Computer Aided Engineering and OPTISHAPE

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Concept of CAE

1. What is CAE ?
2. General Concept of CAE
   and Design Optimization
3. FE Analysis

1. What is CAE ?

- CAE stands Computer Aided Engineering
- The concept of CAE was introduced by Dr. Jasen Lemon in 1980
  - a professor of University of Cincinnati,
  - a founder of SDRC
  - a developer of I-DEAS
- CAD+FE Modeling+FEA+Design
- Utility of the Graphic Display System

CAD

Geometric Modeling

CAM

Process Design
Operating Design
   NC Tape/Data
Manufacturing
   NC Processes
   Assembling
Evaluation

components
assembly & manual
component table
supplying

CAE

Stress, Motion, and Flow Analyses
(FEA/Mutibody Dynamics/FDA)

Toward CIM in 1990s

- 1960s / Batch Drafting System (Plotter)
- 1960s / NC
    - APT(Automatically Programmed Tooling) Language
    - NC Table
- 1970s / Interactive System
    - Interactive CAD and Graphic NC
- 1980s / CADCAM Systems
    - Database and 3D Data

CAD in Automotive

- 1950s / General Motors
  - using Graphic Display System
  - DAC-1 for prototype of a CAD system
  - INCA for NC processing for Master Model
  - CADANCE(70s), CGS(80s) + commercial soft
- 1970s and 80s / in-house CAD System
  - Nissan / CAD-I, CAD-II & GNC / Matuda
  - Integrated CAD/CAM / Toyota
- 1990s / Commercial CAD Soft

CAD in Aerospace

- 1960s and 1970s
  - CADAM / Lockeed for NC tape
  - CADD / McDonnell-Douglass
  - CATIA / Dasseau
  - ICAM Project in USAF
- 1980s : Standardization
  - Network by CIIN
  - IGES(Initial Graphic Exchange Specification) Format
- 1990s : New Standardization

CAE Concept in CAD

- CAD was originally for Computer Aided Drafting, but in 1980s CAD becomes more for Computer Aided Design based on
  - wire frame models
  - surface models
  - three-dimensional solid models
- More toward Design Analysis and Evaluation by FEA

CAE concept in CAM

- CAM is Computer Aided Manufacturing mostly for automated process control of NC machines, but
- Computer Simulation for Process Design and Process Control becomes important in computer aided manufacturing in 1980s
- Sheet Metal Forming, Forging, Molding, Die Design based on Computer Simulation

CAE in 1980s & 90s

- Design Analysis and Evaluation by FEA
  - Linear and Nonlinear Structures
  - Temperature, Magnetic Fields
  - Fluid Flows (Mostly by FDA & FVA)
- Process Simulation
  - Kinematics, Rigid Body Dynamics, Multi-Body Dynamics for Assembly Lines, Robots, ... by ADAMS, DADS, and others
  - Forming Process Simulation by Explicit FEA

Lots of Sophistication and Great Success

Realization of importance and profitability of Geometry Based CAD/CAM and CAE

Market of CAD/CAE

CAD Vendors	CAD Revenue	CAE Vendors	CAE Revenue
COMPUTERVISION	260	MSC	80
CATIA (IBM)	206	PDA	38
PARAMETRIC	167	SWANSON	32
UNIGRAPHICS (EDS)	165	RASNA	17
SDRC	157	HKS	12
AUTODESK	143	MARC	11
OTHERS	199	OTHERS	143

MSC/ Dr. McNeal

Trend In MCAE

- CAE is now widely accepted
  - 1980 J. Lemon / SDRC
  - integration with CAD
    - RASNA-MECHANICA and PRO-E / I-DEAS Master
  - Design Optimization
    - Size/Shape/Topology Optimization
- Automatic Mesh Generation for FEA
- **Modeling Problem**
- Further development is demanded

Cost Reduction

- CAD shows quite the success to make change of engineering, but CAE is still regarded to be expensive, because
  - Modeling is time consuming
  - Analysis results are difficult to be reflected to design change
  - Analysis is limited to Safety/Liability Study
  - Few experts of software
- Link with CAD & CAE is seeking

2. General Concept of CAE

- CAE (Computer Aided Engineering) should not be just for computer aided engineering analysis
- CAE should have large extent of
  - Design Analysis and Evaluation
  - Re-Design and Design Optimization
  - Process Simulation
- CAE is the connector of CAD and CAM

Two Kind of CAE

- MCAE (Mechanical CAE)
  - Structures (Linear and Nonlinear)
  - Explicit FEA (Forming, Crash, ..., Simulation)
  - Multi-Body Dynamics (Simulation)
- FCAE (Fluid CAE)
  - Heat Transfer/Conduction
  - Newtonian and Non-Newtonian Fluid Flow
  - Mold Flow Simulation

MCAE and FCAE

- Two separate CAE groups with two different pre/post processors as well as analysis soft
  - expensive, time consuming, disjointed communication, and difficult management
- CIM requires integrated coupled design study of MCAE and FCAE

Three Types of CAE

- Stand Alone CAE
  - standard FEA based CAE codes
  - special analysts oriented high accuracy
  - independent CAD and Pre-Processing
- CAD Linked CAE
  - present trend / link with CAD
  - automatic mesh generation methods
- CAD Imbedded CAE - ***Design Oriented***

Market Change in MCAE

CAE	1995	1999
CAD Independent	225 M$	175 M$
CAD Linked	100 M$	300 M$
CAD Imbedded	75 M$	400 M$
Total	400 M$	875 M$

Dr. McNeal at MSC

CAD Imbedded MCAE

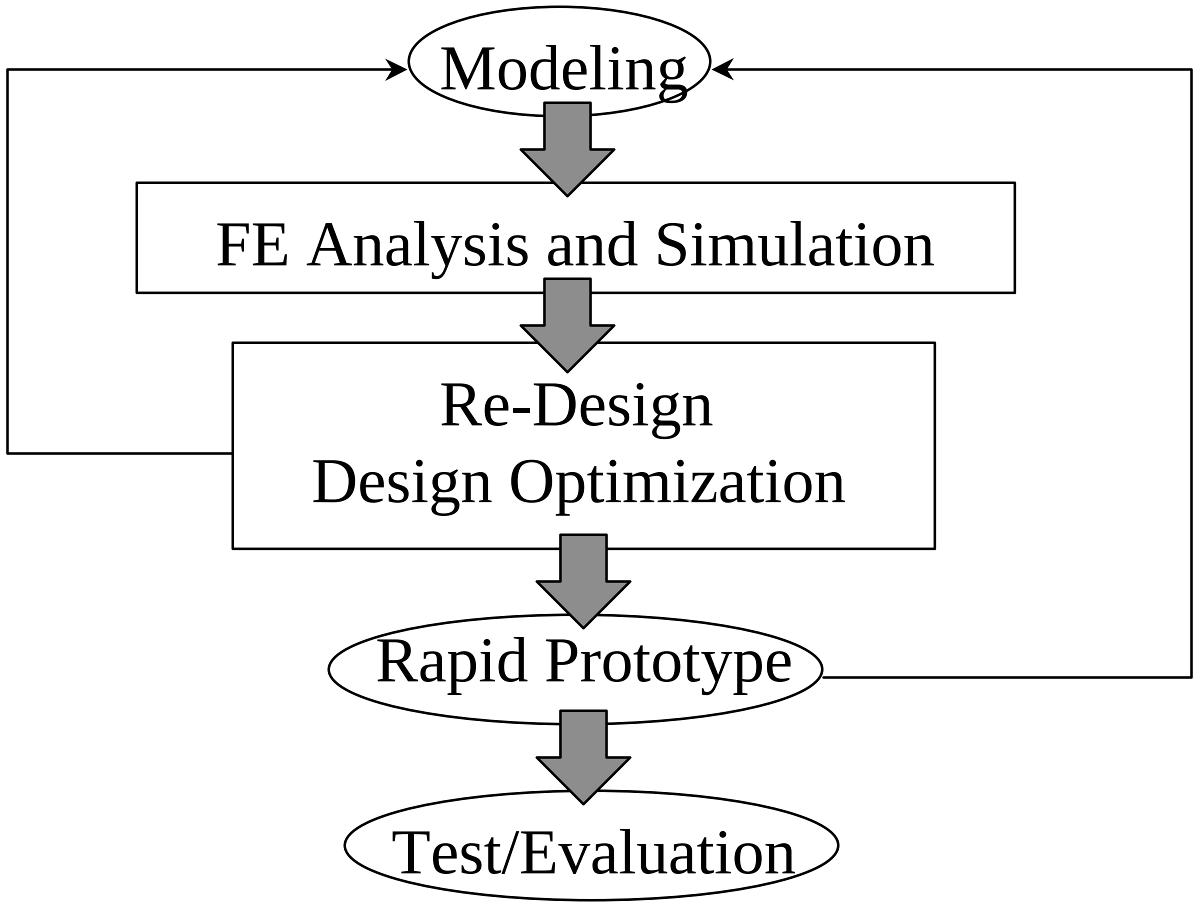
- CAD side takes leadership
- Simulation of design feasibility
  - users are designers rather than analysts
  - less accuracy but user oriented
  - CAD/CAE link must be completed
  - CAE is an icon of CAD menu
- Short Turn Around Time

