Optimal Supernova Ia Search Strategies

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soon on astro-ph

The data are pouring in: CMB anisotropy, SNIa, LSS surveys, galaxy counts, etc.

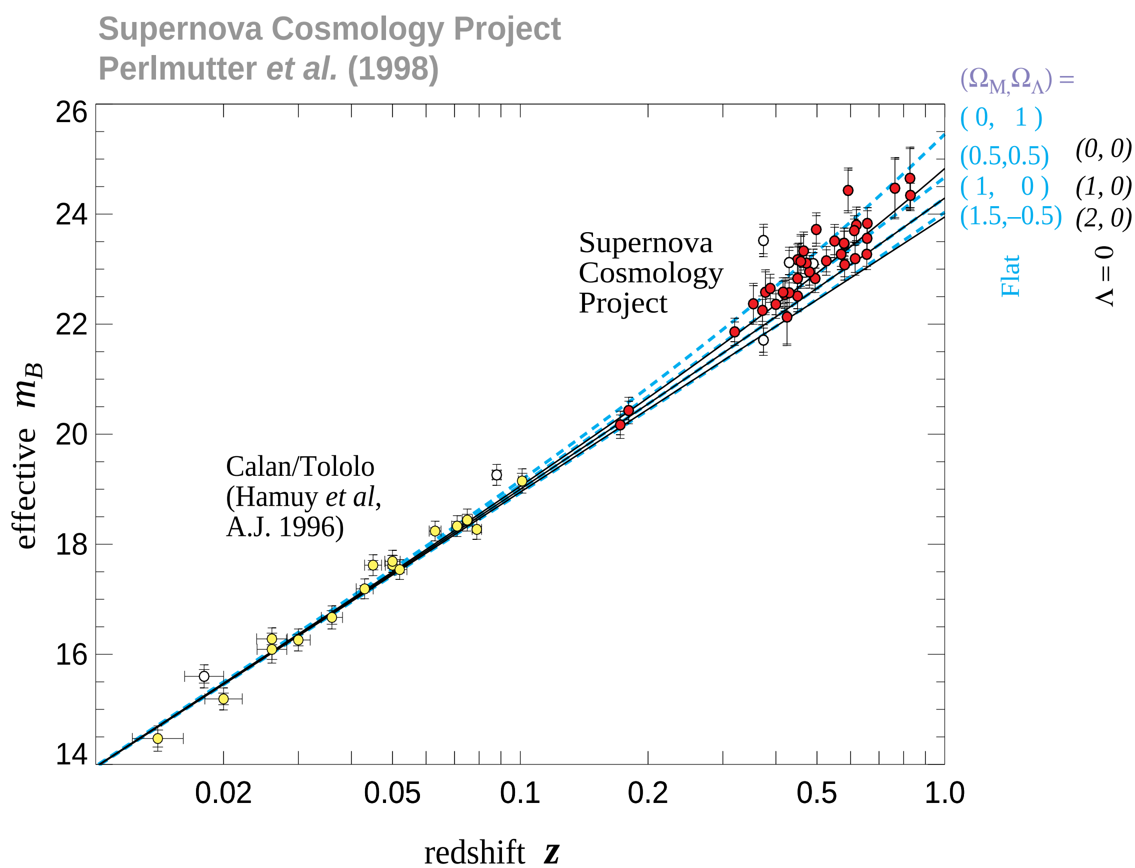
- How to best use them to constrain the cosmological parameters?

- How to best probe the dark energy?

- What redshifts are optimal for probing the dark energy?

At what redshift should we look for supernovae?

