Competition Between Firms that Bundle Information Goods

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Abstract:

Information goods are characterized by high fixed (first-copy) costs, but very low costs for the production of additional copies. Marginal costs of electronically-delivered information goods are further reduced by the remarkable recent decline in computing and digital communication costs. Achieving dynamic efficiency in such a market is difficult because pricing at marginal cost eliminates the incentive to invest in new content creation. When the marginal cost of each good is negligible, bundling numerous goods together can be efficient since individuals face a zero cost for each item consumed. Yet, the producer is able to recover investment costs through bundle sales. This paper models the incentive of firms to bundle information goods. Unlike the previous literature which assumed a monopolist producer, we allow for a multi-firm industry. Two firms produce numerous imperfect substitutes and either sell these goods individually (pure unbundling) or together as an entire collection (pure bundling). When consumers have identical overall valuations for the entire agglomeration of goods even though they differ in their tastes for any particular good, we find that pure bundling results in the efficient distribution of the information goods. Bertrand equilibrium involves firms charging positive prices for their bundles even though the articles they produce are *a priori* identical. Firms' operating profits are substantially less than a monopolist's, yet the incentive to invest in new product creation has been maintained. When consumers are heterogeneous in their overall valuations, a duopoly drastically improves efficiency of a market compared to the monopoly outcome. The duopoly equilibrium is characterized by much lower prices for consumers, much higher social welfare, and only a moderate reduction in firms' profits. In a dynamic version of our model allowing firms to choose collection size, the superiority of the duopoly outcome is dampened due to the lower incentive (relative to a monopolist) to invest in content. Finally, when firms are allowed to mixed bundle (sell items both as a bundle and individually), we find that most revenue will be obtained from bundle sales.

I. Introduction.

Computing and digital communications costs have both decreased at a remarkable rate of about 30% per year since the early 1960s (MacKie-Mason and Varian (1996)). Such a revolutionary shift in information technology costs, coupled with the advent of micropayment systems on the internet now able to support transactions as low as 5 cents, is likely to lead to revolutions in product offerings, access and distribution. For example, it is not economically feasible for the New York Times to sell articles individually when that requires articles to be printed separately and physically delivered to customers through the postal system. However, if all articles are available on a website, it becomes feasible for consumers to download and pay for only those articles that they desire. This suggests that publishers may offer smaller bundles and more individually-tailored products as a result of technology changes. On the other hand, the rapid decline in distribution costs which allows for vast amounts of information to be transported at a very low cost could result in more items being contained in standard bundles. For example, Elsevier Science may choose to sell its entire collection of over 1100 serials as a bundle. Subscribers could read any particular articles from this vast assortment of journals.

The impact of shifts in the delivery of information goods is particularly relevant to the publishing industry, including scholarly publishing, where digital, networked-distribution is now in full swing. Neither publishers nor researchers understand how the transformation in production and communication technologies will affect competition and the sharing of value between authors, readers, and publishers. University presidents and librarians are expressing alarm that the introduction of electronic publication and distribution seems to be increasing prices for scholarly journals and that through mergers the publishers are gaining more control over the market for the research of scholars (Gilpin (1997)). In this paper, we study how the changing economics of information goods affects readers and publishers. Interaction between new technologies and competitive conditions will determine the amount of information value created and how that value is shared.

Information goods are characterized by high fixed (first-copy) costs, but low costs for the production of additional copies. Charging a price for each unit of this type of good cannot result in efficient production or distribution because pricing at (near-zero) marginal cost would not recover the initial fixed costs. This cost structure characterizes print-on-paper publishing too, but is greatly exaggerated in electronic publishing. The flexibility of digital technology also permits a wider range of responses to the cost problem. Consequently, there is great uncertainty about how markets for electronic publishing will evolve.

Several authors have recently begun to study the option of selling numerous information goods as a bundle (Bakos and Brynjolfsson (1998), Chuang and Sirbu (1998), MacKie-Mason and Riveros (1998), and Salinger (1995)). Bundling by a monopolist appears to generate more revenue than per-good pricing when consumers are of relatively the same "average intensity" and/or when there are economies of scale in the distribution of information goods. Consumers disagree on which particular goods are most valuable, but have roughly the same "average intensity" if they have similar overall valuations for the entire agglomeration of goods. Bundling can generate more revenue by capturing the common average value while smoothing out idiosyncratic preferences. In addition, if there are costs per transaction, distribution costs are lowered by selling bundles. Bundling is not always beneficial to either publishers or readers. When consumers vary significantly in average intensity or when there are non-trivial production

or distribution costs, unbundled pricing will tend to be more profitable and thus will be favored by publishers.

These results established in previous literature are based on the assumption of a single publisher (no competition from other publishers). This assumption of a monopolist provider, while common in the literature to date, is not a very reasonable representation of the degree of competition over time nor does it allow for exploration of the interaction between pricing/bundling methods and the degree of market competition.

In this paper, the effects of bundling and unbundling on the intensity of competition between electronic publishers are analyzed. We consider how much curb competition places on the ability of firms to extract value significantly above cost from readers and whether bundling gives firms an effective tool for recovering their original set-up costs. Thus, we are able to explore the effect that electronic publishing is likely to have on incentives for new content creation and for the sharing of value between publishers and readers. A duopoly with firms producing imperfect substitutes is drastically more efficient than a monopoly. A duopoly, regardless of whether the firms decide to bundle or to unbundle their respective collections, will result in much lower prices for the consumers, much higher social welfare, and only a moderate reduction in firms' profits compared to the monopoly outcome. Firms continue to price above marginal cost thus providing an incentive to invest in the creation of information products. However, this incentive is lower than a monopolist's. Thus, when collection size is chosen endogenously, a monopolist will tend to produce more content than a duopoly would. But, a duopoly is still likely to be socially preferred since its lower prices result in greater efficiency in the distribution of the content that is created. Furthermore, we find that when firms are allowed to mixed bundle (sell items both as a bundle and individually), the duopoly outcome is characterized by most revenue being obtained from bundle sales. This occurs due to demand-side and/or competitive reasons. Bundling would become even more favorable if we accounted for cost-side economies of scale in distribution or production. Section II provides a brief literature review and description of our main results. Section III presents the formal model of consumer demand. Section IV describes and interprets the analytical and numerical results for a duopoly. Section V explores several extensions to this model including endogenous bundling choice and investment decisions. Section VI offers concluding remarks.

II. Literature Review and Main Results.

In their seminal paper on bundling, Adams and Yellen (1976) consider a monopolist which produces two goods. Three primary bundling strategies are identified: Pure component selling (each good sold individually), pure bundling (both goods are only sold together as a package) and mixed bundling (goods sold as a package at the bundle price and individually at a per-good price). By example, Adams and Yellen show that bundling can result in higher revenue than component selling, especially if the two goods' demands are negatively correlated. The distribution of resources can differ substantially between the bundling strategies. Specifically, bundling might result in consumption of components for which the marginal cost of producing that component exceeds its value to a consumer. A consumer only faces the bundle price, so she will buy the bundle if the sum of her valuations of the two goods exceed the bundle price. If the majority of this total valuation is attained from the first good, the consumer will buy the bundle

and thus also consume the second good even though her valuation for the second good might be quite low. Violation of this "Exclusion Principle" is socially inefficient. McAfee, McMillan, and Whinston (1989) extend the Adams and Yellen framework to general demand functions. Mixed bundling is shown to dominate either of the two pure pricing strategies. It must perform at least as well as either pure bundling or pure unbundling since mixed bundling contains both pure strategies as sub-cases. The authors also provide conditions for which mixed bundling will strictly dominate either of the two pure strategies. Salinger (1995) shows that as long as valuations for the two goods are not perfectly correlated, the demand for the bundle will be more price elastic than the sum of the individual demands for each component. As a result, bundling is a way for smoothing out idiosyncratic preferences enabling a monopolist to extract more of consumers' surplus than is possible through pure components pricing.

There have been several recent papers which have extended this analysis to N-Goods. A complete N-good bundling model with 2^N bundle combinations as well as N-dimensional consumer preferences quickly becomes intractable. As a result, nearly all previous papers restrict consideration to either pure unbundling (pure components pricing), pure bundling, and/or mixed bundling as will be done in the present paper [1] [2]. Bakos and Brynjolfsson (1998) illustrate that pure bundling is strictly preferred to pure components pricing when marginal cost is very low and consumers have demands of roughly the same average intensity. Chuang and Sirbu (1998) allow for the possibility of mixed bundling, in addition to only pure bundling or pure components pricing and show that it is more profitable than either of the pure strategies. They also incorporate the detail that most readers only value a subset of the entire agglomeration of articles. In the presence of positive marginal costs, bundling will violate the "exclusion" principle and thus it is very possible for pure unbundling to strictly dominate pure bundling.

Although there are a number of papers exploring the merits of bundling by a monopolist, the exploration of bundling under the possibility of competition has been much more limited. Fishburn, Odlyzko, and Siders (1998) consider a duopoly. Each firm produces an identical set of N information goods (perfect substitutes). One firm sells the collection as a bundle while the other utilizes the pure components pricing strategy. In nearly all situations, a price war ensues with both firms' prices falling towards zero (MC) as rounds of the sequential game progress. This situation, although nearing static efficiency, erodes the long-run incentive to provide new content. When a stable equilibrium with strictly positive prices does exist, the firm engaging in pure bundling attains a competitive advantage over the firm that engages in pure components pricing.

Matutes and Regibeau (1992) consider a much different situation under which duopolists each produce two variations of components that are combined to create a system. Under this assumption of imperfect complements, mixed bundling, while being a dominant strategy, often results in the case of a Prisoner's dilemma. A firm offers discounts for a bundle that includes both of its components. Each firm's profits would be higher if they could commit to eliminating such discounts. Such bundle discounts do result in more intense competition between the duopolists and thus lower prices for the consumers. However, these discounts are socially undesirable since they result in a distortion in consumers' choices. Too many consumers are

[1] Hanson and Martin (1990) are an exception as they tackle the full 2^N bundle pricing problem with N = 21.

[2] MacKie-Mason and Riveros (1998) allow for a unique alternative -- generalized subscriptions. Consumers, using pre-paid tokens, are allowed to select those items from the entire collection which are most valuable to them. In effect, individual-specific sub-bundles are created.

swayed to purchase bundles rather than construct a hybrid system which is closer to their "ideal" system, i.e. "excessive bundling" occurs. This suggests that mixed bundling may not be as profitable to firms and valuable to overall welfare in a competitive situation as it is when the producer is a monopolist. [3]

In the model we construct, each duopolist produces multiple imperfect substitutes. These goods are *a priori* identical. Prior to production, each item has the same expected value for any given consumer. Due to the complexity of solving a complete 2^N bundle combinations model, we restrict the firms' bundling possibilities to pure bundling, pure component pricing and mixed bundling. There may be substantial costs incurred in the process of product creation. However, we assume that these costs are taken by firms as being sunk costs so that the final prices set by the firms are independent of these set-up costs. Throughout the paper, we assume that the marginal cost of producing an additional copy of any existing information good is zero. As a result, efficient allocation of these resources would require than a consumer have access to all goods for which she has a strictly positive valuation. A monopoly results in an inefficient distribution of these information goods since the monopolist prices significantly above marginal cost thus discouraging consumers with relatively low, but still positive, valuations from consuming many of the available goods. However, we find that the addition of just a single competing firm producing imperfect substitutes can drastically reduce the efficiency loss incurred. A duopoly, regardless of whether the firms decide to bundle or to unbundle their respective collections, will result in much lower prices for the consumers, much higher social welfare (although still less than the static social optimum), and only a moderate reduction in firms' profits. Firms continue to price above marginal cost which allows for the possibility of recovering initial set-up costs, thus providing an incentive to invest in the creation of new information products.

When consumers are homogeneous in "average intensity" and both firms bundle their products, Bertrand competition results in an efficient allocation of resources. Firm 1 and Firm 2 each produces a sub-collection of information goods and only sells its collection as a whole for a price P_i. Each consumer values the conglomeration of both collections at V_B.[4] Each receives a value of V_i if they are only allowed access to Firm i's sub-collection (i = 1, 2). Since the goods are substitutes: $V_1 + V_2 > V_B$. The unique Bertrand Nash Equilibrium (P_1^*, P_2^*) is ($V_B - V_2$, $V_B - V_1$). Firm i prices at the difference in value to the consumer from having access to both collections versus having access to only the other firm's collection of articles, i.e., each firm gets the incremental value of its own collection. Note: $P_i < V_i$ (the value to the consumer of collection i if he did not have access to the other firm's collection). In equilibrium, firms set positive prices even though marginal cost is zero. The sum of profits to this duopoly ($V_B + V_B - V_1 - V_2$) is substantially less than a monopolist would obtain. A monopolist who controlled both

[3] Carbajo, De Meza, and Seidmann (1990) examine the strategic motivation for bundling when two products are complements for a different market structure. A monopolist in the first good faces a rival which produces an identical second good. If there is Bertrand competition in that second market, bundling by the monopolist can raise both firms' profits since it allows the monopolist to commit to serve a specific sector of the second market. Thus, a price-war can be averted. If there is Cournot competition, the merits of bundling are not quite as robust. However, in many circumstances, bundling does appear to offer a competitive advantage to the monopolist since by committing to sell to a specific sector of the second market, he is able to approach the Stackleberg outcome.

[4] Although the total valuation is identical for each consumer, consumers differ in their preferences over any particular item in the collection. So, consumers get the same value from the entire collection although they have divergent rankings of the particular items.

sub-collections would extract each consumer's entire valuation by selling the entire collection as a bundle for a price V_B, earning a profit of V_B. Although profits are substantially lower for the duopoly, this Bertrand equilibrium may still result in the efficient incentive for new product creation. Suppose there is a fixed cost for creating a collection of a exogenously given number of articles. But, once a collection has been created, there are no marginal costs for creating additional copies of these articles. We show in Section IV that if it is socially optimal for both collections to be provided, then each firm will have the incentive to produce its collection of articles. However, if firms choose the size of their collection, a duopoly will results in a inefficiently low amount of content.

