CRASHWORTHINESS DESIGN USING META-MODELS FOR APRROXIMATING THE RESPONSE OF STRUCTURAL MEMBERS

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ABSTRACT

Vehicle crashworthiness is an important yet difficult design attribute. It can be argued, that an assisting task for the design process would be to pre-compile library databases for the crash behavior of elementary structural members. Such databases would guide designers to quickly select the appropriate dimensions of structural members once estimates of the load requirements are available. Unfortunately, typical structural member cross-sections in the automotive body have complex geometrical shapes and no standard sizes. Thus, generating crash behavior databases for all possible dimensions of even a single shape is an extremely expensive task. A remedy is to generate databases containing relatively few sizes of the structural members, then employ meta-models to approximate the behavior of the structural members over the complete design domain of the dimensions. This paper investigates various meta-modeling techniques for estimating the behavior of structural members subjected to crash conditions. The examined meta-modeling techniques are: i) polynomial response surface fits, ii) cascaded feed forward neural networks and iii) radial basis neural networks. Results of the study show the superiority of the radial basis networks. Furthermore, a design example of a vehicle B-pillar subjected to roof crush is presented to demonstrate the advantage of pre-complied database and meta-model component selection verses direct optimization of a full structure.

KEYWORDS

Vehicle Crashworthiness Optimization, Meta-Models, Surrogate Models, Response Surface Methods, Neural Networks, Taboo Search, Genetic Algorithms.

1. INTRODUCTION

Vehicle crashworthiness is an important vehicle attribute, which designers strive to improve in order to meet governmental regulations and increase market attractiveness. Unfortunately, crashworthiness is a difficult attribute to improve because of the complex nonlinear interactions between structural members, noisy numerical simulations and the requirement of expensive resources for simulating the crash phenomena. A brief review of the general fields of crashworthiness design optimization reveals three main research categories: i) topology optimization, ii) parametric optimization and iii) development of approximate models for the optimization task. Topology optimization may use material homogenization [1, 2]. Material homogenization optimizes material removal from non-critical zones by optimizing size of microstructure voids. An alternative approach to homogenization is material properties interpolation by using a virtual density parameter, which allows a smooth transition between strong material zones (reinforced regions) and softer material zones, which are foam-filled or voided [3-5]. In general, topology optimization is only useful during the early stages of design, that is, during the conceptual design phase, when many of the dimensions and parameters are not yet accurately known or finalized. On the other hand, parametric optimization is used for obtaining the actual final dimensions, which are to be implemented in building the vehicle. Parametric optimization requires prior definition of structural geometry and sets of variables that are allowed to change within limits are defined by the designer. Then, an optimization algorithm is used to estimate the best values for the variables which satisfy pre-set performance targets. Examples of full vehicle parametric optimization are presented in [6, 7] while examples of substructure optimization are presented in [8, 9]. The main difficulties in crashworthiness parametric optimization are the noisiness of the numerical simulation and the requirement of massive computational resources to run the detailed nonlinear finite element models (FEM) of the full vehicles or subsystems. For example, the case considered by Yang *et al*. [6] required the use of 512 processors running in parallel for 72 hours to perform only two local optimization iterations. Such requirement of expensive resources is a challenge that limits the use of full FEM models for optimization.

Due to the massive requirement of computational resources required for crashworthiness optimization, a fair amount of research is dedicated to finding approximate models that can be used for optimization at a less expensive computational cost. Use of such models helps decrease the number of full FEM simulations required to achieve a good design. The most widely used type of approximate models is the response surface method [10, 11] (including several variations). A possible reason for such dominance is that polynomial response surface fits are generally better at providing approximation when few data points are available compared to other meta-models. The main limitation of such approach is that the crash of a full vehicle structure is a very complicated phenomenon, which seriously limits the trust region of virtually any surrogate model [10]. Another, less popular, meta-modeling approach is the use of coarse mesh FEM, lumped parameter or lattice models [12-14]. The main difficulty associated with such models is the realization of lumped parameters into an actual structural design. However, lumped parameter models do provide formalisms for estimating the load and stiffness requirements of substructures and in some cases individual structural components. More recently, a specialized meta-modeling approach based on the identification of the behavior of individual structural components during crash was developed by the authors. The new approach seems to have good potential for design purposes, while retaining the advantages of lumped parameter models [15].

