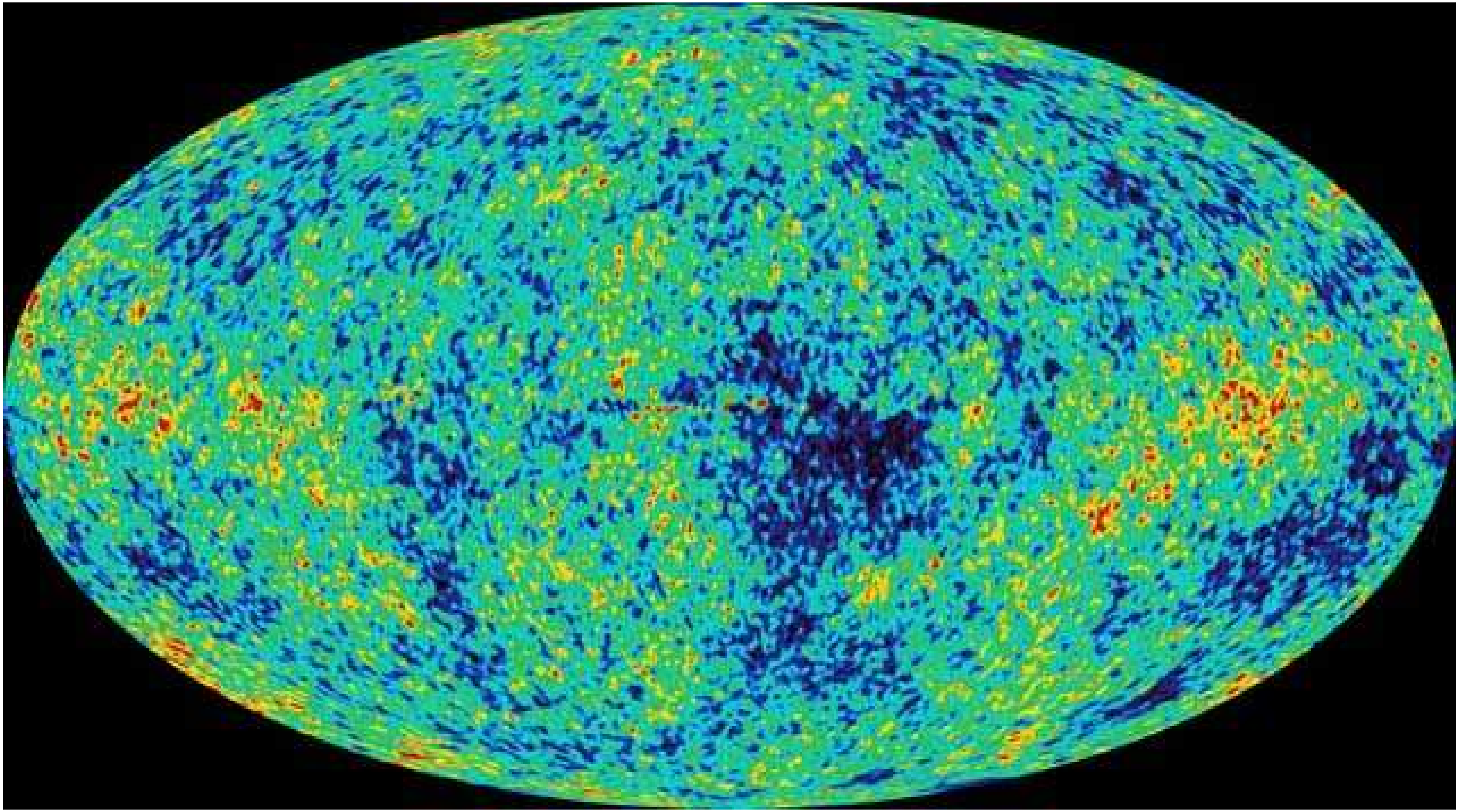
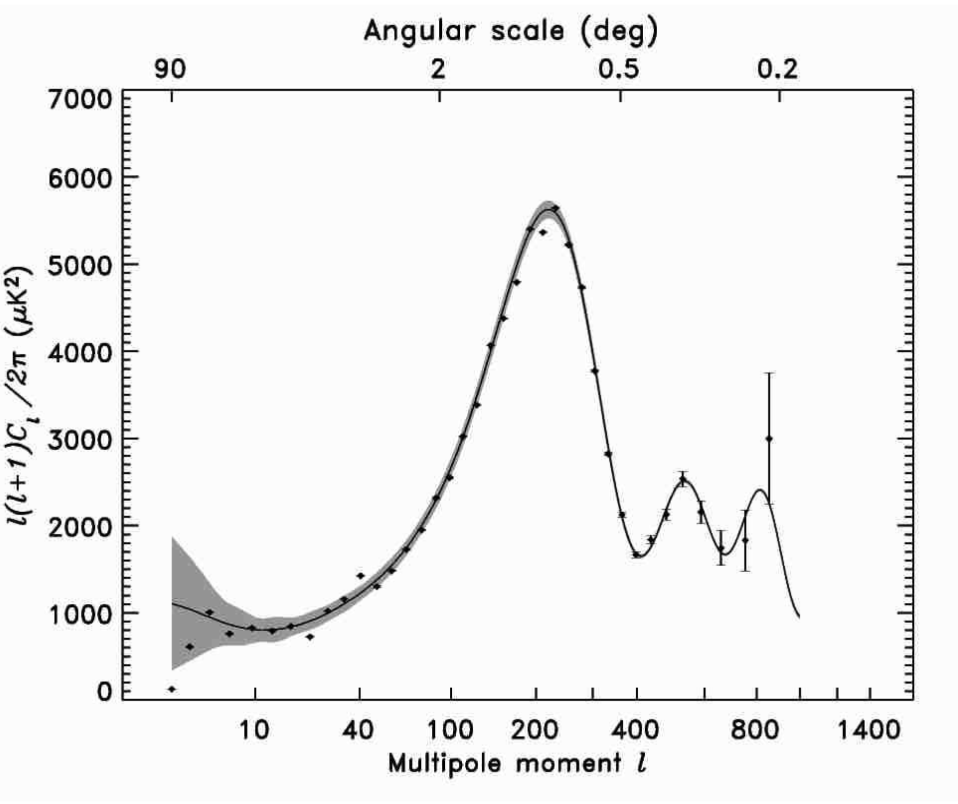
with Craig Copi, Glenn Starkman (astro-ph/0310511)

