Struct Multidisc Optim 27, 89–96 (2004)
DOI 10.1007/s00158-003-0356-x

A sensitivity-based commonality strategy for family products of mild variation, with application to automotive body structures

R. Fellini, M. Kokkolaras, N. Michelena, P. Papalambros, A. Perez-Duarte, K. Saitou, and P. Fenyes

Abstract Identification of the product platform is a key step in designing a family of products. This article presents a methodology for selecting the product platform by using information obtained from the individual optimization of the product variants. Under the assumption that the product variety requires only mild design changes, a performance deviation vector is derived by taking into consideration individual optimal designs and sensitivities of functional requirements. Commonality decisions are based on values of the performance deviation vector, and the product family is designed optimally with respect to the chosen platform. The proposed methodology is applied to the design of a family of automotive body structures. Variants are defined by changing the functional requirements they need to satisfy and/or the geometry of the associated finite element models.

Key words product platform, product family, optimal design, automotive body

1 Introduction

Sharing components within a family of products can be an effective method for corporations to increase cost savings (Meyer and Lehnerd 1997; Ericsson and Erixon 1999). A drawback to commonality is that a performance deviation can be incurred with regard to optimized individual product design. The challenge is to choose which components to share (i.e., define the product platform), and design the product family with minimal deviation from individual optimal designs.

Simpson *et al.* (1999) proposed a method for product platform synthesis and exploration based on a market segmentation grid and leveraging and scaling concepts (Meyer and Lehnerd 1997). They solved the family design problem by using goal programming. This methodology has been built upon in a number of subsequent publications (Messac *et al.* 2000; Conner *et al.* 1999; Nayak *et al.* 2000), where the last method uses robust design principles to aid in selecting the product platform.

Gonzalez-Zugasti *et al.* (1998) presented a method that uses cost gain models as the driving force for designing the product platform while satisfying performance and budget constraints: a priori specified platforms are optimized first; family variants are designed second. Subsequently Gonzalez-Zugasti and Otto (2000) formulated a design optimization problem for modular product architecture that can be solved to determine simultaneously module designs and their combination for the variant instantiations. Fujita *et al.* (2001) proposed a method for simultaneous optimization of module attributes and combinations. The modular architecture of the product family is fixed in both of the latter papers.

Siddique *et al.* (1998) examined the applicability of product variety concepts to automotive design. In particular, they investigated whether product variety design concepts such as standardization, delayed differentiation, modularity, module interfaces, robustness, and mutability can be utilized. They limited their consideration for platform to the underbody structure of a vehicle and came to the conclusion that some of these concepts cannot be applied, mainly because of the integral nature of the product architecture. However, they did mention the possibility of partitioning the underbody platform into major manufacturable modules that can be assembled.

Received: 6 November 2002
Published online: 23 March 2004

R. Fellini[1,✉], M. Kokkolaras[1], N. Michelena[1], P. Papalambros[1], A. Perez-Duarte[1], K. Saitou[1], and P. Fenyes[2]

[1] Department of Mechanical Engineering, University of Michigan, Ann Arbor, MI 48109, USA
e-mail: {rfellini,mk,nestorm,pyp,alexispz,kazu}@umich.edu

[2] General Motors R&D Center, Vehicle Development Research Lab, 30500 Mound Rd., Warren, MI 48090, USA
e-mail: peter.a.fenyes@gm.com

Nelson *et al.* (1999) formulated platform design as a multicriteria optimal design problem. Given a fixed platform, a set of optimal Pareto points is generated based on the importance of the conflicting variant objectives. The designer can identify trade-offs, evaluate multiple platforms, and then make related decisions. Fellini *et al.* (2000) applied this concept to the design of an automotive product family based on a powertrain platform and examined the hierarchical structure of the platform design problem. Kokkolaras *et al.* (2002) extended the target cascading formulation to the design of product families with pre-specified platforms. Both common and individual components, subsystems, and/or systems of the family products were designed optimally with respect to family and variant targets.

In the present work a methodology is proposed for making commonality decisions based on individual optima and sensitivity analysis of functional requirements. Emphasis is put on families of vehicle body structures using modeling approaches proposed by Fenyes (2000). The method assumes only “mild variants”, so that design changes can be guided by sensitivity information reasonably well.

The article is organized as follows: Platform-based design of body structures is introduced in the next section. The mathematical derivation underlying the proposed approach is presented. The methodology based on this derivation is formulated and demonstrated by means of an automobile body structure case study. Results are discussed and conclusions are drawn.