Breaking the optimization problem of a full vehicle structure into a set of smaller problems using methodologies such as in [12-15] significantly reduces the complexity of the problem. However, selecting the dimensions of structural components to meet the target performance can become a tedious task when considering the large number of structural components in a real vehicle. As such, it can be justified that generating databases that describe the behavior during a crash event of different structural cross-sections can make the component selection task quicker, and such databases would be re-useable across different vehicles. Unfortunately, structural member cross-sections used in the automotive industry have difficult geometrical shapes and no standard sizes, which would make generating databases for all possible sizes of even a single shape an extremely expensive task. A remedy is to generate databases containing relatively few sizes of the structural members, then employ meta-models to approximate the behavior of the structural members over the complete design domain of the dimensions. In that case, the behavior of a single component during crash is simpler and easier to approximate using meta-models and more importantly single component FEM simulation is much more reasonable in terms of computational resources requirement, thereby making it possible to generate a reasonably dense set of data points to fit a meta-model upon.

This paper starts with a brief review of vehicle crashworthiness optimization, which motivates the development of a quick and reliable method to estimate the force-displacement characteristics of automotive structural components. The next section describes the main crash loading curve parameters to be identified, followed by a section dedicated to comparing the capability of different meta-modeling techniques to perform the approximation task. The compared meta-models are: i) polynomial response surface fits, ii) cascaded feed forward neural networks and iii) radial basis neural networks. A design example involving the B-pillar of a vehicle subjected to roof crush conditions is presented to demonstrate the advantage of design approaches using meta-models, then the paper ends with a summary of conclusions.

2. CRASH BEHAVIOR PARAMETERS FOR STRUCTURAL MEMBERS

In order to select appropriate performance parameters to describe the behavior of thin walled structural members during a crash event, several nonlinear FEM simulations are performed for different crash conditions and cross-section shapes. The cross-section shapes studied are the thin walled box-section (Fig. 1) and the thin walled hat-section (Fig. 2). Those sections are often used in the automotive industry.

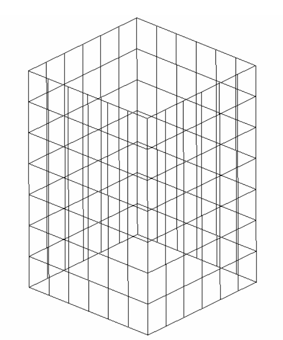

Fig. 1. Model of a Typical Box-Section

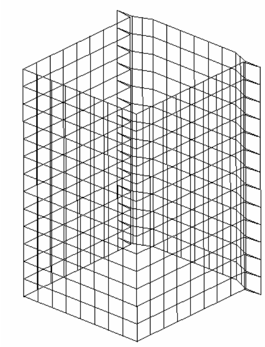

Fig. 2. Model of a Typical Hat-Section

Figures 3 and 4 show dimensionless plots of the typical resistance force (or moment) to deformation for box and hat sections respectively, when subjected to compressive crushing, bending and twisting. The plots were generated by performing nonlinear FEM analyses using the commercial code LS-DYNA [16]. The general deformation resistance behavior is characterized by: i) an increase in the deformation resistance force (or moment) with increase in deformation up till a certain point then ii) a collapse, after which the deformation resistance decreases even as the deformation increases and iii) a steady state, in which the resistance force does not drop below a certain value. Such results of numerical simulation are in general agreement with reported experimental observations [17] as well as with general engineering sense, where one expects to see a near linear behavior when the deformation is small, followed by an occurrence of buckling then steady plastic deformation. It is noted that the considered models are only for short components so that the effect of secondary buckling of long tubes is negligible.