- "Global" parameter determination

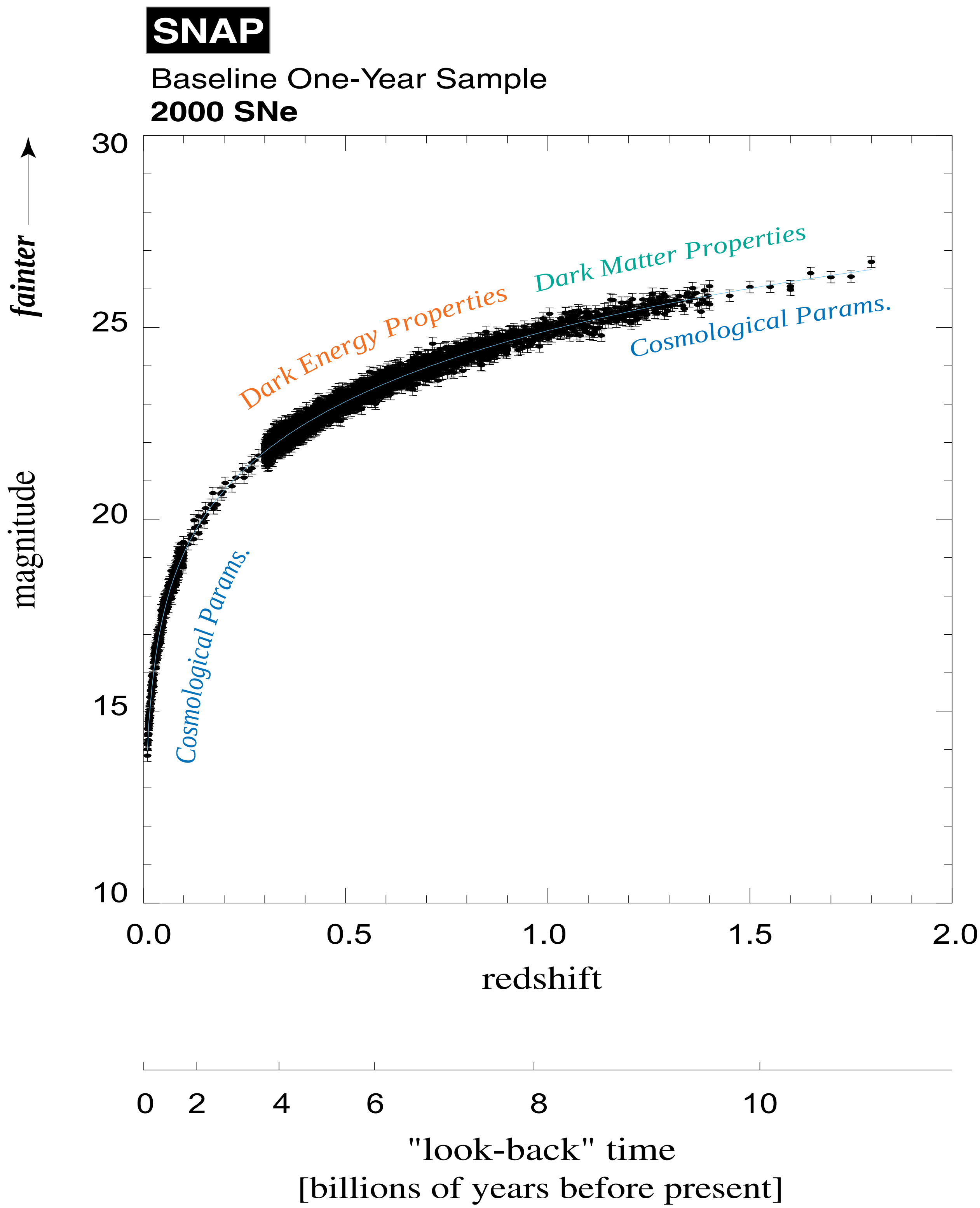


In flat universe: $\Omega_M = 0.28$ [± 0.085 statistical] [± 0.05 systematic]

Prob. of fit to $\Lambda = 0$ universe: 1%

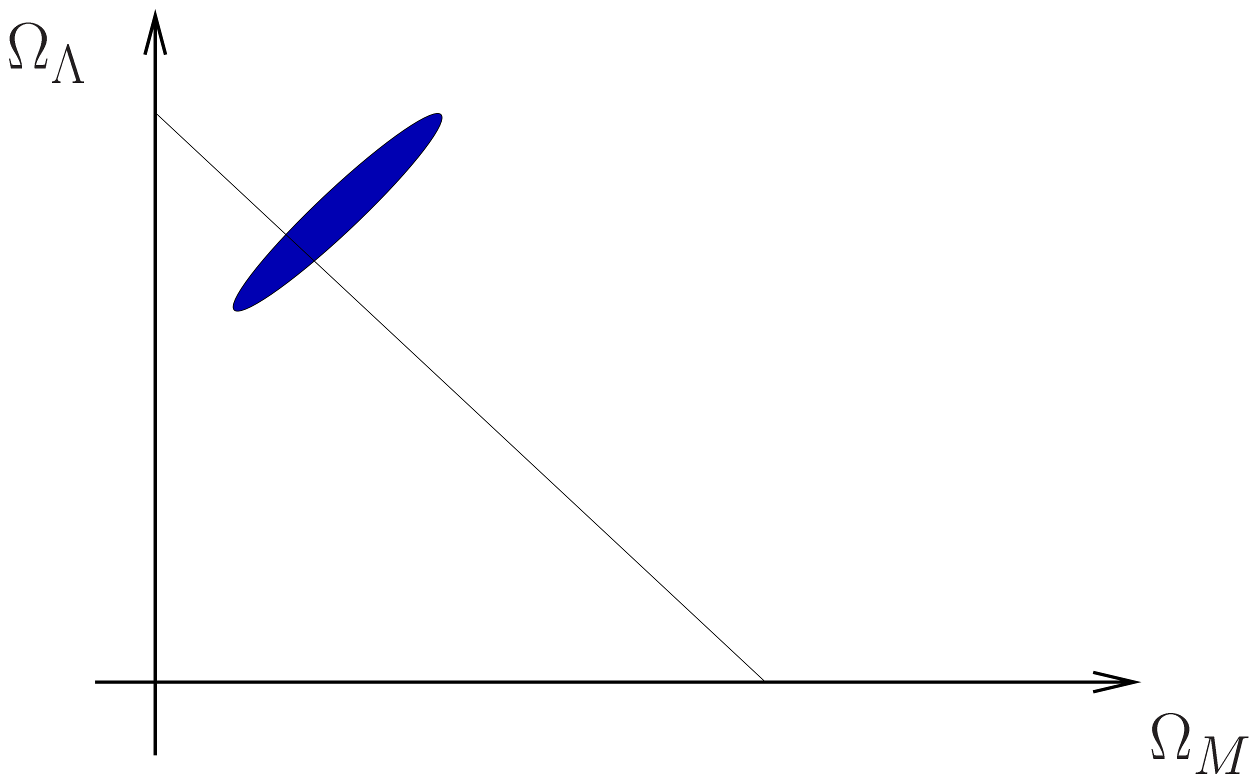
astro-ph/9812133

- Establish that dark energy causes the faintness of SNe



• This talk: Achieving the best <u>local</u> parameter determination.