2 Platform-based design of body structures

A *component* is defined as a manufactured object that is the smallest (indivisible) element of an assembly, and is described by a set of design variables. A *product* is an artifact made up of components. A *product platform* is the set of all components, manufacturing processes, and/or assembly steps that are common to a set of products. A *product family* is the set of products that are built upon a product platform. A family product is also referred to as a *product variant*. Two types of sharing are possible when selecting a product platform that is not based on manufacturing processes or assembly steps. In *component sharing*, one or more components are common across a family of products as shown in Fig. 1. In addition, it is possible to share “scaled” versions of components. Mathematically this can be described as *design variable sharing*, where the components themselves are derived from a platform. The example in Fig. 2 shows the cross-section of two structural beam elements. While the height and width of both parts are the same, the thickness is different. The product containing the “thicker” component variant has higher rigidity requirements. By not sharing the thickness, the other product with lesser rigidity requirements does not have to take on an unnecessarily large deviation from the optimal weight. Possible manufacturing advantages are illustrated by this example. By keeping width and height invariant, the same stamping equipment may be used with different gauge steel. In general, manufacturing considerations should be taken into account in the design of platforms. We do not address this aspect explicitly but we attempt to recognize the associated design impact.

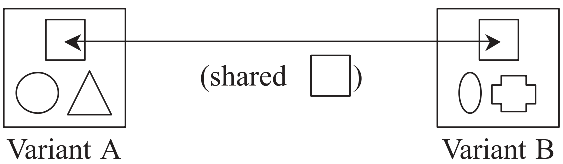

Fig. 1 Platform-based products (component sharing)

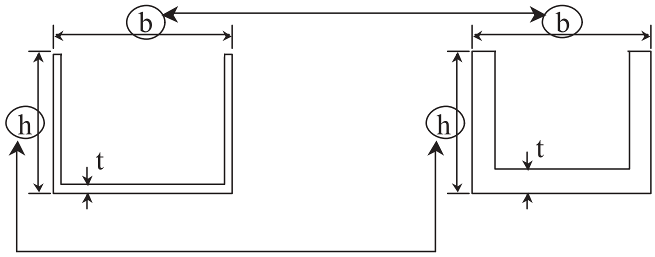

Fig. 2 Platform-based components (design variable sharing)

2.1 Problem formulation

The following definitions are necessary to formulate the variant and family design problems:

- $\mathcal{P} = \{p_1, p_2, \dots\}$: set of m products
- $\mathbf{x}^p$: column vector of design variables for the product $p \in \mathcal{P}$
- $\mathcal{S} = \{s_1, s_2, \dots\}$: set of all permissible platforms, where each $s \in \mathcal{S}$ is a set of indices describing a platform
- s^*: set of indices describing the “optimal” platform
- $\mathbf{x}^{p,\circ}$: null-platform optimal design of product p, corresponding to the individual optimal design solutions of Problem (1) below
- $\mathbf{x}^{p,*}$: family-optimal design of product p, solution of Problem (2) below; because of sharing variables whose indices are in s, for $p, q \in \mathcal{P}$ and $i \in s$, we have $x_i^{p,*} \equiv x_i^{q,*} = x_i^*$
- $\mathcal{G}^p$: set of indices of the active constraints at the null-platform optimum of product p

For convenience, equality constraints are assumed to have been eliminated implicitly or explicitly. The individual optimal design problem for product variant p can be formulated as

$$\min_{\mathbf{x}^p} \quad f^p(\mathbf{x}^p) \qquad (1)$$
$$\text{subject to} \quad \mathbf{g}^p(\mathbf{x}^p) \leq \mathbf{0}\,.$$

The family design problem is then formulated as

$$\min_{\mathbf{x}=[\mathbf{x}^{p1},\mathbf{x}^{p2},\ldots]} \quad \{f^p(\mathbf{x}^p)\} \quad \forall\, p,q \in \mathcal{P}\,,\, i \in \mathcal{S}\,,\, p<q \qquad (2)$$

$$\text{subject to} \quad \mathbf{g}^p(\mathbf{x}^p) \le \mathbf{0}\,,$$
$$x_i^p = x_i^q\,.$$

The platform selection methodology can be summarized as follows: Quantify performance deviations by considering individual optimal designs and sensitivities of functional requirements; decide which components can be shared (i.e., determine the platform) with minimal performance deviation; optimally design the product family around the chosen platform.

3 Commonality decisions

The proposed approach is based on the use of optimality and sensitivity information obtained from individual product optimization to assess the potential deviation from the optimal design incurred by sharing variables. When the products in the family contain a large number of components that are candidates for sharing, platform selection entails the solution of a large combinatorial problem. In the approach proposed, this problem is reduced to a simpler one under the assumptions listed below. The derivation presented in the following section is based on a first order Taylor series approximation. Therefore, in order for the approximation to remain reasonably accurate, the general condition is that the individual optimal designs lie not "too far away" from each other so that the linear approximation is valid in the region between them. The derivation will be presented for a family of two products A and B for simplicity; it can be generalized readily for more products.

