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MODULAR STRUCTURAL COMPONENT DESIGN USING THE FIRST ORDER ANALYSIS AND DECOMPOSITION-BASED ASSEMBLY SYNTHESIS

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ABSTRACT

This paper discusses an automated method for designing modular components that can be shared within multiple structural products, such as automotive bodies for sibling vehicles. The method is an extension of the concept of decomposition-based assembly synthesis. A beam-based topology optimization method, originally developed for First Order Analysis (FOA) of the automotive body structures, is utilized in order to obtain the "base" structures subject to decomposition. It is expected that the method will facilitate the early decisions on module geometry in automotive body structures, by enhancing the capability of the FOA system. Several case studies with two-dimensional structures are reported to demonstrate the effectiveness of the proposed method. The results indicate that two structures optimized for a similar, but slightly different boundary loading conditions are successfully decomposed to contain a component that can be shared by the structures. Several Pareto optimal decompositions are presented to illustrate the trade-offs among multiple decomposition criteria, with different weights for each objective function.

Keywords: Modularity, Structural Optimization, Assembly Synthesis, First Order Analysis (FOA), Genetic Algorithms.

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INTRODUCTION

As the global competition increases rapidly, manufacturing industry struggles to bring well-designed and well-manufactured products to market in a timely fashion. Although product design incurs only a small fraction of the total product cost, the decisions made during the design phase account for a significant portion of this cost and prove crucial to the success or failure of the product. The time and cost involved in making engineering changes, in-process adjustments and the like increase rapidly as the product development process evolves. Early anticipation and avoidance of manufacturing and assembly problems can have a huge impact in reducing the product development time (Gupta *et al.*, 1997; Mantripragada and Whitney, 1998).

Increasing research attention is being directed toward the integration of engineering design and manufacturing to achieve the efficient and timely product development . These attempts have led to the evolution of design for manufacturing (DFM) and design for assembly (DFA) methodologies. These involve simultaneously considering design goals and manufacturing and assembly constraints in order to identify and alleviate problems while the product is being designed; thereby reducing the lead time for product development and improving product quality.

During conceptual design, teams of designers generally begin to develop a new product by sketching its general shape on paper. This "back of the envelope" approach is key aspect of the creative thought process. Often, if the manufacturing engineers noticed any manufacturing-related problems while examining the blueprints of the product design, they would notify the design team and the design would be sent through another iteration. To expedite these time-consuming iterations, as a tool for the engineers during the brain-storming period, this project, based on the earlier work in the Discrete Design Optimization Laboratory at the University of Michigan, aims to achieve a systematic decomposition process to carry out assembly synthesis. Focusing on structural products; which are generally manufactured through assembly of various components which have simpler geometries than the end product; the decision of which components are better to assemble together to achieve a certain end product is defined as *assembly synthesis* (Saitou and Yetis, 2000; Yetis and Saitou, 2000; Cetin and Saitou, 2001).

The presented approach intends to provide the designer with feedback about possible decompositions prior to the detailed design phase. The developed software is capable of evaluating several design problems simultaneously to end up with maximum modularity of the end products, in addition to the optimization of structural strength and assemblability in each design.

A beam-based topology optimization method, originally developed for First Order Analysis (FOA) of the automotive body structures (Nishigaki *et al.*, 2000), is utilized in order to obtain the "base" structures subject to decomposition. It is expected that the method will facilitate the early decisions on module geometry in automotive body structures, by enhancing the capability of the FOA system. First, multiple structures with similar, but slightly different boundary and loading conditions are generated using a FOA with a modified ground structure for topology optimization, which ensures the topology synthesis with non-overlapping beams. A structure is represented as a vector of diameter of beams in the ground structure, and the strain energy in the structure is computed using the Finite Element Method. CONLIN (Convex Linearization) method (Fleury and Braibant, 1986) is used to discover an optimal structure using min-max approach (Suzuki and Kikuchi, 1991) for desired structural criteria such as displacement at a specified location.

This study aims to present the collaborative project to unify the separately established methods of FOA and assembly synthesis to end up with a powerful tool for modularity analysis. The paper is organized so as to summarize each method alone, along with the integration efforts: Section 2 is devoted to the related work on first order analysis and modularity considerations in mechanical design; Section 3 gives the brief mathematical background for FOA and presents assembly synthesis as a formal optimization problem; Section 4 presents the case studies and Section 5 concludes the paper with discussions on the results of the case studies and the future directions of the research.