Specifically, we minimize the area of the uncertainty region.



Assume:

• The total number of available SNe is fixed

• The uncertainty σ_m in each SN is the same

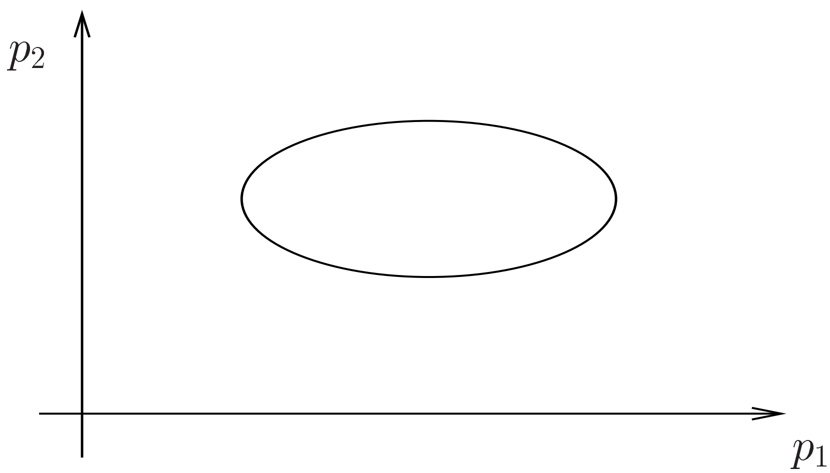
• There is an infinite supply of SNe at each redshift

One can show that $\boxed{V \propto [\det(F)]^{-1/2}}$

where V is volume of uncertainty ellipsoid and

$$F_{ij} = -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_{\mathbf{x}}$$

Example (2 parameters, no correlation):



F is diagonal:

$$F = \begin{bmatrix} F_{11} & 0 \\ 0 & F_{22} \end{bmatrix}$$

Equation of ellipse: $F_{11}\, p_1^2 + F_{22}\, p_2^2 = \text{Const.}$

Area of ellipse: $A = 8\pi (F_{11}\, F_{22})^{-1/2} \propto [\det(F)]^{-1/2}$

So, to minimize V, we need to maximize $\det(F)$.

Calculation of the Fisher matrix for supernovae:

$$m_n = 5\log[H_0 d_L(z_n, \Omega_M, \Omega_\Lambda)] + \mathcal{M} + \epsilon_n,$$

$$\mathcal{M} \equiv M_B - 5\log(H_0) + 25$$

$$\begin{aligned} F_{ij} &= -\left\langle \frac{\partial^2 \ln L}{\partial p_i \partial p_j} \right\rangle_{\mathbf{x}} \\ &= \frac{1}{\sigma_m^2} \sum_{n=1}^{N} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_i} \frac{\partial m_n(z_n, \Omega_M, \Omega_\Lambda, \ldots)}{\partial p_j} \\ &= \frac{1}{\sigma_m^2} \sum_{n=1}^{N} w_i(z_n, \Omega_M, \Omega_\Lambda, \ldots)\, w_j(z_n, \Omega_M, \Omega_\Lambda, \ldots) \qquad \text{(Tegmark et al.)} \end{aligned}$$