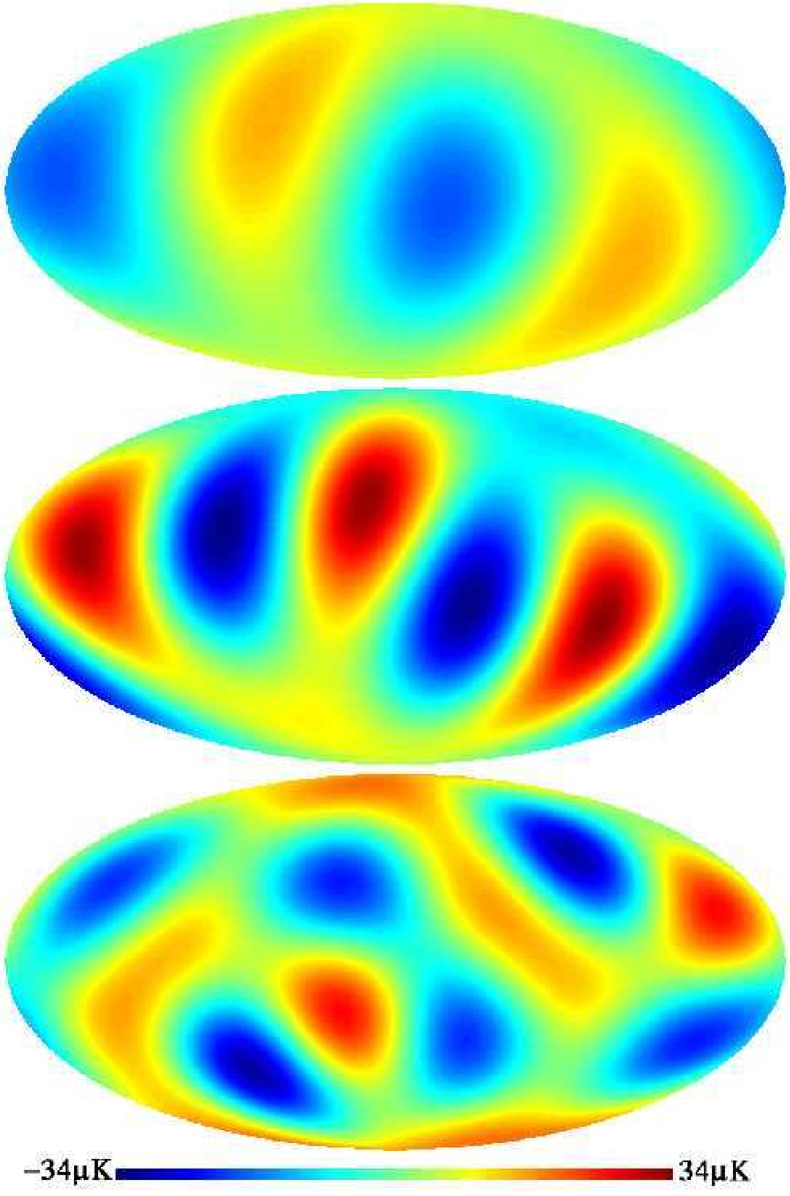
WMAP ILC map



Bennett et al. 2003

“...answered old questions and raised new...”





Bennett et al. 2003, Tegmark et al. 2003

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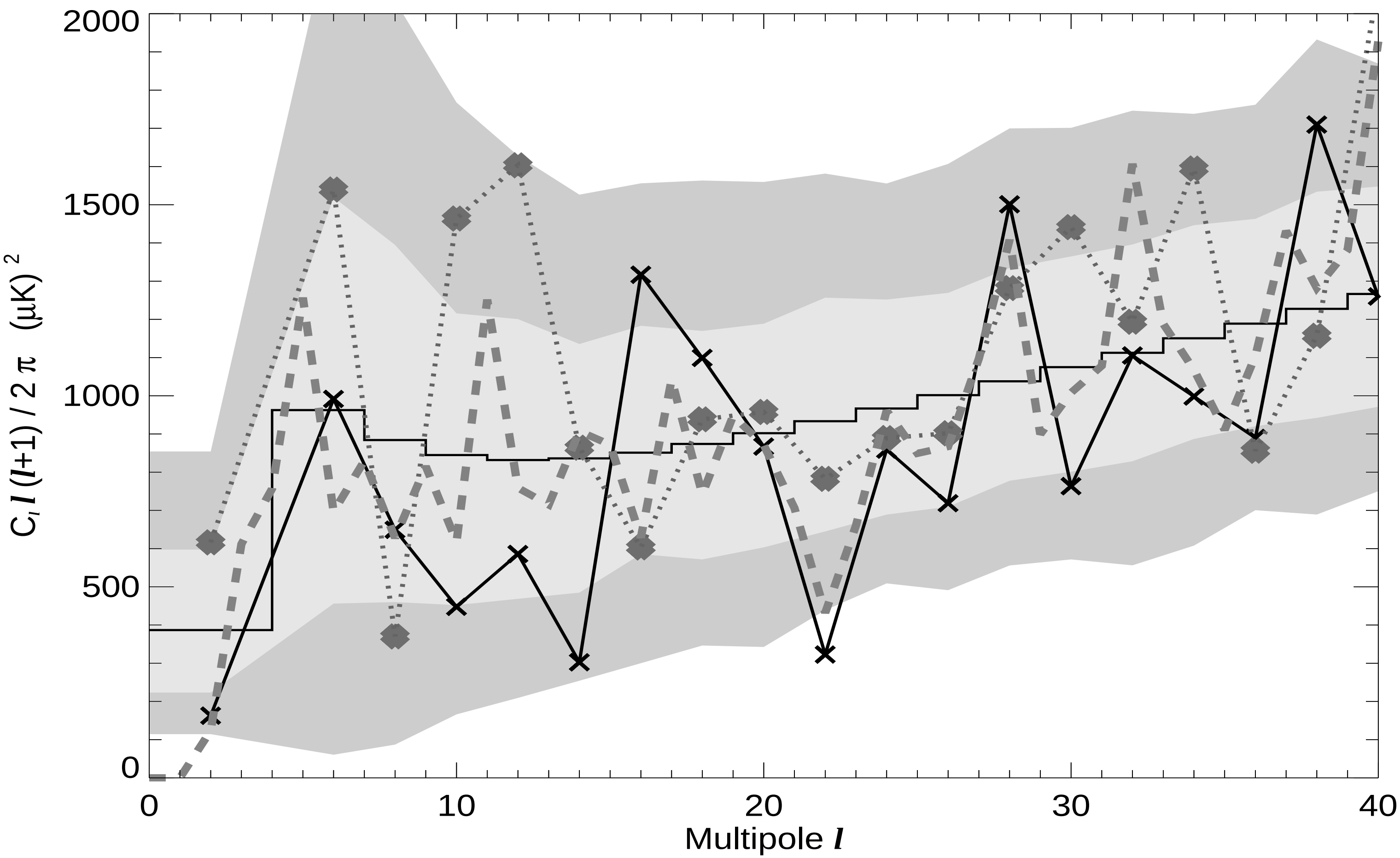
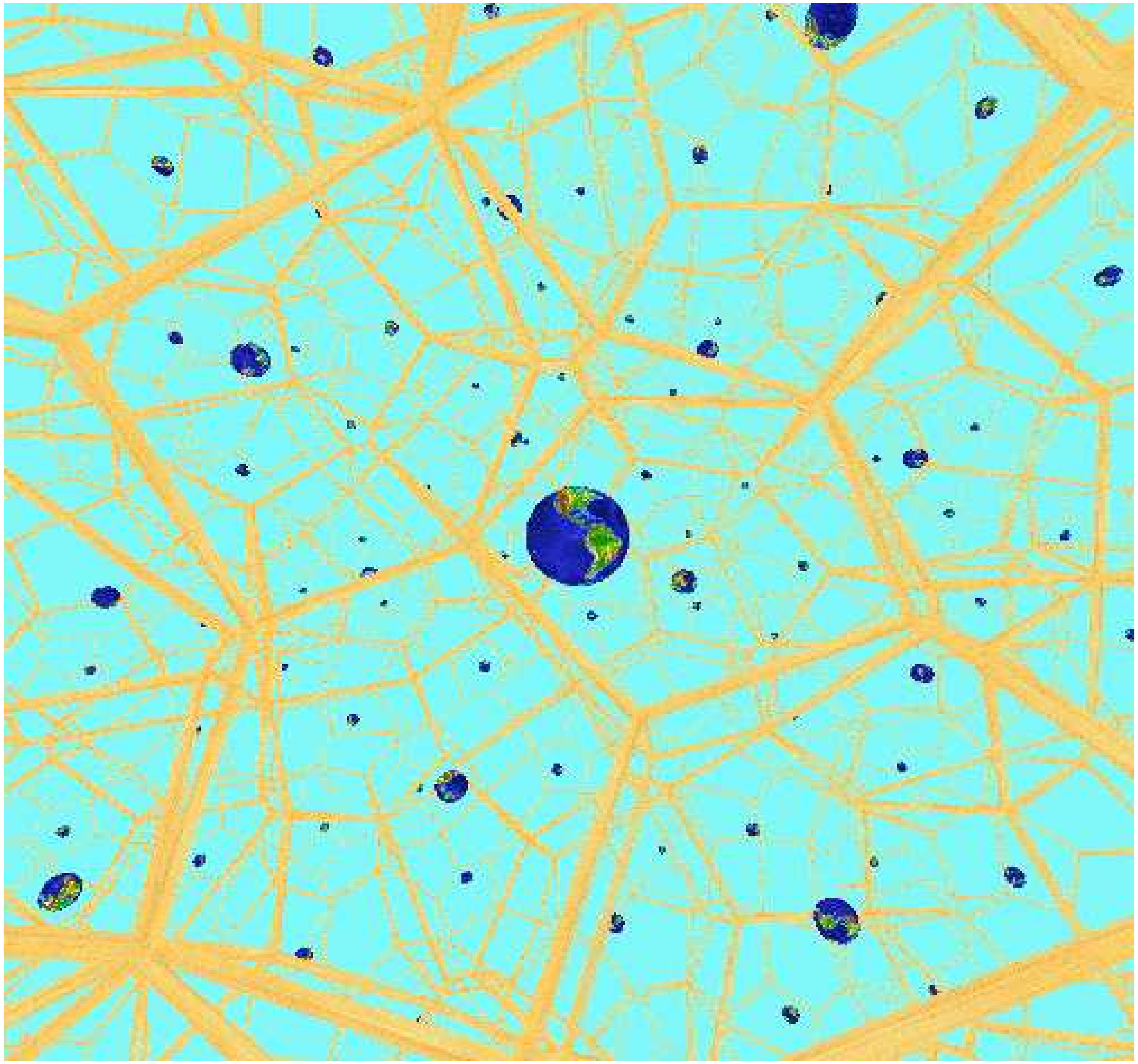


