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A Survey of Structural Optimization in Mechanical Product Development

The widespread availability of affordable high-performance personal computers and commercial software has prompted the integration of structural analyses with numerical optimization, reducing the need for design iterations by human designers. Despite its acceptance as a design tool, however, structural optimization seems yet to gain mainstream popularity in industry. To remedy this situation, this paper reviews past literatures on structural optimization with emphasis on their relation to mechanical product development, and discusses open research issues that would further enhance the industry acceptance of structural optimization. The past literatures are categorized based on their major research focuses: geometry parameterization, approximation methods, optimization algorithms, and the integration with nonstructural issues. Open problems in each category and anticipated future trends briefly are discussed. [DOI: 10.1115/1.2013290]

1 Introduction

Structural analyses in the context of product development refer to the predictions of the product states in response to external loads. In industry settings, they are usually done by conducting finite element analyses (FEA) on the product geometries generated by computer-aided design (CAD) software. Since most mechanical products operate under external loads, structural analyses are essential to modern product development processes, where the use of physical prototype is minimized or even eliminated due to the ever-increasing demands for high quality, low cost products with short development time. In the last decade, the widespread availability of affordable high-performance personal computers and commercial FEA software further prompted the integration of structural analyses with numerical optimization (i.e., structural optimization), reducing the need of the design iterations by human designers.

A very brief history of structural analyses and optimization from an industry perspective can be summarized as follows [1,2]:

(1) **Pre-1980:** Structural analyses became widespread, replacing physical tests. Structural optimization was just not feasible due to high demands on computer resources.
(2) **1980s:** Structural analyses became a tool for design iteration/exploration. Despite growing interests [3], structural optimization was mostly for researchers.
(3) **1990s:** Coupled with desktop 3D CAD, structural analyses became a main driver for design cycle reduction. Structural optimization became an effective option for some product segments.
(4) **2000-present:** Structural analyses completely replaced physical tests in some product segments. Despite its acceptance as a design tool, structural optimization seems yet to gain mainstream popularity.

In light of its potential impact on the product development processes, this paper attempts to provide a bird-eye literature survey of the developments in structural optimization research, with emphasis on its relation to mechanical product development. The past literatures are categorized based on their major research focuses: geometry parameterizations, approximation methods, optimization algorithms, and the integration with nonstructural issues. Since structural optimization is a matured research area with 20 + years of rich history, some issues must be inevitably excluded from the discussion, whose choices are biased by the authors' own experiences. In particular, the following issues are not discussed due to the page limit: CAD/FEA integration, structural optimization in manufacturing process design, composite material design, and biomechanical product design. While the paper attempts to cover representative application areas in mechanical engineering, there is some bias towards automotive applications. Also, the application areas traditionally regarded as outside of mechanical engineering, such as aerospace engineering, civil engineering, naval architectures and shipbuilding, and electrical engineering, have very limited coverage, despite a vast amount of work on the application of structural optimization in these area.

The rest of the papers is organized as follows: Sec. 2 provides an overview of the branches of structural optimization researches as addressed this paper. Section 3 discusses the research focusing on geometry parameterization (sizing, shape, and topology). Section 4 discusses the research focusing on the methods to approximate costly structural analyses for use within optimization loops. Section 5 discusses the optimization algorithms used for structural optimization. Section 6 discusses the integration of structural optimization with nonstructural issues in mechanical product development. Finally, Sec. 7 summarizes open problems in each category and discusses anticipated future research trends.

2 Overview

Structural optimization is a class of optimization problems where the evaluation of an objective function(s) or constraints requires the use of structural analyses (typically FEA). It can be symbolically expressed in a compact form as [4]:

$$\text{minimize } f(\boldsymbol{x})$$

$$\text{subject to } g(\boldsymbol{x}) \leq \mathbf{0}$$

$$h(\boldsymbol{x}) = \mathbf{0}$$

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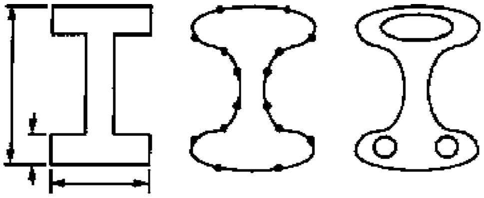

Fig. 1 Types of geometry parameterization: (*a*) sizing, (*b*) shape, and (*c*) topology [5]

$$x \in D, \tag{1}$$

where $\boldsymbol{x}$ is a design variable, $f(\boldsymbol{x})$ is an objective function, $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ are constrains, and D is the domain of the design variable. Note $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ are vector functions. The design variable $\boldsymbol{x}$ is typically a vector of parameters describing the geometry of a product. For example, $\boldsymbol{x}$, $f(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$, and $\boldsymbol{h}(\boldsymbol{x})$ can be product dimensions, product weight, a stress condition against yielding, and constraints on product dimensions, respectively. Depending on the definition of design variable $\boldsymbol{x}$, its domain D can be continuous (e.g., a continuous range of the length of a bar), discrete (e.g., the standard gage thicknesses of a plate or the existences of structural member in a product), or the mixture of both. Also, a variant of structural optimization has multiple objectives, where the objective function is a vector function $\boldsymbol{f}(\boldsymbol{x})$, rather than a scalar function $f(\boldsymbol{x})$.

While there are numerous variations of structural optimization expressed in the form of Eq. (1), they can be roughly classified according to the following three viewpoints:

(1) **Analysis types:** Linear static (stress and displacement), eigenvalues (normal modes and buckling), nonlinear and time-transient (postbucking and cursh), flexible multibody, and multiphysics.
(2) **Application domains:** Generic mechanical, civil, automotive, aerospace, naval architecture and shipbuilding, and microelectromechanical systems (MEMS).
(3) **Research focuses:** Geometry parameterization, approximation methods, optimization algorithms, and the integration with nonstructural issues.

The present paper adopts the third viewpoint, research focuses, as a primary method to classify the past literatures with special emphasis on mechanical product development.