If consumers are homogeneous and Firm 1 bundles while Firm 2 sells only on a per-item basis, no pure-strategy Bertrand equilibrium exists. Firm 2's reaction function is discontinuous. Firm 2 has two pricing options. Aware that consumers will purchase much fewer articles from Firm 2 if they have access to all of 1's collection at zero marginal cost, 2 can either set a relative low price so that consumers will not want to purchase Firm 1's bundle and thus sell a large number of articles. Or, Firm 2 can set a higher price, concede that the consumer will purchase 1's bundle, and sell much fewer articles at this high price. For high P_1's, Firm 2 chooses the first pricing option. However, there is some lower limit of P_1 for which Firm 2 is better off to concede that the consumer will purchase Firm 1's bundle, rather than setting P_2 so low as to discourage the purchase of 1's bundle. Due to this discontinuity, no Bertrand equilibrium exists.

When consumers are homogeneous and both firms sell on a per-item basis, there is a pure-strategy Bertrand equilibrium. Social welfare is lower in this equilibrium versus the equilibrium in which both firms bundled their collections. It is inefficient to discourage consumption of articles by charging consumers for each individual article when there is a zero marginal cost of production. With an example, we also show that total profits to the firms are likely to be lower in this equilibrium since there is less consumers' surplus to split between the firms. As was the case when both firms bundled their sub-collections, a monopolist would charge much higher prices and attain somewhat greater profits.

For the case of consumers with heterogeneous aggregate valuations, the model becomes much more complex. We are often forced to rely on numerical examples rather than analytical results. When both firms bundle their collections, a pure-strategy Bertrand equilibrium does not exist. However, there does exist a pure-strategy equilibrium if one or both of the firms sells on a per-item basis. We also provide the Stackleberg equilibria in a price-setting game for the various bundling strategies and for a variety of firm sizes. This Stackleberg game has firms producing imperfect substitutes and engaging in price competition. As is standard in the Industrial Organization literature when firms produce strategic complements, the follower benefits from being able to observe the leader's action before setting his own price. Still, we find that the Bertrand and Stackleberg outcomes are quite similar to each other and drastically outperform (in terms of social welfare) the monopoly outcome. A firm that bundles its collection does appear to gain a competitive advantage. In a duopoly where one firm bundles and the other does not, the firm that bundles will receive greater revenue per item in its collection versus the firm that only sells its items individually. In the environment in which both firms engage in the mixed bundling, nearly all revenue attained by the firms will be through bundle sales. We also find that larger firms can attain greater surplus per good produced, giving them a competitive advantage over smaller firms.

A final conclusion is that a monopolist never benefits from dividing its collection into smaller, separate ones. Because items are imperfect substitutes with each other, the collections themselves are imperfect substitutes for each other. As a result, dividing a large collection into smaller sub-collections creates artificial competition. The monopolist becomes a competitor with itself and is unable to extract as much surplus from consumers as would otherwise be the case. Consider the following simple example. Assume that there is a single consumer and the monopolist has control over 1000 information goods. If the consumer has access to only the first 500 goods, he values that sub-collection at \$14 (V_1). Conversely, if the consumer only has access to the second 500 items, he also values that sub-collection at \$14 (V_2). However, if the consumer has access to all 1000 goods, he attains a gross surplus of \$20 (V_B). A monopolist who sold all 1000 items together as a single bundle would set a price of \$20 (V_B) and successfully extract the consumer's entire valuation. However, if the monopolist divides his collection into two equal parts, he can no longer attain \$20 in revenue. If each collection is priced at \$10, the consumer will respond by only choosing one of the two collections, earning a net surplus of \$4 (as opposed to zero if he were to purchase both sub-bundles). Thus, the monopolist would earn only \$10 in revenue. In this case, the best the firm can do is offer both bundles at a price of \$14. Only one of the two bundles will be bought with the firm earning \$14 in total revenue. Notice, not only does sub-bundling by a monopolist result in lower profits, it also leads to lower social welfare.[5]

III. The Model.

A. Demand Specification.

A collection that consists of N information goods (articles) is made available to consumers. The collection is either packaged together as a bundle or sold on a per-article basis. We assume that all articles are *a priori* identical. Prior to subscribing, consumers do not know their exact valuation for any particular article. However, they know the distribution of these values. Based upon these *ex ante* expectations, risk-neutral consumers decide whether to buy the bundle, buy on a per-unit basis or to buy nothing. Each article is unique and a consumer would never want more than one unit of any given item. After a mode of purchase has been decided, consumers learn their actual valuation for each article and select which particular items to consume. For example, on-line annual subscriptions to the Wall Street Journal are available. When the reader decides whether to purchase a subscription or not, the articles which will appear in those issues have not yet been written. Once the subscription has been bought, each day the consumer decides whether to read a particular article based on its title and/or its opening lines. When deciding whether or not to purchase a subscription to the Wall Street Journal, the consumer calculates the expected value of having access to these coming articles and compares this to the subscription price. Conversely, archive articles from the New York Times are

[5] Section IV suggests that these conclusions continue to hold for heterogeneous consumers and asymmetric sub-bundle sizes. However, we conjecture that this conclusion could be reversed in some situations. For example, consider the introduction of search costs. A larger collection makes the location of an individual's most valuable items more difficult. In this case, the monopolist would likely want to offer smaller collections in order to make such searches less costly and more manageable.

available for a per-article fee. Consumers decide which articles to purchase based on observable characteristics such as a title and brief description.

Let w(n) represent a consumer's expected valuation of her nth best article from the collection.

$$w(n) = w_o\left(1 - \frac{n}{kN}\right)\left(\frac{kN - n_{tot}}{kN}\right) \qquad [1]$$

where:

w_o is the value of the consumer's best article.
k is the percent of articles that could have positive value to the consumer.
n_{tot} total number of articles consumed.

This formulation borrows heavily from Chuang and Sirbu (1998). Consumers rank the articles in the collection, with a higher n representing a less valuable article. For two persons, A and B, with identical (w_o , k), the expected value of person A's jth best article from the collection is exactly equal to person B's jth best article. However, there is still some heterogeneity between A and B since they do not agree on which is the jth best article. The last term of [1] is unique to this paper. Articles are assumed to be imperfect substitutes for each other. As one consumes a greater number of articles (n_{tot} rises), each available article is less valuable to that consumer (versus the value that article would have had if fewer articles had been consumed).[6]

Now, let the original collection (which consists of *a priori* identical articles) be split into two parts, the first consisting of N_1 articles and the second consisting of N_2 articles. A person who consumes n^1_{tot} articles from collection 1 and n^2_{tot} articles from collection 2 will expect her kth best article from collection i to be valued at:

$$w(n_k) = w_o\left(1 - \frac{n_k}{kN_i}\right)\left(\frac{k(N_1 + N_2) - n^1_{tot} - n^2_{tot}}{k(N_1 + N_2)}\right) \qquad [2]^{7}$$

Notice, n_k is less valuable as either n^1_{tot} or n^2_{tot} is increased.

Let $V(n_1 , n_2)$ represent the total value of consuming the n_1 best articles from collection 1 and the n_2 best articles from collection 2.

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[6] The reason for this addition will soon become obvious. We want to introduce a competitive environment. If $w(n_k)$ did not depend on n_{tot} , then firms producing subsets of this collection would still behave as monopolists. The demand faced by any firm would be completely unaffected by another firm's price (and consumers' responses to that price). This paper focuses on imperfect substitutes since this seems the most plausible interconnection for most information goods. It is likely that information will overlap somewhat between information goods. The Wall Street Journal is likely to convey some of the same information that could also be found in the New York Times. Even more importantly, items on completely unrelated topics are still competing for readers' attentions. Time spent reading article A cannot be used to read article B. As a result, A is less valuable if one has already obtained B. In reality, there exists complimentarities between some articles. However, exploring the impact of these complimentarities is beyond the scope of the present paper. See Matutes and Regibeau (1992) for model of competition between firms bundling two complement goods.

[7] Equation [2] represents consumers' expectations of the sub-collection values if the original collection is randomly divided. Note, *ex post* valuation of articles may not exactly follow this distribution. However, since the valuations of articles are not realized before a consumer makes her purchasing decision, expectations determine her behavior.

$$V(n_1, n_2) = \int_0^{n_1} w_o\left(1 - \frac{n}{kN_1}\right)\left(\frac{k(N_1+N_2)-n_1-n_2}{k(N_1+N_2)}\right)dn + \int_0^{n_2} w_o\left(1 - \frac{n}{kN_2}\right)\left(\frac{k(N_1+N_2)-n_1-n_2}{k(N_1+N_2)}\right)dn$$

$$= w_o\left(n_1 - \frac{(n_1)^2}{2kN_1} + n_2 - \frac{(n_2)^2}{2kN_2}\right)\left(\frac{k(N_1+N_2)-n_1-n_2}{k(N_1+N_2)}\right) \quad [3]\ ^{8}$$

Given prices for each collection (P_1 , P_2), consumers choose n_1 and n_2 to maximize their surplus net of the cost of acquiring those articles**:**

$$\underset{n_1,n_2}{\text{Max}} \quad V(n_1, n_2) - C(n_1, n_2, p_1, p_2) \quad [4]$$

where $C(n_1, n_2, p_1, p_2)$ is the cost to the consumer of purchasing the combination (n_1 , n_2)

If each firm only makes its collection available as a bundle:

$$C(n_1, n_2, p_1, p_2) = d_1 \cdot p_1 + d_2 \cdot p_2 \quad [5]$$

where $d_i = 1$ if $n_i > 0$ ($d_i = 0$ if $n_i = 0$)

If collection 1 is bundled, but collection 2 is sold on per-article basis:

$$C(n_1, n_2, p_1, p_2) = d_1 \cdot p_1 + n_2 \cdot p_2 \quad [6]$$

If both firms charge on a per-article basis, then:

$$C(n_1, n_2, p_1, p_2) = n_1 \cdot p_1 + n_2 \cdot p_2 \quad [7]$$

B. Both Collections offered individually as a bundle.

Suppose that each collection is only available as a bundle. The cost to the consumer is given by [5]. To consume any articles from collection i, one incurs the price P_i regardless of the number of articles chosen. So, once the first item from a given collection has been consumed, the marginal cost of consuming any additional article from that same collection is zero. Let V_1 be the expected value from only having access to articles from collection 1, V_2 be the expected value from only having access to articles from collection 2, and V_B be the expected value from having access to articles from both collections.

[8] Technically, the number of articles chosen must be an integer, so we should be summing over n rather than taking integrals. Chuang and Sirbu (1998) detail an approach that carefully accounts for the discrete nature of the goods. However, we avoid this complication by assuming that n is a continuous variable. As n becomes larger than a few articles, this distinction is trivial.

To calculate V_1 :

$$\frac{dV(n_1,0)}{dn_1} = 0 \qquad \Rightarrow \qquad n_1^*$$

$$V_1 \equiv V(n_1^*, 0) \qquad [8]$$

where: n_1^* is the number of articles one expects to consume from collection 1. The first order condition requires that the marginal utility of an extra article from collection 1 is just equal to the extra cost of that article, zero.

Similarly:

$$\frac{dV(0,n_2)}{dn_2} = 0 \qquad \Rightarrow \qquad n_2^*$$

$$V_2 \equiv V(0, n_2^*) \qquad [9]$$

Solving the simultaneous equations:

$$\frac{dV(n_1^B,n_2^B)}{dn_1^B} = 0$$

$$\frac{dV(n_1^B,n_2^B)}{dn_2^B} = 0 \qquad \Rightarrow \qquad n_1{}^B, \quad n_2{}^B$$

$$V_B \equiv V(n_1{}^B, n_2{}^B) \qquad [10]$$

Note, since the goods are substitutes:

$$V_B < V_1 + V_2 \qquad [11]$$

Also, $n_i^* > n_i{}^B$. Articles from collection i are more valuable to a consumer if that consumer does not also have access to collection j. Also note that n_1^*, n_2^*, $n_1{}^B$, and $n_2{}^B$ do not depend on the bundle prices. One faces a marginal cost per article of zero if the bundle has been purchased regardless of the bundle price. An increase in P_1 will not affect the number of articles consumed from that collection if the bundle is bought although it may discourage one from purchasing collection 1 altogether. The analytical derivation of n_i for this case and the cases that follow are given in the appendix.

Faced with prices (P_1, P_2), a consumer can either:

a) Buy only the bundle of collection 1. Her expected net surplus is:

$$CS_1 = V_1 - P_1$$

b) Buy only the bundle of collection 2. Her expected net surplus is:

$$CS_2 = V_2 - P_2$$

c) Buy both of the collections for an expected surplus:

$$CS_B = V_B - P_1 - P_2$$

d) Buy neither bundle. This yields a surplus of zero.