We represent the measurements as a sum of delta-functions

$$g(z) = \frac{1}{N} \sum_{n=1}^{N} \delta(z - z_n)$$

$$F_{ij} = \frac{N}{\sigma_m^2} \int_0^{z_{\max}} g(z)\, w_i(z)\, w_j(z)\, dz,$$

with

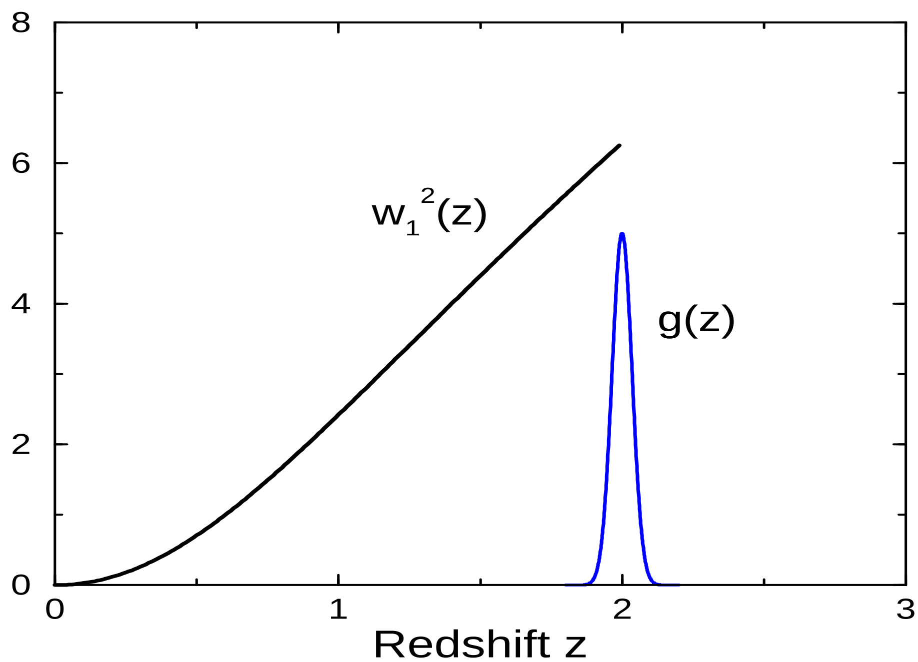
$$\int_0^{z_{\max}} g(z) = 1 \quad \text{and} \quad g_(z) \geq 0$$

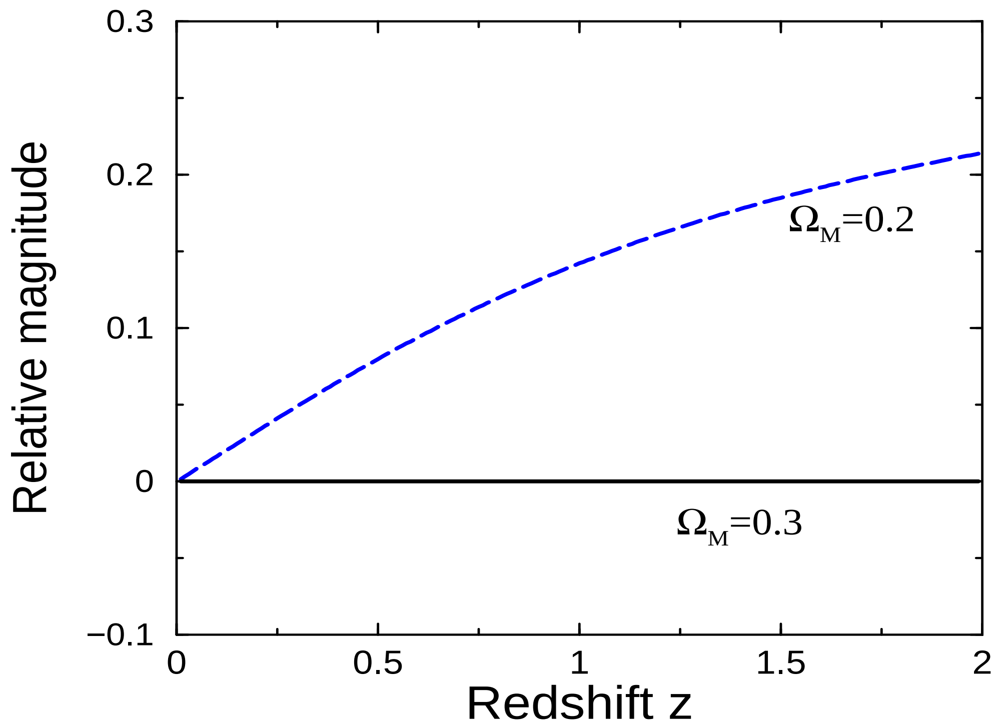
Our goal is to find $g(z)$ such that $\det(F)$ is maximal.

Case of one parameter:

$$\det(F) = F_{11} = \frac{N}{\sigma_m^2} \int_0^{z_{\max}} g(z)\, w_1^2(z)\, dz,$$

is maximized if $g(z) = \delta(z - z_{\max})$.





Case of two parameters

$$\det(F) = \int_{z=0}^{z_{max}} g(z)\, w_1^2(z) dz \int_{z=0}^{z_{max}} g(z)\, w_2^2(z) dz \; - $$

$$\left(\int_{z=0}^{z_{max}} g(z)\, w_1(z)\, w_2(z) dz \right)^2$$

$$\propto \int_0^\infty \int_0^\infty g(z_1)\, g(z_2)\, w^2(z_1, z_2)\, dz_1\, dz_2$$