PREVIOUS WORK

First Order Analysis

The concept of Computer Aided Engineering (CAE) was first proposed by J. Lemon (Lemon, 1980) at SDRC, and has been widely accepted in automotive industries. Recently CAE numerically estimates the performance of automobiles and proposes alternative ideas that lead to the higher performance. However, most automotive designers cannot directly utilize CAE since specific well-trained engineers are required to achieve sophisticated operations. Moreover, CAE requires a huge amount of time and many modelers to construct an analysis model. In order to overcome these problems and to quickly obtain qualitative designs, a new concept of CAE, *First Order Analysis* (FOA) is proposed (Nishigaki *et al.*, 2000). The basic ideas include (1) graphic interfaces for automotive designers; (2) use of sophisticated formulations based on the theory of mechanics of material, (3) the topology optimization method. However, the quantitative and accurate evaluation of the performance criteria must be performed by the usual CAE in the evaluation division.

Topology optimization is useful in the case where the locations of frames are determined in an automotive body. This approach using truss elements is not new (details can be found in (Bendsøe, 1995)). The earliest effort was made by Dorn et al. (Dorn et al., 1963). In the FOA, a beam element is treated as a design structure. This is because (1) a beam element has six degrees of freedom, and it provides an optimal solution in all loading cases including a moment loading, (2) it is practical rather than a truss element, and (3) sizing optimization can simultaneously be performed because the cross-sectional shape in a element can be designed by introducing more than one design variable.

Assembly Synthesis and Modular Design

Assembly synthesis method has some common roots with assembly sequence planning applications, which has been an active research field recently. Most algorithms cited in the literature solve assembly problems by graph searching, but require detailed part geometry information, which is usually not available in the early design stage. A close idea to the assembly synthesis is presented by Wang and Bourne; they describe a formulation to automatically generate some features for a sheet metal bending process as the design progresses. After the designs are complete, an automatic process planning system uses the features and generates new ones to aid the production of plans with near-minimum manufacturing costs (Wang and Bourne, 1997). A more detailed literature survey on the solution of assembly problems was given in an earlier publication (Saitou and Yetis, 2000; Yetis and Saitou, 2000).

Modularity is commonly associated with the division of products into smaller building block, *modules*, and involves architecting a family of products that share inter-changeable components. The benefit of part commonality is that, the effort and resources invested for the design of one module are not considered again, if the component fits another product; resulting in a considerable reduction in lead-times (Stake, 1999).

Ishii discusses the impacts of modularity in product design on every stage of the product life-cycle, as wells as implications such as serviceability and recyclability in terms of disassembly, separation, repair, and reprocessing. He introduces a set of metrics and design charts to be used in design for variety approach (Ishii, 1998). Newcomb *et al.* developed a method employing a commonality table for the entire product family to identify the effects of a product platform (Newcomb *et al.*, 1998). Kota *et al.* follow a similar approach and present an objective measure called the Product Line Commonality Index, to capture the level of component commonality in a product family. They suggest seeking functional design features based on configuration similarities (*e.g.*, geometric shapes), kinematic similarities (*e.g.*, joint types and motion), actuation similarities and the like (Kota *et al.*, 2000). In order to take customer preferences into account, Yu *et al.* introduce a customer need basis for defining the architecture of a portfolio of products (Yu *et al.*, 1999). Nelson *et al.* formulate a multicriteria optimization problem to assess modularity based design decisions quantitatively. They analyze Pareto sets that correspond to various derivative products, to examine the trade-offs that emerge when part commonality conflicts with the individual performances of the products (Nelson *et al.*, 1999).

MODULAR STRUCTURAL COMPONENT DESIGN

The developed method for modular structural component design proceeds in two steps:

1. Multiple structures with similar, but slightly different boundary and loading conditions are generated using a FOA with a modified ground structure for topology optimization, which ensures the topology synthesis with non-overlapping beams.

2. Assuming spot-welding as a joining method, the resulting optimal structures are decomposed to a desired number of components based on graph decomposition with the following criteria: 1) structural strength of each decomposed structure, and 2) similarity of a component in decomposed structures that can be used as a module within structures.

Structural Topology Design via First Order Analysis

The key idea of topology optimization is based on the ground structure approach and the minimization of the mean compliance in order to maximize the global structural stiffness. In the ground structure approach, first, a set of fixed nodal points and all possible connections using beam elements are provided. But to obtain the "base" structures subject to decomposition, a modified ground structure approach that ensures the topology synthesis with non-overlapping beams is used as shown in Figure 1. We seek to find an optimal beam configuration by eliminating unnecessary beams using an optimization scheme.

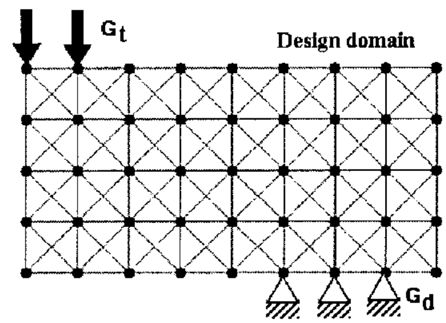


Figure 1. Modified ground structure approach.