Eriksen et al. 2003



$$\frac{\delta T}{T}(\theta,\phi) = \sum_{l,m} a_{lm} Y_{lm}(\theta,\phi), \qquad C_\ell \equiv \frac{1}{2\ell+1}\sum_{m=-\ell}^{\ell} |a_{\ell m}|^2$$

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$$\text{“}a^{(\ell)}_{i_1 \ldots i_l} \;\leftrightarrow\; A^{(l)} \left[\hat{v}^{(\ell,1)} \otimes \hat{v}^{(\ell,2)} \otimes \ldots \hat{v}^{(\ell,\ell)}\right]\text{”}$$

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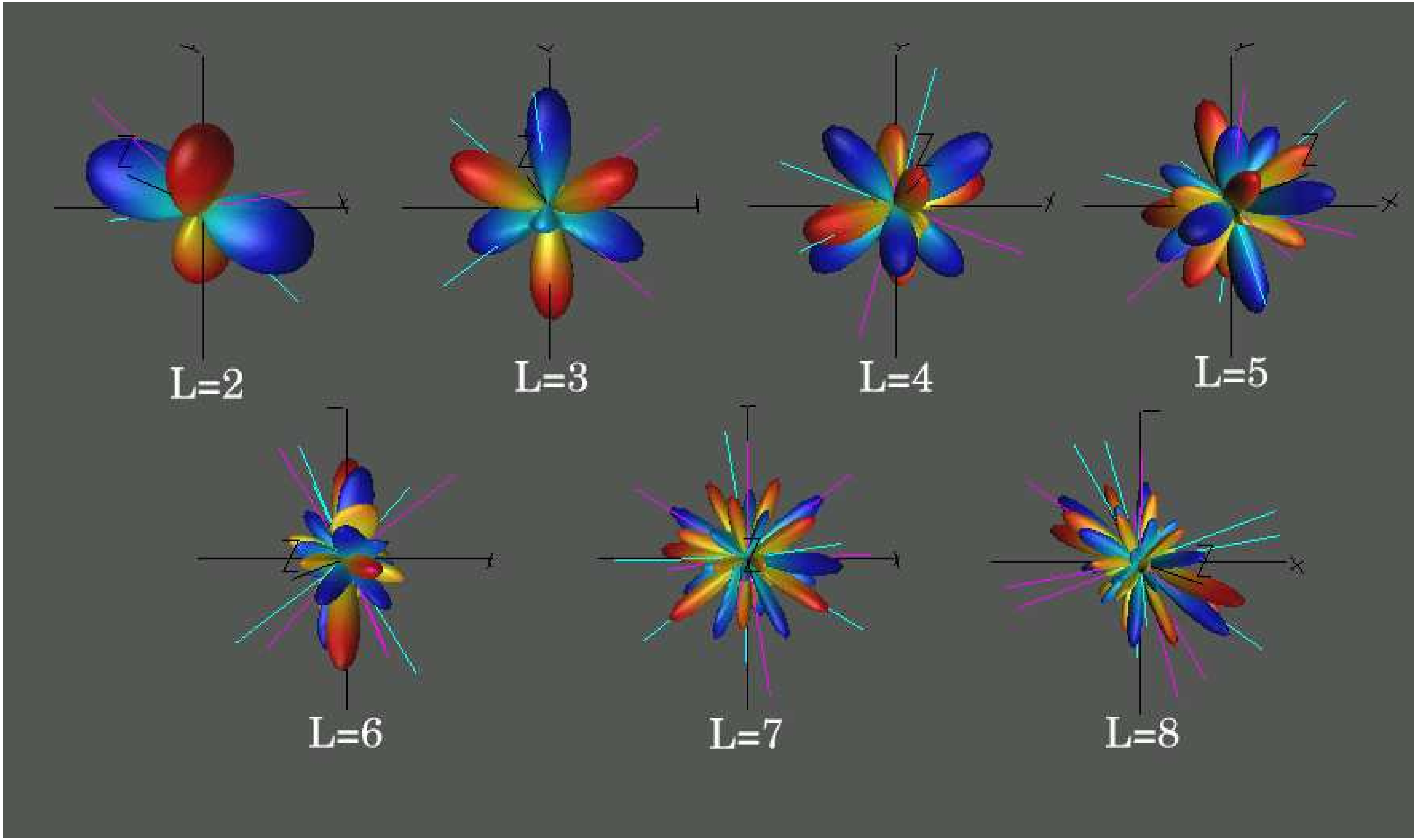
ℓ^{th} order equations?

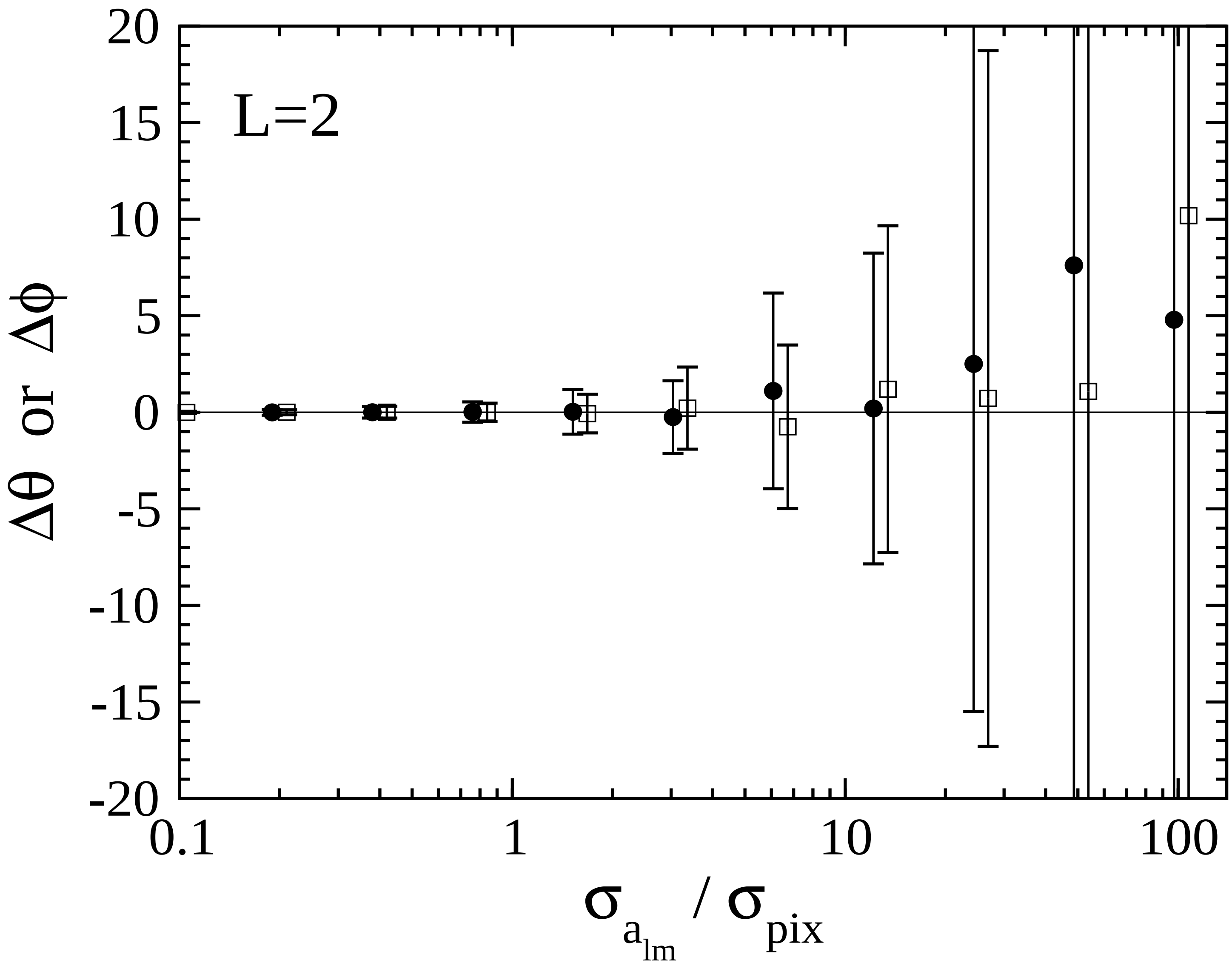
Fortunately, can peel off one vector at a time