$$= \sum_{i,j=1}^{BINS} g_i\, g_j \; w^2(z_i, z_j)$$

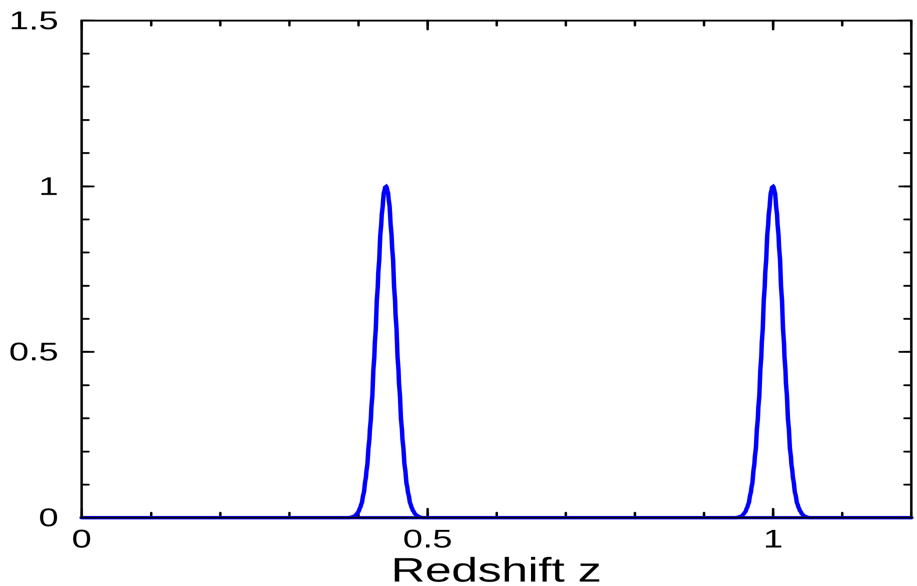
with

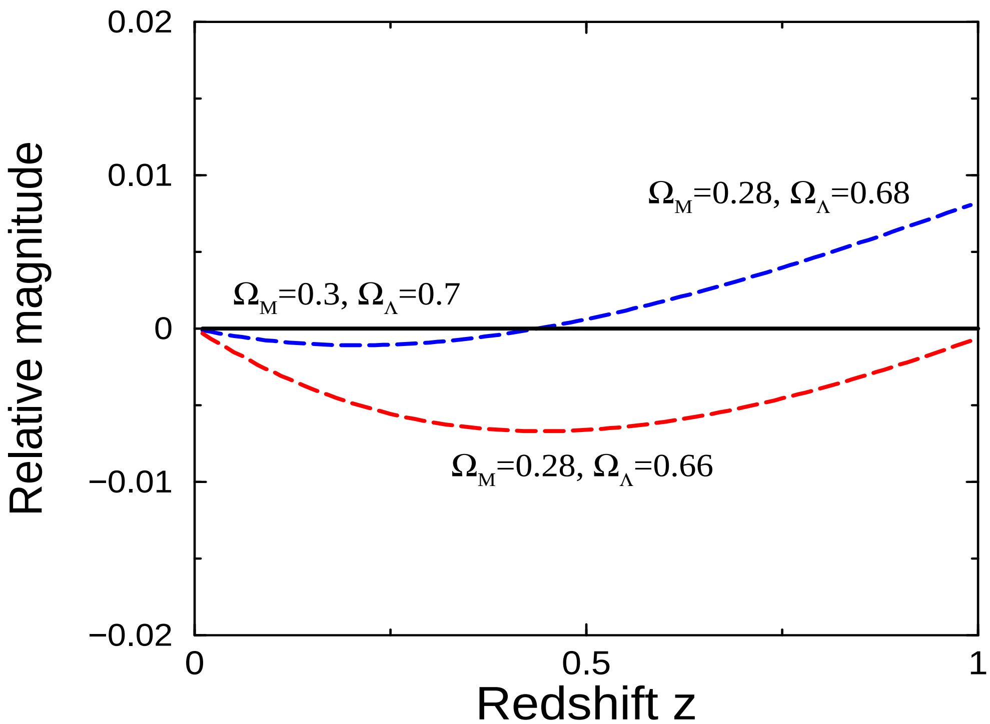
$$\sum_{i=1}^{BINS} g_i = 1 \quad \text{and} \quad g_i \geq 0$$

This is a problem in quadratic programming.
Unfortunately, $w^2(z_i, z_j)$ is not concave, so we maximize using brute force.

For the $\Omega_M - \Omega_\Lambda$ case:

$$\mathbf{g(z) = 0.50\,\delta(z - 0.44) + 0.50\,\delta(z - 1.00)},$$





and for the $\Omega_M - w_Q$ case (flat universe):