Let ρ_{vi} denote the normalized volumetric density of the i-th beam, i=1,...,n, where ρ_{vi} is bounded from 0 to 1, and n is the number of beams. Let us regard it as a design variable of each element. In order to eliminate a beam, ρ_{vi} must be 0, whereas for a solid beam, ρ_{vi} must be 1. Volume V_i of the i-th beam is then represented as,

$$V_i = \rho_{vi} A_i L_i \tag{1}$$

where A_i and L_i are the maximum cross-sectional area and length of the i-th beam, respectively.

The beam properties such as the second moment of inertia depends on the shape of its cross-section. Here, the circular cross-section is assumed. Let d_i denote the diameter of the i-th beam, then we have:

$$d_i = \left(\frac{4\rho_{vi}A_i}{\pi}\right)^{\frac{1}{2}} \tag{2}$$

Therefore, the second moment of inertia with respect to the y and z axis in the local coordinate system, I_y and I_z, is obtained by

$$I_y = I_z = \frac{\pi}{64} d_i^4 = \frac{\pi}{64}\left(\frac{4\rho_{vi}A_i}{\pi}\right)^2 = \frac{\rho_{vi}^2 A_i^2}{4\pi} \tag{3}$$

and the polar moment of inertia, J, is also obtained by

$$J = \frac{\pi}{32} d_i^4 = \frac{\pi}{32}\left(\frac{4\rho_{vi}A_i}{\pi}\right)^2 = \frac{\rho_{vi}^2 A_i^2}{2\pi} \tag{4}$$

Using the physical properties above, the element stiffness matrix is constructed.

Consider that an elastic three-dimensional structure, Ω, is fixed at boundary Γ_d and is subjected to a load representing a force vector $\boldsymbol{f}$ at boundary Γ_t. Body forces applied to the structure are assumed to be ignored for simplicity in the formulation. Let $\boldsymbol{u}$ be the displacement vector due to the applied force. Then, the mean compliance l (Bendsøe and Kikuchi, 1988) defined by:

$$l = \boldsymbol{f} \cdot \boldsymbol{u} = \boldsymbol{f}^T \boldsymbol{u} \tag{5}$$

is interpreted as the measure of stiffness at boundary Γ_t. That is, by minimizing or decreasing l, we can obtain sufficient stiffness. Since the displacement field satisfies the following equilibrium equation:

$$\boldsymbol{Ku} = \boldsymbol{f} \tag{6}$$

where $\boldsymbol{K}$ is the global stiffness matrix, we have

$$l = \boldsymbol{f}^T \boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{Ku} \tag{7}$$

Taking the derivative of Eq. (7), i.e., the sensitivity, with respect to a design variable a, yields,

$$\frac{\partial l}{\partial a} = \frac{\partial \boldsymbol{u}^T \boldsymbol{Ku}}{\partial a} = \frac{\partial \boldsymbol{u}^T}{\partial a} \boldsymbol{Ku} + \boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial a} \boldsymbol{u} + \boldsymbol{u}^T \boldsymbol{K} \frac{\partial \boldsymbol{u}}{\partial a} \tag{8}$$

The equation above is simplified using the differentiation of Eq. (6) as,

$$\frac{\partial l}{\partial a} = -\boldsymbol{u}^T \frac{\partial \boldsymbol{K}}{\partial a} \boldsymbol{u} \tag{9}$$

Suppose that the stiffnesses of a structure due to force vector $\boldsymbol{f}^k$ at boundary Γ_t^k, k=1,..., m are to be maximized with a total volume constraint of beam elements. The structural optimization problem is then formulated as follows:

$$\underset{\rho_{vi}}{\text{minimize}} \; l^k = \boldsymbol{f}^{k^T} \boldsymbol{u} = \boldsymbol{u}^{k^T} \boldsymbol{Ku}^k \quad for\ k=1,...m \tag{10}$$

subject to

$$V = \sum_{i=1}^{n} \rho_{vi} A_i L_i \le V^U \tag{11}$$

$$0 \le \rho_{vi} \le 1 \quad for\ i=1,...n \tag{12}$$

$$\boldsymbol{Ku}^k = \boldsymbol{f}^k \quad for\ k=1,...m \tag{13}$$

where V^U is the total volume constraint of beam elements, $\boldsymbol{u}^k$ is the displacement vector due to $\boldsymbol{f}^k$. Several methods (Koski, 1993) proposed to deal with multi-objective problems. Among them, the weighting method has been employed most commonly because of its convenience of formulation. It is, however, usually difficult for designers to determine an appropriate weighting coefficient for each objective function. In this paper, we formulate the multi-objective function based on the min-max approach (Suzuki and Kikuchi, 1991) as follows:

$$\underset{\rho_{vi}}{\text{minimize}} \; \max_{k=1,...,m} l^k \tag{14}$$

Figure 2 shows a flowchart of the optimization procedure. First, global stiffness matrix $\boldsymbol{K}$ is constructed. Next, the sensitivity of $\boldsymbol{K}$ with respect to ρ_{vi}, i=1,...,n, is calculated. In the third step, the equilibrium equation in Eq. (6) is solved. In the fourth step, the mean compliances in Eq. (10) and the total volume constraint in Eq. (11) are computed. In the fifth step, the sensitivities of mutual compliances and the total volume with respect to design variables are computed if the objective function is not converged. The design variables are updated using these sensitivities by CONLIN (Convex Linearization) (Fleury and Braibant, 1986).