We make the following assumptions:

1. Self-sharing (i.e., component sharing within the same variant) is not possible.
2. Components are either shared by all family members or not at all.
3. Null-platform optimal designs lie "close enough" to each other.
4. The platform design (denoted here by superscript *) lies in the convex hull of the individual solutions (denoted by superscripts $^{A,\circ}$ and $^{B,\circ}$). That is, $\exists\ \lambda_i \in [0,\,1]$ such that $\forall\, i \in s,\ x_i^* = \lambda_i\, x_i^{A,\circ} + (1-\lambda_i)\, x_i^{B,\circ}$.
5. Constraint inactivity remains unchanged between individual and family design problems.

We refer to the design solutions that satisfy these assumptions as "mild variants".

Sharing may cause deviations from individually optimized products designs, which is measured by the responses representing the functional requirements. In the context of the approach introduced in this article, the commonality decision consists of deciding which variables to share. The design variables are arranged in order of increased performance deviation value, and the number n of variables to share is determined by a limit on acceptable design deviations. The optimal platform is determined by minimizing the relative deviation, Δ^p, of the designs based on any platform with n shared variables with respect to the null-platform optimal designs – while remaining in the feasible space for the variants. Formally, this is stated as

$$\min_{s\in\mathcal{S}} \quad \Delta \qquad (3)$$

$$\text{subject to} \quad |s| = n,$$

where $\Delta = \Delta^A + \Delta^B$ and

$$\Delta^p = \left|f^p\left(\mathbf{x}^{p,*}\right) - f^p\left(\mathbf{x}^{p,\circ}\right)\right| + \sum_{j\in\mathcal{G}^p} \max\left(g_j^p(\mathbf{x}^{p,*}), 0\right)$$

for $p \in \mathcal{P} = \{A, B\}$. By the definition of $\mathcal{G}^p$, g_j^p is active at the null-platform optimum $\mathbf{x}^{p,\circ}$; therefore $g_j^p(\mathbf{x}^{p,\circ}) = 0$. Normalization is used to enable the meaningful summation of responses of different nature.

A first order Taylor series approximation of the variation in each response f^p, g_j^p, for $j \in \mathcal{G}^p$ is introduced in agreement with the assumptions described in Sect. 3:

$$f^p(\mathbf{x}^{p,*}) - f^p(\mathbf{x}^{p,\circ}) \approx \left(\nabla f^{p,\circ}\right)^T \left(\mathbf{x}^{p,*} - \mathbf{x}^{p,\circ}\right);$$

$$g_j^p(\mathbf{x}^{p,*}) \approx \left(\nabla g_j{}^{p,\circ}\right)^T \left(\mathbf{x}^{p,*} - \mathbf{x}^{p,\circ}\right),$$

where $\nabla f^{p,\circ}, \nabla g^{p,\circ}$ is the gradient of f^p, g^p evaluated for the null-platform optimal design of product p.

Furthermore, under Assumption 4, the relation between the shared variables, $i \in s$, and the null platform can be rewritten as

$$\left(x_i^* - x_i^{A,\circ}\right) = (1-\lambda_i)\left(x_i^{B,\circ} - x_i^{A,\circ}\right).$$

Consequently, the deviation of the objective f in one variant A due to sharing of the variables x_i, $i \in s$, is approximated by

$$f^A(\mathbf{x}^*) - f^A\left(\mathbf{x}^{A,\circ}\right) \approx \sum_{i\in s} \nabla_i f^{A,\circ}\left(x_i^* - x_i^{A,\circ}\right)$$
$$\approx \sum_{i\in s} (1-\lambda_i)\, \nabla_i f^{A,\circ}\left(x_i^{B,\circ} - x_i^{A,\circ}\right).$$

Letting $\delta_i = |x_i^{B,\circ} - x_i^{A,\circ}|$, an upper bound and an approximation on the total variation in Δ^A is

$$\Delta^A \le \sum_{i\in s} (1-\lambda_i)\left(\left|\nabla_i f^{A,\circ}\right|\delta_i + \sum_{j\in\mathcal{G}^A} \max\left(\nabla_i g_j{}^{A,\circ}\,\delta_i, 0\right)\right). \qquad (4)$$

A similar upper bound can be obtained for Δ^B. We define the performance deviation vector $\mathbf{\Pi}$, whose entries correspond to performance deviations due to sharing:

$$\Pi_i = (1-\lambda_i)\Big(\left| \nabla_i f^{A,\circ} \right| \delta_i + \sum_{j \in \mathcal{G}^A} \max\left(\nabla_i {g_j}^{A,\circ}\, \delta_i, 0\right) \Big) +$$
$$\lambda_i \Big(\left| \nabla_i f^{B,\circ} \right| \delta_i + \sum_{j \in \mathcal{G}^B} \max\left(\nabla_i {g_j}^{B,\circ}\, \delta_i, 0\right) \Big) . \qquad (5)$$

The l_1 norm of the vector $\mathbf{\Pi}$ provides an upper bound on the actual performance deviation Δ:

$$\Delta \leq \|\mathbf{\Pi}\|_1 . \qquad (6)$$

The approach adopted in this article for approximating a solution to the original problem described in Problem (3) is to minimize the upper bound on Δ as given in (6). In this regard, the choice of the parameters λ_i has to be discussed. These parameters are determined theoretically by the position of the family solution for a given platform s relative to the position of the null-platform solutions for the two variants (Assumption 4). In the framework described here, the exact values of λ_i are not known a priori since the solution to the family problem is not available. In this regard, we simply assume that $\lambda_i = 1/2, \forall\, i$. Hence, there is no bias towards one variant or the other with regard to the family design variable values. The choice of this value does not affect the commonality decisions. However, the validity of the "convex-hull" assumption (Assumption 4) needs to be checked after solving the family design problem to ensure that commonality considerations are reasonable for the related component or design variable.