$$\mathbf{g(z) = 0.50\,\delta(z - 0.36) + 0.50\,\delta(z - 1.00)}.$$

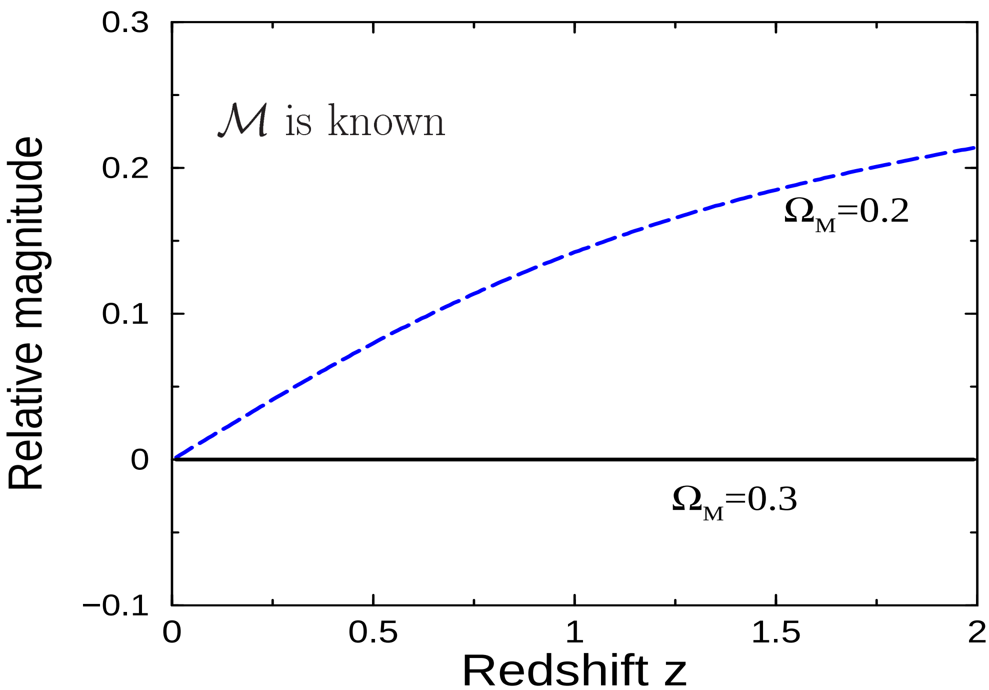
Three parameters:

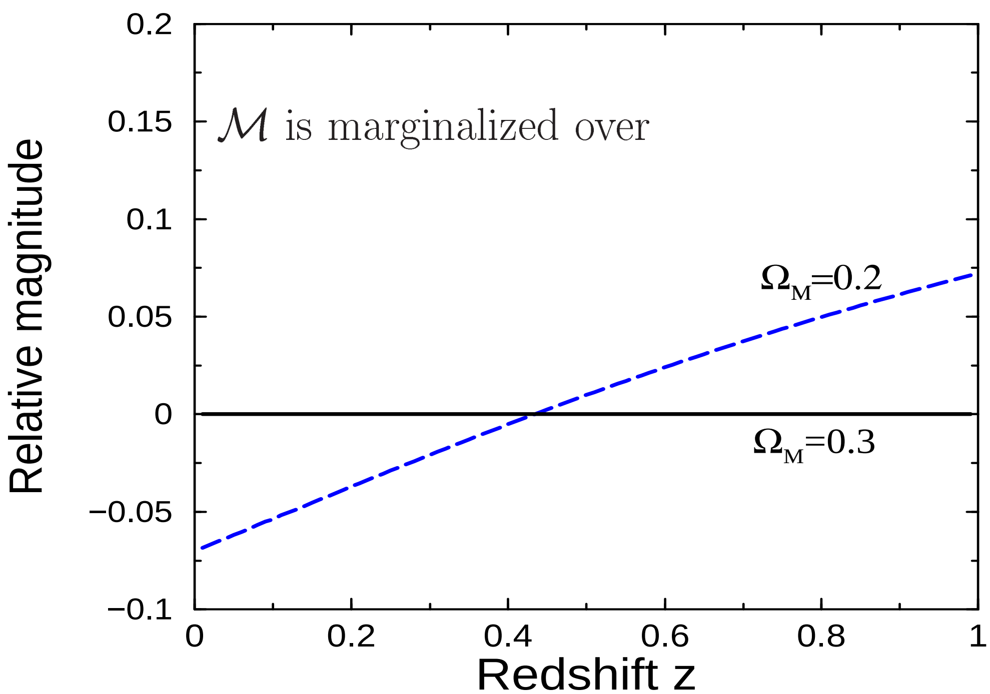
For the $\Omega_M - \Omega_Q - w_Q$ case:

$$\mathbf{g(z) = 0.33\ \delta(z - 0.21) + 0.33\ \delta(z - 0.64) + 0.33\ \delta(z - 1.00)}$$