Geometry parameterization determines the type of geometry changes that can be described by design variable $\boldsymbol{x}$. It can be classified to (1) sizing, (2) shape, and (3) topology parameterizations as illustrated in Figs. 1(*a*)–1(*c*), in the order of increasing freedom in possible geometry changes. In sizing parameterization, the geometry of a product is expressed in terms of a set of dimensions and only the values of these dimensions are allowed to vary. Shape parameterization relaxes this constraint so the boundary of product geometry can be changed more freely using, e.g., parametric curves/surfaces, while topology (the connectivity among geometric subdomains) remains constant. Topology parameterization allows the change in topology in addition to shape, so holes can be placed within product geometry. Due to the amount of design freedom and the difficulty in representing detailed product geometry, topology parameterization is generally suitable for conceptual design, whereas sizing and shape parameterization are effective for both conceptual and detailed designs.

Since optimization is an iterative process, objective function $f(\boldsymbol{x})$ and constraint functions $\boldsymbol{g}(\boldsymbol{x})$ and $\boldsymbol{h}(\boldsymbol{x})$ need to be evaluated many times to obtain a solution. Approximation methods replace costly structural analyses during the optimization iterations, so a solution can be obtained within a reasonable amount of time. It is of a significant importance in product development since computation time is still a large obstacle against the industry acceptance of structural optimization [6]. Major classes of approximation methods are: (1) surrogate modeling, (2) reduced order modeling, and (3) reanalysis methods. Surrogate modeling approximates the input-output relationships of structural analyses by means of implicit or explicit algebraic equations. While surrogate models simply “copy” the input-output relationships with no consideration of the underlined physics, reduced order modeling builds a simplified version of physical model by dropping unimportant details, such as mass-spring-damper models of an automotive suspension. Since most optimization algorithms iterate by slightly modifying a current design, reanalysis methods approximate the analysis result of a new design based on the (nonapproximated) analysis result of the current design.

The third category in research focuses is optimization algorithms, which are classified to (1) nonlinear programming (NLP), (2) metaheuristic, (3) reliability and robustness optimization, and (4) other special purpose algorithms. Since $f(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$, and $\boldsymbol{h}(\boldsymbol{x})$ are usually nonlinear to $\boldsymbol{x}$, nonlinear programming algorithms are often used if $\boldsymbol{x}$ is continuous. In particular, sequential approximation algorithms are successfully applied as they exploit the nature of structural optimization problems whose objective and constraints often exhibit near-monotonic behaviors within small variations of $\boldsymbol{x}$. During the last decade, metaheuristic algorithms such as genetic algorithms (GA) and simulated annealing (SA) have gained popularity in structural optimization due to their global optimization capability, no need of derivative information, the applicability to both continuous and discrete variables, and the ease of computer implementation. While these methods, especially when coupled with topology parameterization, can be effective in exploring many design alternatives, the need of a large number of analyses is limiting their applicability to practical problems. This is also the case for reliability and robustness optimization where one evaluation of $f(\boldsymbol{x})$, $\boldsymbol{g}(\boldsymbol{x})$, and $\boldsymbol{h}(\boldsymbol{x})$ involves many structural analyses due to the stochastic nature of the definition of reliability and robustness. Consequently, special algorithms are developed to efficiently estimate the reliability and robustness, without resorting to expensive Monte Carlo methods. Other special purpose algorithms include the ones that tightly integrate with, and hence are inseparable from the corresponding geometry parameterizations.

Viewed as a tool for effective product development, structural optimization should be an integral part of nonstructural issues in product development processes. Relatively few papers are found in this category, perhaps due to the implicit segregation between the research communities of structural mechanics and of product design. Nevertheless, related literatures are classified as the ones addressing (1) cost, (2) manufacturing and assembly, and (3) product platform design. Although both (2) and (3) are in essence aiming at cost reduction, they include the papers that explicitly deal with the respective issues, beyond the use of simple cost models.

3 Geometry Parameterizations

3.1 Sizing and Shape. In sizing parameterization, design variable $\boldsymbol{x}$ is a predefined set of the dimensions that describe product geometry. The application of sizing optimization, therefore, is mostly limited to detailed designs where only the fine tuning of product geometry is necessary. Sizing optimization is typically done in conjunction with feature-based variational geometry [7] available in many modern CAD software systems. With today’s fast personal computers, sizing optimization is relatively a straightforward task since it typically requires no remshing of finite element models during optimization iterations. A difficulty arises, however, when extremely large finite element models or highly nonlinear phenomena need to be analyzed, in which case surrogate models are typically employed.

Shape parameterization allows the changes in the boundary of product geometry. In shape optimization, boundaries are typically represented as smooth parametric curves, since gunsmith and irregular boundaries deteriorate the accuracy of finite element analysis or even cause the numerical instability of optimization algorithms. Since product geometry can change dramatically during the optimization process, the automatic remeshing of finite element models is usually required. Since shape parameterization is suitable for representing product geometries with smooth external boundaries, many developments have been made in the area of aerospace vehicle design, which is not covered in this paper due to the space limit. Instead, this subsection briefly describes two primal approaches in shape optimization along with major application areas related to mechanical product developments.

Structural optimization based on the shape parameterization of geometry can be classified to (1) direct geometry manipulation and (2) indirect geometry manipulation approaches. In the direct geometry manipulation approaches, design variable $\boldsymbol{x}$ is a vector of parameters representing the boundary of product geometry, e.g., the control points of the boundary surfaces. In the indirect geometry manipulation approaches, design variable $\boldsymbol{x}$ is a vector of parameters that indirectly defines the boundary of the product geometry, e.g., a fictitious load is applied on the boundary.