$\longrightarrow$ coupled quadratic equations.

WMAP's Multipole Vectors



Accuracy in determining MVs



Tests of Isotropy and Gaussianity with MV

Hypothesis:

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle = C_\ell \, \delta_{\ell\ell'} \, \delta_{mm'}$$

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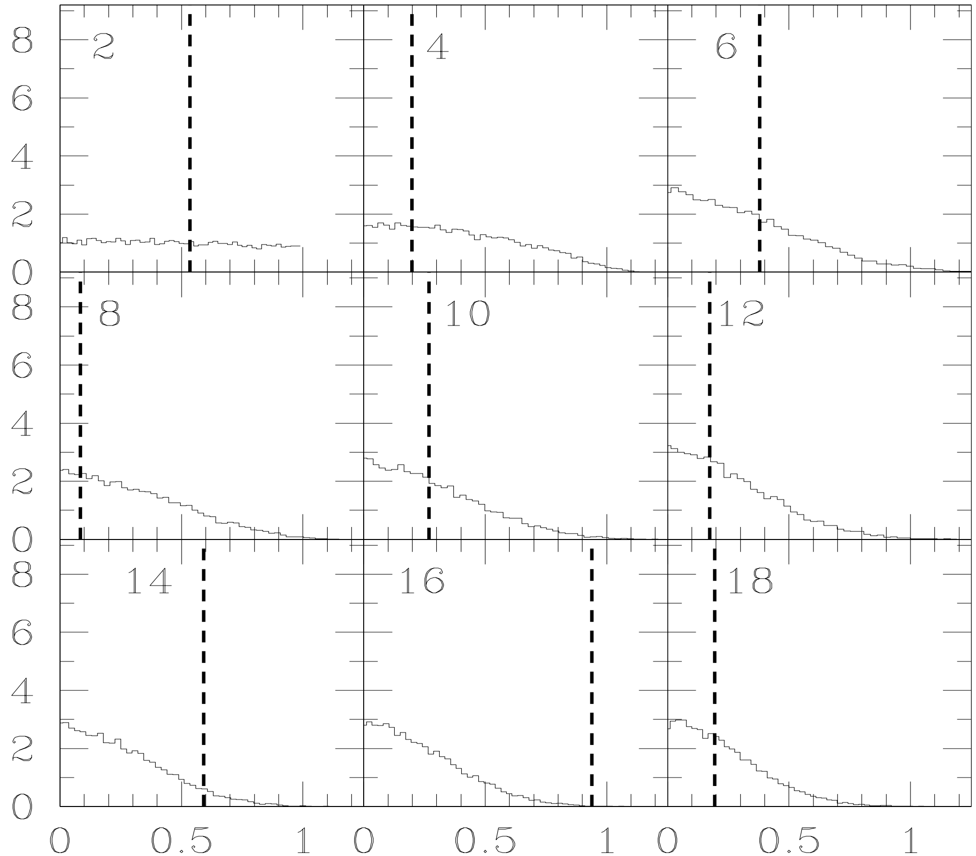

Ferreira, Magueijo & Gorski 1998

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Tests:

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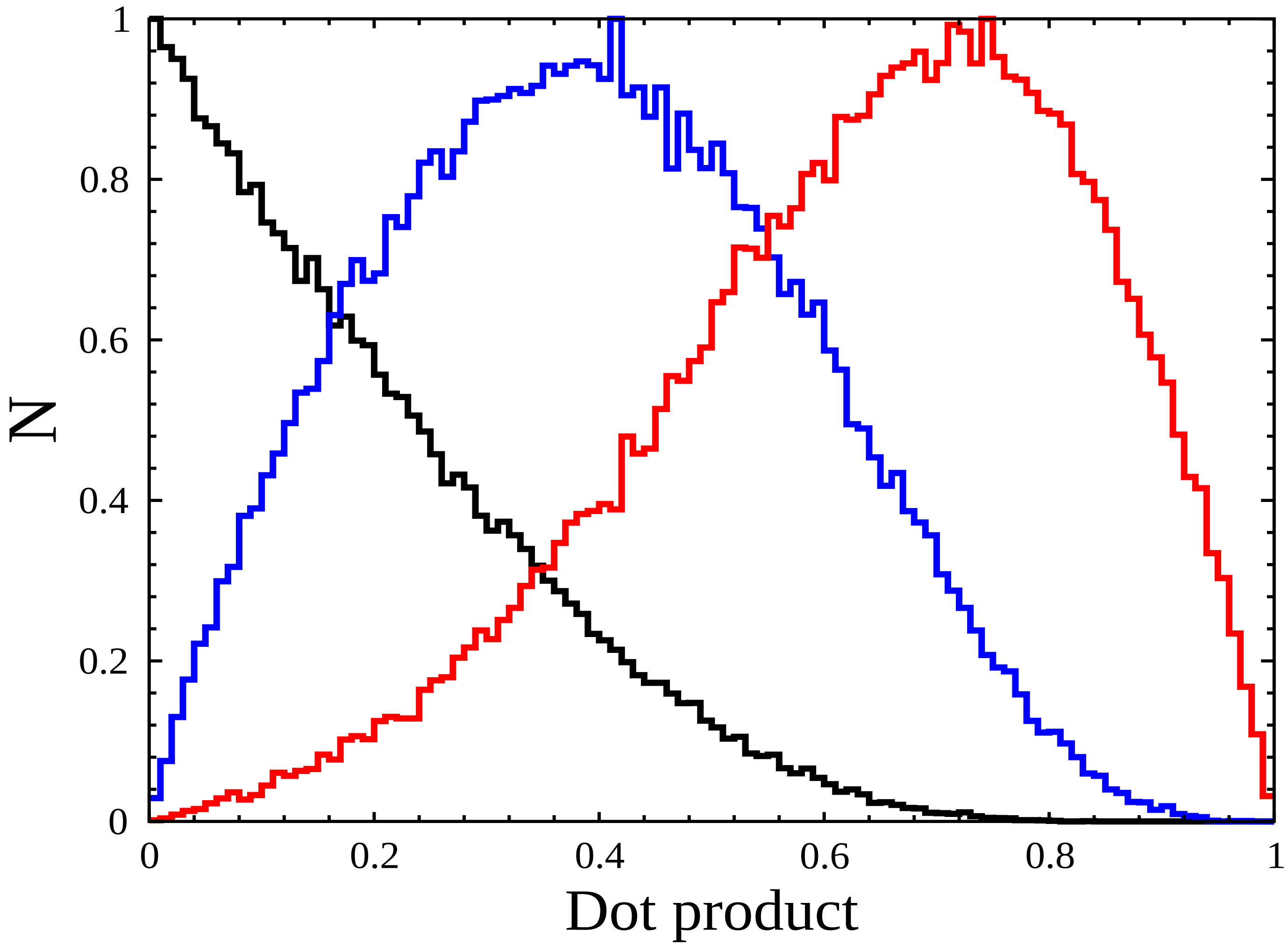
- $\left| \left(\hat{v}^{(\ell_1,i)} \times \hat{v}^{(\ell_1,j)} \right) \cdot \left(\hat{v}^{(\ell_2,k)} \times \hat{v}^{(\ell_2,m)} \right) \right|$