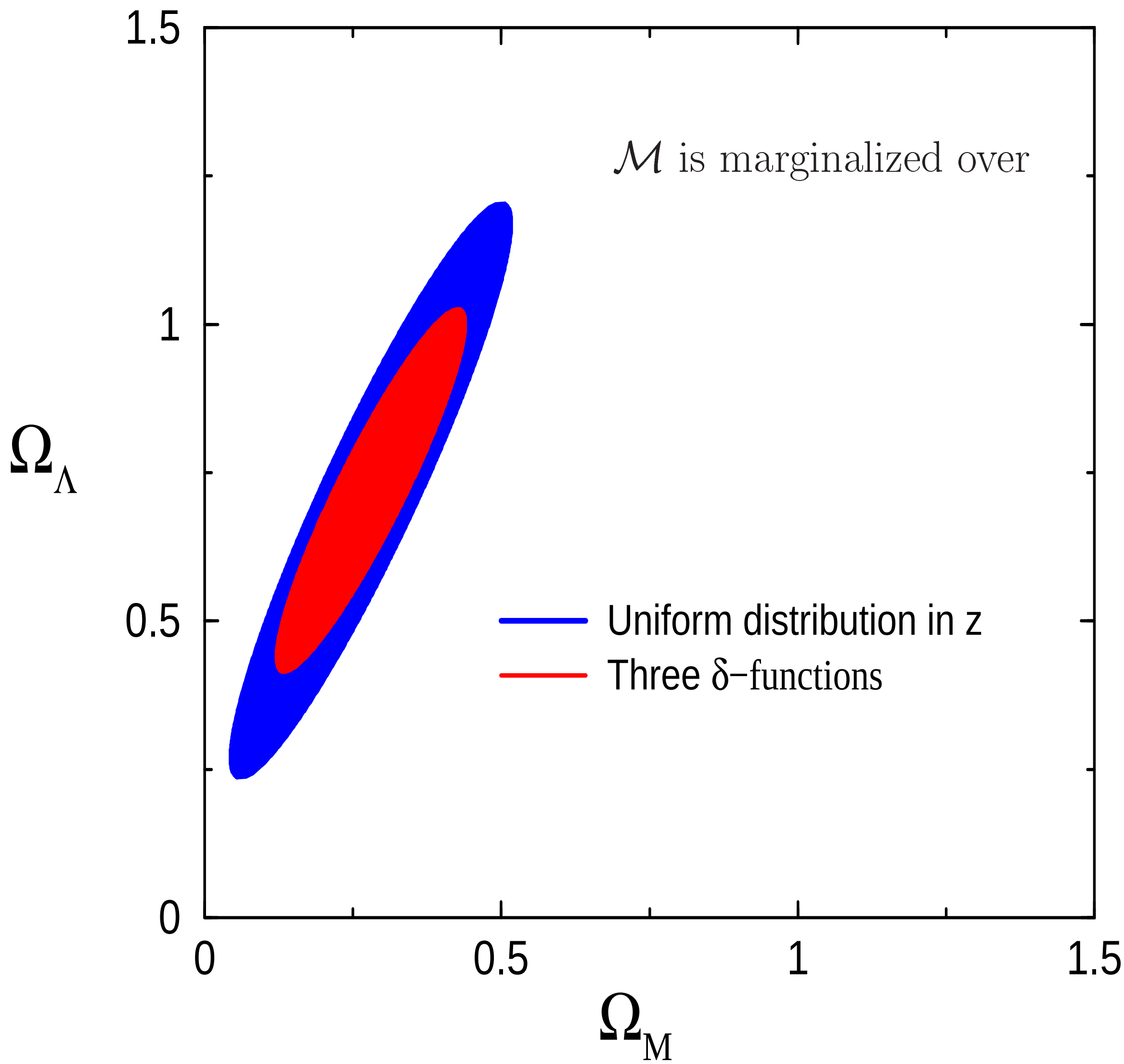
Including marginalization over $\mathcal{M}$

This only adds another delta-function at $z = 0$!