DESIGN ORIENTED

Present Demand

- For Shortening of Turn Around Time by Simplifying FE Modeling Methods
  - **CAD Linked Automatic Mesh Generation**
  - **Adaptive FE Methods (h and p elements)**
  - **Meshless FE Methods (ANALYSIS)**
- Integration for Production Engineering
  - Modeling, Analysis, Design, Manufacturing
  - Paradigm change may be required

Five Step CAE Procedure



Design Optimization

- has been considered mostly in structures under linear elasticity
  - –
  - STRUCTURAL OPTIMIZATION

- recently it is extend to Mechanical Design in more general sense
- few work on heat, fluid flow, multi-bodies

Long Way to Real CAE

- CAE for Computer Aided Engineering Analysis has been well established by using FEM, FDM, FVM, and Multi-Body Dynamics, but
- True sense CAE introduced by Dr. Jasen Lemon is still far away in reality

Bottle Neck is in modeling !

Key Components of CAE

- Modeling / 70%
  - Link with CAD Data
  - Automatic Mesh Generation Methods
  - Input of Load/Support Condition
- FEA (Finite Element Analysis) / 10%
- FES (Finite Element Simulation) / 10%
- Redesign & Optimization / 20%

Modeling

- Pre/Post Processing for FE Modeling was independently developed from CAD

⇨

- This yields a painful and time consuming work for FEA

⇨

- and desire of Full Integration with CAD

Link with CAD in Modeling

- Link is already exists in
  - SDRC I-DEAS Master Series
  - PRO-E and RASNA-MECHANICA
- Link must be established for most of FEA, especially for Pre/Post Software for FEA
  - MSC/PATRAN ----- UNIGRAPHICS (?)
  - HYPERMESH ----- ????
  - Others

Link with CAD

- leads paradigm change in CAD and CAE practice in industry and in education, too
- CAD soft is absorbing CAE, especially, CAD soft must be linked with FEA Pre/Post soft for full integration
- CAD side must take leadership to do so, in order to make real CAD not for drafting
- This movement has already started .....

Major Players

CAD and CAE Link

- UNIGRAPHICS & MSC/NASTRAN
- CATIA & ELFINI
- PRO-E & RASNA MECHANICA
- SDRC I-DEAS

Modeling & Design

- Choice of design variable linked with CAD
  - circle and arc (radius,angle, center location)
  - ellipse
  - control points of Bezier, B-, and NURBS
    - three modules (CAD modeling, Automatic Mesh Generation, and Design Modification) must be integrated ...... very difficult task
    - most of structural optimization software developed in 1970s took this approach .... ELFINI, SAMSEF

Bezier & B-Spline

- Bezier Surfaces
  - P.de Casteljau at Citroen (no publication)
  - P. Bezier at Renaut
  - 1974 conference at the university of Utah
- B-Spline Method
  - Bezier Surfaces + Coons Patch
- NURBS (Non-Uniform Rational B-Spline)

Difficulty

- For shape design optimization, link with CAD system seems to be the most effective, if FE modeling (especially mesh generation) is fully imbedded in the whole system.
  - No CAD system fully support design optimization and FEA, except SDRC/I-DEAS
  - CAD-like Preprocessor for FEA can be utilized for shape design optimization, but it is disjointed with standard CAD systems

1970s & 1980s

- SAMSEF and ELFINI are the most sophisticated structural design optimization software developed in Europe in 1970s
- Many new development for shape design have been organized in Europe in 1980s
- But, they could not be popular because of redundant CAD-like Pre-Processor requirement together with Automatic Mesh Generation Schemes

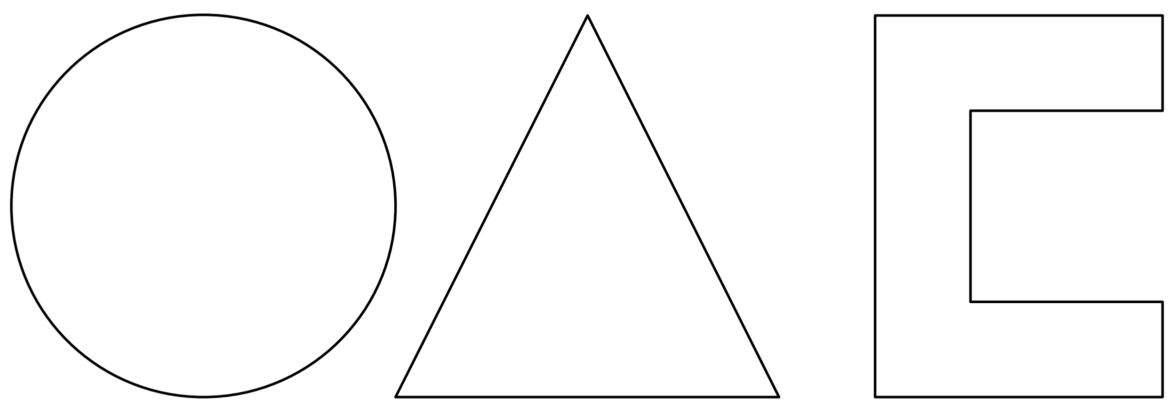
Paradigm Change

Design Optimization

- Design Variables should not be linked with CAD data
  - Sizing Optimization
    - restrict to beam/frame-like structures
  - Shape Optimization
    - GENESYS Approach is most likely choice
  - Topology Optimization
    - density or homogenization design approach