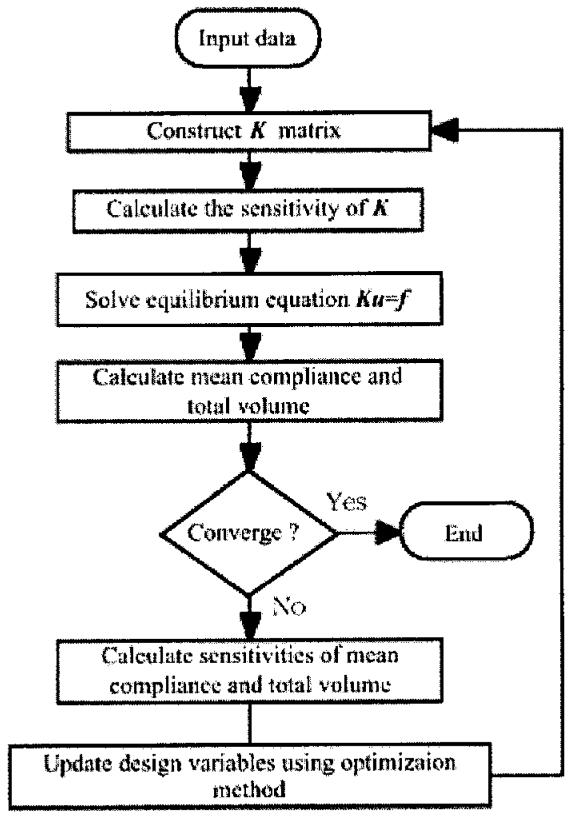
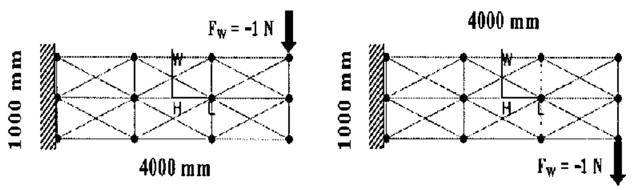


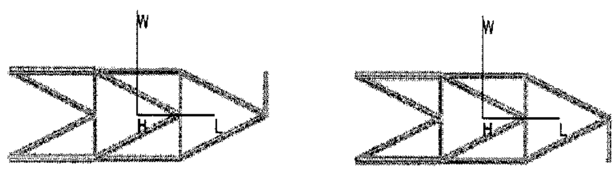
Figure 2. Flowchart of optimization procedure.

As an illustrative example, Figure 3 (a) shows the design domain for a cantilever where boundary conditions and specifications are indicated; in this problem, two loading cases are considered. The total volume constraint of beam elements V^u is set to 20% of the volume of the whole design domain. Figure 3 (b) shows the optimal structures for each loading case.



Case (1) Case (2)

(a) Design domain and boundary conditions



Case (1) Case (2)

(b) Optimal designs

Figure 3. Design of cantilevers for the base structures for decomposition.

Assembly Synthesis via Product Topology Graph Decomposition

In the assembly synthesis stage, structures obtained via FOA are decomposed automatically into an assembly consisting of multiple structural members with simpler geometries. There are two main steps in the process developed:

1. The topology of the problem is examined and the results of the FOA are stored; a product topology graph is then developed automatically.
2. The product topology graph is decomposed into subgraphs by using a genetic algorithm to generate a decomposition of the product with chosen mating features.

The optimal decomposition can be posed as a graph partitioning problem. The members of the structure are mapped to nodes and the intersections are mapped to multiple edges since they can be joining more than two members. The problem can be defined as: given the topology graph of the structure, obtain the partition representing the optimal decomposition and the mating feature for each joint, subject to a cost function evaluating the decomposition quality.

It is decided that joining method at every joint is assigned as spot weld in the current problem and the only joint feature considered is the weld angle which is chosen from discrete set of possible values. When assemblability is considered, the similarity of weld angles and the number of welds in the decomposition are taken into account. Obviously, lower number of welds and similar weld angles result in higher assemblability.