Comparison to Monte-Carlo realizations



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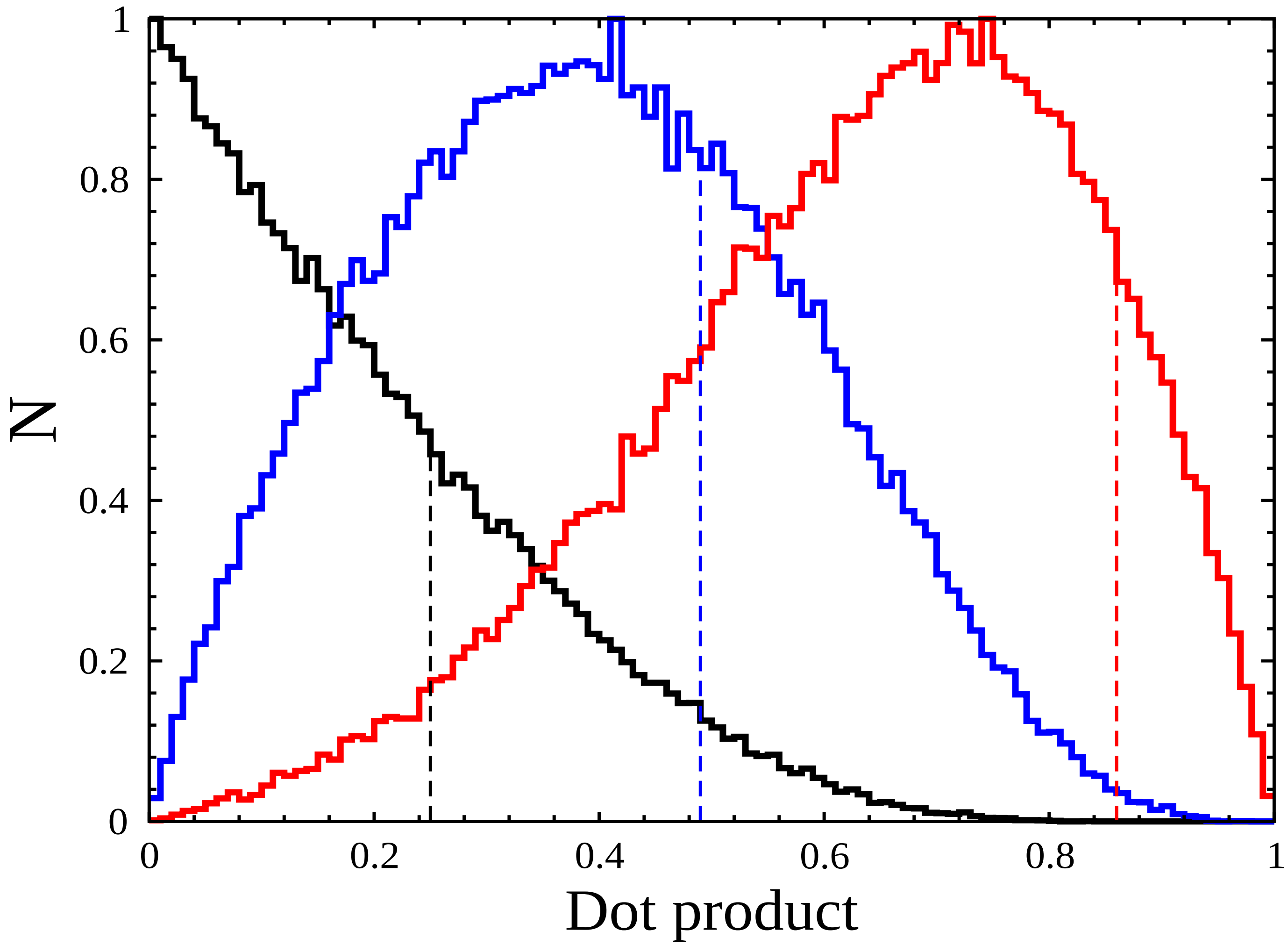

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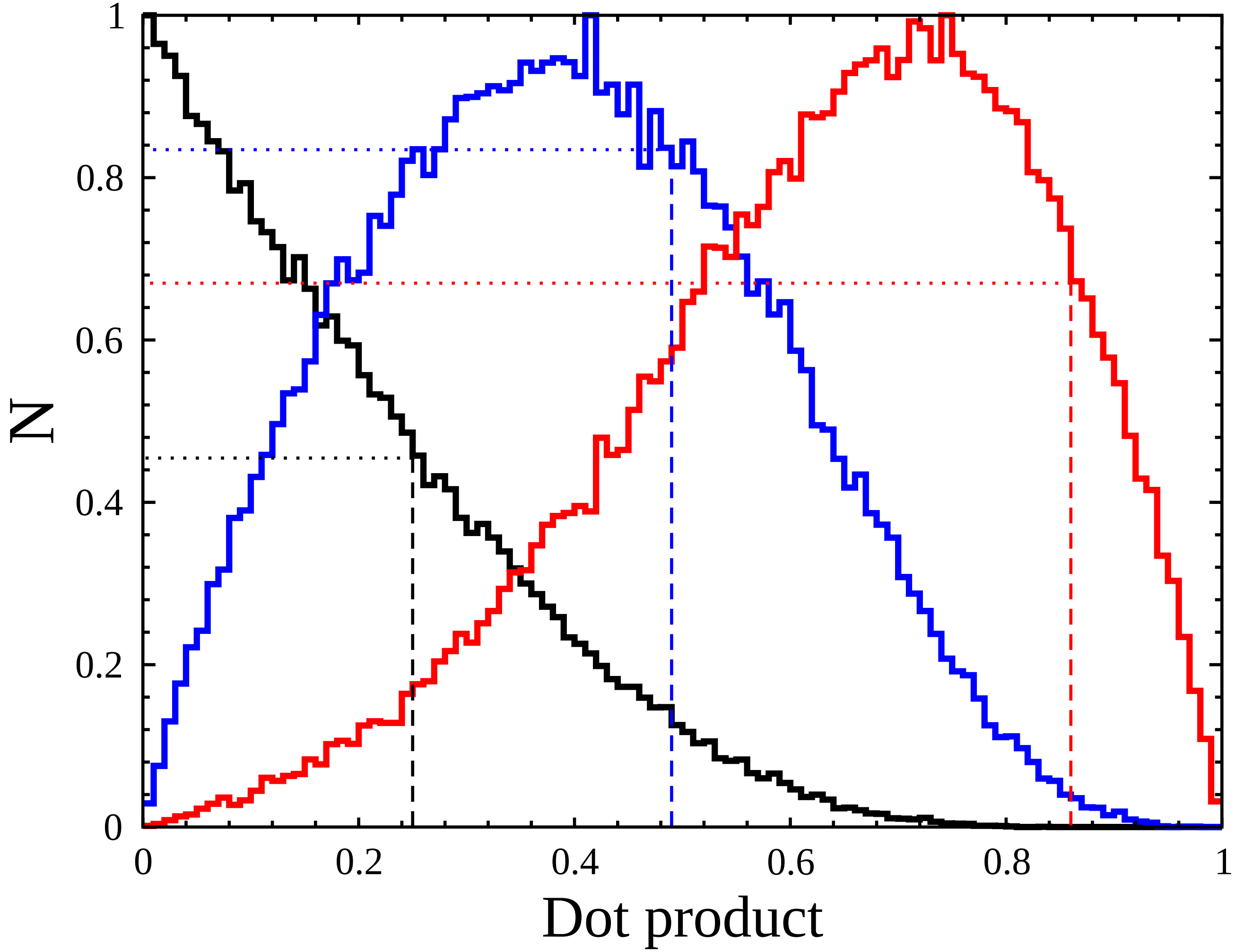

$$\mathcal{L}_{\mathrm{WMAP}} = \prod_{j=1}^{M} \frac{N_{j,\mathrm{WMAP}}}{N_{j,\max}}$$