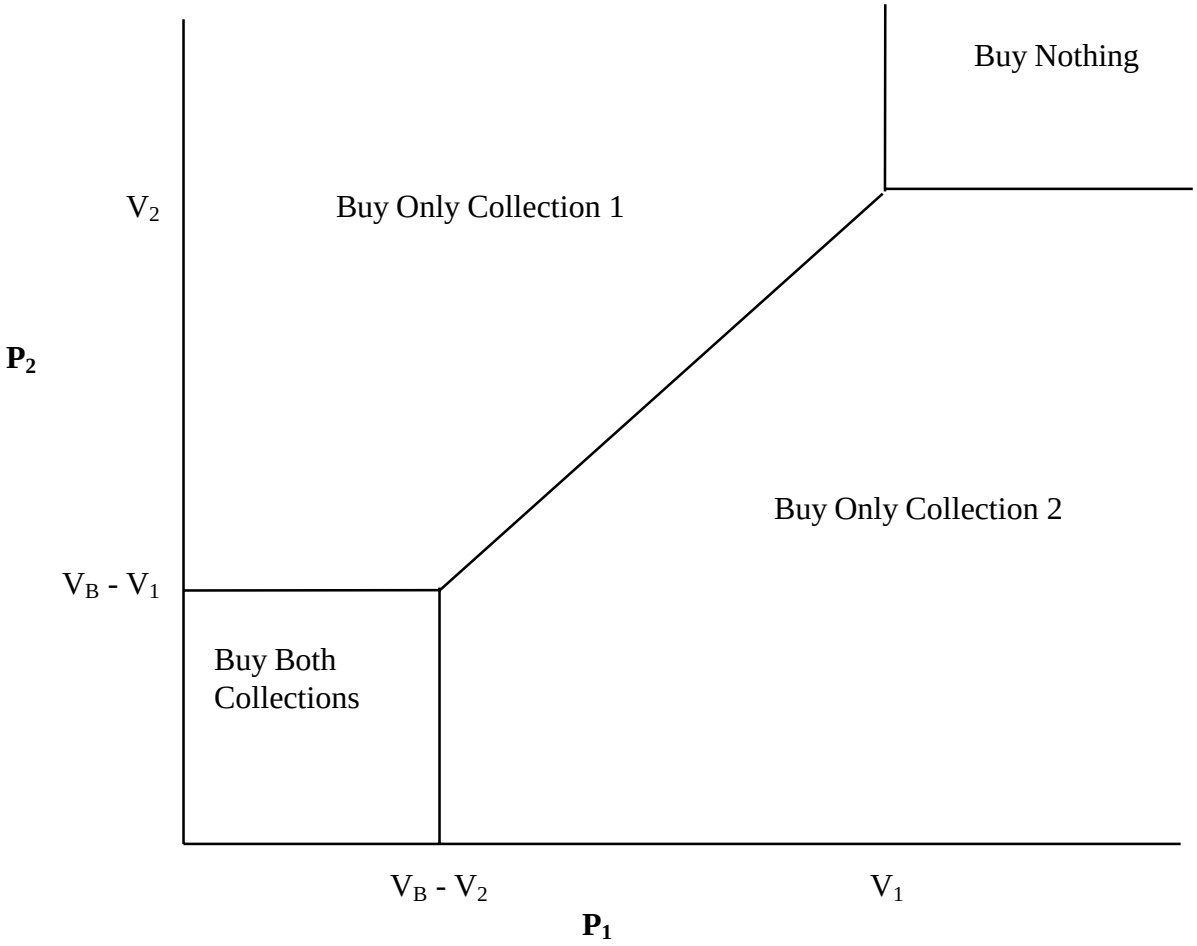
The consumer chooses the action that yields the highest net surplus. If $P_2 > V_2$, then $CS_2 < 0$ and $CS_B < CS_1$. Thus, she will choose to purchase collection 1 if $P_1 \leq V_1$ ($CS_1 \geq 0$). Otherwise, she will buy nothing.

If $V_2 \geq P_2 > V_B - V_1$, then $CS_2 \geq 0$ and $CS_1 > CS_B$. So, she will either choose to only buy collection 1 (if $CS_1 > CS_2$) or only buy collection 2 (if $CS_1 < CS_2$). The first option yields the higher surplus if $P_1 < V_1 - V_2 + P_2$. In the event that $CS_1 = CS_2$ we assume that the person randomly chooses between the two collections. The assumption of how to settle ties has no effect on our results.

If $P_2 \leq V_B - V_1$, either CS_2 or CS_B will be chosen. Both bundles are purchased if $P_1 \leq V_B - V_2$ ($CS_B \geq CS_2$). Otherwise, the consumer will only purchase collection 2.

The optimum choice is illustrated in Figure 1.

Figure 1: Consumer's Decision when both Firms Bundle.



C. Collection 1 sold as a bundle. Collection 2 sold on a per-item basis.

Collection 1 is sold as a bundle at price P_1. But now, articles from collection 2 are sold at a price of P_2 per article chosen. [6] gives the cost function faced by the consumer. V_1 and CS_1 are identical to the previous case ([8]). Suppose the consumer chooses to only buy articles from collection 2. Let $n_2(P_2)$ be the number of articles she would choose to consume. This is found by setting the marginal utility of an article from collection 2 equal to the marginal cost of acquiring that article:

$$\frac{dV(0, n_2)}{dn_2} = P_2 \quad \Rightarrow \quad n_2(P_2)$$

Substituting back into [3]: [12]

$$V_2(P_2) \equiv V(0, n_2(P_2))$$

To find the total gross value of consuming both the bundle of collection 1 and some articles from collection 2, we solve the simultaneous equations:

$$\frac{dV(n_1^B, n_2^B)}{dn_1^B} = 0$$

$$\Rightarrow \quad n_1{}^B(P_2) \ , \ n_2{}^B(P_2)$$

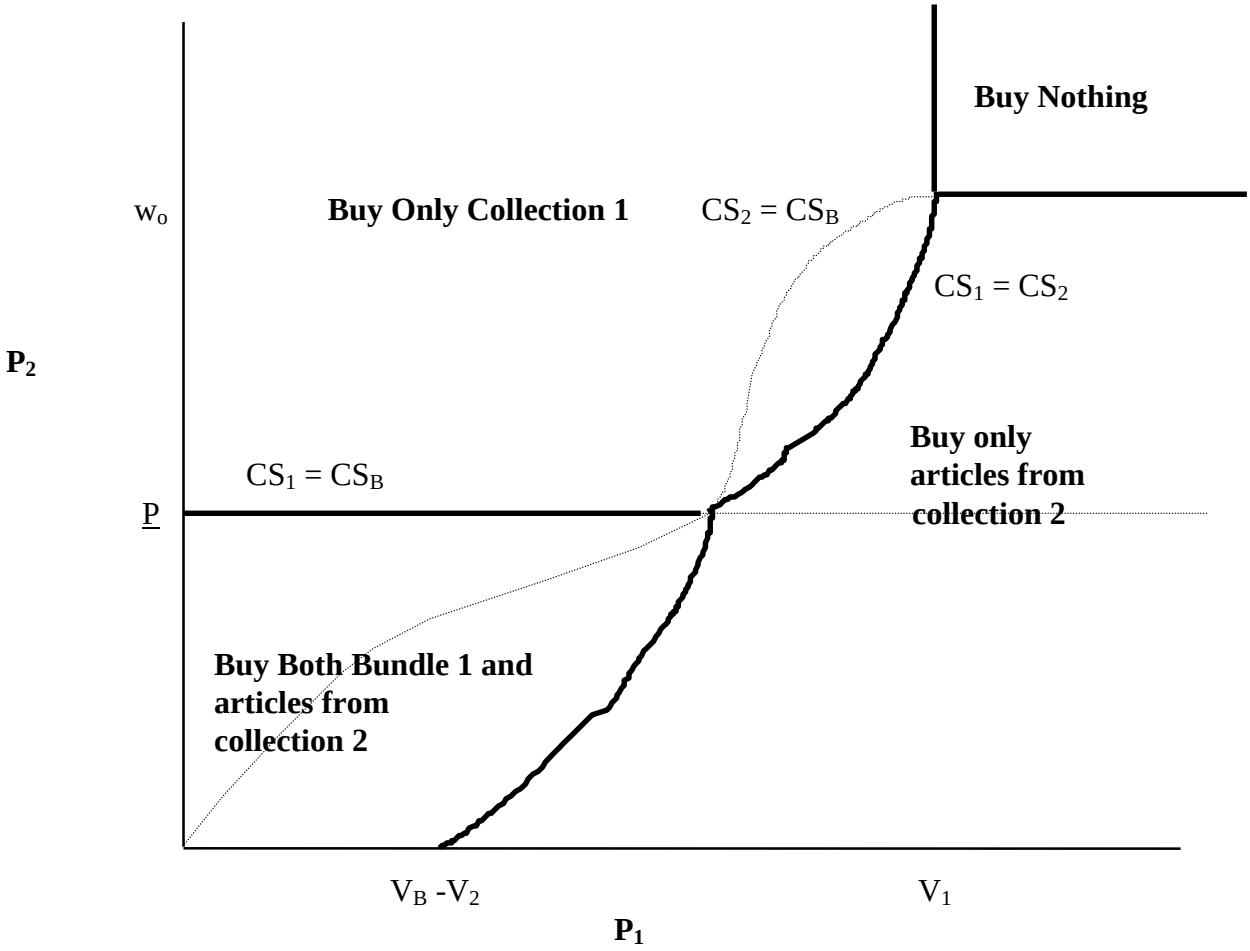
$$\frac{dV(n_1^B, n_2^B)}{dn_2^B} = P_2$$

$$V_B(P_2) \equiv V(\ n_1{}^B(P_2) \ , \ n_2{}^B(P_2)\ ) \qquad [13]$$

The marginal cost to the consumer of acquiring an article from collection 1 is zero, whereas the marginal cost is P_2 for an article from collection 2.

Again, consumers choose the option that gives the greatest surplus net of cost. This decision is described by Figure 2.

Figure 2: Consumer's Decision when Firm 1 Bundles and Firm 2 sells on Per-Article Basis.



$V_B - V_2$ is the total value of collection 1 if all collection 2 articles are given away for free. $\underline{P}$ is the P_2 such that $CS_1 = CS_B$ ($V_1 = V_B(\underline{P}) - \underline{P} \cdot n_2^B(\underline{P})$).

D. Both Collections sold on a per-item basis.

Suppose articles from each collection are only made available on a per-article basis. If the consumer only purchases articles from collection 2, $n_2(P_2)$ and $V_2(P_2)$ are given by [12]. If the consumer only purchases articles from collection 1:

$$\frac{dV(n_1,0)}{dn_1} = P_1 \quad \Rightarrow \quad n_1(P_1)$$

Substituting back into [3]: [14]

$$V_1(P_1) \equiv V(n_1(P_1), 0)$$

To find a consumer's gross value of choosing some articles from each collection, we solve the simultaneous equations:

$$\frac{dV(n_1^B, n_2^B)}{dn_1^B} = P_1$$

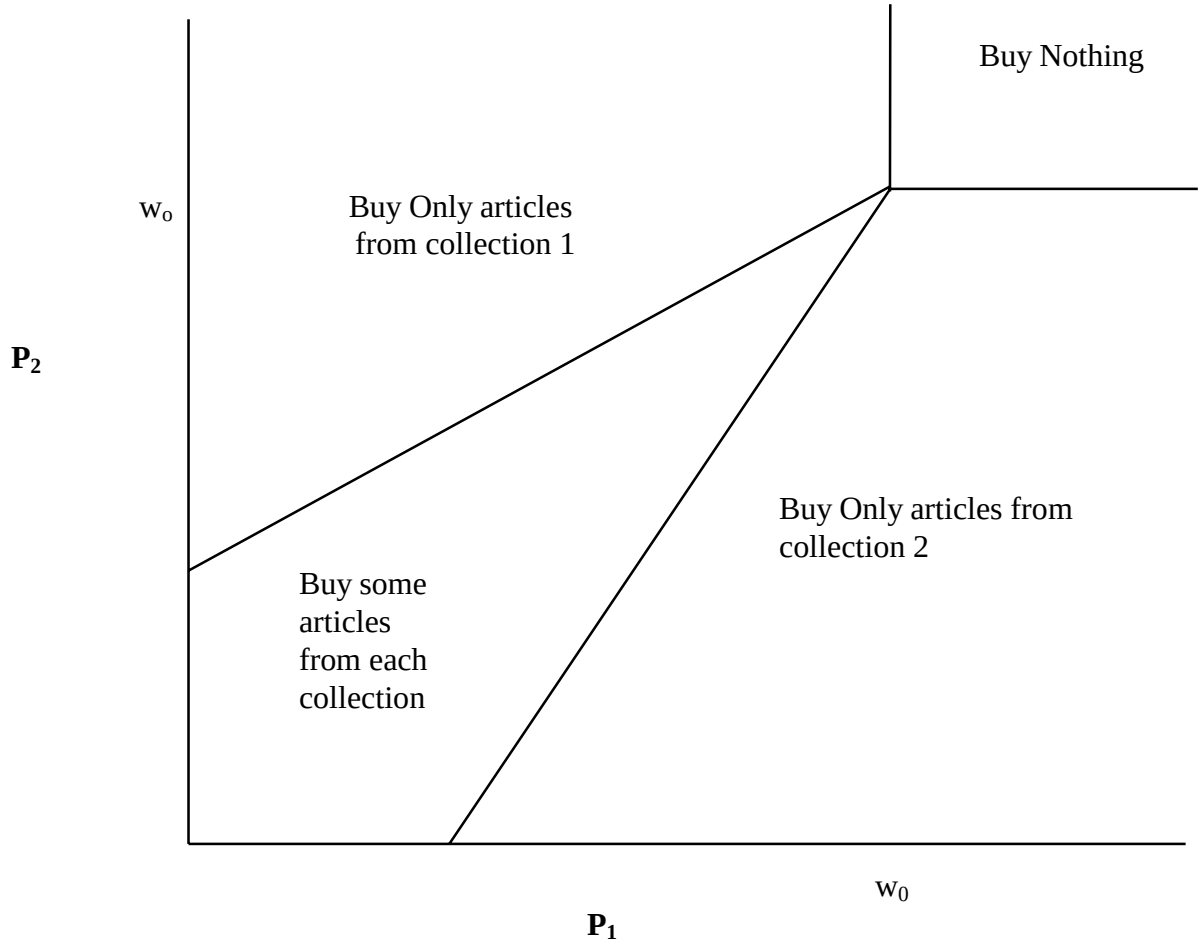
$$\Rightarrow \quad n_1^B(P_1, P_2) \ , n_2^B(P_1, P_2)$$

$$\frac{dV(n_1^B, n_2^B)}{dn_2^B} = P_2$$

$$V_B(P_2) \equiv V(n_1^B(P_1, P_2) \ , n_2^B(P_1, P_2)) \qquad [15]$$

Figure 3 depicts the choice that maximizes a consumer's net surplus for the possible combinations of (P_1, P_2).