An excellent review of shape optimization based on the direct geometry manipulation approaches can be found in [8], where the boundary representations are classified as polynomials, splines, and design elements. In the polynomial representation, design variable $\boldsymbol{x}$ is the coefficients of the polynomials that describe the boundary shape of the product [9–11]. A variant of this approach is to use a linear combination of the nonpolynomial basis functions that can better describe desired product boundaries [12,13]. In the spline representation, design variable $\boldsymbol{x}$ is the control points of spline such as Bezier and B-spline curves [14,15]. In the design element representation, product geometry is discretized to design elements, whose boundaries are represented by a set of key nodes and their interpolation functions [16,17]. Each design element can contain several finite elements, and the design variable $\boldsymbol{x}$ is the coordinates of the key nodes that are allowed to move. The direct geometry manipulation approaches had been implemented in a number of commercial software such as Optistruct [18].

A representative method of the indirect geometry manipulation approaches is the Natural Design Variable method originally developed by Belegundu and Rajan [19], which uses fictitious loads applied on the boundaries of the product geometry as design variables. In each iteration, a new boundary is obtained by adding the displacements induced by these fictitious loads to the original boundary. Since the displacements are calculated by Finite Element Methods based on force equilibrium, the resulting new boundary tends to be smoother and less likely to have heavily distorted meshes, than the ones obtained by the direct geometry manipulation approaches. On the other hand, imposing geometric constraints on product boundary is more complicated than the direct geometry manipulation, since the constraints must be translated to the ones on the fictitious loads. Variants of the Natural Design Variable method have been implemented in a number of commercial software, such as NASTRAN [20], GENESIS [21], and ABAQUS [22].

As a hybrid of the direct and indirect geometry manipulation approaches, Azegami et al. [23,24] proposed the Traction Method, where the boundary sensitivities are replaced with the velocities of boundary changes in response to the fictitious loads that deform the original shape to the target shape defined by the boundary sensitivities. By this replacement, sufficient smoothness of the boundary can be achieved via the direct manipulation of the product boundary.

In addition to stiffness maximization problems, sizing and shape optimization has been applied to numerous areas of concern in mechanical product developments, including vibration [25–27], crashworthiness [28–37], thermomechanical [38], strcture-acorustics [39], structure-electromagnetics [40], fluid-structure [41], compliant mechanisms [42–44], microelectromechanical systems (MEMS) [45–49], and reliability optimization [50–53].

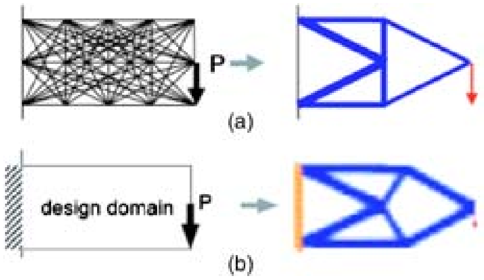

Fig. 2 Two approaches for structural topology optimization. (*a*) Discrete element (ground structure), and (*b*) continuum.

3.2 Topology. Topology parameterization allows the changes in the way substructures are connected within a fixed design domain. Structural optimization based on the topology parameterization of geometry can be classified to (1) discrete element and (2) continuum approaches. In the discrete element approach, (also known as ground structure approach), the design domain is represented as a finite set of possible locations of discrete structural members such as truss, frame, and panels [the left figure of Fig. 2(*a*)]. By varying the width/thickness of each member in the design domain between zero (in this case the element becomes nonexistent) and a certain maximum value, structures with different sizes and topologies can be represented [the right figure of Fig. 2(*a*)]. In the continuum approach, the design domain is represented as the continuum of "void," or material with very low density [the left figure of Fig. 2(*b*)]. By varying the void/material distributions or the material density within an infinitesimally small microstructure at each location in the continuum, structures with different size and topologies can be represented [the right figure of Fig. 2(*b*)]. A survey of structural topology optimization from a product design viewpoint is found in [54].

Since structures are represented as a collection of primitive structural members that allow easy interpretation, the discrete element approach is suitable to conceptual design. However, the feasible topologies for a given design domain is limited by the number and types of possible member locations defined in the design domain. Thanks to the assumption of a continuum design domain, on the other hand, the continuum approach does not have this limitation, with an expense of additional computational costs. The right figures in Figs. 2(*a*) and 2(*b*) illustrate this difference.

While the optimization of structural topology based on the discrete element approach has a long history that dates back to the early 1900s [55], significant developments were made mostly during last two decades. Comprehensive history of the area can be found in [56–58]. Using trusses as structural members, Bendsøe et al. [56] applied the simultaneous analysis and design (SAND), Gil and Andreu [59] conducted simultaneous shape and size optimization, and Pedersen and Nielsen [60] solved problems with eigenfrequencies, displacements, and buckling constraints. Nishigaki et al. [61] presented an Excel-based desk-top tool for the 3D topology optimization of automotive bodies approximated by networks of frame elements with rigid joints, later extended to include flexible joints [62], panel elements [63,64] (Fig. 3), and frame elements with ellipsoidal cross sections [65]. Inspired by the continuum approach, several researchers presented topology optimization methods in discretized (meshed) design domains, such as a binary material approach solved by Genetic Algorithm [5,66–68], Evolutional Structural Optimization (ESO) [69,70], Cellular Automata [71–74], and metamorphic development [75,76].

Independent to these developments, Cagan et al. [77] developed another class of discrete element approaches called shape annealing. Instead of eliminating unnecessary structural members, to-

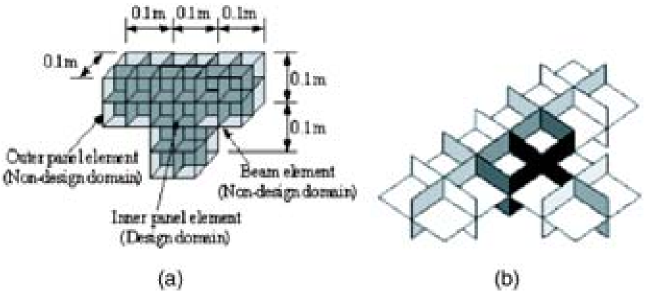

Fig. 3 3D topology optimization with frame and panel elements [64]. (*a*) Design domain, and (*b*) optimal panel configuration.

pologies are "grown" from a primitive member in an optimal fashion, through the repeated applications of a shape grammar using simulated annealing [78–80].