The modularity criteria proposed in this work is implemented by analyzing two structures at a time, and assessing the similarity of the disconnected components to point at a probable part commonality. A term is added to the objective function to favor the decompositions that result at: a) components with similar stress states, represented by the joint angles, b) components that are geometrically similar to each other, by considering the lengths and thicknesses of their corresponding members, or by using an equivalent measure of shape similarity. Also, before evaluating the cost function component related to modularity, it is certified that the subgraphs of the components to be shared are *isomorphic*; note that this is a necessary but not sufficient condition for two structures to be assembled in the same way.

Thus the final objective function attempts to find a solution that brings about two structures with maximum structural strength, maximum assemblability, and one or more components that can be shared by the both designs. In this project, the assembly synthesis method will be tested by using two structural design problems, as given in Figure 3: note that the only difference between (a) and (b) is the application point of the concentrated force F_w.

Definition of the design variables

Let the members of the structure be mapped to the nodes of the product topology graph and the intersections be mapped to the edges[1]. So the whole structure can be represented as $G=(V, E)$ with a node set V and an edge set E. The problem of optimal decomposition becomes one of finding a partition P of the node set V such that the objective function, $c(P)$, is maximized. Let F be a set of possible mating angles at the welded joints. For the ease of formulation, a partitioning of G can be represented by a vector $\boldsymbol{x}=(x_i)$ of a binary variable x_i representing the presence of edge e_i in the decomposition defined by the partitioning P. It is obvious that $i=1,\ldots,|E|$ since there are $|E|$ edges in the topology graph. Similarly, another vector $\boldsymbol{y}=(y_i)$ is defined to store the mating features for each edge e_i; note that domain of $\mathbf{y}$ depends on the model of the joint represented by the edge, which in this case is F.

Definition of the constraints

The constraint on the vector $\boldsymbol{x}$, which represents the presence of edges, is the following:

$$\text{COMPONENTS}(\text{GRAPH}(\boldsymbol{x})) = k \tag{15}$$

where

[1] LEDA library developed at the Max-Planck Institute of Computer Science (http://www.mpi-sb.mpg.de/LEDA/) is used for the graph algorithms.

- GRAPH(x) returns the graph after the edges with $x_i = 0$ in vector x, have been removed from the original topology graph,
- COMPONENTS(G) returns the number of disconnected components in graph G,
- k denotes the desired number of components specified by the user.

The constraint on vector **y** is as follows:

$$y_i \in F \tag{16}$$

where F is the set of mating angles at which spot welds can be applied at the joints. One element of set F represents the case for no weld at the corresponding joint.

Another constraint is imposed on the combination of the vectors $\boldsymbol{x}$ and $\boldsymbol{y}$ in the following way:

$$\text{IS\_CONNECTED(COMBINED\_GRAPH}(\boldsymbol{x}, \boldsymbol{y})) = 1 \tag{17}$$

where

- IS_CONNECTED(G) is a function which returns 1 if the graph G is connected and returns 0 otherwise.
- COMBINED_GRAPH($\boldsymbol{x}$,$\boldsymbol{y}$) is a function that returns a graph which consists of the nodes of the original graph and the edges in vectors $\boldsymbol{x}$, $\boldsymbol{y}$. This constraint ensures that the combination of the decomposition given by vector $\boldsymbol{x}$ and the mating angles given by vector $\boldsymbol{y}$ constitutes a structure which has the same connectivity as the original disconnected structure.

Definition of the objective function

Objective function will evaluate a decomposition according to the following criteria:

- Reduction of structural strength due to introduction of joints to be spot-welded.
- Assemblability of the decomposed structures.
- The maximum modularity of the structures.

To evaluate the decomposition according to the structural strength criteria, the normal stress at the joints and the area on which the normal stress acts are calculated.

While assessing the decomposition with respect to the assemblability criteria, the similarity of weld angles and the number of welds in the decomposition are taken into account. Obviously, lower number of welds and similar weld angles result in higher assemblability.

These criteria result in the following objective function component for structural considerations:

$$f_S(x, y) = w_1 \sum_{i=1}^{N_{welds}} \left(F_i\right) + w_2 \sum_{i=1}^{N_{welds}} \sum_{j=i+1}^{N_{welds}} \left(\theta_i - \theta_j\right)^2 + w_3 N_{welds} \tag{18}$$

The variables are defined as follows:

$x=(x_i)$	x_i is a binary variable representing the presence of edge e_i in subset x
$y=(y_i)$	y_i is discrete variable representing the choice of weld angle at joint i
w_i	weight of i^{th} criteria in the objective function
N_{welds}	total number of welds in the decomposed structure
θ_i	weld angle with respect to vertical direction at joint i
F_i	the force normal to the weld area at joint i

As the second part of the objective function, the cost function for modularity is incorporated to evaluate the following two attributes of the components to be shared between the structures:

1. Similarity in stresses which the components are subject to: this condition is simply implemented by maintaining that joint angles of the components should be close to each other.