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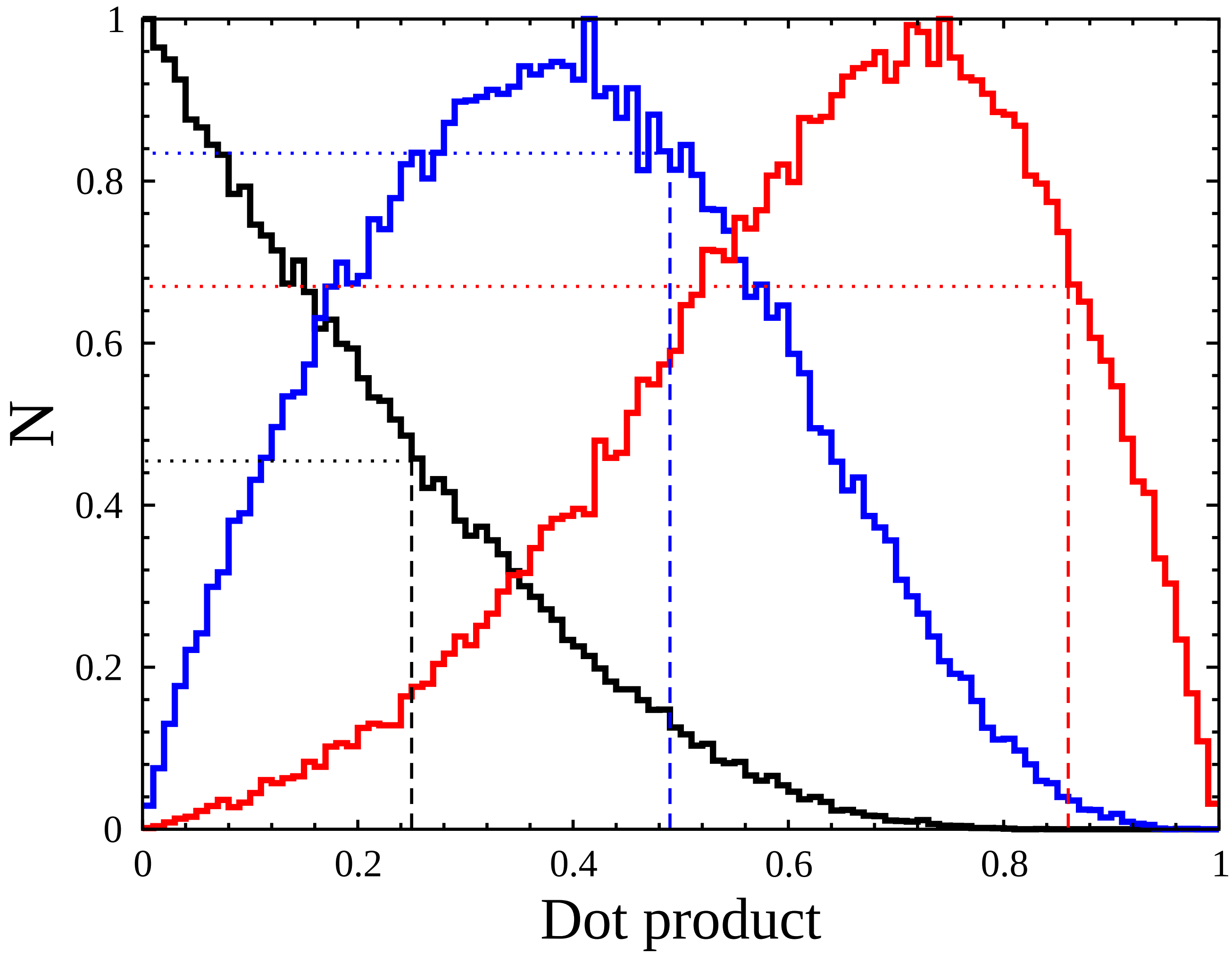


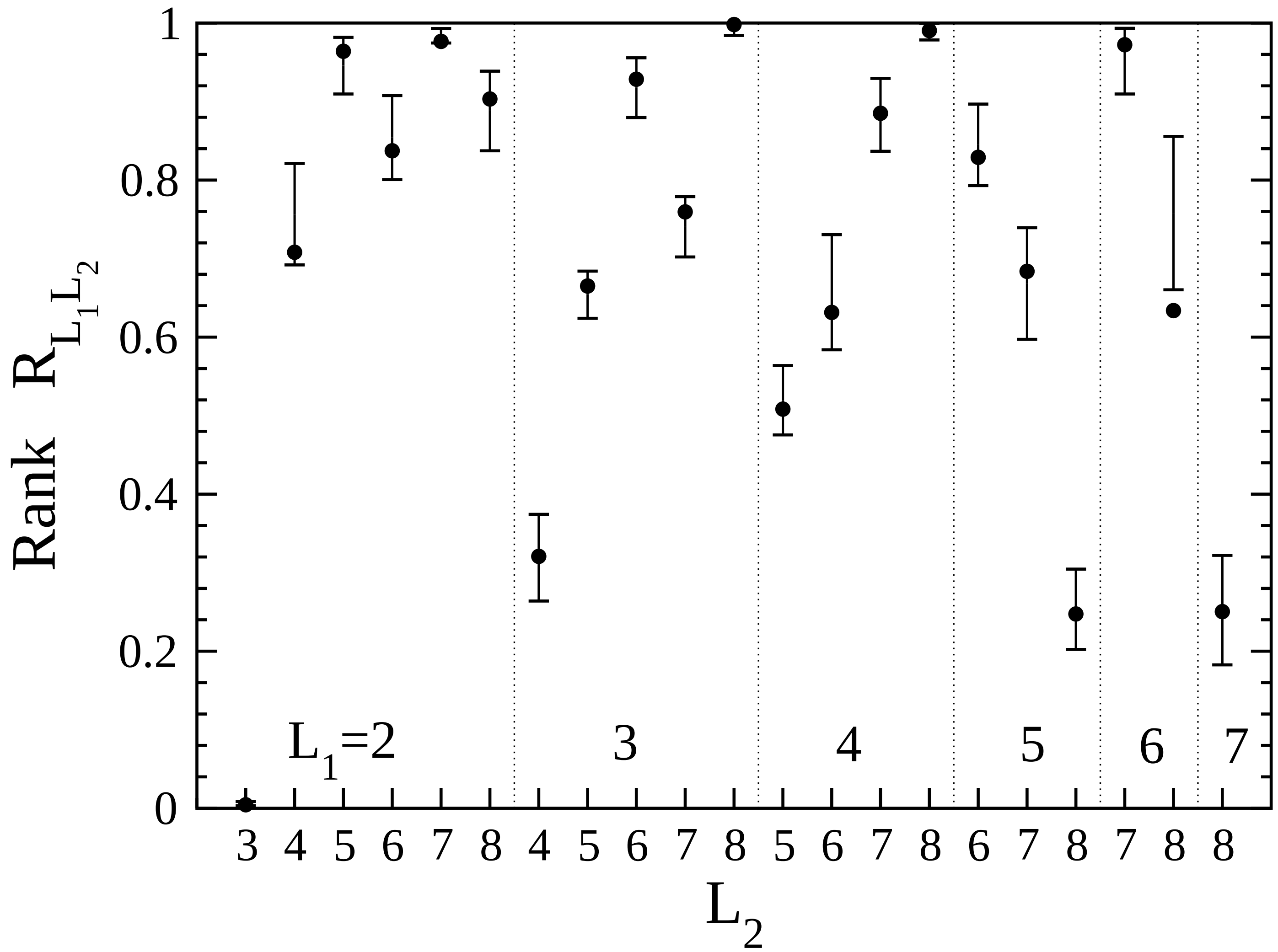
$$\mathcal{L}_{\mathrm{WMAP}} = \prod_{j=1}^{M} \frac{N_{j,\mathrm{WMAP}}}{N_{j,\max}}$$

$$\mathcal{L}_{\mathrm{MC}} = \prod_{j=1}^{M} \frac{N_{j,\mathrm{MC}}}{N_{j,\max}}$$