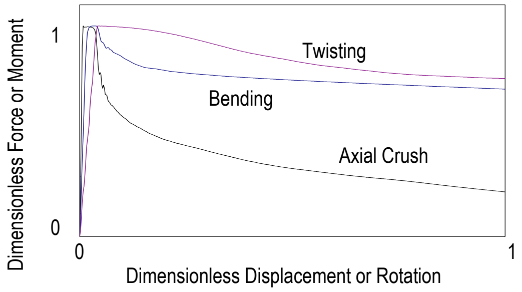

Fig. 3. Typical Box-Section Crash Curves

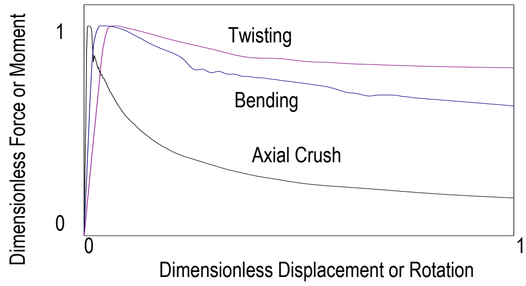

Fig. 4. Typical Hat-Section Crash Curves

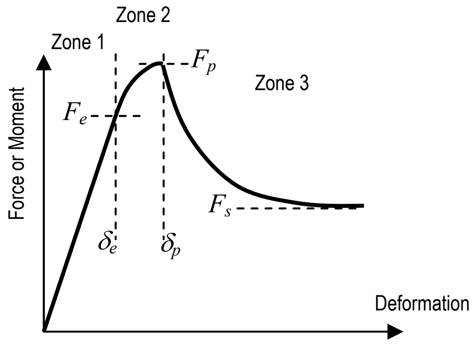

Fig. 5. Crash Curves Main Parameters

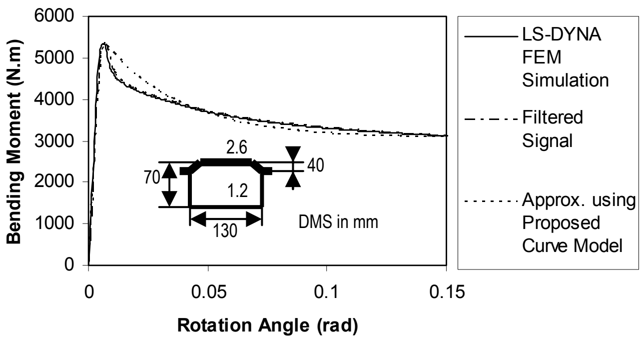

Fig. 6. Example Fit of a Crash Curve

Following the approach in [15], the deformation resistance curve (force or moment) is approximated by dividing the curve into three main zones (Fig. 5). Zone #1 represents a purely linear elastic behavior, while in zone #2, the curve follows a quadratic equation that leads up to the peak value, then in zone #3, the curve follows an exponential decay that asymptotically approaches a steady value. To avoid discontinuities at the interconnections between zones, log-sigmoid functions [18] are used to provide a smooth blend. For the details of the equations, the reader is referred to [15].

For every crash resistance curve, there are three main parameters (Fig. 5), namely the peak load (F_p), steady load (F_s) and the location of the peak load (δ_p). There are also three tuning parameters which allow better matching between the approximated and actual crash resistance curves. The tuning parameters are: extent of the elastic zone (δ_e), maximum elastic force (F_e) and an exponential parameter (μ) that dictates how rapidly the fitted curve collapses to the steady value.

Due to the inherent existence of numerical noise in the FEM crash simulations, prior to the identification of the parameters F_p, F_s, δ_p, δ_e, F_e and μ, the force or moment curve is filtered through a Butterworth digital filter [19] with a cut-off frequency of about 300 Hz. Several plots using different dimensions are generated to confirm that those six parameters are capable of capturing the overall deformation resistance curve (force-displacement or moment-rotation) with fairly good accuracy. However, due to space limitations, only one such plot is shown in Fig. 6, which gives the bending resistance curves of a hat section. It is noted that the bending behavior for unsymmetrical sections such as hat sections depends on the bending direction because of the differences in size and location of the compression flanges.

For any given structural cross-section, there are six behavior curves, namely; axial compression, bending in two directions about two section axes and torsion. Each curve has six identifiable parameters.

Building a good database of sections typically requires about 1000 to 2000 data points for every cross-section shape, which requires many LS-DYNA runs (about 10-15 days run time on a 2.4 GHz PC). However, it is a well invested computational cost since the database is re-useable.