Figure 3: Consumer's Decision when both Firms sell on a per-article basis.



IV. Duopoly Competition.

Two firms compete by simultaneously choosing prices. Here, we assume that it has been exogenously determined how many articles are in each firm's collection (N_i) and whether the firm will sell these articles as a bundle or purely as components. In section V, we will endogenize bundling choice as well as collection size. Firm 1 sets its price (P_1) taking Firm 2's price (P_2) as given. Firm 2 sets its price assuming P_1 is given. A pure strategy Bertrand equilibrium exists if there is a (P_1*, P_2*) such that P_1* is Firm 1's best response to P_2* and P_2* is Firm 2's best response to P_1*.

A. Homogeneous Consumers:

The number of consumers is normalized to one. Assume that (w_o , k) is the same for all consumers. This is analogous to the Bakos and Brynjolfsson (1998) formulation of consumer heterogeneity. Consumers' rankings vary so that their *ex post* valuations of articles differ. However, articles are *ex ante* identical and all consumers have the same total valuations for the aggregate bundles. Thus, consumers' demands are identical.

1. Both firms bundle.

Suppose Firm 1 sells N_1 articles as a bundle for a price of P_1. Firm 2 sells N_2 articles as a bundle for a price P_2. Does a pure-strategy Bertrand equilibrium exist? It turns out the unique Bertrand equilibrium (P_1^* , P_2^*) is ($V_B - V_2$, $V_B - V_1$). Here, we construct Firm 1's reaction function. The demand faced by Firm 1, $D(P_1)$, will depend on Firm 2's price. Refer to Figure 1.

a) If $P_2 > V_2$, no one buys collection 2 regardless of Firm 1's price. $D(P_1) = 1$ if $P_1 \leq V_1$. Otherwise, $D(P_1) = 0$. Firm 1 maximizes Profit ($P_1 \cdot D(P_1)$) by setting a price equal to V_1.

b) If $V_2 \geq P_2 > V_B - V_1$

$$D(P_1) = \begin{cases} 1 & \text{if } P_1 < V_1 - V_2 + P_2 \\ .5 & \text{if } P_1 = V_1 - V_2 + P_2 \\ 0 & \text{if } P_1 > V_1 - V_2 + P_2 \end{cases}$$

Profit is maximized at $P_1 = V_1 - V_2 + P_2 - \varepsilon$ where ε is the smallest increment needed to get all consumers to strictly prefer Firm 1's collection to Firm 2's collection.

c) If $P_2 \leq V_B - V_1$, $D(P_1) = 1$ if $P_1 \leq V_B - V_2$. Else, $D(P_1) = 0$. Firm 1 maximizes its profit by setting a price equal to $V_B - V_2$.

Thus, Firm 1's reaction function, $P_1(P_2)$, is given by:

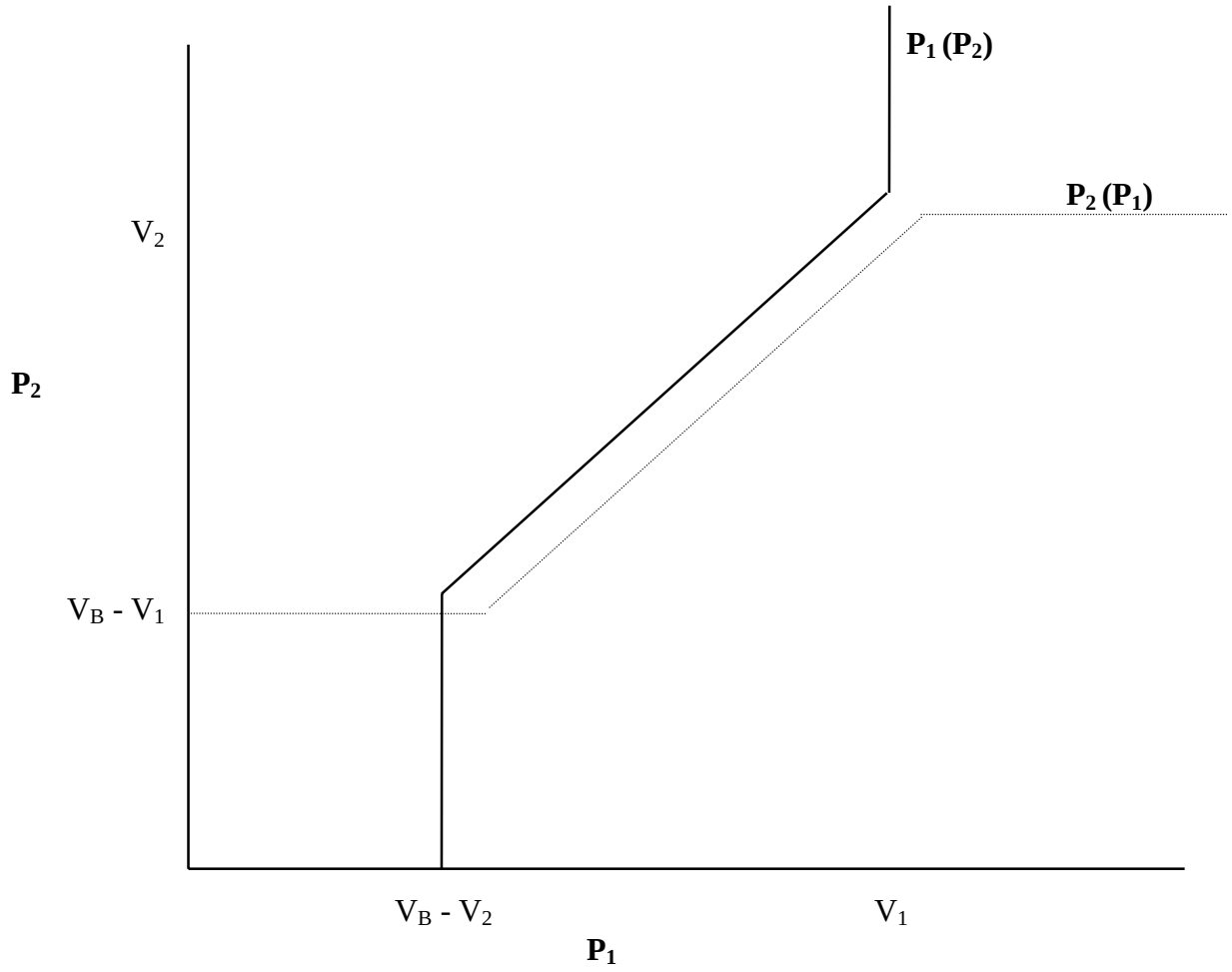
$$P_1(P_2) = \begin{cases} V_1 & \text{if } P_2 > V_2 \\ V_1 - V_2 + P_2 - \varepsilon & \text{if } V_2 \geq P_2 > V_B - V_1 \\ V_B - V_2 & \text{if } P_2 \leq V_B - V_1 \end{cases} \qquad [16]$$

Similarly, we can derive Firm 2's reaction function, $P_2(P_1)$:

$$P_2(P_1) = \begin{cases} V_2 & \text{if } P_1 > V_1 \\ V_2 - V_1 + P_1 - \varepsilon & \text{if } V_1 \geq P_1 > V_B - V_2 \\ V_B - V_1 & \text{if } P_1 \leq V_B - V_2 \end{cases} \qquad [17]$$

These reaction functions are illustrated in Figure 4. When Firm B sets a very high price, Firm A responds with its monopolist price which extracts the entire consumers' surplus. If B establishes a moderate price, consumers can never been convinced to purchase both bundles. As a result, consumers are just going to buy from one of the two firms. Thus, A wants to charge a price just low enough to attract all of these homogeneous consumers. When B charges a relatively low price, consumers are always going to buy from B. So, A's best response is to charge the highest price possible that gets consumers to also purchase A's bundle.

Figure 4: Reaction Functions when both Firms Bundle: Homogeneous Consumers



As is apparent from the graph, the reaction functions cross at a single point: $(V_B - V_2 , V_B - V_1)$. This is the unique Bertrand equilibrium. Neither firm can increase its profits by unilaterally deviating from these prices.

This equilibrium has a number of appealing characteristics. The outcome is socially optimal. All consumers have access to both collections. After the subscription has been bought, the price to the consumer of reading any article is zero -- the same as its marginal cost of production. A monopolist would also reach the social optimum in this scenario. If the monopolist controlled all $N_1 + N_2$ articles, he would sell them altogether as a package for a price of V_B, extracting all consumers' surplus. Notice, however, that the duopolists' total profits ($V_B + V_B - V_1 - V_2$) is lower than V_B (from [11]), the monopolist's profit. Still, the duopolists are acquiring strictly positive producer's surplus. This may seem surprising considering that the articles produced are *ex ante* identical to the consumers. Suppose firms are of equal size. Then, collection 1 is expected to be of the same value as collection 2. Even if firms have different collection sizes, an article randomly selected from collection 1 is expected to be of the same value as an article randomly selected from collection 2. Usually, when firms are producing homogeneous products, we would expect prices to be bid down to MC (assumed to be zero) in a price game. However, in this model, that does not happen. Why not? Consumers, at most, want

one of each article. A consumer is never better off reading an article again rather than reading a different article. In addition, a reader's most valuable articles are sprinkled throughout the two collections -- the 10th best article from collection 1 is less valuable than her very best article from collection 2. Readers are willing to pay for access to the other collection. No bundle price reduction by Firm 1 can completely eliminate this demand for Firm 2's product, even if collection 1 is given away for free. Thus, firms retain some degree of market power.

Table 1 illustrates these results with numerical examples of the Bertrand Equilibrium for the demand specification given in [3]. Recall that in equilibrium, (P_1*, P_2*), all consumers purchase both bundles. Since we normalized the number of consumers to one, Firm i's operating profit is equal to P_i*. Π_{tot} is the total operating profit earned by the duopolists. V_B is the total value of having access to both collections. It also represents the price and profits of a monopolist, as well as the Social Welfare (Profits plus Consumers' Surplus) that results with either a monopoly or a duopoly.

Table 1: Bertrand Equilibrium for Homogeneous Consumers when Both Firms Bundle.

$w_o = 1, k = .3$

N_1	N_2	V_1	V_2	V_B	P_1*	P_2*	Π_{tot}
500	500	44.44	44.44	57.74	13.29	13.29	26.59
400	600	39.39	48.39	57.74	9.35	18.35	27.70
200	800	24.37	53.99	57.74	3.75	33.37	37.12

$w_o = 1, k = .1$

N_1	N_2	V_1	V_2	V_B	P_1*	P_2*	Π_{tot}
500	500	14.81	14.81	19.25	4.44	4.44	8.87
400	600	13.13	16.13	19.25	3.12	6.12	9.24
200	800	8.12	18.00	19.25	1.25	11.13	12.38

Competition has substantially eroded the market power of firms. Even if the competitor is only a quarter of the size of the larger firm, total profits are less than 65% as much as a monopolist would earn producing the same articles. Furthermore, note that when there are heterogeneous collection sizes, the larger firm earns higher profits per article suggesting that larger firms may have a competitive advantage over smaller ones.

Endogeneity of product creation:

Information goods have negligible marginal costs of production. However, they are characterized by significant fixed costs which are incurred during initial product creation (first-copy costs). If competition significantly reduces operating profits, then we may be concerned that a less than efficient amount of investment in new products will take place in a competitive environment. Consider the following two-stage game. Discounting is ignored. Let F_i be the fixed costs incurred by Firm i in order to create N_i articles, where N_i is given exogenously (i = 1, 2).[9] In the first stage, the duopolists simultaneously decide whether to undertake the investment

[9] In section V, we allow N_i to be determined endogenously. We find that a duopoly will produce less than the efficient number of articles.

project. In the second stage, if both firms invest, then the Bertrand Equilibrium (P_1*, P_2*) will result. If only one firm invests, that firm will charge the monopoly price (V_i) in the second period. Figure 5 presents the normal form of this game.

Figure 5: Normal Form Representation of Information Good Creation

		Firm 2	
		Invest	**Do not Invest**
Firm 1	**Invest**	(V_B -V_2 - F_1 , V_B -V_1 - F_2)	(V_1 - F_1 , 0)
	Do not Invest	(0, V_2 - F_2)	(0, 0)

payoffs = (Firm 1's net profit, Firm 2's net profit)

The Social Optimum is:

$$\text{Max}[V_B - F_1 - F_2 , V_1 - F_1, V_2 - F_2 , 0]$$

The following propositions can be established:

1) *If is efficient for both firms to invest, Invest is the dominant strategy*.

Proof: Since (Invest, Invest) is social optimum we have:

$$V_B - F_1 - F_2 > V_1 - F_1 \qquad [18]$$

$$V_B - F_1 - F_2 > V_2 - F_2 \qquad [19]$$

Rearranging [19]:

$$V_B - V_2 - F_1 > 0 \qquad [20]$$

Combining [11] with [20] and simplifying:

$$V_1 - F_1 > 0 \qquad [21]$$

From [20] and [21], we see that Firm 1's dominant strategy is to invest. Following the same steps will show that Firm 2's dominant strategy is to invest.

2) *If $V_i < F_i$ (it is not efficient for Firm i to invest even if were the sole source of articles), then Firm k will invest if and only if it is efficient to do so ($V_k \geq F_k$).*[10]

Since $V_i < F_i$, we have $V_i - F_i < 0$ and combining with [11] see that $V_B - V_k - F_i < 0$. So, Firm i's dominant strategy is to not invest. Firm k will respond by investing if and only if $V_k - F_k \geq 0$.

[10] There is one other possibility: $V_i > F_i$ and $V_B - V_k < F_i$ ($\forall$ i = 1, 2 i ≠k) The social optimum is for the firm which maximizes $V_i - F_i$ to invest, while the other firm does not. If this is the case, there are two Nash Equilibria: (Invest, Do Not Invest) and (Do Not Invest, Invest). One is the social optimum. The other is not since it has the wrong firm undergoing the investment project.