The design variables are arranged in order of increasing Π_i. The variables to be shared are the first n variables below some threshold. This minimizes the upper bound on Δ according to (6) as an acceptable alternative to solving Problem (3).

4 Proposed methodology

The proposed general methodology for selecting the product platform and designing the product family is as follows:

1. Generate product variants based on:
   a. design requirements,
   b. geometry (no topological changes) of the model(s),
   c. or both.
2. Develop appropriate analysis models and identify inputs and outputs.
3. Formulate and solve the optimal design problem (1) for each variant, i.e., find the null-platform optimal designs.
4. Use optimal design variables and sensitivity information to compute the performance deviation vector $\mathbf{\Pi}$ by means of (5).
5. Sort the variables in order of increasing Π_i.
6. Using the performance deviation vector $\mathbf{\Pi}$, decide which components to share based on how much performance deviation is acceptable.
7. Formulate and solve the family design problem (2) for the chosen platform.
8. Compare family-optimal designs to individual variant optimal designs and evaluate the actual performance deviation, Δ; iterate if necessary.

5 Application study

A family of automotive body structures is considered. A variant is defined as a structure associated with specific dimensional properties (lengths) and functional requirements.

5.1 Model description

The structures are modeled using finite elements in MSC Nastran according to modeling approaches described in Fenyes (2000). Modal and static load cases (torsion on the front and rear shock towers, and bending) are considered, as shown in Fig. 3. It is assumed that these load cases give access to the properties that the designer wishes to tailor, and therefore are valid as a basis of the design.

The finite element analysis outputs mass (m) and natural frequencies (ω), in addition to displacements and stress responses for static load cases of front torsion, rear torsion, and bending (denoted d_{ft}, d_{rt}, d_b, respectively) along with corresponding sensitivity information for all the design variables. These are the cross-sectional dimensions of the beams (width b, height h, and thickness t) and thicknesses t of the shells. There are 66 design variables.

We used the SCPIP algorithm (Zillober 2001) for solving the optimization problems, which is an implementation of the method of moving asymptotes (MMA), tailored to solve large-scale structural optimization problems efficiently.

As mentioned, variants are generated either by implementing dimensional changes or by imposing different design requirements. We examine these two cases next.

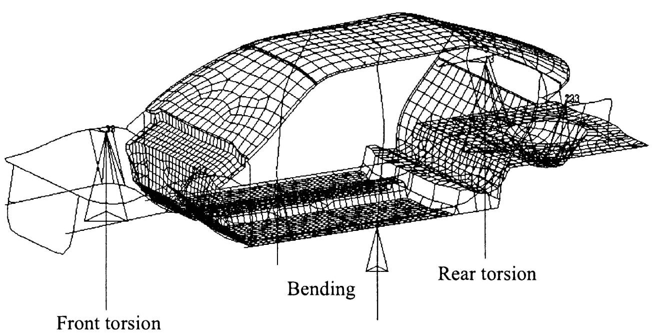

Fig. 3 Automotive body structure model

5.2 Dimensional variants

The individual optimal design problem is formulated as

$$\min_{\mathbf{b},\mathbf{h},\mathbf{t}} \quad m \tag{7}$$

$$\text{subject to} \quad \omega_1 \geq 21\,\text{Hz}\,,$$
$$\omega_2 \geq 24\,\text{Hz}\,,$$
$$d_{ft} \leq 2.9\,\text{mm}\,,$$
$$d_{rt} \leq 2.9\,\text{mm}\,,$$
$$d_b \leq 0.2\,\text{mm}\,,$$
$$\sigma_{max} \leq 25\,\text{MPa}\,.$$

Here we consider a family of two variants based on dimensional changes (cf. Figure 4) having the same objective functions and constraints.

As shown in Fig. 4, a second variant is generated by stretching the wheelbase and trunk of the baseline vehicle. The engine compartment is shortened, and therefore a smaller engine (and lumped mass representing the engine) is assumed. The models will be correspondingly referred to as the *short* and *long* wheelbase body models. The null-platform optima are summarized in Table 1.

The performance deviation vector $\mathbf{\Pi}$ is computed according to (5), and the platform is determined as described in Sect. 4. Using a threshold value of 0.01, 59 variables are selected for sharing and the family problem (2) is solved. The family problem is solved also considering a "total" platform – in which all variables are shared – to assess the usefulness of the approach. Family optima obtained for both platforms are given in Table 2.