GENESYS Approach

- Design change is considered to be a linear combination of basis design shapes



$$d = \sum_{k=1}^{m} a_k d_k$$

d_k = kth pattern of design

a_k = design variable

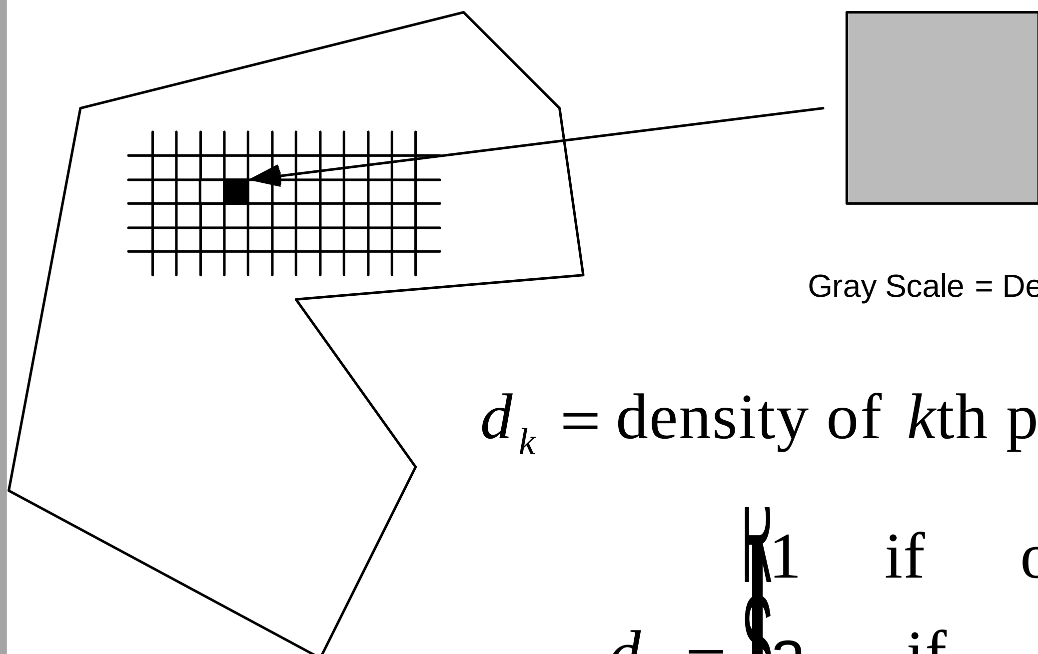
Characteristics

- FE Meshes are subordinated to the base shape design so that automatic remeshing methods need not be integrated into the design optimization system, but
- This may lead excessive mesh distortion during the design process, and then some automatic distortion correction scheme is desirable

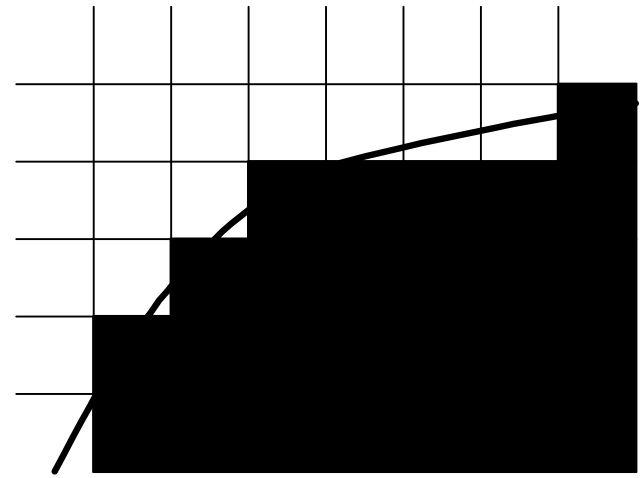
CAD independent

Density and Homogenization

Shape is recognized by a set of on pixels



Gray Scale = Density



$$d_k = \text{density of } k\text{th pixel / voxel}$$

$$d_k = \begin{cases} 1 & \text{if} \quad \text{occupied by solid structure} \\ \alpha & \text{if} \quad \text{structure is perforated} \\ 0 & \text{if} \quad \text{no structure is placed} \end{cases}$$

Characteristics

- Design optimization is completely decoupled with any sort of mesh adaptation
- Shape and topology design variables are transformed into the density of material or elasticity matrix of material which is assigned in each finite element of a fixed FE model, at least a fixed FE mesh generated at the initial time.