2. Similarity in shapes of the components in a given (user-specified) tolerance: this attribute is checked by comparing the components with respect to their areas.

Note that this procedure requires that all components that come out of the decomposition process of one structure be compared with the components in the second design problem. However, probably only a few of the components at each iteration will have the same number of members assembled in a similar manner. Thus, before evaluating how similar two components are, it is convenient to test if the corresponding subgraphs are *isomorphic*: the modularity cost function should return a large number if no components are found to be *isomorphic*, and if this check is passed, then the similarity measure can be applied. Considering the computational overhead of this check, a simple approximation, actually a necessary but not sufficient condition is utilized in the software: it is required that the components have an equal number of nodes and edges to be shared. A fast *graph isomorphism* check algorithm will be employed for more complex design problems in the future work.

Thus the modularity component of the objective function is defined conditionally to be:

$$
f_m(x_1,y_1,x_2,y_2)=\begin{cases}\textbf{if } \text{Is\_Isomorphic}(g_1,g_2)=\text{FALSE}, \\ \textbf{return } (\text{a large number}). \\ \\ \textbf{if } \text{Is\_Isomorphic}(g_1,g_2)=\text{TRUE}, \\ \textbf{return } w_5 \sum_{i=1}^{N^c_{welds}} ((\theta_1)_i-(\theta_2)_i)^2+w_6 h(g_1,g_2). \end{cases} \tag{19}
$$

where

- g_1 and g_2 are two subgraphs representing components resulting from the decomposition of structure 1 and structure 2 respectively,
- w_5 and w_6 are the weights for the corresponding criteria,
- $(\theta_1)_i$ and $(\theta_2)_i$ are the weld angles at joint i of each component,
- N^c_{welds} is the number of welds in the shared components,
- Is_Isomorphic(g_1,g_2) is a function that returns TRUE if subgraphs g_1,g_2 are *isomorphic*, FALSE otherwise. For the time being the function only checks if the two subgraphs have the same number of nodes and edges.
- h(g_1,g_2) is a function that returns a measure of geometric similarity between the components. This measure is realized by the calculation of first moments of component areas with respect to the centroids; so the similarity is assessed in a rotationally invariant way.

Note that before $f_m(x_1,y_1, x_2,y_2)$ returns a cost at an iteration, all components, i.e. all subgraphs are examined, and only if none of them are *isomorphic* a large number is returned to introduce a penalty for lack of part commonality. In a similar manner, if more than one component in each structure match with others, the similarity measures are added up to favor the sharing of several components among the products.

The constraints and objective function combine to give the following optimization problem:

$$
\text{minimize } f(x_1,y_1,x_2,y_2)=f_s(x_1,y_1)+f_s(x_2,y_2)+f_m(x_1,y_1,x_2,y_2)
$$

subject to

$$
\begin{aligned}
&(x_1)_i \in \{0,1\}, \quad i=1,\ldots,|E_1| \\
&(x_2)_i \in \{0,1\}, \quad i=1,\ldots,|E_2| \\
&(y_1)_i \in F, \quad i=1,\ldots,|E_1| \\
&(y_2)_i \in F, \quad i=1,\ldots,|E_2| \\
&\text{COMPONENTS}(\text{GRAPH}(x_1))=k_1 \\
&\text{COMPONENTS}(\text{GRAPH}(x_2))=k_2 \\
&\text{IS\_CONNECTED}(\text{COMBINED\_GRAPH}(x_1,y_1))=1 \\
&\text{IS\_CONNECTED}(\text{COMBINED\_GRAPH}(x_2,y_2))=1
\end{aligned} \tag{20}
$$

Optimization Method

Since graph partitioning problems are NP-complete, an exact solution requires exponential computation. Noting the computational overhead, and taking into account the high non-linearity of the cost function as well, genetic algorithms (GA), which are regarded as a compromise between random and informed search methods, and which have proved very efficient in the solution of discrete optimization problems, is conveniently used in this project.

The decomposition problem is to be solved by using a steady-state GA. Empirical advantages of steady-state GA are that it prevents premature convergence of population and reaches an optimal solution with fewer number of fitness evaluations (Saitou and Yetis, 2000).

Each solution is encoded in a chromosome in the following way: The chromosome is of length 2|E| where |E| is the number of the edges in the graph. First |E| genes carry binary information about which edges of the topology graph are kept and which are removed to produce a decomposition. If the i^{th} element of the chromosome is 0, it means that this edge has been cut in this particular decomposition represented by this chromosome.

For this study, the possible mating angles have been chosen as −45, 0, 45, 90 degrees from the vertical and map to gene values of 1, 2, 3, 4, respectively, as given in Figure 4. A gene value of zero means no weld at that intersection.