A continuum approach to topology optimization was first introduced by Bendsøe and Kikuchi [81] in their Homogenization Design Method (HDM). Bendsøe [82] also proposed another approach called SIMP (Solid Isotropic Microstructure with Penalization), which is further developed by Rozvany et al. [83] and Yang and Chuang [84]. The idea of a material-based interpolation within the SIMP model was introduced in [85]. A textbook by Bendsøe and Sigmund [86] describes the recent developments, applications, and details of the SIMP methods. A comprehensive list of literatures on HDM and SIMP are found in [87].

The continuum approach has been applied to various design problems such as stiffness problems [88], eigenfrequency problems [89–91], automotive crashworthiness problems [92–98], and reliability optimization problems [99,100]. However, these applications have been limited to high-level, conceptual designs due to the difficulty in presenting detailed geometries and smooth boundaries. In the situation where representing detailed geometries or smooth boundaries are crucial (e.g., when checking stress concentration and local bucking and failure), the results of topology optimization need to be postprocessed to a parametric boundary representation for subsequent sizing/shape optimizations [101].

Compliant mechanisms are mechanisms which gain some or all of their motion from the relative flexibility of their members rather than from rigid-body joints [102]. During the last decade, topology optimization has been extensively applied to the design of compliant mechanism. Based on the discrete element approach, Frecker et al. [103] posed compliant mechanism design as a multicriteria optimization. Hetrick and Kota [104] introduced a concept of efficiency to the problem formulation. Saxena and Ananthasuresh [105] developed a method to enforce a prescribed nonlinear output deflection. Compliant mechanism design based on the continuum approach was introduced by Sigmund [106] and Nishiwaki et al. [107,108] under the linear elastic assumption, which was later extended for large displacements [109,110] and snap-through behavior [111]. Tai and Chee [112] and Tai et al. [113] proposed a design method for compliant mechanisms based on the ESO approach.

These developments lead to the successful applications of optimal compliant mechanism designs for piezoceramic-driven flextensional actutators [114,115], piezoresistive sensors [116], thermal [117] and electrothermal [118–120] actuators, micromechanical resonators [121,122], and micromechaical bioprobes [123].

Open research issues on topology optimization may include: (1) nonlinear problems, (2) large-scale problems, and (3) topology optimization based on level set theory. Despite several attempts, topology optimization of structures considering nonlinear phenomena has shown limited successes due to underlying numerical difficulties. A similar problem exists on the use of topology optimization in a large scale, multidisciplinary problems. Further research efforts are desired in these areas, including the integration with surrogate or reduced order models. Recently, a new class of shape optimization methods based on the level set theory was proposed [124] and later extended to allow limited topological changes [125,126]. Since the level set representation allows topological changes with smooth boundary, it is expected to overcome the difficulty of the conventional topology parameterization in representing smooth product boundaries without postprocessing and the subsequent sizing/shape optimization. Considering the independent development of level-set based geometry modeling [127,128], the research in this area may lead to a seamless integration of geometric modeling and shape and topology optimization, which would have a profound impact on product development processes.

4 Approximation Methods

The computation time required for structural analyses was a major obstacle against structural optimization in the 1980s due to the limited computer power in those days. Surprisingly, however, it is still a problem after 20 years, even with the dramatic increase in the computer power in the last decades. This paradoxical situation is due to the increased demands in high accuracy (higher DOFs with finer meshes), complex analyses (e.g., nonlinear and multiphysics), and in large design spaces (e.g., topology parameterizations). The approximation methods, therefore, are key technologies for the industry acceptance of structural optimization [6].

4.1 Surrogate Models. Representative surrogate models empirically capture the input-output relationship of structural analyses for evaluating the objective functions and constraints. They are utilized for two reasons, the first of which is to obtain global the behavior of the original functions that have complex local noises. The second is to shorten optimization calculation time by using surrogate functions that can quickly return approximate values, instead of relying on the time-consuming function calls such as crashworthiness analysis [129–134], and optimization under uncertainties [135,136]. Artificial Neural Network (ANN) [137], Polynomial Regression, Kriging Method [134,138], and Radial Basis Function [139,140] are popularly used.

The formulations of the approximation methods are described as follows. The Polynomial Regression approach represents a surrogate function that uses a polynomial expression. For instance, a second-order polynomial model can be formulated as the following equation:

$$\hat{y} = \beta_0 + \sum_{i=i}^{k} \beta_i x_i + \sum_{i=i}^{k} \beta_{ii} x_i^2 + \sum_{i} \sum_{j} \beta_{ij} x_i x_j \tag{2}$$

where $\hat{y}$ is the estimated function value and k is the number of design variables, $\beta_i (i=1,2,\ldots,k)$ are the coefficients to be obtained via the surrogate model construction processes.

The formulation of the Kriging Method is the following equation:

$$\hat{y} = \sum_{j=1}^{k} \beta_j Y_j(\boldsymbol{x}) + Z(\boldsymbol{x}) \tag{3}$$

$$\mathrm{cov}[Z(x_i), Z(x_j)] = \sigma^2 R(x_i, x_j) \tag{4}$$

where $Y_j(\boldsymbol{x})$ is a known fixed function for the global approximation of design space, and $Z(\boldsymbol{x})$, which represents the localized deviations of the approximation model, is the realization of a normally distributed Gaussian random process with mean zero and variance σ, and R is the correlation function. The Kriging method provides good approximation models while its implementation is relatively complex and time-consuming.

Radial basis functions, formulated as the following equation, is a popular approximation method since it offers good approximation models for various functions and is easy to be implemented.

$$\hat{y} = \sum_i a_i \|X - X_{0i}\| \tag{5}$$

where a_i is a coefficient and X_{0i} is the observed input.