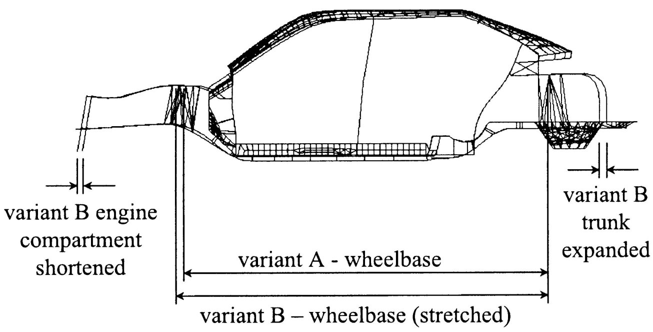

Fig. 4 Automotive body structure dimensional variants

Table 1 Null-platform optima (dimensional variants)

	short	long
mass (kg)	715.13	703.36
ω_1 (Hz)	21.00	22.06
ω_2 (Hz)	24.82	27.00
d_{ft} (mm)	2.158	2.170
d_{rt} (mm)	1.905	1.909
d_b (mm)	0.200	0.200

Table 2 Optima for 59-variable and total platforms (dimensional variants)

	59 var. platform		total platform	
variant	short	long	short	long
mass (kg)	715.17	703.54	725.65	703.37
ω_1 (Hz)	21.00	22.06	21.00	22.24
ω_2 (Hz)	24.82	27.00	25.83	27.00
d_{ft} (mm)	2.158	2.170	2.082	2.171
d_{rt} (mm)	1.905	1.909	1.837	1.911
d_b (mm)	0.200	0.200	0.191	0.200

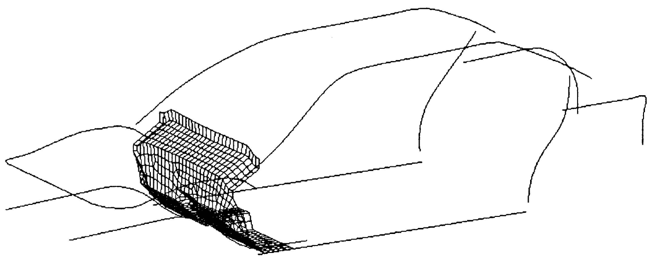

Fig. 5 Dimensional variants: Non-shared components

Overall, the family based on the 59-variable platform is close to the null platform: The optimal masses of both the short and long wheelbase variants are almost identical to the corresponding null-platform designs. The long wheelbase variant using the total platform is still close to the corresponding null-platform variant, compared with a 10.5-kg difference in mass in the short wheelbase variants. The components that are not completely shared among the variants are shown in Fig. 5.

For each of these components the material thickness is the variable that varies. Overall a large number of variables may be shared with negligible performance deviation, considering that the performance deviation of each variant is less than 1.5% compared to the corresponding null-platform variants. This can be traced to the fact that the variants do not have competing design objective functions, and that their geometric configurations are very similar (cf. Fig. 4). The combination of these two factors results in relatively close individual optima and family optima. The next study was subsequently devised to test the proposed methodology on a problem that does not present these features, and it is discussed more thoroughly.

5.3 Performance variants

We now look at variants based on the same geometric model (the short wheelbase model) having different design objectives and constraints. Two variants with competing objectives are designed, denoted "stiff" and "light weight", respectively. In the former the designer aims at maximizing the stiffness of the structure to improve ride quality, while in the latter the goal is to minimize weight to improve fuel economy.

The flexibility φ is defined as a weighted sum of the displacements d_{ft}, d_{rt}, d_b in the three load cases considered, namely, front torsion, rear torsion, and bending, respectively. The weights approximate the ratios of the expected displacements (cf. null-platform optima in Table 1) in each load case; hence flexibility is computed as follows:

$$\varphi = d_{ft} + d_{rt} + 10 d_b \,. \tag{8}$$

The optimal design problem statement for the light weight variant is

$$\min_{\mathbf{b},\mathbf{h},\mathbf{t}} \quad m \tag{9}$$

subject to $\omega_1 \geq 15\,\mathrm{Hz}$,

$\omega_2 \geq 17\,\mathrm{Hz}$,

$d_{ft} \leq 2.9\,\mathrm{mm}$,

$d_{rt} \leq 2.9\,\mathrm{mm}$,

$d_b \leq 0.2\,\mathrm{mm}$,

$\sigma_{max} \leq 25\,\mathrm{MPa}$,

while for the stiff variant the statement is as follows:

$$\min_{\mathbf{b},\mathbf{h},\mathbf{t}} \quad \varphi \tag{10}$$

subject to $\omega_1 \geq 21\,\mathrm{Hz}$,

$\omega_2 \geq 24\,\mathrm{Hz}$,

$m \leq 822\,\mathrm{kg}$,

$\sigma_{max} \leq 25\,\mathrm{MPa}$.