3. META-MODELS FOR CRASH CURVES PARAMETER ESTIMATION

Since the number of data points in a generated database is not enough to cover all design possibilities, it is important to have an accurate way of estimating the crash behavior for dimensions unavailable in the database. In this section, general purpose meta-models (also called surrogate models in the literature) are tested for such estimation tasks. The broad categories of general purpose meta-models in the literature are: i) Response Surface methods [10], ii) Neural Networks [18] and iii) Kriging approaches [20].

Response surface methods, which are available in many variations generally has have the same basic principle [21], which is summarized in the following steps:

1. Assume a symbolic expression that relates outputs to inputs. The functions and variables in the expression do not change, but a set of parameters (like for example the coefficients of a polynomial) could be tuned.
2. Perform tuning of the parameters in the expression to minimize an error function (usually least squares error) for the set of given data points (also called *training points*).
3. Substitute in the symbolic expression to estimate the output due to a given set of inputs.

A key issue for successful implementation of response surface methods is the selection of the symbolic expression in step 1. A popular choice is polynomial expressions [21]. While a high order polynomial generally gives a good fit at the database data points, it may have poor interpolating capability. As such, polynomials beyond the second order are rarely used in practice. In this paper both second order and third order polynomials are considered.

Neural networks represent a family of methods inspired by the way the human brain cells work [18]. In this paper, feed forward neural networks (FFNN) and radial basis neural networks (RBNN) are examined. Other, more sophisticated neural networks (such as neuro-fuzzy networks) are not considered in this paper because they usually involve much user intervention in order to properly tune the network. For the same reason, Kriging models are not considered in this paper.

Since the hat-section shape presents sufficient complexity, it is used as a basis for comparison between the selected meta-models. The five dimension variables (which represent the inputs to a meta-model) are shown in Fig. 7 and the ranges and numbers of data points of every variable are given in Table 1. The outputs of a meta-model are the crash curve parameters F_p, F_s, δ_p, δ_e, F_e and μ for each of the six loading conditions (crushing, twisting and four cases of bending). A total of 1500 data points were generated, however 14 points were excluded because of failures in their nonlinear FEM simulations. Only 1470 of the data points are used for training (or tuning) of the meta-models. The rest of the points are used for performance evaluation. The normalized error for comparing the different meta-models is given as:

$$Err_{Normalized} = \frac{\sum_{samples} \sum_{outputs} Abs(Output - \text{Target}) / outputRange}{nSamples \times nOutputs} \tag{1}$$

Table 2 presents a comparison of the normalized error value for: second and third order polynomial response surface fits, 70 and 120 inner neurons sigmoid-linear FFNN [18] and a regular RBNN. The normalized error values in Table 2 are computed using all six crash curve parameters for the six loading conditions. Thus a meta-model produces thirty six outputs given five inputs (dimension variables). Table 3 provides a similar comparison, but only the main outputs F_p, F_s and δ_p are considered, thus a meta-model produces eighteen outputs. Tables 2 and 3 show how well the meta-model fits the original training set, as well as three random samples at data points not included in the training set. Due to the computational cost, the random sample sizes were fairly small (5 data points per sample). However, the overall performance per sample was found to be consistent, therefore larger sample sizes were not perused.

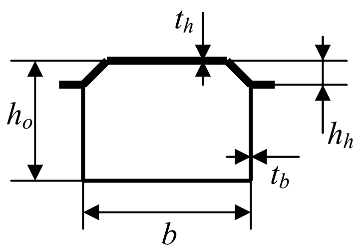

Fig. 7. Hat-Section Dimension Variables

Table 1. Hat Section Variable Ranges

Variable	Ranges*		# of steps in DB
	Minimum	Maximum	
h_o	60.0	100.0	5
b	70.0	160.0	4
h_h	10.0	50.0	5
t_h	2.4	3.2	5
t_b	0.8	1.2	3

* All dimensions are in mm

It is observed from Tables 2 and 3 that the RBNN has less error measure in almost all sample tests than polynomial response surface and FFNN. The observation that polynomial response surface cannot achieve as good performance as RBNN is the inherent limitation on the capability of low order polynomials to fit regions where rapid localized changes occur in the fitted data. Increasing the polynomial order gives better chance at fitting such regions but incurs other difficulties such as ill-conditioned matrices when performing the least squares fit as well as excessive sensitivity to errors in estimating the polynomial constants.