4.2 Reduced Order Models. Since the construction of surrogate models requires the sample data of the input-output pairs calculated by the structural analyses of detailed product models, they cannot be used during conceptual design stages when no such models are available. For an effective concept exploration via optimization, approximation models must be based on physics to allow the interpretation of the resulting designs by human designers, with an reasonable accuracy and a computational speed [141]. Reduced order models have been developed to meet these goals for various applications, such as compliant mechanisms and automotive structures.

Since members in compliant mechanisms often experience large elastic deformations, geometric nonlinearly becomes significant. However, such large deformations are highly localized to the "hinged" ends of slender members, allowing most portions of the members to be seen as rigid links. Pseudorigid body models [142–146] exploit this fact and model a compliant mechanism as rigid links connected by nonlinear torsional and translational springs.

For static analyses of structures with general geometries, flavors of Guyan reduction can be applied to obtain reduced-order models. Guyan reduction is essentially a substructuring method which reduces the problem to a smaller one by relating certain degrees of freedom to certain others by means of constraint equations, thereby reducing the size of the problems. Using this method, Sugiura et al. [147] and Tsurumi et al. [148] presented reduced order models of an automotive torsion bar suspension and spot-weld joints of thin-walled members in automotive bodies, respectively.

Due to the massive computational requirements, there is a significant need for reduced-order models for automotive crashworthiness optimization. As such, coarse mesh, lumped parameter, and lattice models [149–151] have been developed. While these models can be computationally inexpensive and also bear some physical roots in underlying crash phenomena, they are too abstract for the examination of crash modes (sequences of collapse events), which are essential for early design iterations to identify effective energy-absorbing strategies [153]. Since most crash energy is absorbed by beam-like structural frames in a body structure, techniques similar to pseudorigid body models for compliant mechanisms have been successfully applied to automotive crashworthiness [152–157]. With a "right" level of abstraction preserving general geometries of automotive bodies, these models would be capable of simulating the crash mode of a body structure with complex topologies. Hamza and Saitou demonstrated that their equivalent mechanism (EM) models (Figs. 4 and 5) could in fact simulate the crush modes with a reasonable accuracy [153], and the crash mode of the optimized EM model could be used to guide design iterations with a detailed FE model to dramatically reduce the total number of FE crash simulations [154,155].

With the common use of structural analyses during conceptual design, reduced-order models are becoming increasingly more important in product development. The methodologies for building reduced order models with right abstraction levels in various application domains will continue to be a challenging research area [141]. In particular, advancements are desired on reduced-order modeling for multidomain and multiphysics analyses, due to their increased use in the development processes of many mechanical products.

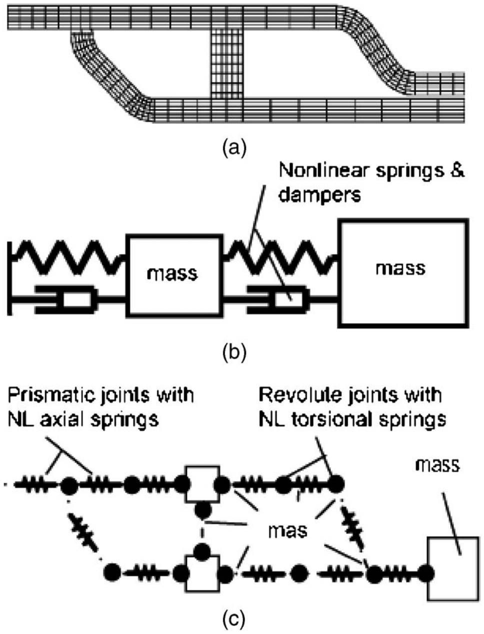

Fig. 4 (*a*) Finite element, (*b*) lumped parameter, and (*c*) equivalent mechanism models of a vehicle front subframe [153]

4.3 Reanalysis Methods. Reanalysis is a class of methods for approximating the structural responses of modified designs, based on the analysis results of a single original design. In contrast to surrogate models that require multiple analyses (multipoint approximation), reanalysis methods only utilize a single analysis result (single point approximation). Due to this character, reanalysis methods are most suitable for design exploration within a small neighborhood of a nominal design, and post-optimization sensitivity analyses.

Of most notable in structural reanalysis is the Combined Approximations (CA) method [158–160], which is based on the reduced-basis approximation of the nodal displacement vector in terms of a binomial series. Initially developed for static linear analyses, the method is later extended to nonlinear static analyses, eigenvalue analyses, and also applied to calculate design sensitivities [161,162]. Another class of reanalysis methods is based on Taylor series approximation [163–165], where the derivatives of desired structural responses with respect to design variables are calculated by using Adjoint Method [163,164] or Direct Method [165].

Since reanalysis methods efficiently and accurately estimate the structural responses of a modified design within a small neighborhood of the original design, their integration with robust/reliability optimization seems a very promising research direction.

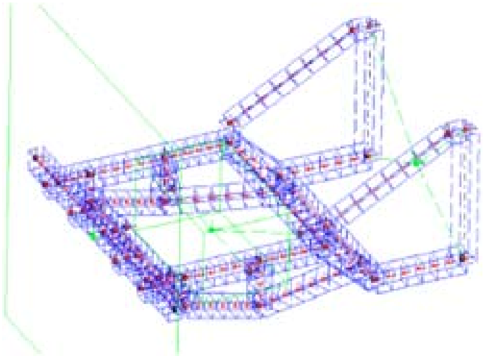

Fig. 5 Equiavlent mechanism model of vehicle front half-body [155]. Boxlike outlines are shown only to provide a visual clue to human designers.

5 Optimziation Methods

5.1 Nonlinear Programming Algorithms. Typical mathematical approaches for solving nonlinear optimization problems such as structural optimization can be represented as iterative processes of approximated optimizations. Sequential Linear Programming (SLP) [166], also known as Successive Linear Programming, approximates nonlinear optimization problems as linear optimization problems. SLP can handle a range of objective functions and a large number of design variables, while the computational efficiency is sensitive to the values of the move limit parameters. Kumar developed an adaptive move limits setting method to stably obtain good solutions [167].