Each variant is optimized individually to obtain a null-platform design. The optimal objective function values for the light weight and stiff variants are 691.87 kg and 4.4049 mm, respectively. The null-platform optimal designs and sensitivities are used to compute the performance deviation vector $\mathbf{\Pi}$.

The design variables are arranged in order of increasing performance deviation. Figure 6 depicts a plot of the sorted performance deviation vector.

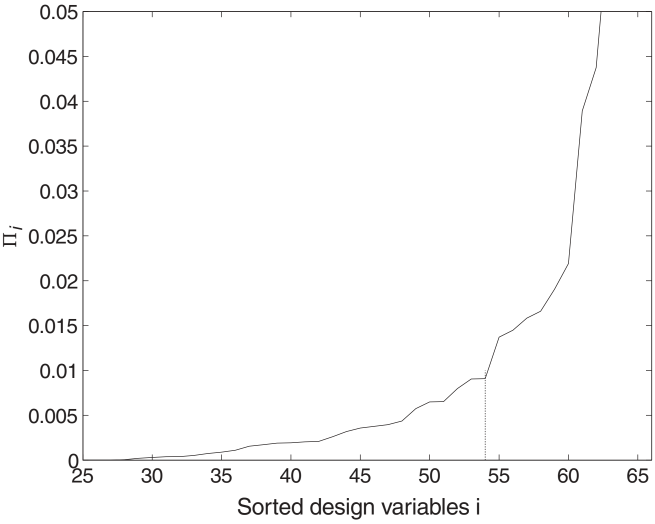

Fig. 6 Sorted performance deviation vector $\mathbf{\Pi}$ (performance variants)

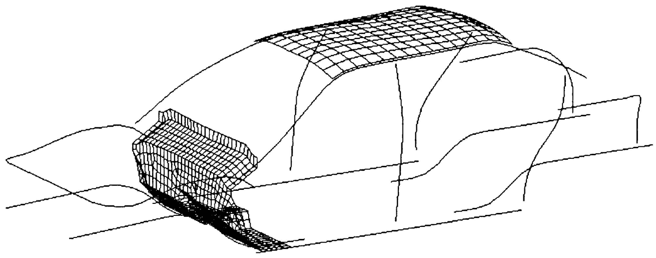

Fig. 7 Performance variants: Non-shared components

The graph shows that the performance deviation remains low for the first 50 variables approximately, and then begins to increase sharply. We chose a 54-variable platform based on the fact that the curve exhibits a sharp increase after 54 variables (cf. Fig. 6). The components that are not shared among the variants are shown in Fig. 7. As in the previous example the material thickness is most often the dimension that varies. One exception is that the rocker panels differ in width, height, and thickness. We solved the family problem for the 54-variable platform and the total platform by minimizing the distance to the null-platform optimum. The results are shown in Table 3, where the variant objectives have been underlined.

Figure 8 shows the Pareto sets for the 54-variable and total platforms. The 54-variable platform shares all but

Table 3 Optima for null, 54-variable, and total platforms (performance variants)

	null platform		54-variable platform		total platform	
variant	stiff	l. weight	stiff	l. weight	stiff	l. weight
mass (kg)	822.00	691.87	822.00	699.90	822.00	822.00
d_{ft} (mm)	1.581	2.429	1.595	2.270	1.607	1.607
d_{rt} (mm)	1.396	2.148	1.409	1.007	1.419	1.419
d_b (mm)	0.1427	0.2922	0.1429	0.2829	0.1443	0.1443
flexibility(mm)	4.405	7.499	4.433	7.107	4.468	4.468