The slightly surprising observation that FFNN show worse performance than polynomial response surface can be attributed to the large number of required outputs. Such a large number of outputs automatically increase the number of inner weights inside the FFNN. For the considered problem, FFNN used to generate Table 2 had approximately 2520 weights (70 inner neurons) to 4320 weights (120 inner neurons), and 1260 to 2160 weights for the networks used to generate Table 3. Such a large number of tunable weights present a difficult task for the network training algorithms. Another observation is that both the 70-inner neurons and 120-inner neurons are exhibiting exactly the same performance, which implies that saturation in terms of performance has been reached. Reaching saturation makes adding more neurons to the FFNN ineffective in improving its performance. RBNN on the other hand are insensitive to the number of required outputs, and they retain the local zone adaptability of neural networks, and therefore it is no surprise that their performance is observed as superior.

Good fitting of the crash curve parameters is a key step towards the use of pre-compiled cross-section databases and meta-models for crashworthiness design, as required in strategies in [15]. Moreover, such databases and meta-models can be used for preliminary design purposes, as demonstrated in the next section.

Table 2. Normalized Error in Meta-Models – 36 Outputs Considered

Meta-Model	Test Data Samples			
	Not within Training Set			Within Training Set
	Sample #1 – at Corners	Sample #2 – at Edges	Sample #3 – Inner Points	Whole Set
Polynomial 2nd Order	0.2011	0.1126	0.2291	0.0522
Polynomial 3rd Order	0.1698	0.1287	0.1031	0.0557
FFNN – 70 Inner Neurons	0.6428	0.3967	0.7885	0.1472
FFNN – 120 Inner Neurons	0.6428	0.3967	0.7885	0.1472
RBNN	0.1172	0.0805	0.1031	0.0022

Table 3. Normalized Error in Meta-Models – 18 Outputs Considered

Meta-Model	Test Data Samples			
	Not within Training Set			Within Training Set
	Sample #1 – at Corners	Sample #2 – at Edges	Sample #3 – Inner Points	Whole Set
Polynomial 2nd Order	0.0936	0.1031	0.2660	0.0266
Polynomial 3rd Order	0.1057	0.1293	0.2578	0.0288
FFNN – 70 Inner Neurons	0.9152	0.4863	1.2577	0.1462
FFNN – 120 Inner Neurons	0.9152	0.4863	1.2577	0.1462
RBNN	0.0968	0.0784	0.0978	0.0014

4. DESIGN EXAMPLE: VEHICLE B-PILLAR SUBJECTED TO ROOF CRUSH

4.1. Problem Description

In the automotive industry, the term *B-pillar* refers to the vertical member on the side of the vehicle, which is located to the rear of the front passenger door (Fig. 8). The B-pillar plays essential roles in protecting the vehicle occupant during i) side crash and ii) roof crush if the vehicle gets overturned. Where side crash requirements usually dictate special reinforcements of the lower part of the B-pillar, such reinforcements rarely extend till the top portion of the B-pillar. Thus by assuming that additional reinforcements in the lower zone of the B-pillar would take care of the side crash requirement, selection of the all-through cross-section of the B-pillar can be done based on roof crush requirements alone. It is noted that an actual B-pillar is usually slightly tapered. However, in the current problem, which is intended for earlier design stages, the taper is ignored.

The roof crush safety is tested by crushing the structure from the roof (Fig. 9), while recording the crush resistance. A vehicle passes the roof crush test if the recorded crush resistance force exceeds 1.5 times the weight of the vehicle (which is a good indication that the vehicle can get overturned without sever roof collapsing). In accordance with such safety requirement, the load carrying capacity requirement for the B-pillar model in this paper is set at a value of 39.2 kN. (or 4000 kg) according to load sharing estimates. Although the force value and dimensions are not those of a particular vehicle model, these values are typical for a compact sized car. The design problem of the hat-section B-pillar is summarized as:

Minimize:	$f = PillarMass$	(2)
Subject To:	$MaxCrushForce > 39240$	(3)
Continuous Variables:	h_o , b, h_h (Fig. 7, ranges given in Table 1)	
Discrete Variables:	t_h , t_b (Fig. 7, ranges given in Table 1, step is 0.1mm)	