This approach leads 1990s

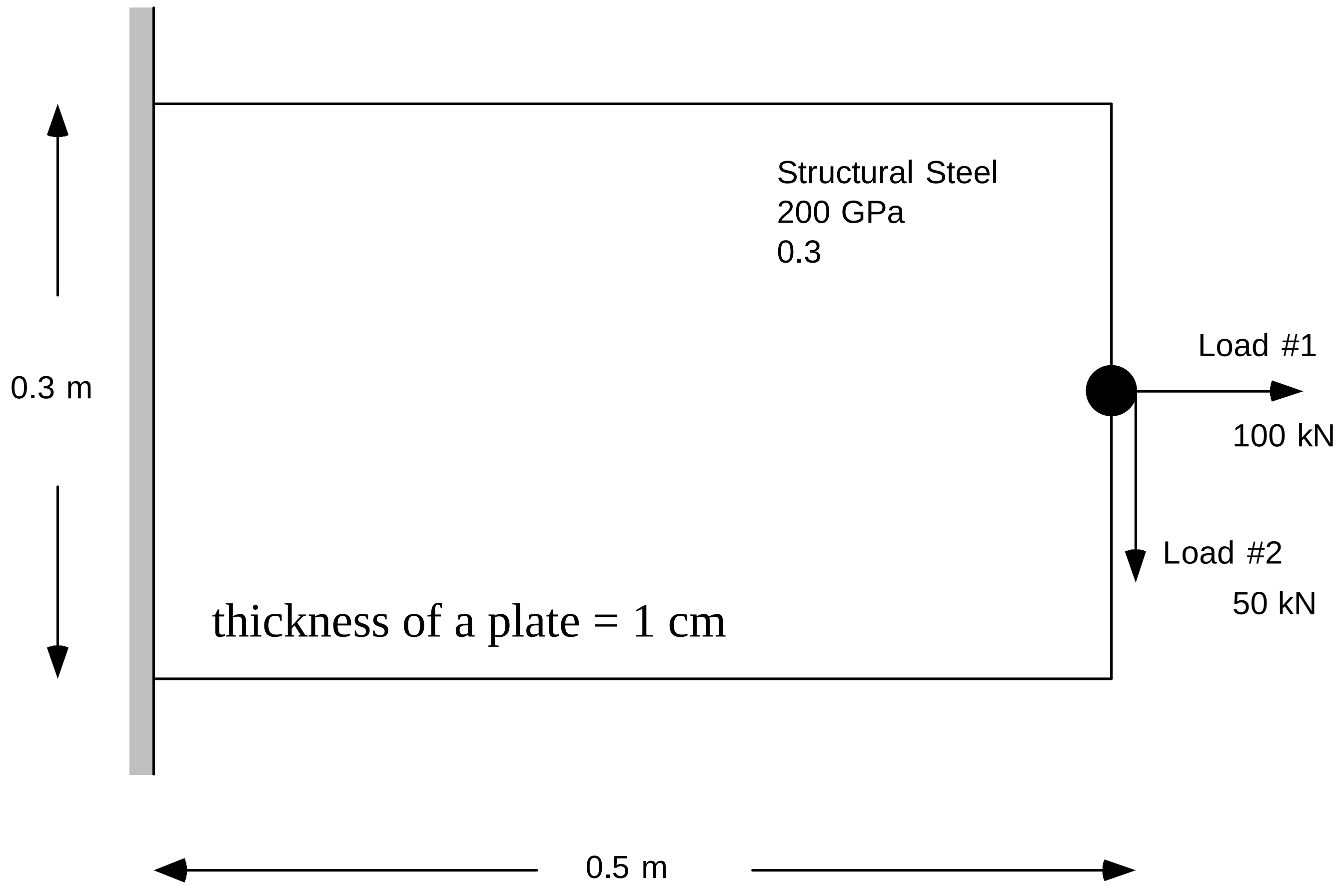
OPTISHAPE from QUINT

OPTISTRUCT from ALTAIR

ANSYS-Topology

MSC/NASTRAN-Topology

Exercise #1 : OPTISHAPE

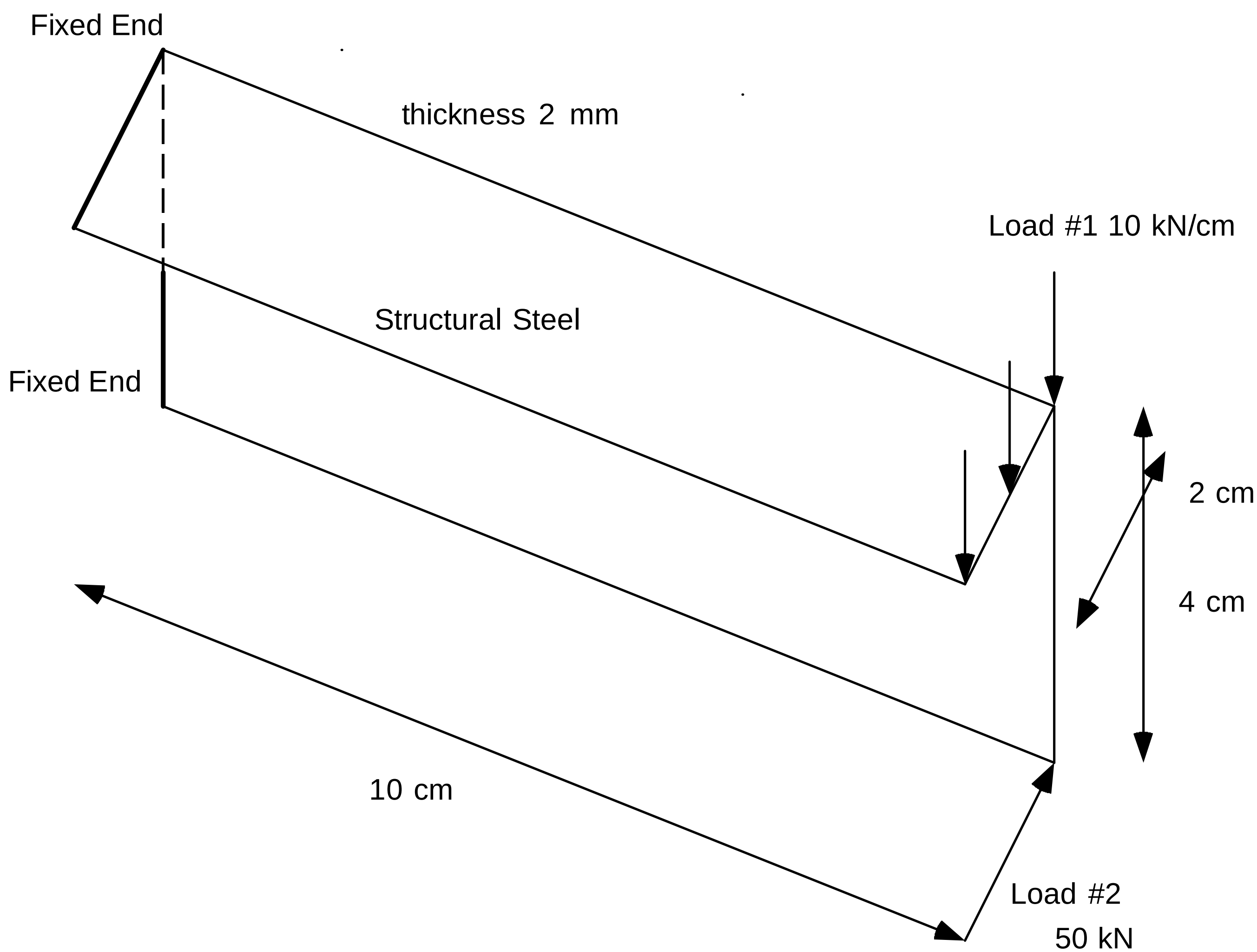


Two loads are considered : Load #1 is a tensile force, and Load #2 is a bending force. Apply OPTISHAPE with the volume constraint (25% of the rectangular design domain) to the following three cases :

(1) Load #1 is applied at the center of right edge
(2) Load #2 is applied at the center of right edge
(3) Load #1 and #2 are applied at the same time
(4) Load #1 and Load #2 are applied independently

Find the nature of the optimum structures to these loading conditions. Especially, observe the difference between (3) and (4).

Exercise #2


Fixed End
thickness 2 mm
Load #1 10 kN/cm
Structural Steel
Fixed End
2 cm
4 cm
10 cm
Load #2
50 kN

Two loads, a distributed edge load #1 and a point load #2 are considered for a 2 mm thick L-shape folded plate as shown in the figure. Consider reinforcement of this L-shape folded plate by adding 2 mm high ribs in the 20% of the design domain for the case that two loads are applied independently.

You may solve this by using OPTISHAPE, but if you have other software for FE analysis and/or structural optimization, solve this by using those software, and make comparison the results obtained by both methods.

Optimum Structural Design in CAE

1. CAE and Design Optimization
2. Redesign and Optimization
3. Size and Shape Design Optimization

CAE and Design Optimization

1. Structural Optimization
2. Typical Setting of Design Problems
3. Characteristics
4. General Remarks on Stresses

Structural Optimization

- A small portion of Mechanical Design Optimization which involves mechanical systems, multi-body mechanisms/structures, and individual structural components
- The concept of Multi-Disciplinary Optimization is required in mechanical system design, but this is far from the reality of structural design optimization

Many Design Problems

- Design in Linear Elastic Structures
  - Global Stiffness Maximization
  - Strength Maximization (Composite Laminates)
  - Frequency Response Problem
  - Dynamic Stiffness Maximization
  - Frequency Control Problem
  - Buckling Load Maximization
- Design in Nonlinear Structures & Processes

Mechanical Design

- Maximizing formability of sheet metals

$$\min_{\text{design}} \frac{\varepsilon_1 - \varepsilon_2}{\varepsilon_2} \quad , \quad \{\varepsilon_1 \quad \varepsilon_2\} = \text{pricipal strains}$$

- Minimization of holding forces of sheet metals
- Maximization of quality of sheet forming

$$\min_{\text{design}} \left| (1 - \varepsilon_1 - \varepsilon_2) - \frac{1}{|\Omega|} \int_{\Omega} (1 - \varepsilon_1 - \varepsilon_2) \, d\Omega \right|$$

Very Complex !

Typical Setting

$$\min_{\substack{l(\boldsymbol{u}) \le l_{\max} \\ a(\boldsymbol{u},\boldsymbol{v}) = l(\boldsymbol{u}) \quad \forall \boldsymbol{v}}} \quad \text{Total Weight}$$

internal virtual work

$$a(\boldsymbol{u},\boldsymbol{v}) = \int_\Omega (\partial \boldsymbol{v})^T \boldsymbol{E} (\partial \boldsymbol{u}) d\Omega - \omega_0{}^2 \int_\Omega \boldsymbol{v}^{\boldsymbol{T}} \rho \boldsymbol{u} \boldsymbol{d}\Omega$$

external virtual work

$$l(\boldsymbol{v}) = \int_\Omega (\partial \boldsymbol{v})^T \sigma_0 d\Omega + \int_\Omega \boldsymbol{v}^{\boldsymbol{T}} \rho \boldsymbol{b} \boldsymbol{d}\Omega + \int_{\Gamma_{\boldsymbol{t}}} \boldsymbol{v}^{\boldsymbol{T}} \boldsymbol{t} d\Gamma$$

Internal Virtual Work

$$a\left(\boldsymbol{u},\boldsymbol{v}\right)=\int_{\Omega}\left(\partial\boldsymbol{v}\right)^{T}\underbrace{\boldsymbol{E}}_{Elasticity\ Matrix}\underbrace{\left(\partial\boldsymbol{u}\right)}_{Strain\ Vector}d\Omega-\underbrace{\omega_0^2}_{Shifted\ (Excited)\ Frequency}\int_{\Omega}\boldsymbol{v}^{T}\underbrace{\rho}_{Mass\ Density}\underbrace{\boldsymbol{u}}_{\substack{Displacement\ Vector\\ in\ Equilirium}}\boldsymbol{d}\Omega$$

Strain-Displacement Relation

$$\boldsymbol{\varepsilon}(\boldsymbol{v}) = \begin{Bmatrix} \varepsilon_x(\boldsymbol{v}) \\ \varepsilon_y(\boldsymbol{v}) \\ \varepsilon_z(\boldsymbol{v}) \\ \gamma_{yz}(\boldsymbol{v}) \\ \gamma_{zx}(\boldsymbol{v}) \\ \gamma_{xy}(\boldsymbol{v}) \end{Bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} & 0 & 0 \\ 0 & \frac{\partial}{\partial y} & 0 \\ 0 & 0 & \frac{\partial}{\partial z} \\ 0 & \frac{\partial}{\partial z} & \frac{\partial}{\partial y} \\ \frac{\partial}{\partial z} & 0 & \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} & \frac{\partial}{\partial x} & 0 \end{bmatrix} \begin{Bmatrix} v_x \\ v_y \\ v_z \end{Bmatrix} = \partial \boldsymbol{u}$$