In summary, if consumers are homogeneous in aggregate valuations and both firms, competing in a price game, offer their collections as a bundle, the efficient outcome is reached. Firms still charge positive prices for their bundles even though the articles they produce are *a priori* identical. Firms' operating profits are substantially less than a monopolist's, yet the incentive to efficiently invest in new product creation has been maintained.

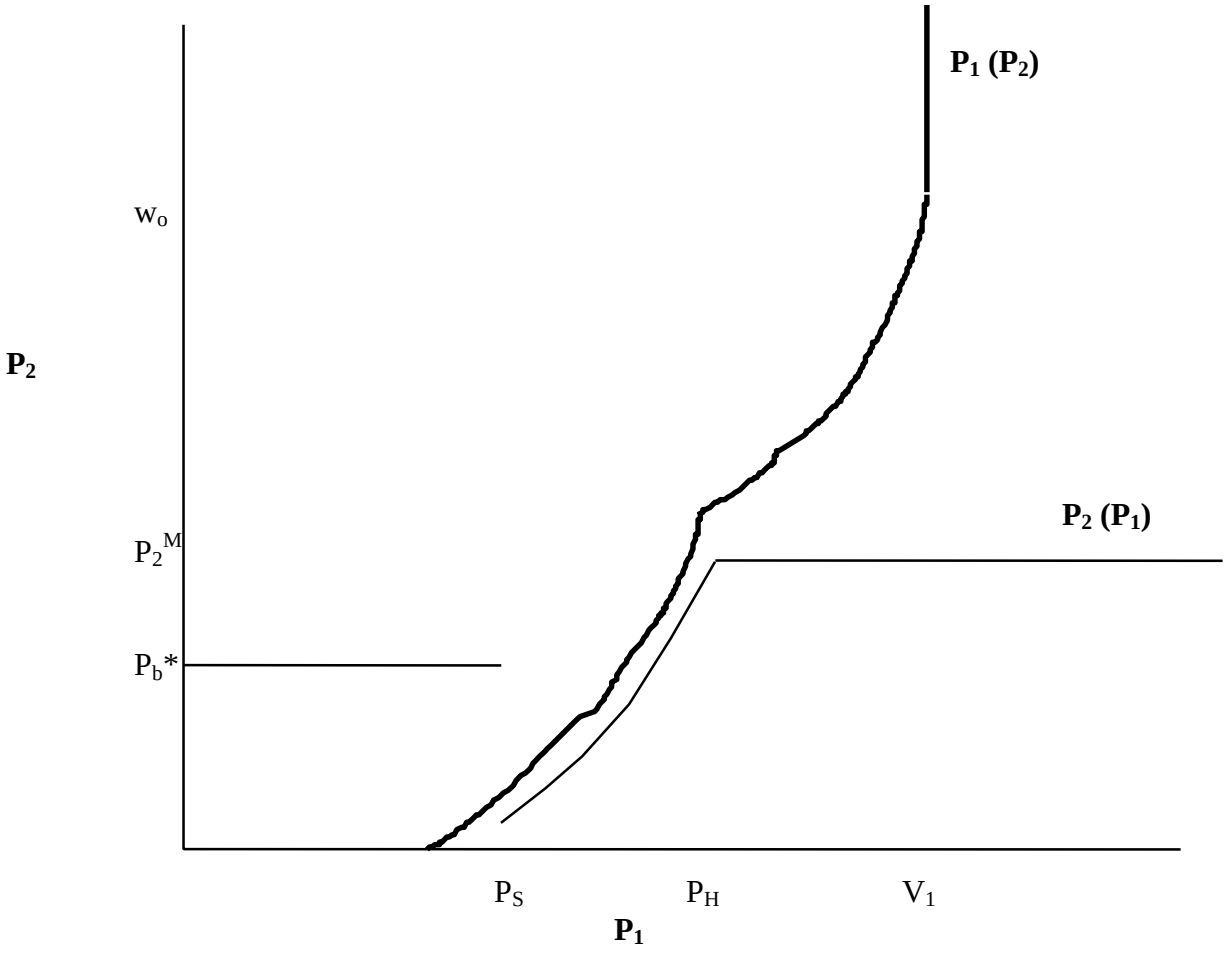
2. Firm 1 bundles. Firm 2 sells on per-item basis.

Now, suppose Firm 1 continues to sell N_1 articles as a bundle for a price of P_1, but Firm 2 sells each article (of the N_2 total articles) for a price P_2. In this case, with homogeneous consumers, there does not exist a pure strategy Bertrand equilibrium.

Figure 5 illustrates the firms' reaction functions for this scenario. Firm 1 sells its bundle for a price P_1 . If consumers choose to purchase the bundle, Firm 1's profit will be P_1 . If consumers do not buy this bundle, Firm 1's profit is zero. (Since consumers are homogeneous in aggregate preferences, they either all buy the bundle or all do not buy the bundle.). Clearly, Firm 1 will set P_1 as high as possible that still gets consumers to purchase its bundle. See Figure 2. Firm 1's reaction function, $P_1(P_2)$, will be just to the left of the boundary to the area where the consumer only buys articles on a per-item basis from collection 2. By pricing P_1 slightly below these cut-off prices, Firm 1 is able to convince all consumers to buy its bundle. Firm 1's reaction function is continuous in P_2 , responding to a slight decrease in P_2 with a slight price reduction of its own.

Firm 2 is selling articles. Thus, its profits are: $P_2 \cdot [(1 - b) \cdot n_2(P_2) + b \cdot n_2^B(P_2)]$, where $n_2(P_2)$ and $n_2^B(P_2)$ are defined in [8] and [9] respectively. b is a dummy variable equal to 0 if consumers do not purchase Firm 1's bundle. b = 1 if consumers do purchase Firm 2's bundle. Both $n_2(P_2)$ and $n_2^B(P_2)$ are decreasing in P_2. A higher price discourages article consumption. For any P_2' , $n_2(P_2') > n_2^B(P_2')$. Consumers will buy more articles from Firm 2 if they do not have access to articles from collection 1 at a marginal cost of zero. Also, if $P_2' > w_o$, $n_2(P_2') = 0$. Let P_2^M be the price that maximizes $P_2 \cdot n_2(P_2)$. P_b^* maximizes $P_2 \cdot n_2^B(P_2)$. Let Π_B^* be Firm 2's profits if is charges P_B^* and consumers also purchase Firm 1's bundle. Note P_B^* is less than P_2^M. If Firm 2 were a monopoly, it would charge a price P_2^M. Refer to Figures 2 and 5. If $P_1 > P_H$, then Firm 2 can charge a price of P_2^M and still discourage all consumers from purchasing Firm 1's bundle. Since P_2^M maximizes $P_2 \cdot n_2(P_2)$, this is Firm 2's best response. When $P_1 < P_H$, if Firm 2 charges P_2^*, consumers would buy Firm 1's bundle. But, we know that P_B^* maximizes profits in this case. So, Firm 2 is faced with two options: 1) Price at P_B^* and earn a profit of Π_B^* or 2) set a price P' where P' is the highest P_2 that still gets consumers to only purchase articles from Firm 2 (to not buy Firm 1's bundle). The second option earns a profit of $P'(P_1) \cdot n_2(P'(P_1))$. P' is a function of P_1 since as P_1 declines, so must P'. Define P_S such that $P'(P_S) \cdot n_2(P'(P_S)) = \Pi_B^*$. For $P_S < P_1 < P_H$, the second option yields higher profits. For $P_1 < P_S$, the first option is preferred by Firm 2. Thus, there is a discontinuity in Firm 2's reaction function.

Figure 5: Reaction Functions: Firm 1 Bundles and Firm 2 sells on Per-Article Basis (Homogeneous Consumers).

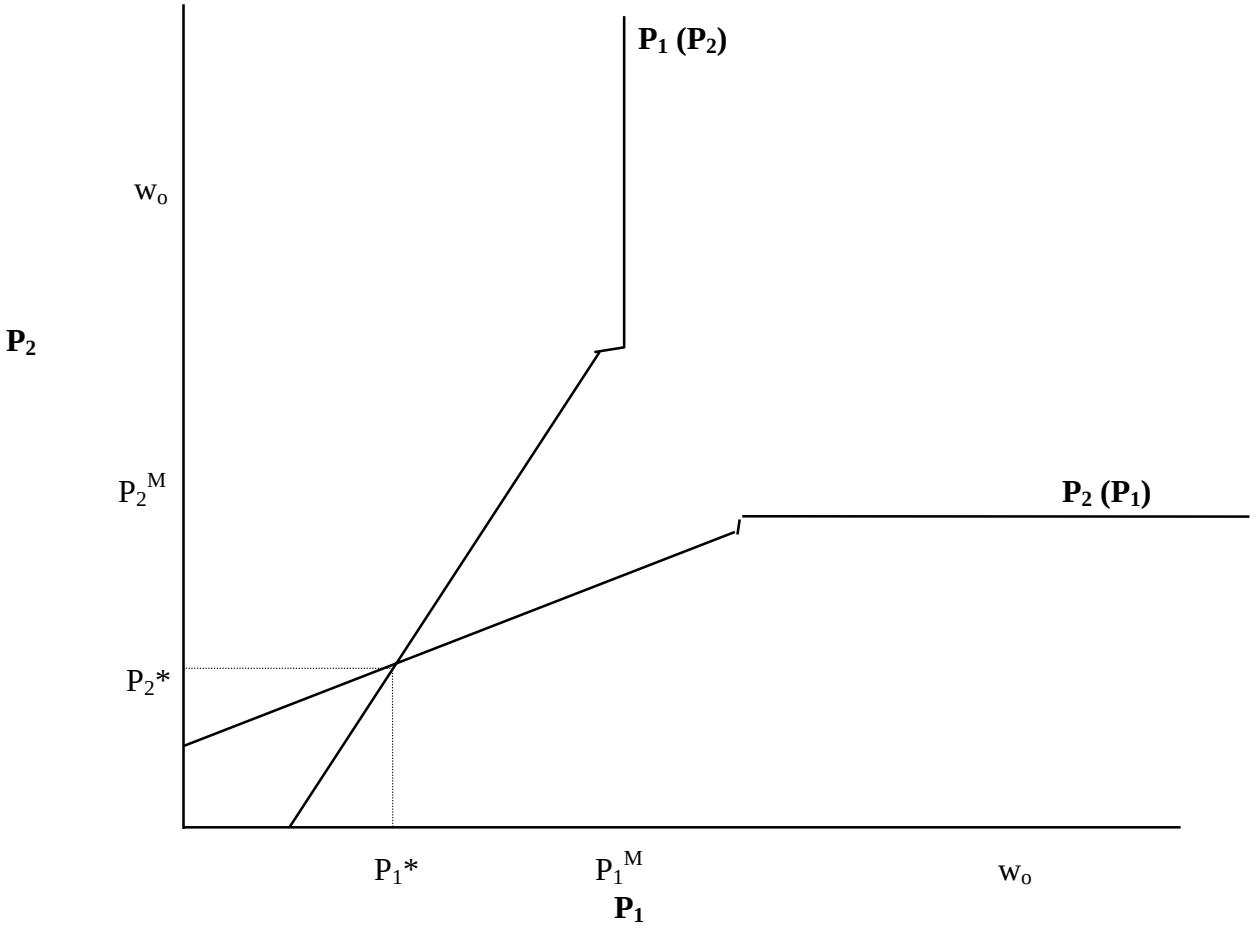


There is no intersection in between these reaction functions. Thus, a pure strategy Bertrand equilibrium does not exist in this specification.

3. Both firms engage in pure components pricing.

Suppose both firms sell articles from their respective collection on a per-article basis. Figure 6 illustrates the firms' reaction functions. P_i^M is the price that maximizes $P_i \cdot n_i(P_i)$ where $n_i(P_i)$ is defined in [14]. If Firm k charges an extremely high per-article price, then Firm i can set $P_i = P_i^M$, still keep consumers from purchasing any articles from the other firm, and earn the same profit that a monopolist would. However, for lower P_k, Firm i sets P_i to maximize its profits when consumers buy some articles from each firm: $P_i \cdot n_i^B(P_1, P_2)$ where $n_i^B(P_1, P_2)$ is given in [15]. If Firm k's price is reduced, Firm i will face less demand for its product. As a result, it will offer a lower price. Thus, we have upward-sloping reaction curves. There is a small kink in each firm's reaction function, where it is in Firm i's interest to slightly reduce P_i in order to eliminate consumption of Firm k's collection altogether.

Figure 6: Reaction Functions: Both firms do not bundle (Homogeneous Consumers).



The unique Bertrand Equilibrium is at (P_1*, P_2*). Each consumer purchases articles from both firms: $n_i^B(P_1^*, P_2^*)$, i = 1,2. Since consumers face a cost on the margin for each article consumed while the marginal cost of production is zero, this outcome is not efficient. For example, suppose $w_o = 1$, k = .3, $N_1 = 400$ and $N_2 = 600$. The Bertrand equilibrium is (.285, .31). Profits to the firms are \$9.92 and \$14.19 respectively. Consumers' Surplus is \$26.71 and Social Welfare is \$50.82. Compare this to line 2 of Table 1 which lists the outcome for the same preferences and collection sizes if firms bundle their products instead of selling on a per-item basis. Bundling achieves higher profits, Consumers' Surplus, and Social Welfare (13%, 9%, and 12% higher respectively). Selling articles on a per-item basis is inefficient. Less value is created so less surplus is available to be either extracted by the firms or retained by consumers.