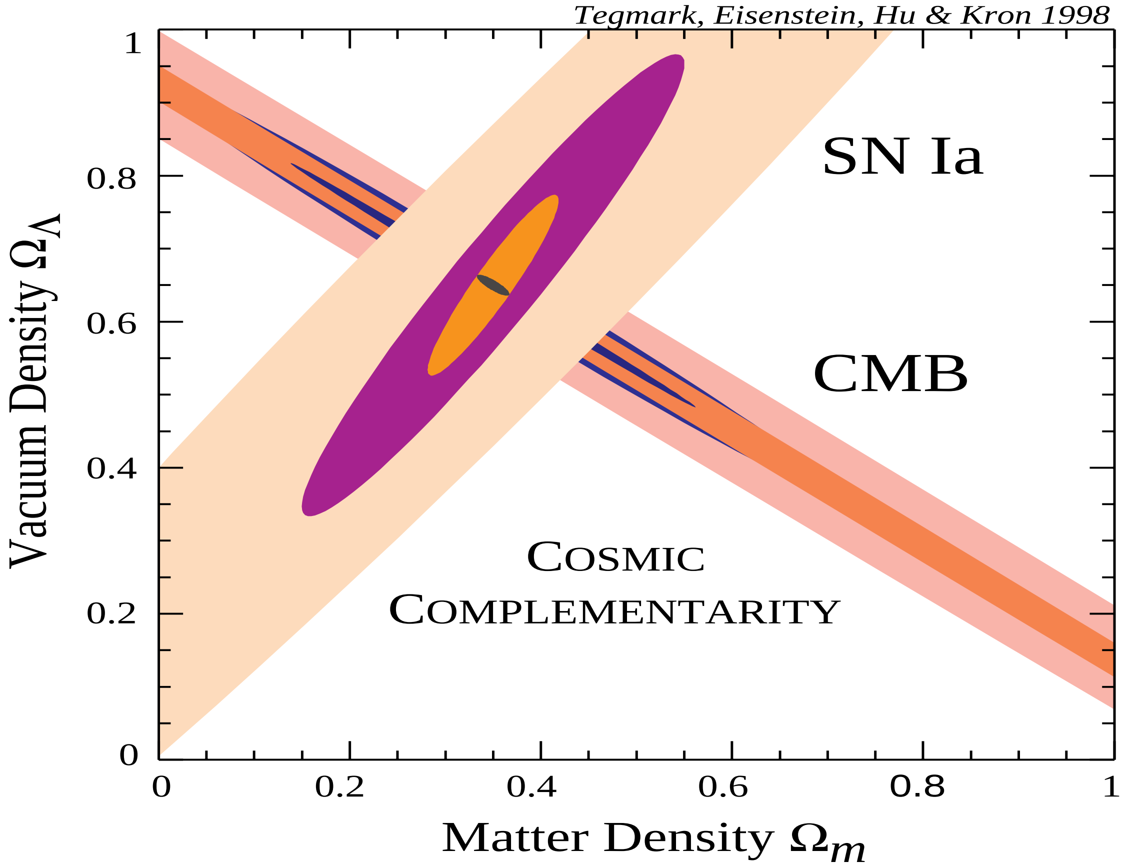




Improvement due to this choice of redshifts:



Finding the "thinnest" ellipse

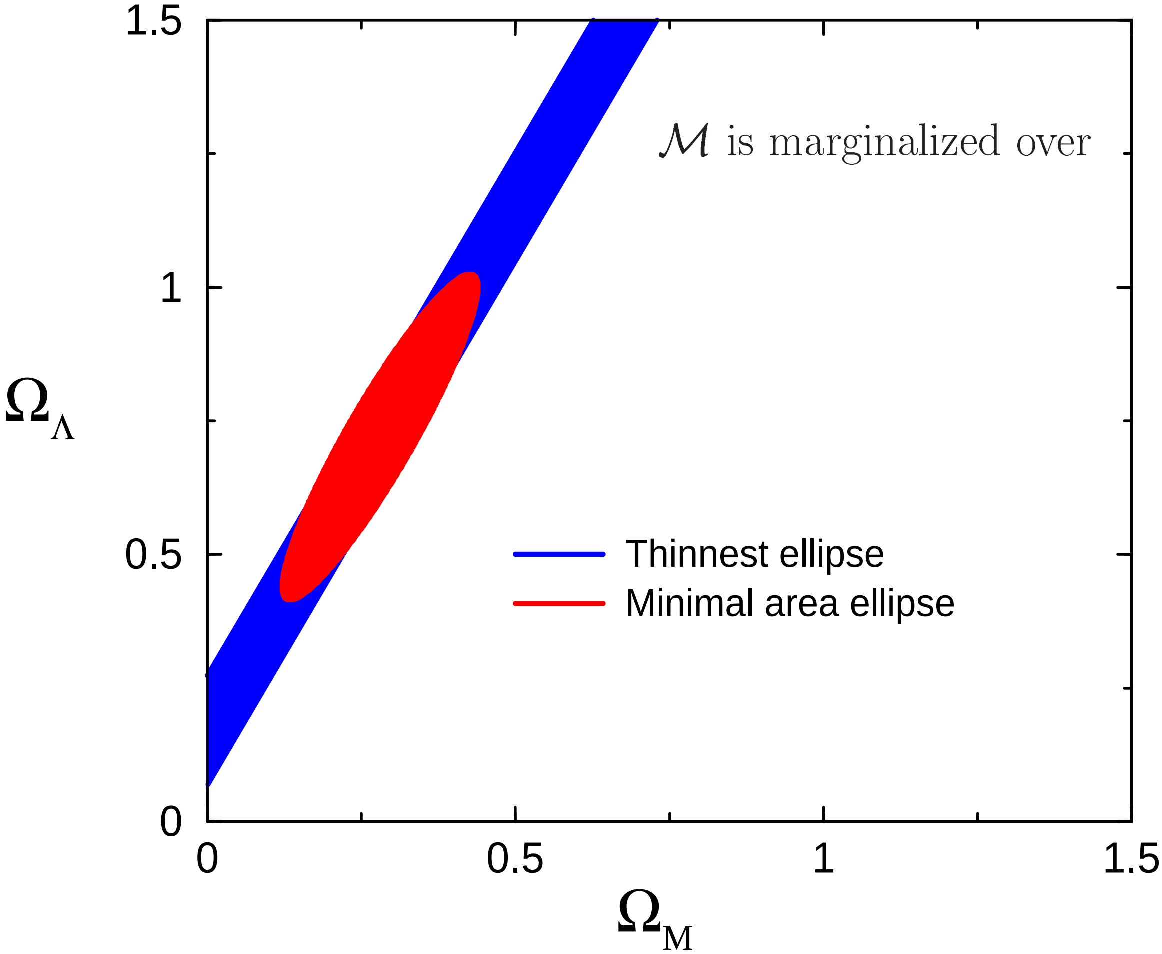


- We need to maximize the bigger eigenvalue of the Fisher matrix.

- Result: all SNe should be at $z_{\max}$.

But then the ellipse is infinitely long!

- For all purposes, the minimum-area ellipse is the "thinnest" ellipse.



Conclusions:

• For the tightest constraint on N parameters, supernovae should ideally be located at N specified redshifts (one of them $z_{\max}$)

• The same strategy works to obtain the "thinnest" ellipse in the Ω_M–Ω_Λ plane

• This procedure can be tailored to specifications for any given SN search

• There are many other things to consider in answering the question of how to best constrain the parameters/dark energy.