Stress-Strain Relation

$$\sigma = \boldsymbol{E}\varepsilon - \sigma_0$$

$\boldsymbol{E}$ = Elasticity Matrix

σ_0 = initial stress

External Virtual Work (Work potential)

$$l(\mathbf{v}) = \underbrace{\int_\Omega (\partial \mathbf{v})^T \sigma_0 \, d\Omega}_{\textit{Work Done by Initial Stress}}$$

$$\sigma_0 = \boldsymbol{E}\Delta T \alpha$$

$$+ \underbrace{\int_\Omega \boldsymbol{v}^T \rho \boldsymbol{b} \boldsymbol{d}\Omega}_{\textit{Work Done by Body Force}} + \underbrace{\int_{\Gamma_t} \boldsymbol{v}^T \boldsymbol{t} d\Gamma}_{\textit{Work Done by Traction}}$$

$$l(\boldsymbol{u}) = \text{mean compliance by } \{\sigma_0 \quad \rho \boldsymbol{b} \quad \boldsymbol{t}\}$$

Mean Compliance

If the thermal stresses, body forces, and tractions are specified, if the displacement resulted by such applied forces is small, it means that the structure is stiff in its global response.

Minimization of the Mean Compliance
= Maximization of the Global Stiffness

If constrained displacement is specified on the boundary, then the resulted stress (that is traction) on the boundary must be large if the structure is stiff. In this case, we have to

Maximize the Mean Compliance

Discrete Form / FEM

$$\min_{\substack{\boldsymbol{u}^T \boldsymbol{f} \le l_{\max} \\ \boldsymbol{Ku}=\boldsymbol{f}}} \quad \text{Total Weight}$$

Shifted Stiffness Matrix

$$\boldsymbol{K} = \underbrace{\int_\Omega \boldsymbol{B}^T \boldsymbol{E}\boldsymbol{B}\, d\Omega}_{\textit{stiffness matrix}} - \omega_0^{\,2} \underbrace{\int_\Omega \boldsymbol{N}^T \rho \boldsymbol{N} \boldsymbol{d}\Omega}_{\textit{mass matrix}}$$

Generalized Load Vector

$$\boldsymbol{f} = \int_\Omega \boldsymbol{B}^T \sigma_0\, d\Omega + \int_\Omega \boldsymbol{N}^T \rho \boldsymbol{b}\boldsymbol{d}\Omega + \int_{\Gamma_t} \boldsymbol{N}^T \boldsymbol{t}\, d\Gamma$$

Equivalent Formulation

$$\min_{\substack{\boldsymbol{u}^T \boldsymbol{f} \le l_{max} \\ \boldsymbol{Ku} = \boldsymbol{f}}} \int_\Omega \rho \, d\Omega$$

Minimizing the volume with the mean compliance constraint

$\Longleftrightarrow$

They are the dual problems and are equivalent.

$$\min_{\substack{\boldsymbol{Ku} = \boldsymbol{f} \\ \int_\Omega \rho \, d\Omega \le W_0}} \boldsymbol{u}^T \boldsymbol{f}$$

Minimizing the mean compliance with the volume constraint

Compliance and Energy

Mean Compliance = Twice of the Total Strain Energy

$$\boldsymbol{u}^T \boldsymbol{f} = \mathit{work\ done}$$

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \quad \Rightarrow \quad \boldsymbol{u}^T \boldsymbol{K}\boldsymbol{u} = \boldsymbol{u}^T \boldsymbol{f}$$

$$\min_{\boldsymbol{v}} I(\boldsymbol{v}) = \underbrace{I(\boldsymbol{u})}_{\substack{\text{minimum potential energy} \\ \text{at equilibrium}}} = \frac{1}{2}\boldsymbol{u}^T \boldsymbol{K}\boldsymbol{u} - \boldsymbol{u}^T \boldsymbol{f} = -\frac{1}{2}\boldsymbol{u}^T \boldsymbol{f}$$

Equivalent Formulation

Using the relation

$$\min_{design} \; \boldsymbol{u}^T \boldsymbol{f} = \min_{design} \left[-2 \min_{\boldsymbol{v}} I(\boldsymbol{v}) \right] = -2 \max_{design} \; \min_{\boldsymbol{v}} I(\boldsymbol{v})$$

we can define the optimum design problem by

$$\max_{design} \; \min_{\boldsymbol{v}} I(\boldsymbol{v})$$

by using the total potential energy

Design Optimization

The most fundamental structural design problem can be stated as the maximization of the minimum total potential energy of a structural system with respect to designs and admissible displacements

$$\max_{\boldsymbol{d}} \min_{\boldsymbol{v}}$$

Stress and Compliance

$$\bar{\sigma}^2 = \begin{Bmatrix} \sigma_1 & \sigma_2 & \sigma_3 \end{Bmatrix} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & 1 & -\frac{1}{2} \\ -\frac{1}{2} & -\frac{1}{2} & 1 \end{bmatrix} \begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_3 \end{Bmatrix}$$

Mises Equivalent Stress

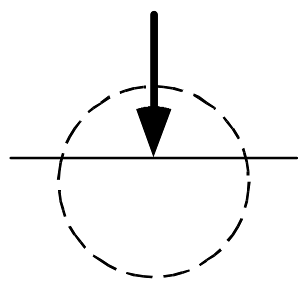
$$\varepsilon^T \boldsymbol{E} \varepsilon = \sigma^T \boldsymbol{C} \sigma = \sigma^T \frac{1}{E} \begin{bmatrix} 1 & -\nu & -\nu \\ -\nu & 1 & -\nu \\ -\nu & -\nu & 1 \end{bmatrix} \sigma = \sigma^T \boldsymbol{C} \sigma$$

strain energy density

$$\frac{1}{E}\frac{2(1+\nu)}{3}\bar{\sigma}^2 \le \sigma^T \boldsymbol{C} \sigma \le \frac{1}{E}\frac{1}{\nu}\bar{\sigma}^2$$

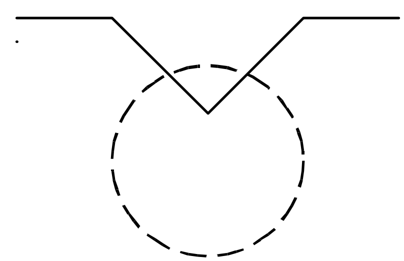
relation

Stress Singularity



Stresses become infinite
as well as the strain energy density
(Essential Singularity)

Stresses become infinite, but the
strain energy density is finite in
the sense that it is integrable
(Normal Singularity)



Local Stress

- Stresses can be infinite in continuum structures (Plates/Shells, Solids), while stresses are finite for trusses, beams, and frames.
- Thus, making the upper bound of the local stress value itself does not make sense.
- Some sort of integral (average) form of stresses should be constrained.