B. Heterogeneous Consumers.

Our demand specification, given in [3], allows for two dimensions of heterogeneity. w_o represents the magnitude of value of a person's best article. k represents intensity of use. We run simulations to explore the effects of heterogeneity in both of these dimensions. In one set, k

is fixed at .3, and w_o is uniformly distributed along the interval [0, 1]. So, consumers find the same percentage of articles valuable, but the magnitude of these values differs across persons. In the second set of simulations, w_o is fixed at 1, while k is uniformly distributed between zero and one. Every consumer's best article is of the same value, but persons differ in the fraction of articles they find valuable and how steep of decline in value they experience as more articles are selected. We focus on two potential pricing strategies for each firm: pure bundling and pure unbundling. Tables 2-5 present the results of these simulations. P_i is the price charged by Firm i. Q_i is the quantity sold by Firm i. When the firm bundles its product, Q_i refers to the fraction of consumers that purchase the bundle. If Firm i sells on a per-item basis, Q_i refers to the number of articles sold. (If all people consumed all 1000 articles, Q_i = 1000). CS is the value of Consumers' Surplus. Welfare is social welfare = CS + total profits. We calculate the Bertrand Equilibria (when they exist) as well as the Stackleberg Equilibria. Also presented are the socially optimum outcome and the monopolist outcome for the various pricing options and collection sizes.

Key Observations:

1. Monopoly Outcomes.

Consider a monopolist who produces 1000 articles and then sells them either through pure bundling, pure unbundling or mixed bundling. The profit-maximizing results for these three pricing strategies and for the two dimensions of consumer heterogeneity are listed in Table 2. When $w_o \sim U[0,1]$, pure bundling achieves the profit-maximizing outcome (mixed bundling does not perform any better). However, when $k \sim U[0,1]$, mixed bundling is strictly preferred to pure bundling. In the first case, the consumers who would not purchase a bundle have relatively low per-item valuations. To get them to purchase any articles on a per-item basis would require a low price. However, if the monopolist does this, high-valuation consumers would choose to buy on a per-article basis rather than to buy the relatively high-priced bundle. In the second scenario, the consumers that value only a few articles (and thus are not willing to purchase the bundle) do highly value some of the articles in the collection. So, the monopolist can sell some articles (at a high price) without giving high-k consumers the incentive to switch away from the bundling option. Note, per-article sales make up almost 30% of the total revenue from mixed bundling in this case.

Tables 3 - 5 contain the monopoly outcome if a monopolist prices sub-collections rather the collection as a whole. The primary conclusion is that a monopolist does not want to sub-bundle its products. Profits to the monopolist are always higher (provided he is either pure bundling or mixed bundling) when the collection is priced as a whole (N = 1000) versus if the collection were sub-divided into two parts ($N_1 = N_2 = 500$ or $N_1 = 400$, $N_2 = 600$) regardless of the pricing strategy implemented. The goods produced are substitutes. By introducing sub-bundles, the monopolist is providing competition to itself and will not be able to capture as much of consumers' valuations. Note, mixed bundling also results in a higher total social welfare than for any of the other monopoly outcomes. Intuition of these results can be illustrated with a simple example. Assume that there are homogeneous consumers and the entire collection is equally divided into two parts. If a consumer has access to only the first 500 goods, she values that sub-collection at \$14 (V_1). Conversely, if the consumer only has access to the second 500 items, she also values that sub-collection at \$14 (V_2). However, if the consumer has access to all

1000 goods, she attains a gross surplus of \$20 (V_B). A monopolist who sold all 1000 items together as a single bundle would set a price of \$20 (V_B) and successfully extract the consumer's entire valuation. However, if the monopolist divides his collection into two equal parts, it can no longer attain \$20 in revenue. If each collection is priced at \$10, the consumer will respond by only choosing one of the two collections, earning a net surplus of \$4 (as opposed to zero if he were to purchase both sub-bundles). Thus, the monopolist would earn only \$10 in revenue. In this case, the best the firm can do is offer both bundles at a price of \$14. Only one of the two bundles will be bought with the firm earning \$14 in total revenue. Not only does sub-bundling by a monopolist result in lower profits, it also leads to lower social welfare.

2. Duopoly Outcomes.

Tables 3 - 5 list the duopoly outcomes for a variety of firm sizes. Throughout the various simulations, all duopoly equilibria (Bertrand or Stackleberg) significantly outperform (in terms of social welfare) their monopoly counterparts.

a) Both Firms Bundle.

Table 3 lists the outcomes when both firms bundle their respective collections. The most evident feature of this case is that there is no (pure strategy) Bertrand equilibrium. For any set of prices (P_1 , P_2), there will be three categories of consumers: (a) Those that will buy both firms' bundles; (b) Those that will buy only one of the firm's bundles; and (c) Those that will buy nothing. If Firm 2 charges a relatively high price, Firm 1 maximizes profits by undercutting Firm 2 so that of those in category (b), all choose Firm 1 over Firm 2. However, if Firm 2's price is low enough, Firm 1 is better off ignoring those in category (b) and instead switching to a higher price which maximizes profits on sales to consumers in category (a). The appendix provides a proof of this result in the case where firms are of equal size.

In all Stackleberg Equilibria, unlike the homogeneous consumers case, profit per article is higher for the firm with the smaller collection (For example, bottom row of Table 3 -- Stackleberg (2). *Profit 1* / 400 = \$.034, *Profit 2* / 600 = \$.026). Also notice that for both distributions, both firms' profits are higher when the larger firm (2) is the leader, versus when the smaller firm is the leader.

b) Firm 1 Bundles, Firm 2 sells per-item:

Table 4 lists the duopoly outcomes when only the first firm bundles its collection. In this case, a Bertrand equilibrium does exist. Recall that when there were homogeneous consumers, this was not the case. The Bertrand, Stackleberg (1 is leader) and Stackleberg (2 is leader) equilibria are all surprisingly similar to each other for a given distribution of collection sizes and consumer types.

A striking result of these simulations is that the firm that bundles is at a decided advantage over the firm that sells per-item. See the second set of results. Firm 1 gets higher profits than Firm 2 even though the second firm has the larger collection. Even in the next-to-last set of results, where Firm 2's profits exceed Firm 1's, the profit-per-article is higher for Firm 1 which bundles its products. This suggests that bundling offers a competitive advantage over unbundling.

c) Both Firms sell on Per-Item Basis.

Table 5 lists the duopoly outcomes when both firms do not bundle their collections. Notice that prices per article are substantially lower for any duopoly outcome than a monopolist would charge. Industry profits experience a much less steep decline. As would be expected, Stackleberg competition results in slightly higher prices and profits versus Bertrand. A leader can commit to being a little less aggressive than would result if the leader's pricing choice is not observable to the other firm. Both firms benefit from the less fierce competition but the follower receives a greater proportion of this benefit then the leader does.

V. Model Extensions

A. Endogenous Bundling Decisions

To this point, we have taken firms' decisions of whether or not to bundle its collection as exogenous. We now allow bundling decisions to be determined endogenously. Two approaches are used: 1) Firms simultaneously commit to a bundling strategy and after observing these choices, engage in price competition; 2) Firms are allowed to use the mixed bundling strategy so they are allowed to simultaneously sell articles individually as well as sell their entire collection as a bundle.

1. Firms Simultaneously Commit to Pure Bundling or Pure Unbundling .

Consider the following Game:

Stage I: Each firm is *endowed* with N_i articles.
Stage II: Having observed I, each firm *simultaneously* decides whether to bundle or to unbundle its collection.
Stage III: Having observed I and II, Firm 1 chooses P_1.
Stage IV: Having observed I-III, Firm 2 chooses P_2.

N_i is given exogenously. Bundling decisions are made endogenously under the realization that Stackleberg competition with Firm 1 as the leader will follow. Ideally, we would prefer to use Bertrand equilibria in order to avoid giving firms the power to undermine industry competition by committing to prices. However, no Bertrand equilibrium exists if both firms choose to bundle their collections. So, we use the Stackleberg equilibria instead.

Consider the following example. Suppose each firm produces a collection that contains 500 articles. Also assume that all consumers have $k = .3$ and w_o is uniformly distributed from zero to one. Using our results from Tables 3 - 5, the following normal form game (Stage II) can be constructed:

		Firm 2	
		Bundle	Unbundle
Firm 1	Bundle	($3.33, $3.33)	($5.53, $3.83)
	Unbundle	($3.42, $5.35)	($4.83, $5.29)

Payoffs = (Firm 1's Profit, Firm 2's Profit)

This game has 2 pure strategy Nash Equilibria: (Bundle, Unbundle) and (Unbundle, Bundle). But, (Bundle, Bundle) would result in a higher social surplus and (Unbundle, Unbundle) would result in a higher total profit. Both firms do not bundle their collections even though this is the efficient outcome. The appendix provides several other examples. Our primary conclusion is that any outcome can result, with no apparent connection to achieving either the highest total profit or the highest social welfare. Bundling does not appear to be firms' predominant strategy.

2. Mixed Bundling

The first model of endogenous bundling choices contains a large amount of pre-commitment. Firms announce a bundling strategy prior to setting prices. Firm 1 commits to a price before Firm 2 responds with its own price. As we are aware, such a sequential game can often result in perverse and unsatisfactory results. Thus, in this section, we introduce a less restrictive approach. We allow firms to engage in mixed bundling. To solve this problem analytically for a monopolist quickly becomes unmanageable, as the previous literature attests (see Chuang and Sirbu (1998)). Allowing for duopoly competition in mixed bundle pricing drastically adds to the complexity of this problem. As a result, we are forced to rely on numerical simulations instead of analytical results.

Firm 1 sets prices P_i^A and P_1^B simultaneously as Firm 2 sets P_2^A and P_2^B. P_i^A is the price of consuming any individual article from collection i. P_i^B is the price of attain the bundle of collection i. A Bertrand equilibrium exist if there is a set of prices ($P^*_1{}^A$, $P^*_1{}^B$, $P^*_2{}^A$, $P^*_2{}^B$) such that ($P^*_1{}^A$, $P^*_1{}^B$) is Firm 1's best response to ($P^*_2{}^A$, $P^*_2{}^B$) and ($P^*_2{}^A$, $P^*_2{}^B$) is Firm 2's best response to ($P^*_1{}^A$, $P^*_1{}^B$). Table 6 lists the Bertrand equilibria for a variety of firm sizes. Notice, when firms are of similar size, no Bertrand Equilibrium exists. The rationale is that mixed bundling is predominantly competition in bundle prices. Suppose a firm is trying to sell to a certain sector of the population. Selling bundles to these persons is the best way to capture as much surplus as possible. As a result, when firms are allowed to mixed bundle, the greatest margin of competition occurs in the setting of bundle prices. But, section IV.B showed that an equilibrium does not exist when firms simultaneously choose bundle prices. For heterogeneous firm sizes, a Bertrand equilibrium can exist when firms target different sectors of the population --- the small firm sells to consumers that do not wish to purchase the high-priced large bundle that the other firm is offering. Table 7 illustrates the choices that consumers make in equilibrium. 1A (2A) indicates the percentage of the population that purchases articles individually from Firm 1 (2) , 1B (2B) is the fraction that buys Firm 1 (2)'s bundle in equilibrium. We find that, when an equilibrium exists, most revenue will be attained through bundle sales. For the first set of simulations (w_o is uniformly distributed from zero to one), article sales account for less than .5% of total revenue regardless of firm size. In the second set of results, the smaller firm attains less

than 5% from article sales, while the larger firm can obtain up to 30% from article sales. It seems that bundling is the primary revenue generator for information goods.

B. Endogenous Collection Size.

To this point we have primarily focused on static efficiency. Once an information good exists, since its marginal cost is at or near zero, social welfare is maximized by distributing each article to everyone who has a positive value for it. Although a duopoly competing in price does not achieve this objective completely, it does greatly outperform a monopolist. Competing firms bid down prices to a level lower than a monopolist would set. As a result, more consumers have access to the information goods. However, information goods are also characterized by significant first-copy costs. At the end of section IVA1, we briefly considered the incentive to provide content when both duopolists bundle their collections, sell to a homogeneous consumer base, and collection size is exogenous. In this section, we more fully model the incentive to provide new content by allowing firms not only to chose whether or not to invest in a collection but also to adjust how large of a collection to build. We assume articles have convex first-copy costs. Firms simultaneously choose the number of articles to produce. Then, firms simultaneously choose prices given these collection sizes. We consider both homogeneous and heterogeneous consumers, allow for various bundling and unbundling strategies, and compare these duopoly outcomes to the corresponding monopoly outcomes.

Assume the investment cost of producing N_i articles is given by:

$$C(N_i) = a\,N_i + b\,(N_i)^2 \qquad [22]^{11}$$
$$a, b > 0$$

Having chosen (N_1, N_2), firms compete in a pricing game (taking bundling decisions as exogenous). Let $\Pi_1(N_1, N_2)$ and $\Pi_2(N_1, N_2)$ be the firms' respective operating profits from this price game. Thus, Firm i chooses N_i to maximize $\Pi_i(N_1, N_2) - C(N_i)$ taking N_k as given. An equilibrium (N_1^*, N_2^*) exists if N_1^* is Firm 1's best response to N_2^* and N_2^* is Firm 2's best response to N_1^*. Table 8 provides several examples of the resulting equilibria for both homogeneous and heterogeneous consumers for those bundling strategies that have a pure-strategy Bertrand equilibrium for the pricing game. For comparison, the outcomes which maximize social welfare and monopoly profits are also presented. For the monopoly outcome, a firm is allowed to choose N_1, N_2, P_1 and P_2 in order to maximize total profit (revenue less investment costs) from the two collections.