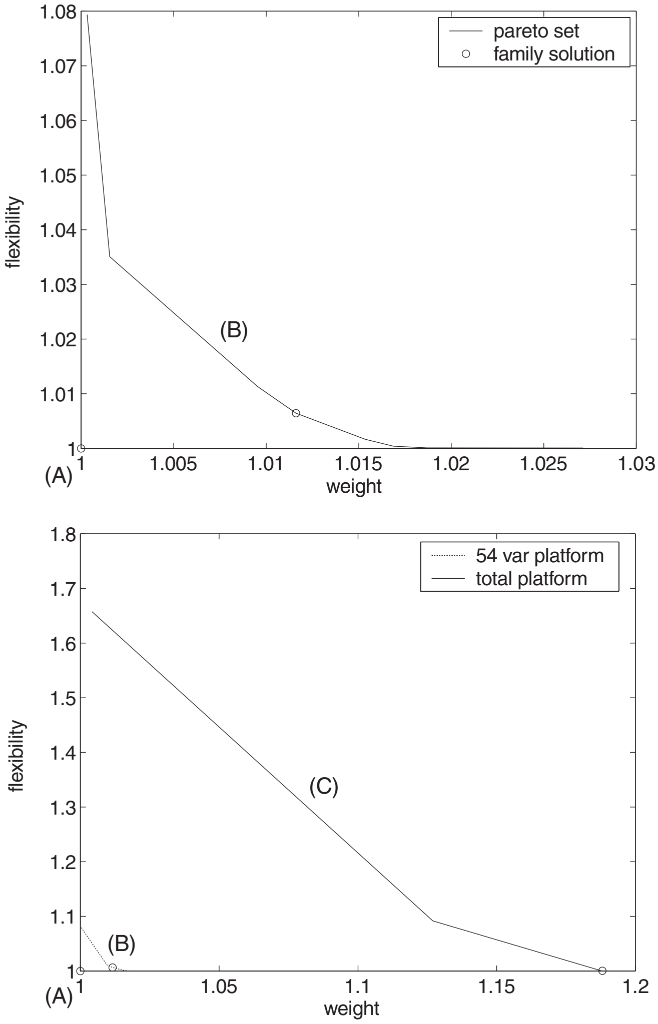

Fig. 8 Pareto sets for the 54-variable and total platforms, with normalized objectives (performance variants). The plot on the top contains the null platform point (A) and the Pareto set for the 54-variable platform (B). The plot on the bottom includes also the Pareto set for the total platform (C).

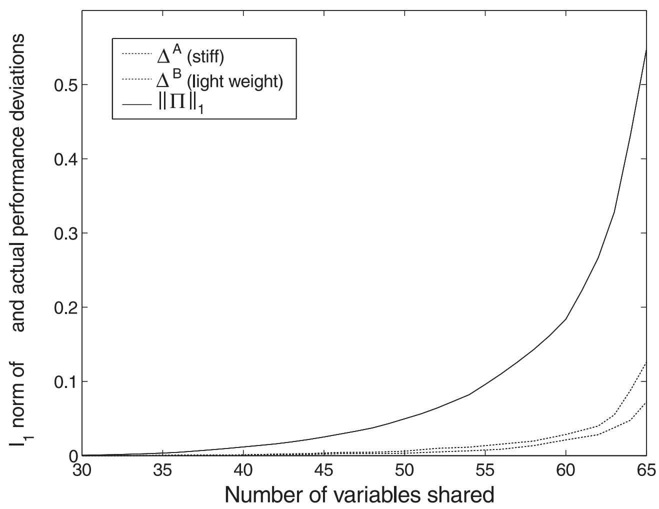

Fig. 9 Normalized actual performance deviation vs. upper deviation bound as computed by means of the performance deviation vector for different platforms, i.e., number of shared variables (performance variants)

18% of the variables, with a deviation of 0.6% for the stiff variant and 1.16% for the light weight variant. In contrast, the total platform has a 1.4% deviation for the stiff variant and an 18.8% deviation for the light weight variant.

5.4 Discussion

The validity of some of the assumptions described in Sect. 3 can only be checked a posteriori, i.e., after solving the individual optimization problems and the family design problem. We checked the assumptions for both case studies.

Assumptions 1 and 2 are automatically satisfied by the implementation of the methodology for this case study.

The distance between the null-platform designs was relatively small for both cases (Assumption 3).

Assumption 4 is satisfied; by inspecting the results obtained from solving the family design problems, $\exists\, \lambda_i$ such that $x_i^* = \lambda_i\, x_i^{\text{stiff},\circ} + (1-\lambda_i)\, x_i^{\text{l.weight},\circ}\, \forall i$ in all cases. This assumption holds for this problem but may not hold for other problems. It is rather strong and further research is needed in order to relax it.

Assumption 5 is designed to avoid the case in which a constraint that is inactive in the individual solution becomes active in the family solution, a case which is not taken into account in the current derivation of Sect. 3.2. Here this assumption is validated with no additional constraints becoming active. In fact, one of the active constraints became inactive in the performance variants case. This is expected, since adding equality constraints in the family design problem (the commonality constraints), it is likely that some constraints may become inactive.

If the assumptions required by the approximation are satisfied, the performance deviation vector $\mathbf{\Pi}$ provides an upper bound on the potential performance deviation. To illustrate the fact that the performance decision vector is a conservative metric, the family problem was solved for several platforms, with a number of variables ranging from 30 to 65. In Fig. 9, the actual objective deviations for each variant are compared with the upper bound given by the l_1 norm of $\mathbf{\Pi}$ for different platforms (number of shared variables).

6 Conclusions

The proposed methodology uses first-order information, obtained from individual design optimizations to compute a metric for performance deviations attributed to component sharing. Under the assumption of mild variations among family products, this analysis can be used to identify which components should or should not be part of the product platform, and becomes essential when the number of sharing combinations becomes too large to search exhaustively. The methodology has been applied to the design of a family of automotive body structures. Results obtained for the two case studies demonstrate the usefulness of the proposed approach. Future work focuses on integrating this methodology with the commonality decision formulation presented by Fellini *et al.* (2002).