Candidates

Average Stress Bound in a Finite Element

$$\sigma_e = \left\{ \int_{\Omega_e} \bar{\sigma}^2 d\Omega \right\}^{\frac{1}{2}} \leq \sigma_{\max}$$

The finite element model must be fixed during the optimization

Noting that

$$\int_{\Omega_e} \bar{\sigma}^2 d\Omega \leq \frac{3E}{2(1+\nu)} \int_{\Omega_e} \sigma^T \boldsymbol{C} \sigma d\Omega \leq \sigma_{\max}$$

the element strain energy can be used for stress constraint

Note 1

$$\int_{\Omega_e} \sigma^T \boldsymbol{C} \sigma \, d\Omega = \int_{\Omega_e} \varepsilon^T \boldsymbol{E} \varepsilon \, d\Omega = \boldsymbol{u}^T \boldsymbol{K} \boldsymbol{u}$$

can be calculated much accurately than

$$\int_{\Omega_e} \overline{\sigma}^2 \, d\Omega$$

since the first derivatives of the displacement must be calculated to evaluate the Mises stress

Note 2

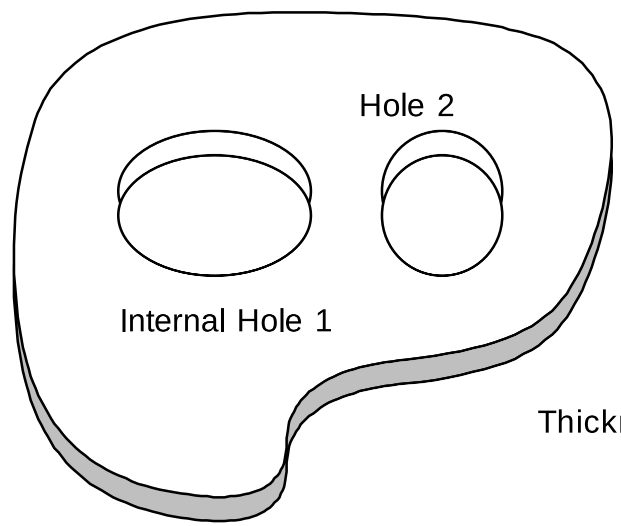
- Mean compliance was introduced by Prager and Taylor to define structural optimization for continuum solids and structures
- Weight minimization with stress and displacement constraints was introduced for trusses, beams, and other space frame type structures in aerospace and civil engineering

⇐ stresses are bounded in these frame structures

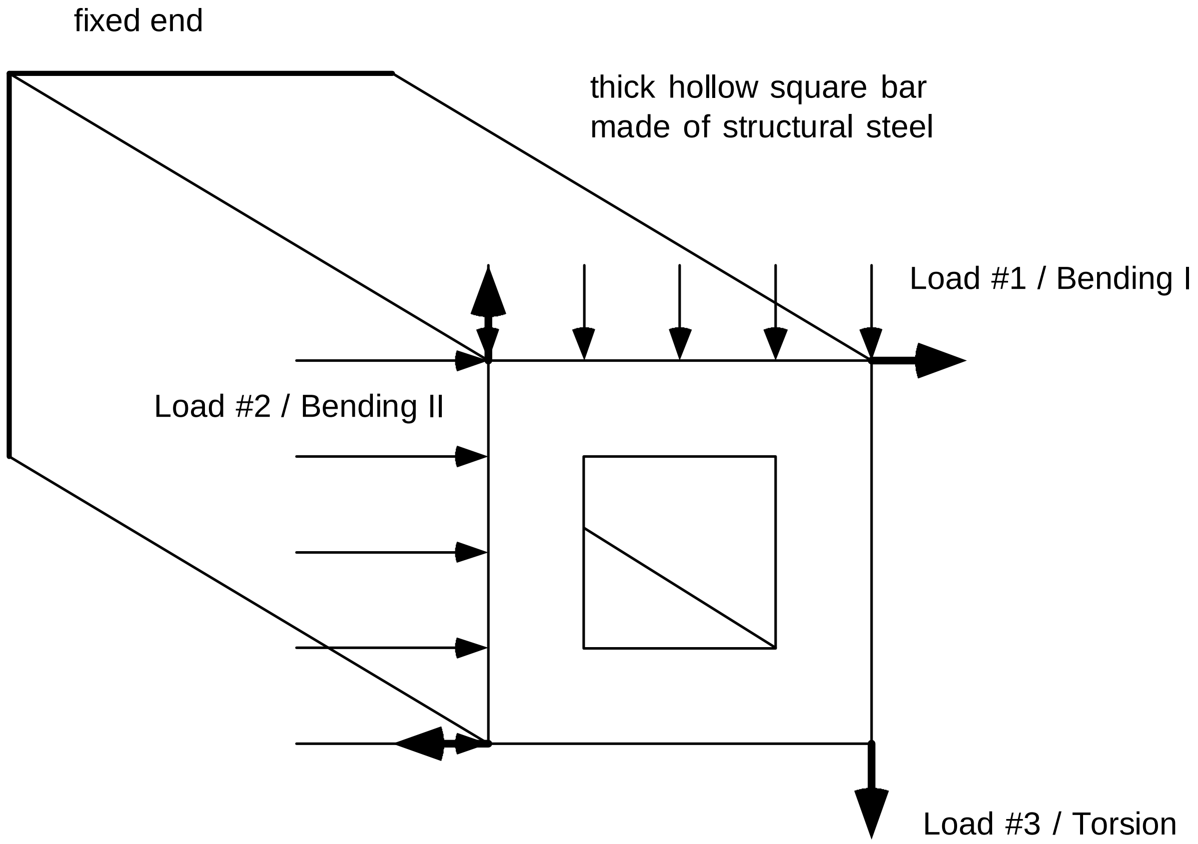
Indirect Stress Control

Three Major Design Problems

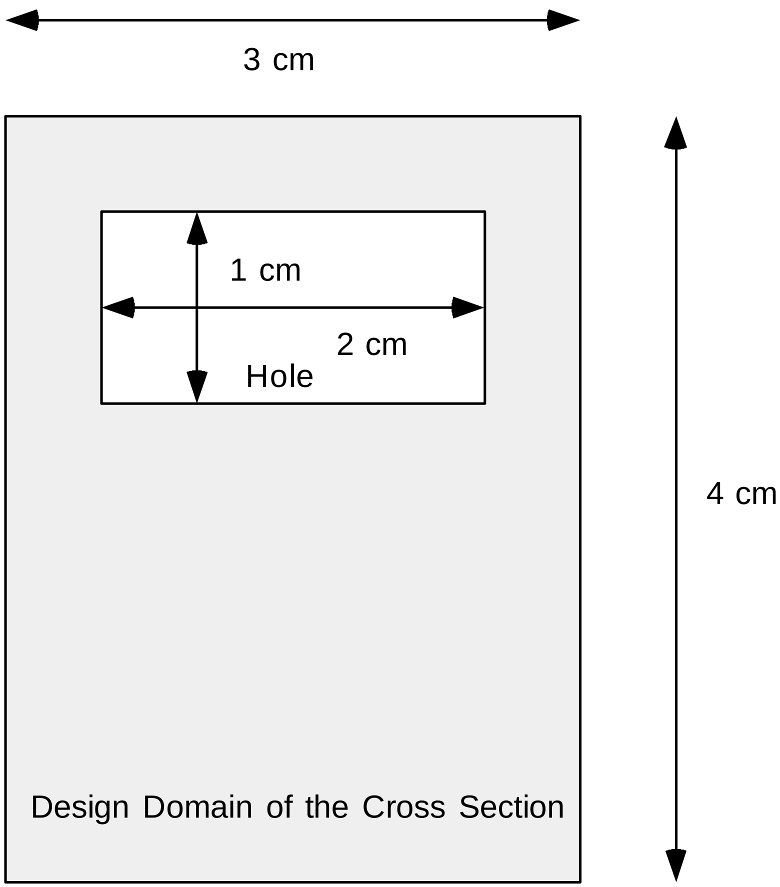
- Sizing Optimization
  - thickness and cross sectional properties
- Shape Optimization
  - Location of holes/arcs
  - Radii of holes/arcs
  - control points of splines
- Topology Optimization
  - number of holes
  - shape of holes