WMAP Ranks relative to MC maps

Ranks of Products of Multipole Vectors										
	Vector-Vector		Vector-Cross			Cross-Cross		Oriented Area		
(ℓ_1, ℓ_2)	M	Rank	M	(ℓ_1, ℓ_2) Rank	(ℓ_2, ℓ_1) Rank	M	Rank	M	Rank
(2, 3)	6	0.30085	6, 3	0.03881	0.02663	3	0.02851	3	0.00437
(2, 4)	8	0.44821	12, 4	0.40617	0.28697	6	0.64002	6	0.70802
(2, 5)	10	0.86384	20, 5	0.91952	0.12703	10	0.34211	10	0.96408
(2, 6)	12	0.49091	30, 6	0.39465	0.88406	15	0.79848	15	0.83726
(2, 7)	14	0.26945	42, 7	0.58516	0.74222	21	0.90614	21	0.97656
(2, 8)	16	0.52924	56, 8	0.47201	0.80074	28	0.94728	28	0.90320
(3, 4)	12	0.21693	18, 12	0.77938	0.27641	18	0.16803	18	0.32075
(3, 5)	15	0.18173	30, 15	0.37847	0.80930	30	0.36049	30	0.66510
(3, 6)	18	0.29579	45, 18	0.27379	0.54152	45	0.74466	45	0.92840
(3, 7)	21	0.99128	63, 21	0.86604	0.54870	63	0.57494	63	0.75950
(3, 8)	24	0.74998	84, 24	0.98974	0.64684	84	0.99994	84	0.99800
(4, 5)	20	0.44103	40, 30	0.28163	0.83108	60	0.54514	60	0.50828
(4, 6)	24	0.71024	60, 36	0.57400	0.87122	90	0.74174	90	0.63144
(4, 7)	28	0.86308	84, 42	0.52634	0.93646	126	0.52980	126	0.88500
(4, 8)	32	0.49009	112, 48	0.55776	0.85588	168	0.84710	168	0.99034
(5, 6)	30	0.82896	75, 60	0.41045	0.90076	150	0.66392	150	0.82894
(5, 7)	35	0.86748	105, 70	0.55802	0.95898	210	0.59690	210	0.68370
(5, 8)	40	0.85286	140, 80	0.97216	0.33791	280	0.30357	280	0.24739
(6, 7)	42	0.03659	126, 105	0.14563	0.02445	315	0.19767	315	0.97232
(6, 8)	48	0.93586	168, 120	0.91310	0.77020	420	0.73614	420	0.63374
(7, 8)	56	0.03285	196, 168	0.04175	0.02303	588	0.09505	588	0.25041

WMAP Ranks relative to MC maps



How high are the ranks?

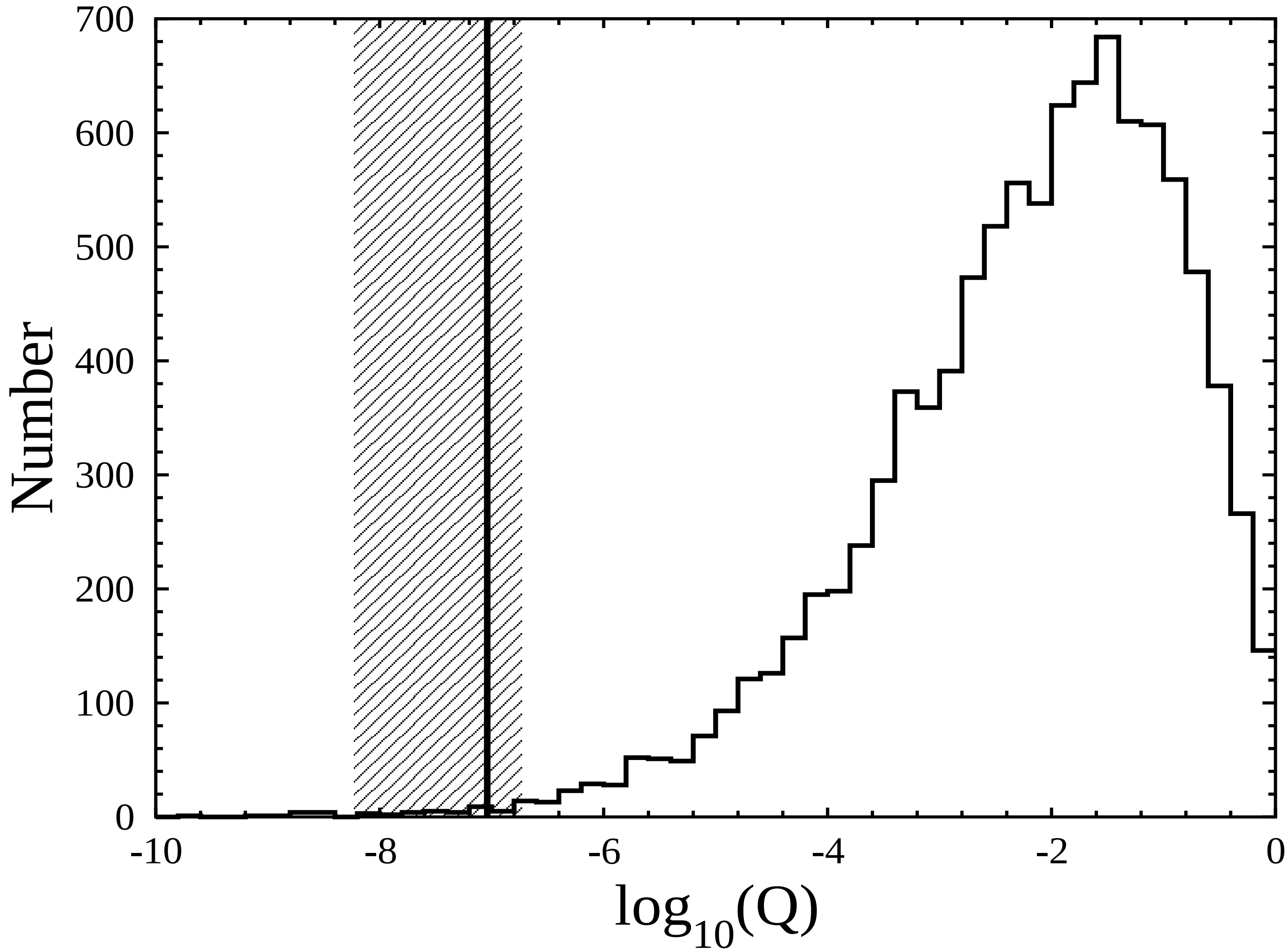
We use the following statistic:

$$Q(x_1,\ldots,x_N) = N! \int_{x_1}^{1} dy_1 \int_{x_2}^{y_1} dy_2 \ldots \int_{x_N}^{y_{N-1}} dy_N$$

For uniform random y's, this is equal to

$$\text{Probability}\left[(y_1 > x_1) \text{ AND } (y_2 > x_2) \text{ AND } \ldots \text{ AND } (y_N > x_N)\right]$$

Final Probability



Systematic Tests

- ranks $2 \leq (\ell_1, \ell_2) \leq 8$ are unusual at the level of 4 parts in a 1000 (12 in a 1000 for ILC map)

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Varying the Multipole Coverage		
$\ell_{\rm min}$	$Q_{\rm WMAP}$	$f(Q_{\rm MC} < Q_{\rm WMAP})$
2	9.00×10^{-8}	$4/1000$
3	2.11×10^{-6}	$10/1000$
4	3.18×10^{-4}	$70/1000$
$\ell_{\rm max}$	$Q_{\rm WMAP}$	$f(Q_{\rm MC} < Q_{\rm WMAP})$
8	9.00×10^{-8}	$4/1000$
7	1.26×10^{-5}	$20/1000$
6	3.42×10^{-3}	$193/1000$