Acknowledgements This research was partially supported by the Automotive Research Center, a US Army Center of Excellence in Modeling and Simulation of Ground Vehicles, by a US Army Dual-Use Science and Technology Project, and by the General Motors Collaborative Research Lab at the University of Michigan. This support is gratefully acknowledged. We would like to thank Christian Zillober for providing the SCPIP algorithm, and Joe Donndelinger, Devadatta Kulkarni, and Robin Stevenson for their many helpful ideas and suggestions. The opinions expressed in this paper are solely those of the authors and do not reflect acknowledgement or acceptance by the sponsors.

References

Conner, C.G.; De Kroon, J.P.; Mistree, F. 1999: A product variety tradeoff evaluation method for a family of cordless drill transmissions. In: *Proceedings of the 1999 ASME Design Engineering Technical Conferences* (held in Las Vegas), Paper No. DAC-8625

Ericsson, A.; Erixon, G. 1999: *Controlling design variants: modular product platforms*. Dearborn: Society of Manufacturing Engineers

Fellini, R.; Kokkolaras, M.; Papalambros, P.; Perez-Duarte, A. 2002: Platform selection under performance loss constraints in optimal design of product families. In: *Proceedings of the 2002 ASME Design Engineering Technical Conferences* (held in Montreal), Paper No. DAC-34099; also submitted to *J. Mech. Des.*

Fellini, R.; Papalambros, P.; Weber, T. 2000: Application of a product platform design process to automotive powertrains. In: *Proceedings of the 8th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization* (held in Long Beach), Paper No. AIAA-2000-4849

Fenyes, P.A. 2000: Multidisciplinary design and optimization of automotive structures – a parametric approach. In: *Proceedings of the 8th AIAA/NASA/USAF/ISSMO Symposium on Multidisciplinary Analysis and Optimization* (held in Long Beach), Paper No. AIAA-2000-4706

Fujita, K.; Yoshida, H. 2001: Product variety optimization: simultaneous optimization of module combination and module attributes. In: *Proceedings of the 2001 ASME Design Engineering Technical Conferences* (held in Pittsburg), Paper No. DAC-21058; also to appear in *Concurrent Eng.: Res. Appl.*

Gonzalez-Zugasti, J.P.; Otto, K.N. 2000: Modular platform-based product family design. In: *Proceedings of the 2000 ASME Design Engineering Technical Conferences* (held in Baltimore), Paper No. DAC-14238

Gonzalez-Zugasti, J.P.; Otto, K.N.; Baker, J.D. 1998: A method for architecting product platforms with an application to interplanetary mission design. In: *Proceedings of the 1998 ASME Design Engineering Technical Conferences* (held in Atlanta), Paper No. DAC-5608; also appeared in 2000, *Res. Eng. Des.* **12**(1), 48–60

Kokkolaras, M.; Fellini, R.; Kim, H.M.; Michelena, N.F.; Papalambros, P.Y. 2002: Extension of the target cascading formulation to the design of product families. *Struct. Multidisc. Optim.* **24**(4), 293–301

Messac, A.; Martinez, M.P.; Simpson, T.W. 2000: Effective product family design using physical programming and the product platform concept exploration method. In: *Proceedings of the 2000 ASME Design Engineering Technical Conferences* (held in Baltimore), Paper No. DAC-14252; also appeared in 2002, *Eng. Optim.* **34**(3), 245–261

Meyer, M.H.; Lehnerd, A.P. 1997: *The power of product platforms*. New York: The Free Press

Nayak, R.U.; Chen, W.; Simpson, T.W. 2000: A variation-based methodology for product family design. In: *Proceedings of the 2000 ASME Design Engineering Technical Conferences* (held in Baltimore), Paper No. DAC-14264; also appeared in 2002, *Eng. Optim.* **34**(1), 65–81

Nelson, S.A.; Parkinson, M.B.; Papalambros, P.Y. 1999: Multicriteria optimization in product platform design. In: *Proceedings of the 1999 ASME Design Engineering Technical Conferences* (held in Las Vegas), Paper No. DAC-8676; also appeared in 2001, *J. Mech. Des.* **123**(2), 199–204

Siddique, Z.; Rosen, D.W.; Wang, N. 1998: On the applicability of product variety design concepts to automotive platform commonality. In: *ASME Design Engineering Technical Conferences* (held in Atlanta), Paper No. DTM-5661

Simpson, T.W.; Maier, J.R.A.; Mistree, F. 1999: A product platform concept exploration method for product family design. In: *Proceedings of the 1999 ASME Design Engineering Technical Conferences* (held in Las Vegas), Paper No. DTM-8761; also appeared in 2001, Product platform design: method and application. *Res. Eng. Des.* **13**(1), 2–22

Zillober, C. 2001: Software manual for SCPIP 2.2. Technical Report Nr. TR01-2, Informatik, Universitaet Bayreuth