Exercise #3 : OPTISHAPE



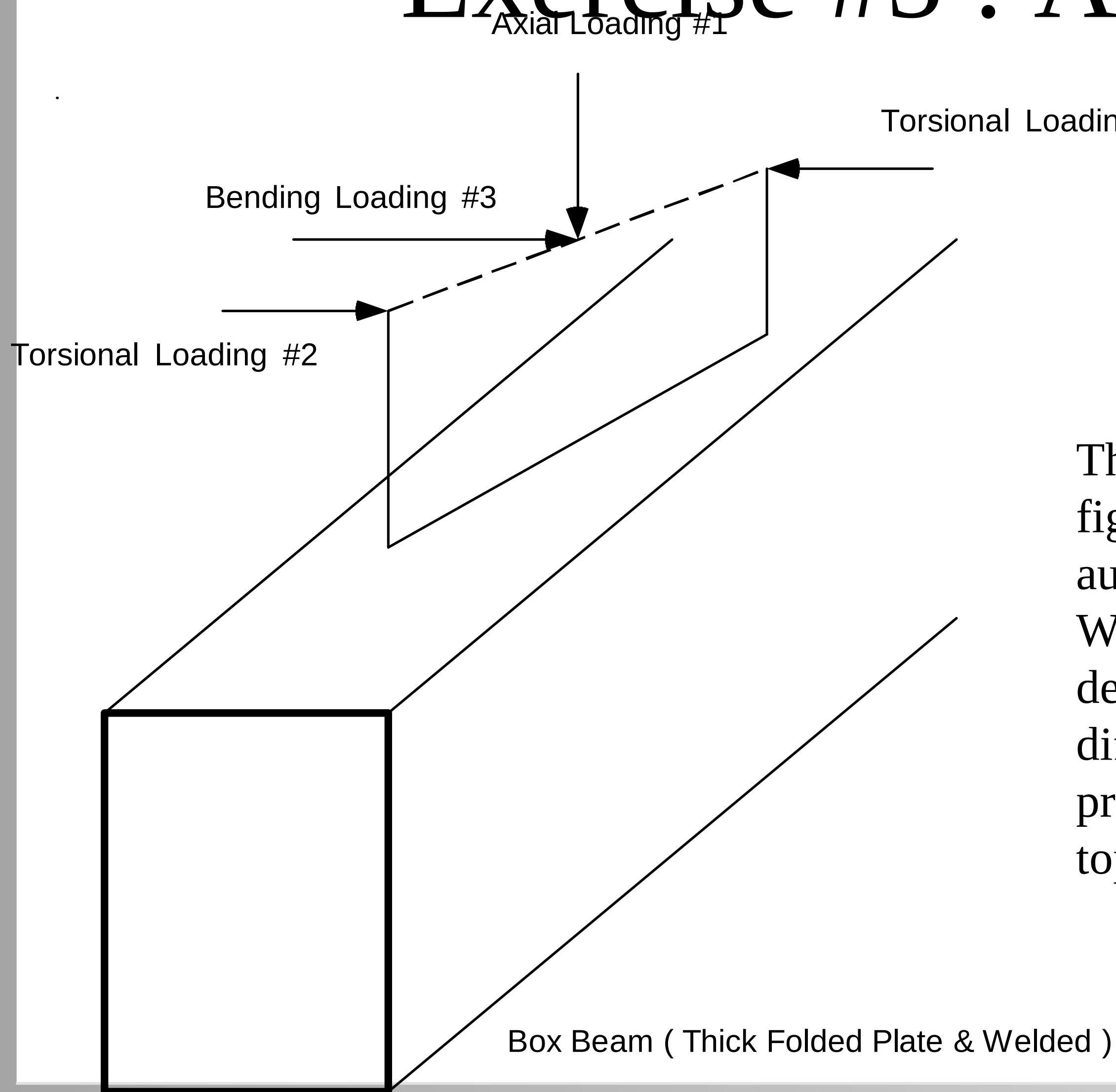
Exercise #4 : A Cross Section



Using 30% area of the outer rectangle, design the cross section with a specified rectangular hole that can maximize