In Sequential Quadratic Programming (SQP), an approximation is made of the Hessian of the Lagrangian functions in each iteration to generate a quadratic subproblem [4]. While SQP provides a good searching efficiency when the number of design variables is relatively small [168], the calculation of the Hessian becomes cumbersome in large-scale problems.

Sequential Convex Programming (SCP) is based on a convex approximation. Convex Linearization (CONLIN) [169] uses the following intervening variables to obtain better approximations:

$$\dot{x}_i = \begin{cases} x_i & \text{if } \partial h/\partial x_i > 0 \\ 1/x_i & \text{if } \partial h/\partial x_i < 0 \end{cases} \quad \text{for } i = 1,2, \ldots ,n \tag{6}$$

where $h(\boldsymbol{x})$ are objective or constraint functions. When using the Method of Moving Asymptotes (MMA) [170], the following intervening variables are used for the convex approximation:

$$\dot{x}_i = \begin{cases} 1/(U_i - x_i) & \text{if } \partial h/\partial x_i > 0 \\ 1/(x_i - L_i) & \text{if } \partial h/\partial x_i < 0 \end{cases} \quad \text{for } i = 1,2, \ldots ,n \tag{7}$$

Using artificial intermediate variables, MMA provides an improved convexity of approximation. Recently, better convex approximation techniques have been developed [171–173], however parameters the values of U_i and L_i cannot be determined *a priori*.

Instead of sequentially solving an optimization problem locally approximated in each iteration, the Method of Feasible Directions (MFD) determines a direction of the next iterate by solving an auxiliary optimization problem based on the gradients of the objective function and the active constrains. Vanderplaats [174] greatly improved the numerical stability of MFD, which later evolved to commercial software DOT and GENESIS [21].

Semidefinite Programming (SPD) [175] is an effective algorithm that can solve optimization problems with the constraints represented as positive semidefinite matrices. Since SPD does not require explicit sensitivity coefficients, eigenvalues and buckling loads in structural problems can be easily handled [176,177].

Hierarchical optimization techniques have been proposed for efficiently solving multidisciplinary optimization problems, techniques such as Bi-level Integrated Synthesis (BLISS) [178] and Collaborative Optimization (CO) [179,180], which provide multidisciplinary optimization environments where optimization processes in each engineering discipline are hierarchically connected using linking variables, facilitating the achievement of comprehensive optimal solutions. The Analytical Target Cascading (ATC) method [181,182] includes a representation scheme for engineering design problems of hierarchical systems. In the ATC method, a given mechanical system can be partitioned into multiple layered components, each of which corresponds to a subsystem that are interconnected using linking variables. A system level optimal solution is obtained through iterative optimization procedures [183].

Thanks to these developments, commercial software can now handle many practical structural optimization problems using continuous design variables [184]. However, the design optimization of large-scale structures still remains a time-consuming matter. To reduce computational time, parallel computing methods have been applied [185,186].

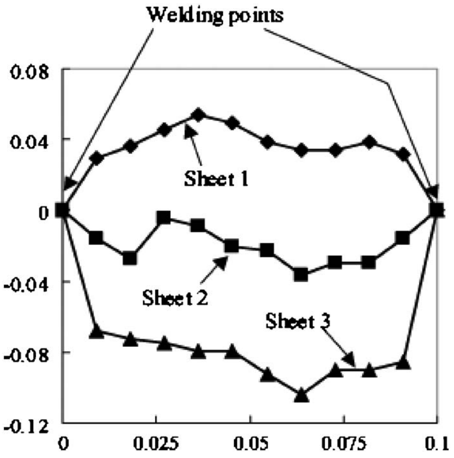

Fig. 6 Optimization result of cross-sectional shape of automotive body frame using genetic algorithm [192]

5.2 Metaheuristic Optimization Methods. Metaheuristic approaches that imitate natural phenomena have been applied to complex optimization problems that cannot be solved using conventional mathematical methods.

Genetic algorithms seem the most widely used metaheuristic methods in structural optimization, and numerous applications, such as shape optimization [187,188] and topology optimization [68], have been developed. Genetic algorithms can be applied to a range of design optimization problems since GAs allow a variety of design variable representations using various types of gene structures. A mixed discrete/continuous design variable problem in mechanical component design was solved by Deb and Goyal [189]. Structural optimization using real-coded GAs has been carried out by Deb and Gulati [190]. Yoshimura and Izui proposed a genotype representation method of hierarchical design alternatives for machine tool structural optimization problems [191], and Yoshimura et al. applied GAs to cross-sectional shape optimization problems of automotive body frames involving mixed discrete/continuous design variables [192] (Fig. 6). Another important advantage of using GAs is that multiobjective optimization problems can be solved and non-dominated solutions can be obtained in a single optimization run [193–195].

One crucial problem in the use of GAs is their high computational cost, since GAs are population-based algorithms, and GAs that use parallel processing methods have been developed [196,197] to try and mitigate this. However, the handling of constraints still remains difficult, though some improved methods have recently appeared [198,199].

Many GA-like metaheuristic methods also have been applied to structural optimization such as new crossover [200] and mutation [201] schemes, Immune Algorithms (IAs) [202], and Pareto-frontier differential evolution (PDE) [203] to enhance computational efficiencies for obtaining an optimal solution or multiple Pareto optimal solutions.

While GAs are population-based optimization methods utilizing multiple searching points in a single iteration, simulated annealing (SA) methods [204] use a single searching point. SA can be applied to discrete design variable problems [205], and SA computational requirements are relatively modest, particularly when compared with genetic algorithms. However, parallel-computing methods for SA are required since the computational cost of SA is higher than mathematical approaches in large-scale problems [206]. Furthermore, parameter settings for temperature control remains problematic since the optimal cooling schedule depends on optimization problems.

Particle Swarm Optimization (PSO) is another metaheuristic method, one that mimics the social behaviors of groups of living organisms. In certain structural applications, PSO offers better searching efficiencies for obtaining global optimal solutions than genetic algorithms, especially for problems that include numerous continuous design variables [207,208].

