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Natural Units Conversions and Fundamental Constants

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Abstract: Conversions are listed between basis units of natural units system where $\hbar = c = 1$. Important fundamental constants are given in various equivalent natural units based on GeV, seconds, and meters.

Natural units basic conversions

Natural units are defined to give $\hbar = c = 1$. All quantities with units then can be written in terms of a single base unit. It is customary in high-energy physics to use the base unit GeV. But it can be helpful to think about the equivalences in terms of other base units, such as seconds, meters and even femtobarns.

The conversion factors are based on various combinations of $\hbar$ and c (Olive 2014). For example

$$1 = \hbar \quad = \quad 6.58211928(15) \times 10^{-25}\,\mathrm{GeV\,s}, \quad \text{and} \tag{1}$$

$$1 = c \quad = \quad 2.99792458 \times 10^{8}\,\mathrm{m\,s^{-1}}. \tag{2}$$

From this we can derive several useful basic conversion factors and

$$1 = \hbar c \quad = \quad 0.197327\,\mathrm{GeV\,fm}, \quad \text{and} \tag{3}$$

$$1 = (\hbar c)^2 \quad = \quad 3.89379 \times 10^{11}\,\mathrm{GeV^2\,fb} \tag{4}$$

where I have not included the error in $\hbar c$ conversion but if needed can be obtained by consulting the error in $\hbar$. Note, the value of c has no error since it serves to define the meter, which is the distance light travels in vacuum in 1/299792458 of a second (Olive 2014). The unit fb is a femtobarn, which is 10^{-15} barns. A barn is defined to be $1\,\mathrm{barn} = 10^{-24}\,\mathrm{cm}^2$. The prefexes letters, such as p on pb, etc., mean to multiply the unit after it by the appropropriate power: femto (f) 10^{-15}, pico (p) 10^{-12}, nano (n) 10^{-9}, micro (μ) 10^{-6}, milli (m) 10^{-3}, kilo (k) 10^{3}, mega (M) 10^{6}, giga (G) 10^{9}, terra (T) 10^{12}, and peta (P) 10^{15}.

In the next section we list the natural units conversions of several fundamental constants, but as one example of the utility of the above equation we can convert an instantaneous luminosity of $\mathcal{L} = 10^{34}\,\mathrm{cm^{-2}\,s^{-1}}$, which is LHC design luminosity, into an integrated luminosity

$\int \mathcal{L} dt$ over a year.

$$\begin{aligned}\mathcal{L}\cdot(1\text{ yr}) &= (10^{34}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1})\cdot(1\,\mathrm{yr})\cdot\left(\frac{3.154\times 10^{7}\,\mathrm{s}}{1\,\mathrm{yr}}\right)\cdot(1.97327\times 10^{-14}\,\mathrm{GeV\,cm})^2 \\ &\quad \times\left(\frac{1}{3.89379\times 10^{11}\,\mathrm{GeV}^2\,\mathrm{fb}}\right) = 315\,\mathrm{fb}^{-1} \end{aligned} \tag{5}$$

Of course no collider runs a full year at its design peak luminosity and it is conventional to assume that full year lumonisity is about a third of integrated peak luminosity, $\sim 100\,\mathrm{fb}^{-1}$. It is for this reason one sometimes hears an experimentalist say, "One collider year is 10^7 seconds."

Fundamental constants in natural units

It is convenient at times to know the following fundamental constants in natural units with basis GeV, seconds, and meters: electron mass m_e, proton mass m_p, Planck scale $M_{\mathrm{Pl}} = G_N^{-1/2}$, Fermi scale G_F, and the critical density of the universe scale Λ_c.

$$M_{\mathrm{Pl}} = 1.2209\times 10^{19}\,\mathrm{GeV} = \frac{1}{1.616\times 10^{-35}\,\mathrm{m}} = \frac{1}{5.391\times 10^{-44}\,\mathrm{s}} \tag{6}$$

$$\begin{aligned} G_F &= 1.1663797(6)\times 10^{-5}\,\mathrm{GeV}^{-2} = \left(\frac{1}{292.81\,\mathrm{GeV}}\right)^2 \\ &= (6.739\times 10^{-4}\,\mathrm{fm})^2 = 4.542\,\mathrm{nb} \end{aligned} \tag{7}$$

$$m_e = 0.511\,\mathrm{MeV} = \frac{1}{386\,\mathrm{fm}} = \frac{1}{1.288\times 10^{-21}\,\mathrm{s}} \tag{8}$$

$$m_p = 0.938\,\mathrm{GeV} = \frac{1}{0.210\,\mathrm{fm}} = \frac{1}{7.017\times 10^{-25}\,\mathrm{s}} \tag{9}$$

$$\begin{aligned} \Lambda_c &= 1.05375(13)\times 10^{-5}\,h^2\,\mathrm{GeV\,cm}^{-3} = h^2(3.00\times 10^{-12}\,\mathrm{GeV})^4 \\ &= h^2(3.00\times 10^{-3}\,\mathrm{eV})^4 = h^2\left(\frac{1}{65.78\,\mu\mathrm{m}}\right)^4 = h^2\left(\frac{1}{2.194\times 10^{-13}\,\mathrm{s}}\right)^4 \end{aligned} \tag{10}$$

where $h = 0.673(12)$ is reflective of the uncertainty in the Hubble constant (Olive 2014).

References

K.A. Olive et al. (Particle Data Group). *Chin. Phys. C*, 38, 090001 (2014).