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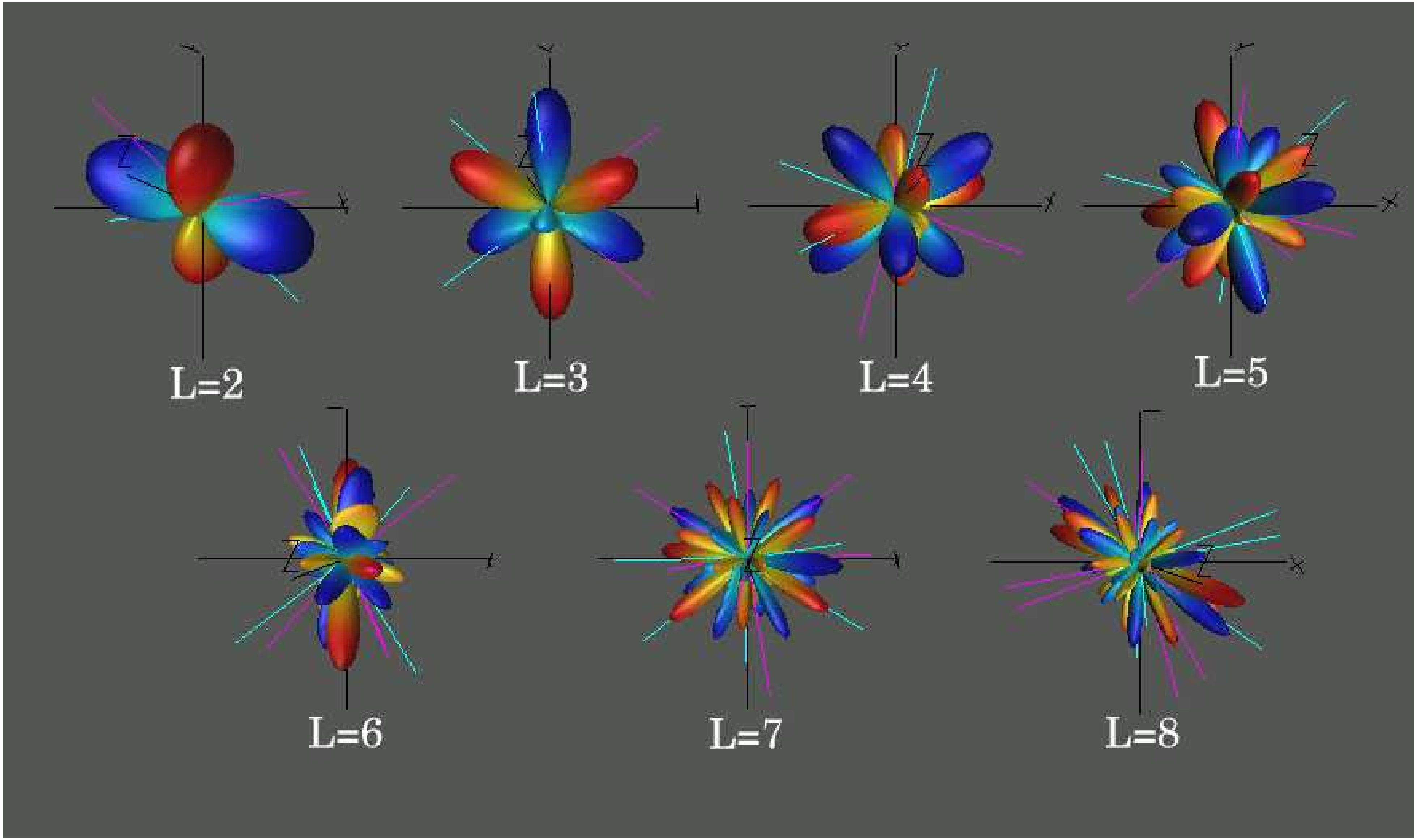
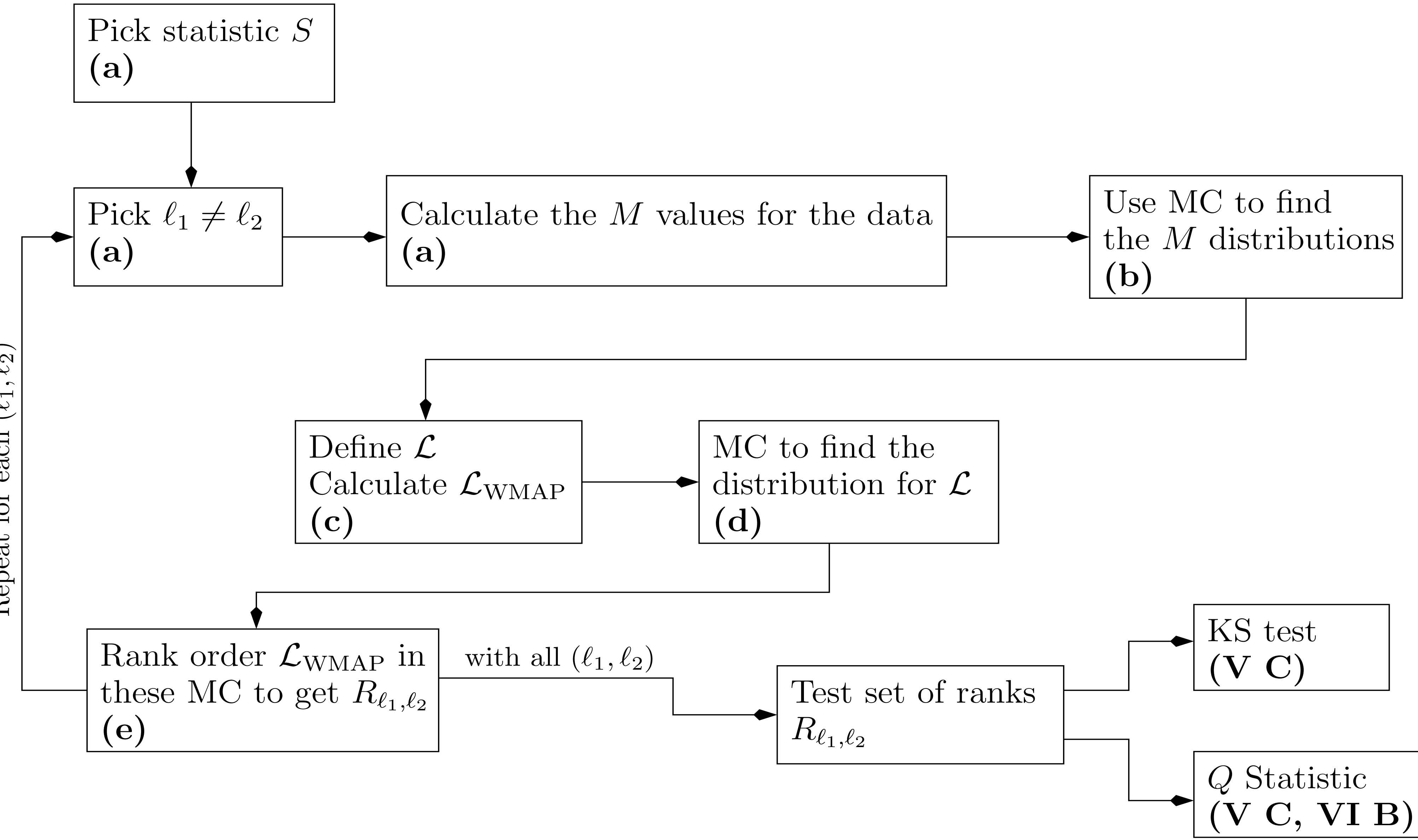
- Dust, random-gaussian contamination
- Cut skies
- Other work (Park, Eriksen *et al.*), but relation unclear

http://www.phys.cwru.edu/projects/mpvectors/




Pick statistic S **(a)**
Pick $\ell_1 \neq \ell_2$ **(a)**
Calculate the M values for the data **(a)**
Use MC to find the M distributions **(b)**
Define $\mathcal{L}$ Calculate $\mathcal{L}_{\mathrm{WMAP}}$ **(c)**
MC to find the distribution for $\mathcal{L}$ **(d)**
Rank order $\mathcal{L}_{\mathrm{WMAP}}$ in these MC to get R_{ℓ_1,ℓ_2} **(e)**
Repeat for each (ℓ_1, ℓ_2)
with all (ℓ_1, ℓ_2)
Test set of ranks R_{ℓ_1,ℓ_2}
KS test **(V C)**
Q Statistic **(V C, VI B)**