5.3 Reliability and Robustness Optimization Methods. The general optimization techniques described above often can provide the optimal solutions for practical optimization problems set by designers. However, the premises of optimization problems occasionally differ from practical design cases due to the uncertainties in material forming, product assembly and so on. Therefore, optimization methods have been developed that take such uncertain factors into consideration.

Reliability analysis is a method for quantifying the relations between the variations in parameter values and the resulting performance insufficiencies or failures. A variety of design optimization techniques based on reliability analysis have been proposed [209]. Such techniques are called Reliability-Based Design Optimization (RBDO) techniques [50].

Murotsu et al. proposed a topology optimization method based on reliability analyses using truss elements [51]. Recently, Kharmanda et al. clarified that a reliability-based design optimization method can be applied to a topology optimization problem based on continuum mechanics [100].

To date, the Reliability Index Approach (RIA) and the Performance Measure Approach (PMA) [50,52] are used as basic algorithms for the RBDO. These algorithms require double loop calculations, where the reliability index is calculated in the inner loop and the design optimization is conducted in the outer loop. However, such double loop calculation approaches result in an extremely high computation cost, but accurate reliability indexes can be calculated for multiple failure-mode problems.

The Single Loop Single Variable (SLSV) method, the Safety-Factor Approach (SFA), and the Sequential Optimization and Reliability Assessment (SORA) method are popular techniques in which the double loop problems are transformed to single loop problems to reduce calculation time. The SLSV method proposed by Chen et al. [53] enables the single loop calculation while avoiding the need to calculate the reliability index during the optimization process. The SFA proposed by Wu and Wang [210] introduced the concept of safety-factor in reliability design problems into optimization problems, and used approximately equivalent deterministic constraints. Du and Chen [211] proposed the SORA method that transforms a probabilistic design problem to an equivalent deterministic optimization problem using an inverse reliability assessment for checking the constraint feasibility. In the SFA and SORA methods, the design optimization process and the reliability analysis can be strictly partitioned and conducted in reduced calculation time [212].

Robust optimization methods aim to ensure that the objective function is insensitive to variations in design variables. Taguchi methods in experiments using orthogonal arrays representing various combinations of control factor magnitudes can obtain conditions, where the effect of uncertain noise is minimized, to evaluate the robustness of the objective function, based on a signal to noise (S/N) Ratio [213]. Uncertain variations can also be handled using Fuzzy sets. Arakawa et al. represented the design variable variations using Fuzzy sets and provided a robust optimal solution that considers the correlations of design variables [214]. Furthermore, fuzzy sets can be used in multiobjective problems to represent designers' preferences [215,216]. In multidisciplinary design, uncertainties in different disciplines often show interrelationships where unwanted effects are a factor and may be compounded. Gu et al. introduced robust optimization concepts into multidisciplinary hierarchical optimization systems to evaluate interdisciplinary uncertainty propagations [217].

5.4 Other Methods. Cellular automaton approaches [71–74], evolutionary optimization methods [69,70], and metamorphic development approaches [75,76] are heuristic methods that can obtain structural optima using only simple local rules. These methods can provide very similar solution to mathematical programming methods with a lower computational cost. These methods are expected to be more widely used in practical design cases if complex structural optimization problems such as large-scale, multiphysics problems can be solved using simple local rules.

6 Integration With NonstructruaL Issues

6.1 Cost. In most structural optimization work, the product weight is often assumed as an implicit representation of product cost. This assumption is valid when the material cost in the structure accounts for a majority of the total cost of a product. When manufacturing and assembly costs depend much on product geometry specified by design variable $\boldsymbol{x}$, however, the minimization of product weight does not necessarily imply the minimization of product cost. Therefore, the explicit inclusion of cost is of a practical importance from a product development viewpoint.

Cost is typically incorporated as an only objective, subject to the constraints on weight and structural responses [218,219], or as one of multiple objectives in addition to weight and structural responses [220–222]. While the single-objective formulation is simple, the multiobjective formulation is attractive when the examination of multiple pareto-optimal designs for cost-performance trade-off is of interest, e.g., during conceptual design.

6.2 Manufacturing and Assembly. Despite the recognition of its potential, the use of structural optimization in practice has been limited by the lack of manufacturing considerations. As a result, the "optimized" design must usually go through significant (manual) design changes since they are initially not economically manufacturable. With the recent emphasis on concurrent engineering [223], attempts have been made on the integration of structural optimization with design for manufacturing (DFM). Primal approaches have been the utilization of feature-based variational geometry representations in sizing and shape optimizations [7,224,225]. More recently, Zhou et al. [18,226] developed a simple mathematical constraint for ensuring the manufacturability of topology optimization results via casting processes. José et al. [227] presented topology optimization of components manufactured by fused deposition processes.

Since complex product geometry is often realized by assembling multiple components with simpler geometries, the optimization of multicomponent structural products based on product-level design objectives is of a significant engineering interest. Some research addressed this problem as the single-objective optimization of component topologies within the predefined component boundaries [228–231], while others addressed as the multiobjective optimization of component boundaries (i.e., joint locations) and joint attributes in a given product geometry, with the consideration of the manufacturability and assemblablity of each component [232,233]. These approaches are later relaxed to allow variable component sizing [234,235], and variable product topologies [236]. Figure 7 shows representative Pareto optimal multicomponent topologies of a simplified automotive floor frame, optimized for stiffness against multiple loading conditions and the manufacturability and assembleability of each component [236].

As an extension of [235], Fig. 8 shows one of the pareto optimal solutions in [237], where the space frame structure of an automotive body is optimally decomposed for the overall structural stiffness, the manufacturability of each component, and the adjustability of desired dimensions (indicated as KC: key characteristics) during the assembly process [Fig. 8(*a*)], and the corresponding assembly sequence to realize the adjustability [Fig. 8(*b*)].