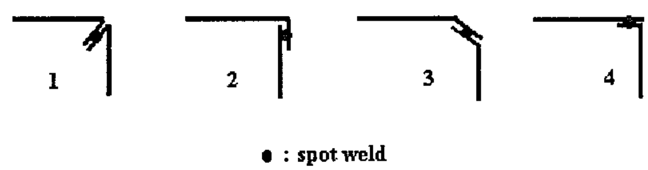


Figure 4. Possible mating angles at the joints

The resulting optimal partitioning and the corresponding decomposition for the cantilevers are given in Figure 5 as an example. The optimal assembly for 4 components is aimed, and the results indicate that the 3-component substructure at the right is shared by the configurations.

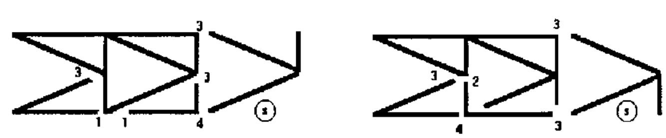


Figure 5. Optimal decomposition for the example cantilevers.

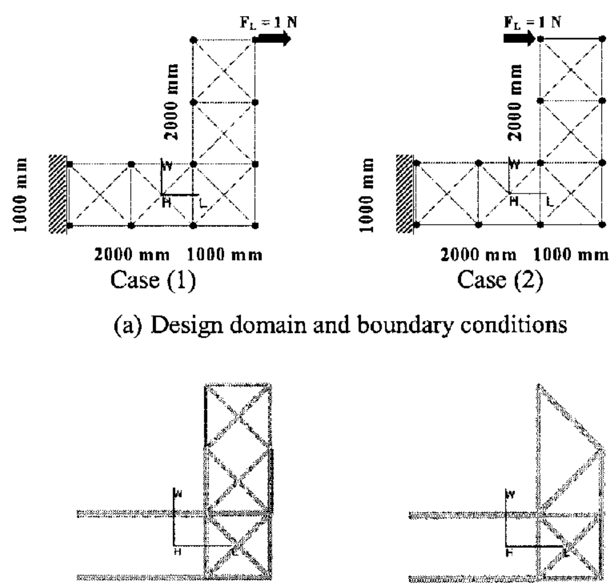
CASE STUDIES

Two case studies with two-dimensional structures are presented to demonstrate the effectiveness of the proposed method. For each example, multiple structures with similar, but slightly different boundary and loading conditions are generated to form the base structure for decomposition. In these examples, the initial value of ρ_{vi}, i=1,..., the number of beams, is set to 0.0001, and the upper limit of diameter d_i, i=1,..., the number of beams, is set to 100. For the first example, the total volume constraint of beam elements V^u is set to 30% of the volume of the whole design domain. For the second example, V^u is set to 20%.

L-lever

Figure 6 (a) shows the design domain for L-lever where boundary conditions and specifications are indicated. Figure 6 (b) shows the optimal structures for each loading case. In these figures, the unnecessary beams whose diameters are below 30% of the upper limit of diameter are eliminated.

Assembly synthesis software is run to simultaneously evaluate these two problems to end up with 4 component design. Figure 7 (a) is achieved when only structural criteria are used and the weight for modularity is equated to zero. In Figure 7 (b), modularity is assigned a sufficiently large weight, and it is observed that two substructures are shared in the optimal decomposition. Note that Case 1 for Figure 7 (b) has 3 components, instead of the desired value; this indicates that the software estimated the trade-off of keeping less number of components and thus made such a decision. Apparently, if the number of components is strictly determined, the corresponding weight can be increased to an extent that will force the system to have the desired decomposition at all times.



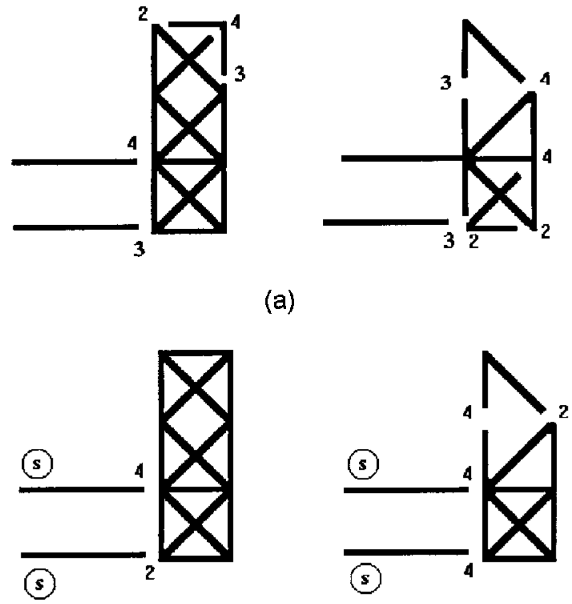
Case (1) Case (2)

(a) Design domain and boundary conditions

Case (1) Case (2)

(b) Optimal designs

Figure 6. Design of L-levers for the base structures for decomposition.