When consumers are homogeneous (first 4 lines of Table 8), a duopoly results in less than the optimum amount of content. Recall, that a monopolist would be able to extract all of consumers' surplus by setting a bundled price of V_B for the entire pool of articles. As a result, the monopolist's objective function would be the same as a central planner's. With a = .006, b = .0003, social welfare is maximized at $44.56 by having two collections, each containing 862 articles. A monopolist would maximize profits by producing 862 articles in each collection,

[11] Thus, first-copy costs are convex. If there were constant returns to scale (or economies of scale) in the production process, then no pure-strategy equilibrium would exist. Larger collections allow firms to extract slightly more revenue per article. With constant marginal costs of production, the optimal number of articles will either be zero or infinite. Only with increasing marginal costs will an interior solution exist.

charging a bundle price of $99.56 to consumers for access to both collection. Thus, the efficient outcome is achieved. However, a duopoly would produce less than the efficient number of articles. If each firm sets a bundle price for its individual collection, in equilibrium each will choose to produce 492 articles. Consumers purchase both bundles so there is efficient distribution of the existing content. However, less than the efficient amount of content has been created (and welfare is 81.5% of the efficient level). If each firm is pricing per-article, welfare is further reduced. Since per-unit pricing extracts less consumers' surplus than bundling does, firms have a lower incentive to invest in content. As a result, only 282 articles are published. Furthermore, these articles that are produced will not be distributed efficiently since consumers face a price of $ .30 on the margin. If a consumer values an article between zero and thirty cents, the article will not be consumed even though it would be efficient to do so since its marginal cost of production is zero. Due to these two sources of inefficiency, per-unit pricing results in social welfare of only 46% of its optimum level.

When consumers are heterogeneous, the story is a bit different. A duopoly still results in a less than efficient amount of content being produced since firms are unable to capture all of the surplus created by that content. But now, the same is true for the monopolist since consumers differ in their aggregate valuations. Since a monopolist is able to capture more of this surplus than a duopoly can, the monopolist has the greater incentive to invest in content. However, the monopolist increases prices to a level that is greater than is possible for a duopoly. As a result, a monopolist increases the distortion of the distribution of those articles that are produced. The examples from Table 8 indicate that this second effect dominates. For example, using the same cost parameters as before, assuming all consumers have $w_o = 1$ and that k is uniformly distributed between zero and one, the efficient outcome is to produce 1504 articles in each collection, achieving a social welfare of $135. A monopolist who charged a per-article price for articles from each collection would set a price at $ .47 and produce 670 articles in each collection. Each firm in a duopoly would price at $ .30 and produce 528 articles. The duopoly achieves a higher social welfare ($67 > 60$). Notice that the same is true if we allow collection 1 to be bundled instead of being sold on a per-unit basis. There is a greater incentive to produce content for collection 1 since bundling extracts greater surplus than unbundling -- this is especially true when we allow w_o to vary across consumers. Even though a duopoly results in less articles being produced, those that are produced are distributed more efficiently because a monopolist would set much higher prices. Throughout the paper, we stressed the gains in distribution efficiency that a duopoly achieves over a monopoly. When we allow for endogenous choices in the amount of content, this dominance by the duopoly is narrowed but not eliminated.

VI. Concluding Remarks.

This paper begins to explore the interaction between competition and the decision to bundle electronically-delivered information goods. Some of the results from the previous literature based on a monopoly supplier continue to hold for a duopoly: When consumers have homogeneous total valuations for the aggregate collection and there are no marginal costs of production for existing information goods, pure bundling is more profitable than pure unbundling; For heterogeneous preferences, this superiority of pure bundling is no longer guaranteed to hold. However, in a static model, we find that a duopoly, when compared to a monopoly, yields much lower prices, much higher social welfare, and slightly less profits

regardless of the bundling strategies employed. These lower profits reduce the incentive to invest in content. Thus, a monopoly is likely to produce closer to the efficient amount of content. However, the higher prices on existing content that a monopolist charges leads to efficiency losses in distribution that outweigh the efficiency gains from having more content. As a result, a duopoly is welfare-improving even though it is likely to result in fewer total information goods. Bundling remains a powerful tool for a firm. When a competitor is pricing per-article, bundling one's collection results in higher profits for the firm. If firms engage in mixed bundling, the margin with the most intense competition is in the setting of bundle prices.

There are a number of extensions left for future work. In this current model, a random article selected from one firm has the same expected value as a random article selected from any other firm. In practice, most firms produce goods that are differentiated either by quality or by some other distinguishing characteristics. Either all people agree that Firm A produces better products than Firm B. Or, because persons differ in their tastes, some are more likely to favor articles from Firm A while others will favor Firm B. In either of these cases, the degree of competition between firms is weakened. We would expect this to result in higher equilibrium prices and thus greater distribution inefficiency.

The cost structure assumed in this paper ignores several characteristics of most information goods. Although marginal costs of production are quite low, these cost are still positive. For non-negligible marginal costs, Chuang and Sirbu (1998) show that pure unbundling will be more profitable than pure bundling for a monopolist in the absence of large economies of scale. It remains to be shown that this result extends to the case of multiple firms. In addition, we have ignored all transaction costs. Each time a consumer purchases a product, payment to the seller must be made. This process is costly. Bundling, which reduces the number of transactions that must be made, is likely to result in efficiency gains and greater profits to the firm compared to unbundling which requires repeated transactions. Conversely, search costs are likely to reduce the incentive to bundle. If a firm offers a large, varied bundle, consumers will have a difficult time locating those items which are most valuable to them. This may induce firms to offer smaller, more niche-orientated bundles. Or, as Kephart *Et Al* (1998) explore, automated search agents might be used to reduce such costs.

Finally, one would like to explore a richer set of bundling possibilities. Like the previous literature, we have focused on pure bundling, pure unbundling, and mixed bundling. With N goods, firms could form up to 2^N bundles and price each one individually. Such a model would be infeasible to solve. However, it would be interesting to understand how competition is affected as firms begin to use more complex bundling/pricing strategies.

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Appendix

Analytical Derivations of n_i* :

Both firms bundle, consumer buys both bundles ($n_1 > 0$, $n_2 > 0$):

$$\frac{dV()}{dn_1} = 0, \quad \frac{dV()}{dn_2} = 0$$

$$n_1^* = \frac{3-\sqrt{3}}{3} kN_1 \qquad n_2^* = \frac{3-\sqrt{3}}{3} kN_2$$

$$V(n_1^*, n_2^*) = \frac{N_1 + N_2}{3\sqrt{3}} kw_0$$

If consumer only purchases only per-item from Firm i ($n_i > 0$, $n_k = 0$):

$$\frac{dV()}{dn_i} = P_i \implies n_i^* = \frac{k}{3}\left(3N_i + N_k - \sqrt{\frac{6N_iN_kP_i + (N_k)^2 w_o + 3(N_i)^2(2P_i + w_o)}{w_o}}\right)$$

If consumer only purchases Firm i's bundle ($n_i > 0$, $n_k = 0$):

Same n_i* as above substituting $P_i = 0$.

If consumer buy from each firm at the per-article prices:

$$\frac{dV()}{dn_1} = P_1, \quad \frac{dV()}{dn_2} = P_2$$

$$n_1^* = kN_1 - \frac{B\sqrt{6}\left(A + N_1(2P_1 + w_o) - N_2(2P_2 + w_o)\right)}{12w_oN_2(N_1 + N_2)(w_o + 3P_2 - P_1)}$$

$$n_2^* = kN_2 - \frac{B\sqrt{6}}{6w_o(N_1 + N_2)}$$

where:

$$A = \sqrt{(N_1)^2(2P_1 + w_o)^2 + (N_2)^2(2P_2 + w_o)^2 + 2N_1N_2\left[(w_o)^2 - 6(P_1)^2 - 6(P_2)^2 + 2P_2w_o + 2P_1(8P_2 + w_o)\right]}$$

$$B = \sqrt{k^2w_oN_2(N_1 + N_2)\left[3(N_1)^2(2P_1 + w_o) + N_1(N_2(18P_2 - 10P_1 + 4w_o) - 3A) + N_2(N_2(2P_2 + w_o) + A)\right]}$$

If consumer buy's 1's bundle and per-article from Firm 2:

Same (n_1*, n_2*) as above substituting in $P_1 = 0$.

Proof of nonexistence of Bertrand equilibrium when consumers are heterogeneous and both firms pure bundle:

We can derive of the reaction curves for duopolists when $N_1 = N_2 = 500$, $w_o = 1$ and $k \sim U[0, 1]$. [12]

$V_1 = V_2 = 148.15\ k$
$V_B = 192.456\ k$
$V_B - V_i = 44.3\ k$

Demand for Firm 1:

<u>if $P_2 > 148.15$</u> (no one will ever buy from Firm 2)

$$D(P_1) = \int_{\frac{P_1}{148.15}}^{1} f(k)dk$$

$$\Pi_1 = P_1 \left(1 - \frac{P_1}{148.15}\right) \Rightarrow \quad P_1^* = \$74.075,\ \Pi_1^* = \$37.04$$

<u>if $44.3 < P_2 \leq 148.15$</u> (No consumer will ever want to buy both firms' bundles)

if $P_1 > P_2 \Rightarrow D(P_1) = 0$

if $P_1 < P_2 \Rightarrow D(P_1) = \int_{\frac{P_1}{148.15}}^{1} dk$

$P_1^* = \text{Min}[\$74.075, P_2 - \varepsilon]$

<u>if $P_2 \leq 44.3$</u> (High k consumers may want to purchase both bundles)

a) if $P_1 > P_2 \Rightarrow D(P_1) = \int_{\frac{P_1}{44.3}}^{1} dk$ (only those k who buy both)

$$\Pi_1 = P_1 \left(1 - \frac{P_1}{44.3}\right) \Rightarrow \quad P_1^* = \$22.15,\ \Pi_1^* = \$11.08$$

b) if $P_1 < P_2 \Rightarrow D(P_1) = \int_{\frac{P_1}{148.15}}^{1} dk$,would like to price at \$74.075, but must

[12] This proof also holds for the first type of heterogeneity (and $w_o \sim U[0, 1]$) when firms are of identical size.

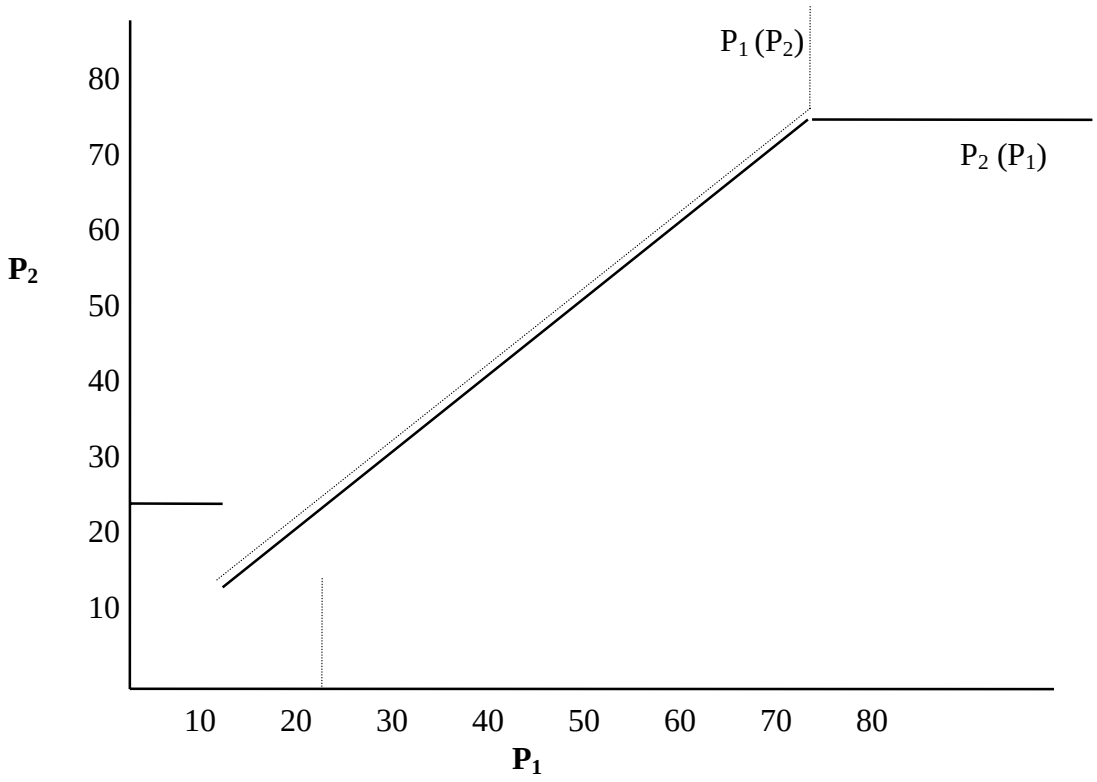
charge below P_2, so $P_1^* = P_2 - \varepsilon \Rightarrow \Pi_1 = (P_2 - \varepsilon)\ (1 - \frac{P_2 - \varepsilon}{148.15})$ [A1]

So, $P_1^* = P_2 - \varepsilon$ if [A1] > \$11.08 (as $\varepsilon \to 0$, $P_2 > \$12.06$)

Reaction function: $P_1(P_2)$ =

\$74.075	$P_2 > 74.075$
$P_2 - \varepsilon$	$12.06 < P_2 \leq 74.075$
\$22.15	$P_2 \leq 12.06$

(Here, the reaction functions of Firms 1 and 2 are symmetric since $N_1 = N_2$)



Since these reaction functions never cross, there does not exist a pure strategy Bertrand equilibrium.

Examples of Outcomes from Endogenous Bundle Choice Game:

a) $N_1 = N_2 = 500$, $k \sim U[0, 1]$

		Firm 2	
		Bundle	Unbundle
Firm 1	Bundle	($11.08, $11.08)	($18.62, $14.60)
	Unbundle	($15.04, $20.25)	($20.33, $20.93)

There is only one NE (Unbundle, Unbundle). It also results in the highest total profit. (Bundle, Unbundle) would result in a slightly higher social surplus.

b) $N_1 = 400$, $N_2 = 600$, $w_o \sim U[0, 1]$

		Firm 2	
		Bundle	Unbundle
Firm 1	Bundle	($3.44, $4.60)	($4.51, $3.66)
	Unbundle	($2.87, $6.59)	($4.01, $5.68)

There is one NE (Bundle, Bundle). It also maximizes social surplus. (Unbundle, Unbundle) would result in the highest total profit.

c) $N_1 = 400$, $N_2 = 600$, $k \sim U[0, 1]$

		Firm 2	
		Bundle	Unbundle
Firm 1	Bundle	($11.56, $15.32)	($15.36, $18.94)
	Unbundle	($11.50, $23.41)	($16.79, $24.70)

There is one NE (Unbundle, Unbundle). It also results in the highest total profit. (Bundle, Bundle) would result in the highest social surplus.