(Source: American Iron and Steel Institute: www.steel.org)

Fig. 8. B-Pillar Location in Vehicle

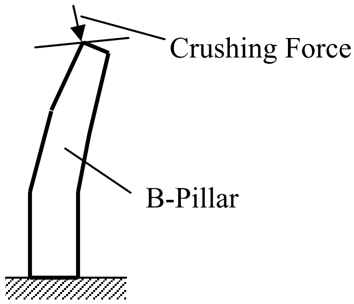

Fig. 9. Model of Roof Crush Test

4.2. Best Known Solution of the Problem

The design problem of a B-pillar subjected to roof crush conditions is discussed in [22] using Mixed Reactive Taboo Search[1] (MRTS). In general, taboo search techniques have a good chance at finding good designs without requiring too many objective function evaluations [23-25]. Like Genetic Algorithms (GA) [26, 27], MRTS does not stop upon encountering a local optimum design, but has the capability to continue searching for even better designs. The basic principle of operation of MRTS pivots upon searching the local *neighborhood* of a current design, while *memorizing* its previous moves and preventing entrapment in local optima through imposing taboo conditions. MRTS has the advantage of allowing the user to pre-dictate the maximum number of model evaluations and when properly applied, it is guaranteed to do no worse than a corresponding local search algorithm. A reference solution of the problem is sought by performing several runs of each of MRTS [22] and real coded GA [27] (Table 4). The runs use a FEM model of the B-pillar for simulating the roof crush test conditions and the maximum number of FEM simulations per run is set at 1500. The best known solution to the problem is the one found by run #5 of GA.

[1] For details of the MRTS algorithm, the reader is referred to [22]

Table 4. Results of Global Optimization Runs for the FEM Model of the B-Pillar

Algorithm	GA					MRTS				
Run ID	#1	#2	#3	#4	#5	#1	#2	#3	#4	#5
h_o (mm)	60.1	61.0	61.2	61.3	61.2	61.1	61.1	62.3	61.2	61.0
b (mm)	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0	70.0
h_h (mm)	12.5	11.6	11.4	11.5	11.4	11.5	11.5	12.4	11.5	11.6
t_h (mm)	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4	2.4
t_b (mm)	1.2	0.8	1.0	0.9	0.8	0.9	0.8	0.8	1.2	0.8
Mass (Kg)	4.590	4.581	4.579	4.580	4.579	4.579	4.579	4.621	4.582	4.581
Con. Force (kN)	39.2	39.2	39.2	39.2	39.2	39.2	39.2	40.7	39.2	39.2

4.3. Problem Solution Employing the RBNN Meta-Model

Having access to a pre-compiled database of cross-sections crash properties can be valuable to designers. Instead of performing costly FEM simulations to evaluate the performance during crash, it is possible to use the meta-model for a rough estimate of the suitable section size then further tune the estimate through a short local optimization run. Only RBNN is used in this design example since it seemed to have superior performance. After thoroughly searching the design alternatives using the RBNN (which is quick task), a short local optimization run is performed to fine tune the design. Table 5 shows the success of this approach in finding a design that is almost as good as the best known design (only 0.001% difference in the structural mass), while utilizing only 9% of the computational resources.

Table 5. Results and Run Time Comparison – Direct vs. RBNN Assisted Optimization

	Best Known Solution	RBNN – Assisted
h_o (mm)	61.2	61.1
b (mm)	70.0	70.0
h_h (mm)	11.4	11.5
t_h (mm)	2.4	2.4
t_b (mm)	0.8	0.8
Mass (Kg)	4.57905	4.57910
Constraint Force (kN)	39.2	39.2
Number of Full FEM Simulations	1500	120
Total Run Time on a 2.4GHz PC	36 hours	3 hours

5. CONCLUSIONS

This paper motivates generating pre-compiled databases and suitable meta-models for approximating crash properties of structural cross-sections. Measures for approximating the crash behavior of structural members are introduced and a comparison of meta-models is performed. The comparison showed that radial basis neural networks have better performance in estimating such crash behavior compared to polynomial response surface fits and feed forward neural networks. A design example of a vehicle B-pillar demonstrated how proper implementation of meta-models can significantly improve the efficiency of the design task.

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