(a)

(b)

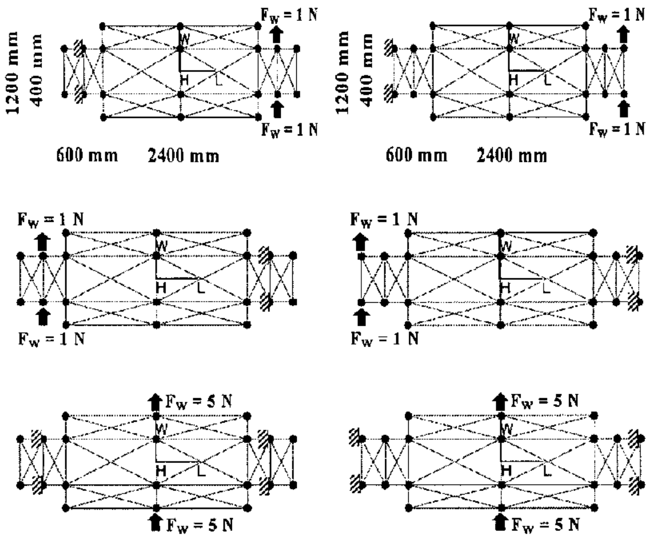
Figure 7. Optimal decompositions of L-levers: a) when no modularity between two structures is considered, and b) when modularity is considered.

Multi-loading

Figure 8 (a) shows the design domain for the floor where boundary conditions and specifications are indicated. In this problem, two loading cases are considered: Case (1) and Case (2) as shown in Fig. 8. Three loads are simultaneously applied at the floor for each loading case.

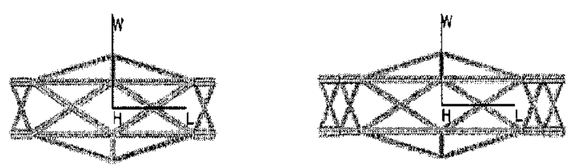
Figure 8 (b) shows the optimal structures for each loading case. In these figures, the unnecessary beams whose diameters are below 30% of the upper limit of diameter are eliminated.

The optimal decomposition for this case study is given in Figure 9. The software is run for structural considerations alone, and then with modularity terms again, results given in Figure 9 (a) and (b) respectively. For this example there is only a minor difference between the two results: the middle complex network is preserved, while the outer members are decomposed and to be welded with different weld angles for each case. Though the software points to a single sharing for Figure 9 (b), it turns out that multiple elements can be held common; coincidentally, similar part commonality traits are observed in Figure 9 (a), where there is no reward given in the objective function for shared substructures.



Case (1) Case (2)

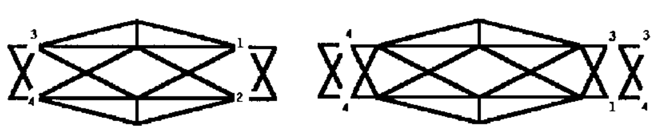
(a) Design domain and boundary conditions



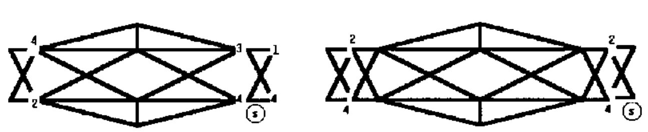
Case (1) Case (2)

(b) Optimal designs

Figure 8. Design of floor frames for the base structures for decomposition.



(a)



(b)

Figure 9. Optimal decompositions for floor frames: a) when no modularity between two structures is considered, and b) when modularity is considered.

DISCUSSION AND FUTURE WORK

This study presents a successful integration of the First Order Analysis and assembly synthesis methods for modular structural component design. It is observed that the algorithm manages to find an acceptable solution, allowing the sharing of one component by both end products and still maintaining a good structural strength and assemblability. It may be necessary, however, to carry out the synthesis with different objective function weights in a systematic way to have a complete understanding of the design. Depending on the problem, changing the weights of the corresponding criteria could be quite effective on the convergence of the solutions. Same argument is valid for the number of components determined by the user; this variable is usually not known *a priori*, and the structures at hand can be sensitive to a change in this number.

The approximation used instead of a formal *graph isomorphism* check seems to be working well, obviously introducing a faster evaluation of the objective function. Note that the real evaluation for similarity is actually done by area comparison, and the graph check helps effectively to select candidate substructures.

Future work will include more detailed modeling of joint features to achieve more accurate evaluation of effect of joints on structural strength. Extension of the method to 3-D structures will also be considered in the near future.

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