Table 2 Numerical Simulations: Monopoly owns the entire collection

N = 1000, k = .3, w_o ~U[0,1]

Description	P_b	P_a	Quantity	Profit	CS	Welfare	% of Optimum
Social Optimum	0	0	1 (126.8)	0	28.9	28.9	100
Pure Bundling	29.1		0.5	14.46	7.12	21.58	74.67
Pure Unbundling		0.27	42.36	11.44	8.95	20.39	70.55
Mixed Bundling	29.1	.51+	.5(0)	14.46	7.12	21.58	74.67

N = 1000, w_o =1 , k ~ U[0,1]

Description	P_b	P_a	Quantity	Profit	CS	Welfare	% of Optimum
Social Optimum	0	0	1 (211.3)	0	96.23	96.23	100
Pure Bundling	96.23		0.5	48.13	24.06	72.19	75.02
Pure Unbundling		0.47	98	46.06	25.04	71.1	73.89
Mixed Bundling	112.4	0.66	.35, 25.6	56.12	20.51	76.63	79.63

Table 3 Numerical Simulations: Both Firms Bundle

$N_1 = N_2 = 500$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	1	0	0	28.9	28.9	100
Monopoly	22.23+	22.22	0	0.5	0	11.11	5.56	16.67	57.68
Bertrand			No pure strategy equilibrium exists					0	
Stackleberg (1)	3.62	6.65	0.92	0.5	3.33	3.33	20.44	27.1	93.77

$N_1 = 400$, $N_2 = 600$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	1	0	0	28.9	28.9	100
Monopoly	19.7+	24.2	0	0.5	0	12.1	6.06	18.16	62.84
Bertrand			No pure strategy equilibrium exists					0	
Stackleberg (1)	3.8	9.1	0.9	0.51	3.44	4.6	18.42	26.46	91.56
Stackleberg (2)	4.7	9.1	0.87	0.5	4.07	4.66	17.63	26.36	91.21

$N_1 = N_2 = 500$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	1	0	0	96.23	96.23	100
Monopoly	74.1+	74.08	0	0.5	0	37.04	18.52	55.56	57.74
Bertrand			No pure strategy equilibrium exists					0	
Stackleberg (1)	12.06	22.15	0.92	0.5	11.08	11.08	68.06	90.22	93.75

$N_1 = 400$, $N_2 = 600$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	1	0	0	96.23	96.23	100
Monopoly	65.8+	80.8	0	0.5	0	40.4	20.2	60.6	62.97
Bertrand			No pure strategy equilibrium exists					0	
Stackleberg (1)	12.8	30.4	0.9	0.5	11.56	15.32	61.28	88.16	91.61
Stackleberg (2)	15.6	30.5	0.88	0.5	13.7	15.37	58.73	87.8	91.24

Table 4 Numerical Simulations: Firm 1 bundles, Firm 2 sells articles individually

$N_1 = N_2 = 500$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	50	0	0	28.9	28.9	100
Monopoly	17.2	0.25	0.5	12.71	8.6	3.18	8.96	20.74	71.76
Bertrand	8.2	0.09	0.65	38	5.35	3.42	17.89	26.66	92.25
Stackleberg (1)	9.5	0.1	0.58	38.2	5.53	3.83	16.71	26.07	90.21
Stackleberg (2)	8.2	0.09	0.65	38	5.35	3.42	17.89	26.66	92.25

$N_1 = 400$, $N_2 = 600$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	54.3	0	0	28.9	28.9	100
Monopoly	14.3	0.25	0.49	18.41	6.99	4.6	9.08	20.67	71.52
Bertrand	5.9	0.09	0.72	43.67	4.24	3.93	18.8	26.97	93.32
Stackleberg (1)	5.2	0.14	0.87	26.14	4.51	3.66	17.74	25.91	89.65
Stackleberg (2)	7.92	0.15	0.75	27.4	5.96	3.97	15.38	25.31	87.58

$N_1 = 600$, $N_2 = 400$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	44.17	0	0	28.9	28.9	100
Monopoly	24.3	0.48	0.5	0.04	12.1	0.02	6.01	18.13	62.73
Bertrand	10.8	0.09	0.61	31.9	6.59	2.87	16.78	26.24	90.80
Stackleberg (1)	12.4	0.1	0.54	32.3	6.75	3.23	15.54	25.52	88.30
Stackleberg (2)	10.8	0.09	0.61	31.9	6.59	2.87	16.78	26.24	90.80

Table 4 cont.

$N_1 = N_2 = 500$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	166.7	0	0	96.32	96.32	100
Monopoly	85.1	0.66	0.36	19.08	30.55	12.5	16.32	59.37	61.64
Bertrand	36.6	0.23	0.51	63.6	18.56	14.62	51.55	84.73	87.97
Stackleberg (1)	36.5	0.23	0.51	63.6	18.62	14.6	51.47	84.69	87.93
Stackleberg (2)	40.9	0.275	0.5	54.7	20.25	15.04	46.6	81.89	85.02

$N_1 = 400$, $N_2 = 600$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	181	0	0	96.32	96.32	100
Monopoly	66.3	0.435	0.23	59.46	14.98	25.87	25.8	66.65	69.20
Bertrand	30.75	0.25	0.49	75.4	15.25	18.86	50.45	84.56	87.79
Stackleberg (1)	31.1	0.255	0.51	74.3	15.36	18.94	49.9	84.2	87.42
Stackleberg (2)	32.97	0.285	0.5	67.33	16.35	19.19	46.86	82.4	85.55

$N_1 = 600$, $N_2 = 400$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	1	147.25	0	0	96.32	96.32	100
Monopoly	90.6	0.675	0.39	14.24	35.6	9.61	17.62	62.83	65.23
Bertrand	44.5	0.205	0.5	54.78	22.16	11.23	50.66	84.05	87.26
Stackleberg (1)	48.4	0.215	0.46	55.44	22.36	11.92	48.24	82.52	85.67
Stackleberg (2)	47.3	0.24	0.5	47.92	23.41	11.5	47.48	82.39	85.54

Table 5 Numerical Simulations: Both Firms Sell on Per-item Basis

$N_1 = N_2 = 500$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	50	50	0	0	28.9	28.9	100
Monopoly	0.27	0.27	21.19	21.19	5.72	5.72	8.96	20.4	70.59
Bertrand	0.134	0.134	35	35	4.69	4.69	16.45	25.83	89.38
Stackleberg (1)	0.17	0.15	28.4	35.2	4.83	5.29	14.75	24.87	86.06

$N_1 = 400$, $N_2 = 600$, $k = .3$, $w_o \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	44.17	54.3	0	0	28.9	28.9	100
Monopoly	0.27	0.27	16.95	25.43	4.58	6.87	8.96	20.41	70.62
Bertrand	0.133	0.15	30.15	37.6	4.01	5.64	15.85	25.5	88.24
Stackleberg (1)	0.135	0.15	29.7	37.87	4.01	5.68	15.79	25.48	88.17
Stackleberg (2)	0.145	0.175	30.14	33.03	4.37	5.78	14.6	24.75	85.64

$N_1 = N_2 = 500$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	166.7	166.7	0	0	96.32	96.32	100
Monopoly	0.47	0.47	49	49	23.03	23.03	25.04	71.1	73.82
Bertrand	0.296	0.296	67.94	67.94	20.11	20.11	45.33	85.55	88.82
Stackleberg (1)	0.32	0.305	63.53	68.62	20.33	20.93	43.14	84.4	87.62

$N_1 = 400$, $N_2 = 600$, $w_o = 1$, $k \sim U[0,1]$

Description	P_1	P_2	Q_1	Q_2	Profit 1	Profit 2	CS	Welfare	% of Optimum
Social Optimum	0	0	147.25	181	0	0	96.32	96.32	100
Monopoly	0.47	0.47	39.2	58.82	18.42	27.64	25.04	71.1	73.82
Bertrand	0.287	0.314	58.15	76.15	16.69	23.91	44.42	85.02	88.27
Stackleberg (1)	0.31	0.32	54.16	77.19	16.79	24.7	42.67	84.16	87.38
Stackleberg (2)	0.295	0.335	58.81	71.94	17.35	24.1	42.4	83.85	87.05

Table 6 Numerical Simulations: Bertrand Equilibrium for Mixed Bundling

$k = .3$, $w_o \sim U[0,1]$

N_1	N_2	P^*_1	P^*_2	Profit 1	Profit 2	CS	Welfare
500	500		No pure strategy equilibrium				
460	540		No pure strategy equilibrium				
400	600	($.16, $4.20)	($.14, $6.60)	2.99	4.84	19.58	27.41
300	700	($.18, $3.20)	($.25, $11.10)	2.6	6.41	17.23	26.24
200	800	($.19, $2.30)	($.32, $15.80)	1.72	8.62	14.68	25.02

$w_o = 1$, $k \sim U[0,1]$

N_1	N_2	P^*_1	P^*_2	Profit 1	Profit 2	CS	Welfare
500	500		No pure strategy equilibrium				
400	600		No pure strategy equilibrium				
300	700		No pure strategy equilibrium				
200	800	($.45, $8.00)	($.49, $62.00)	6.64	32.91	46.1	85.65
100	900	($.45, $4.00)	($.57, $86.00)	3.27	43.24	34.5	81.01

Table 7 Consumer Choice and Revenue Mix for Mixed Bundling Equilibrium

$k = .3$, $w_o \sim U[0,1]$

		Consumer Choices				Bundling Revenue (BR)		BR as % of Tot. Rev.	
N_1	N_2	1A	1B	2A	2B	Firm 1	Firm 2	Firm 1	Firm 2
400	600	0.04	0.71	0	0.74	$ 2.98	$ 4.84	100	100
300	700	0	0.8	0.02	0.58	$ 2.60	$ 6.41	100	100
200	800	0	0.74	0.01	0.55	$ 1.72	$ 8.62	100	100

$w_o = 1$, $k \sim U[0,1]$

		Consumer Choices				Bundling Revenue (BR)		BR as % of Tot. Rev.	
N_1	N_2	1A	1B	2A	2B	Firm 1	Firm 2	Firm 1	Firm 2
200	800	0.18	0.79	0.61	0.39	$ 6.32	$ 24.18	95	73
100	900	0.16	0.77	0.64	0.35	$ 3.10	$ 30.10	95	70

Table 8 Numerical Examples: Endogenous Collection Size

Cost: $C(N_i) = a N_i + b (N_i)^2$

Bertrand Outcomes

a	b	~U[0,1]	Bundle? Firm 1	Bundle? Firm 2	N_1*	N_2*	P_1	P_2	$Profit_1$	$Profit_2$	Consumers' Surplus	Welfare	% of Optimum
0.006	0.00003	no	yes	yes	492	492	13.08	13.08	2.86	2.86	30.65	36.37	81.5
0.006	0.00003	no	no	no	282	282	0.3	0.3	2.73	2.73	15.28	20.74	46
0.008	0.00003	no	yes	yes	459	459	12.19	12.19	2.21	2.21	28.56	32.98	80
0.006	0.00005	no	yes	yes	295	295	7.84	7.84	1.72	1.72	18.38	21.82	81.5
0.006	0.00003	k	no	no	528	528	0.3	0.3	9.73	9.73	47.77	67.23	49.6
0.008	0.00003	k	no	no	500	500	0.3	0.3	8.64	8.64	45.2	62.48	48.2
0.006	0.00005	k	no	no	318	318	0.3	0.3	5.84	5.84	28.74	40.42	49.7
0.006	0.00003	w_o	yes	no	125	30	2.9	0.1	0.33	0.05	1.97	2.35	27
0.006	0.00003	k	yes	no	495	448	36.6	0.22	7.76	4.32	48.51	60.59	44.7

Social Optimum

a	b	~U[0,1]	N_1*	N_2*	$Profit_1$	$Profit_2$	Consumers' Surplus	Welfare
0.006	0.00003	no	862	862	-27.5	-27.5	99.56	44.56
0.008	0.00003	no	829	829	-27.25	-27.25	95.7	41.2
0.006	0.00005	no	517	517	-16.5	-16.5	59.7	26.7
0.006	0.00003	k	1504	1504	-76.9	-76.9	289	135.2
0.008	0.00003	k	1470	1470	-76.6	-76.6	283	129.8
0.006	0.00003	w_o	381	381	-6.64	-6.64	22	8.72
0.008	0.00003	w_o	348	348	-6.41	-6.41	20.08	7.26

Monopoly Outcomes

a	b	~U[0,1]	Bundle? Firm 1	Bundle? Firm 2	N_1*	N_2*	P_1	P_2	Profits Firm 1	Profits Firm 2	Consumers' Surplus	Welfare	% of Optimum
0.006	0.00003	k	no	no	670	670	0.47	0.47	13.38	13.38	33.55	60.31	45
0.008	0.00003	k	no	no	635	635	0.47	0.47	12.08	12.08	31.8	55.96	43
0.006	0.00003	k	yes	no	770	480	113.6	0.69	23.49	1.21	22.3	47	35
0.006	0.00003	w_o	no	no	91	91	0.27	0.27	0.247	0.247	1.63	2.124	24
0.008	0.00003	w_o	no	no	57	57	0.27	0.27	0.1	0.1	1.02	1.22	17
0.006	0.00003	w_o	yes	no	145	45	5.1	0.48	1.02	-0.33	1.24	1.93	22