With the increasing emphasis on "front loading" in product development processes, the integration of structural optimization with design for manufacturing and assembly (and design for X in

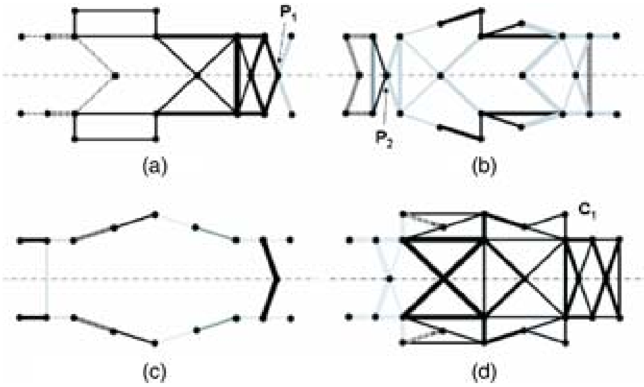

Fig. 7 Pareto-optimal multicomponent topologies of a simplified automotive floor frame subject to multiple loading conditions, considering stiffness, manufacturability, and assembleability [236]

general) became a key issue for enhancing its use in concept generation, and will continue to be an active research area.

6.3 Product Platform. The recent adoption of platform architectures in automotive industry motivated the integration of structural optimization with product platform design, based on the quantification of the trade-off between cost gain and performance loss by sharing a platform among multiple vehicle designs. Fellini et al. [238] presented a sensitivity-based method for identifying potentially sharable design variables among multiple optimization problems with mild differences, and applied the method to the platform identification in a family of automotive body structures. Cetin and Saitou [239–241] developed a method for optimally decomposing multiple structures to maximize sharable components while maximizing structural stiffness and minimizing manufacturing and assembly costs.

Since structural products such as automotive bodies are often homogeneous, the identification of a potential platform(s) requires the determination of the location of platform boundaries in each structure and the design of the joints common to all structures, which prevents the direct application of generic platform design methods. Although these works demonstrated the potential of optimization-based product platform design to structural products, much needs to be done to improve its practical applicability. Due to its potential impact on cost reduction, the optimal structural platform design will keep drawing much attention from industry.

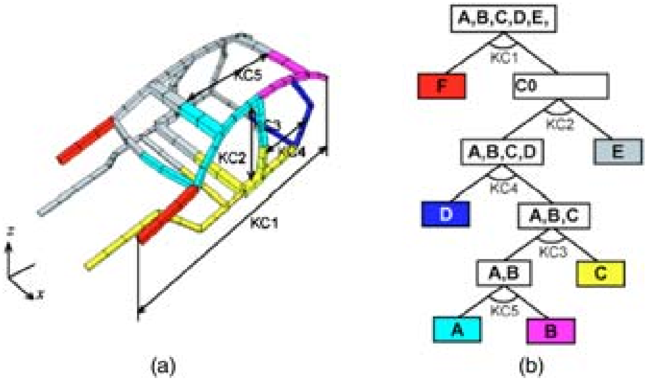

Fig. 8 (*a*) Automotive space frame optimally decomposed for overall stiffness, component manufacturability, and the adjustability of dimensions K1–K5 during assembly process, and (*b*) the corresponding assembly sequence to realize the adjustability [237]

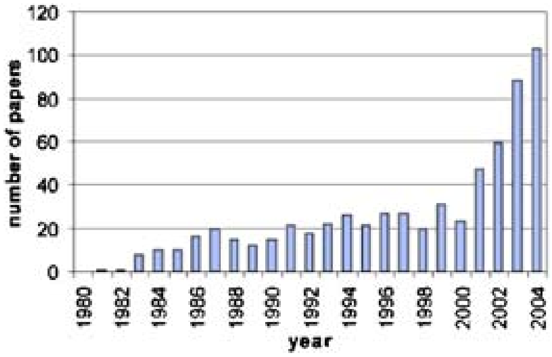

Fig. 9 Number of conference and journal papers (written in English) in Compendex between 1980 and 2004 with classification code 6* and containing "structural optimization" and "product development" in subject, title, or abstract

7 Future Trends

This paper attempted to provide a bird-eye survey of the structural optimization, with a special emphasis on its relation to product development. The past literatures are categorized based on their major research focuses: geometry parameterizations, approximation methods, optimization methods, and the integration with nonstructural issues. While the structural optimization is one of the most extensively researched areas in optimization, its use in product development is relatively new and seems yet to gain a mainstream popularity in industry.

Figure 9 shows the number of conference and journal papers (written in English) appeared in Compendex between 1980 and 2004 with classification code 6*, and containing "structural optimization" and "product development" in subject, title or abstract. The classification code 6* includes areas such as mechanical, aerospace, automotive, and marine engineering. It can be seen that the number has been rising rapidly since 2000, indicating the increasing interests in the topic. This trend is likely to continue, and will contribute to more industry acceptance of structural optimization in product development.

The future research in this area will continue to address some or all of open research issues discussed in the earlier sections, which are summarized as:

(1) **Technologies for structural optimization in conceptual design:** method for generating innovative design concepts through optimization, reduced order modeling of multidomain, multiphysics analyses for rapid concept evaluation, and integration to design-for-X.
(2) **Technologies for large-scale structural optimization:** efficient approximation methods for large-scale, nonlinear, and robust/reliability optimization problems, accurate and stable coordination schemes for multidisciplinary optimization problems, and efficient global optimization algorithms to these problems.

While not covered in this paper, the CAD/FEA integration is also an important area of research. Due to the limited interoperability between commercial CAD and FEA/Optimization software, design iterations involving large geometry changes are currently very difficult to automate as an optimization process. Therefore, research and development are highly desired to realize seamless integration of initial geometry creation with CAD software, design optimization with FEA/Optimization software, and further design detailing with CAD software including the generation of manufacturing data.

In addition to the advancements in the above areas, the education of engineers knowledgeable in all of structural analyses, optimization, and product development would be essential for further industry acceptance of structural optimization.

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