1. Bending Rigidity
2. Torsional Rigidity
and
3. Shear Rigidity

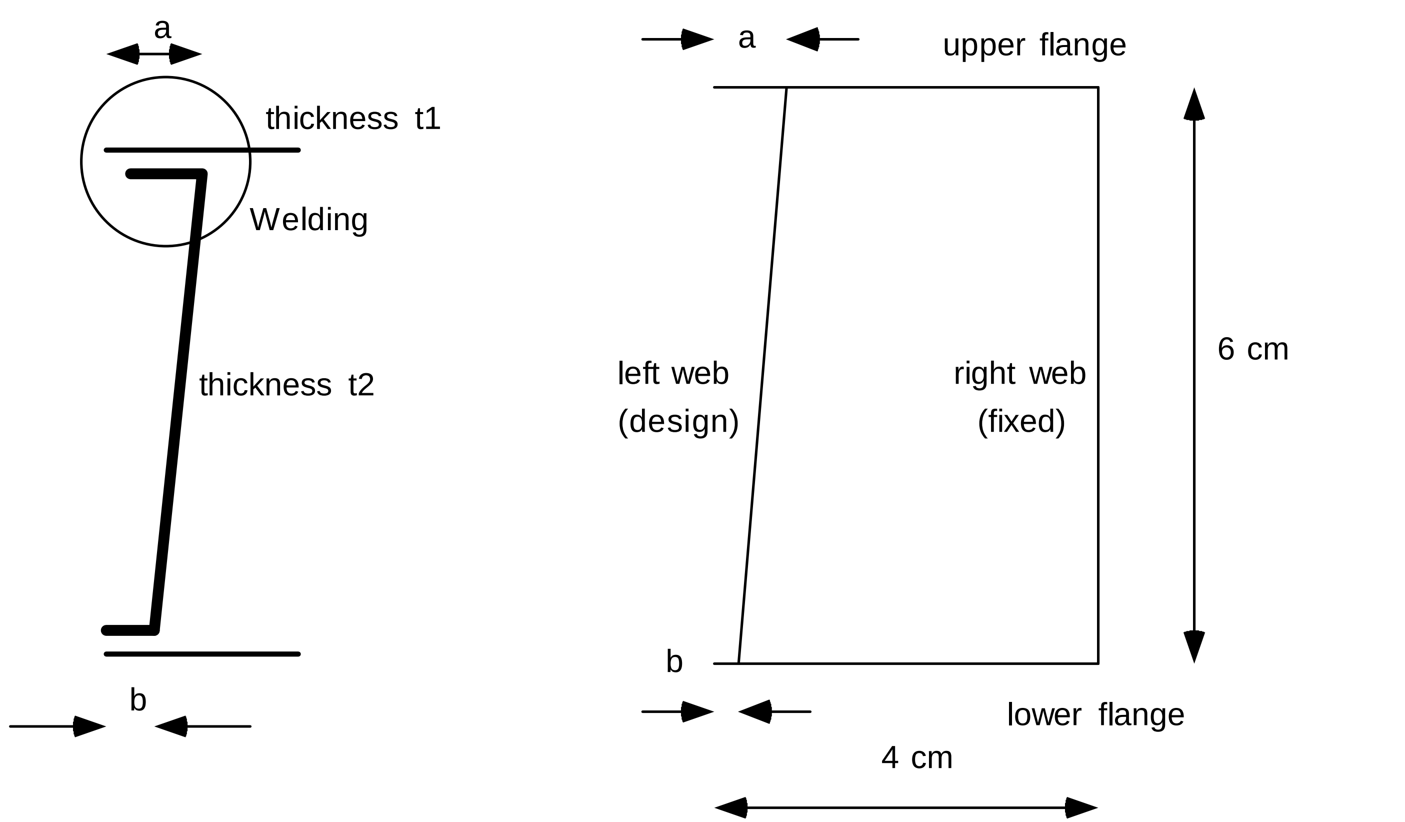
Exercise #5 : A Joint



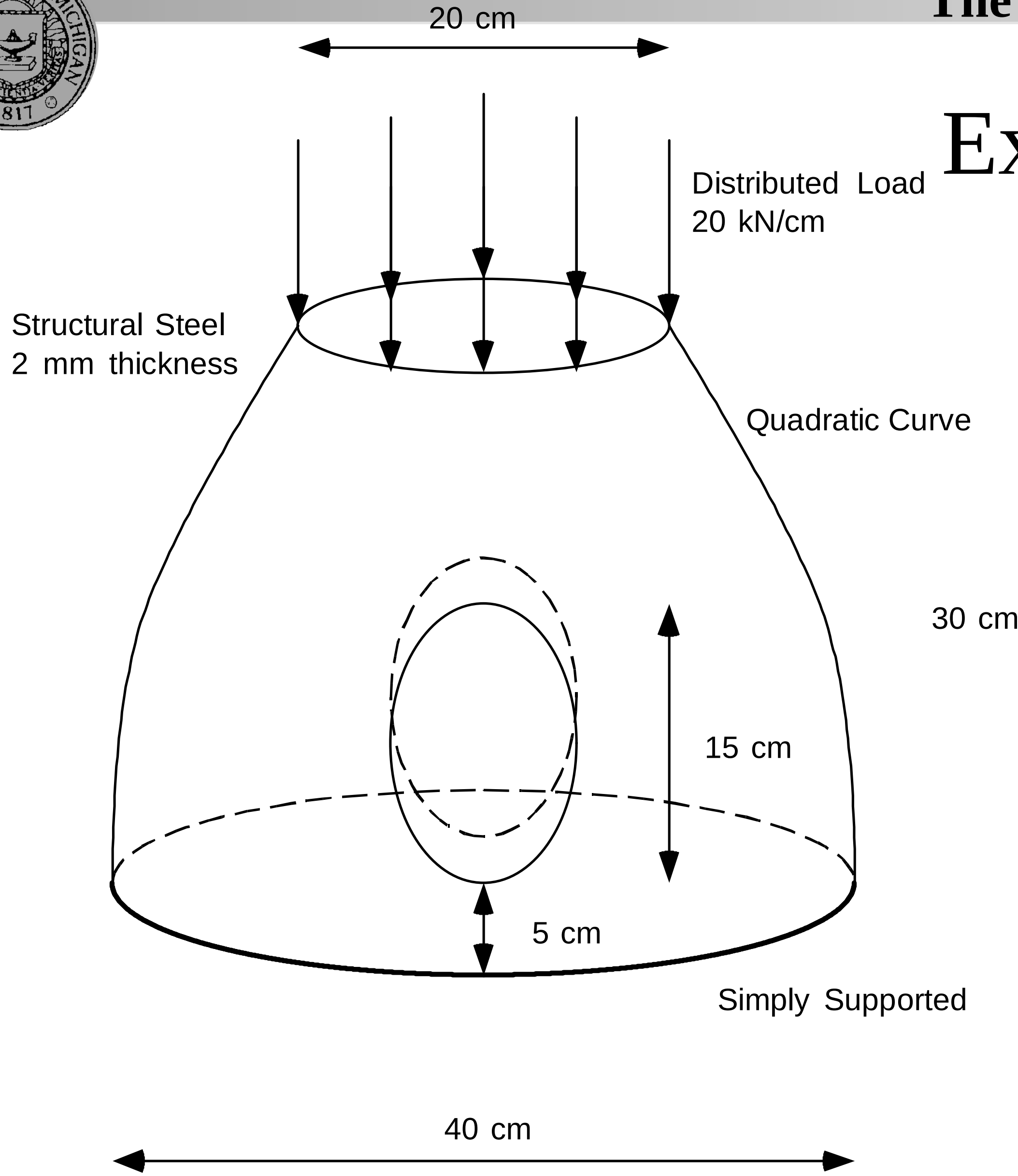
This is a conceptual abstract figure of a joint portion of an automotive body structure. When the thick box beam is designed, state possible three different structural optimization problems : sizing, shape, and topology problems.

Additional Design Problem

Assuming the rigid welding at the joints,
find the optimum location (a,b) of the left web
as well as the thickness of the flange and web (t1,t2).

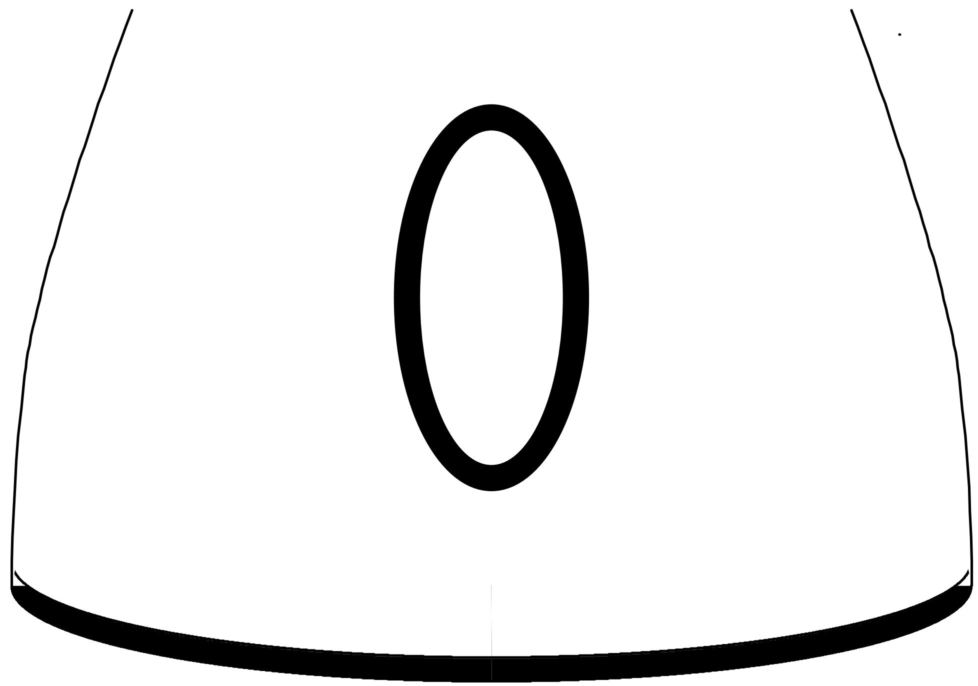


Exercise # 6 : Shell



Consider a shell structure which has two circular holes, whose thickness is 2mm made of steel. When it is subjected to a uniformly distributed load at the top circular edge, find the optimum reinforcement by using 30% of the total area of the shell. Here the bottom circular hole is simply supported.

When OPTISHAPE is applied to this shell structure, reinforcement should be always placed along the top, bottom and internal hole edges with 5 mm wide.



Redesign and Optimization - Fully Stressed Design -

Sizing Design Optimization

Optimality Condition

Fully Stressed Design

Redesign Method

1st Generation Software

Sizing Optimization

- 1960s : Prof. L.Schmit's Leadership
  - Mathematical Programming (Minimization)
  - Finite Element (Matrix Structural) Method
- Design Sensitivity Analysis : Fox 1967

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f} \quad \Rightarrow \quad \frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}}\boldsymbol{u} + \boldsymbol{K}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{d}} = \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{d}}$$

$$\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{d}} = \boldsymbol{K}^{-1}\left(-\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}}\boldsymbol{u} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{d}}\right)$$

Design Sensitivity

Performance Functions
Objective Function & Constraints

$$\boldsymbol{g}\left(\boldsymbol{u},\boldsymbol{d}\right) \leq \boldsymbol{g}_{\max}$$

Design Sensitivity (Direct Method)

$$\frac{D\boldsymbol{g}}{D\boldsymbol{d}} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{d}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{d}} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{d}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\boldsymbol{K}^{-1}\left\{-\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}}\boldsymbol{u} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{d}}\right\}$$

Dual Method

Defining the dual (conjugate) problem

$$\underbrace{\boldsymbol{K}^T}_{n\times n}\underbrace{\boldsymbol{H}}_{n\times m} = \underbrace{\left[\frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\right]^T}_{m\times n} \quad \Rightarrow \quad \boldsymbol{H}^T\boldsymbol{K}\boldsymbol{K}^{-1} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\boldsymbol{K}^{-1}$$

Design sensitivity can be computed by

$$\frac{D\boldsymbol{g}}{D\boldsymbol{d}} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{d}} + \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{u}}\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{d}} = \frac{\partial \boldsymbol{g}}{\partial \boldsymbol{d}} + \boldsymbol{H}^T\left(-\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}}\boldsymbol{u} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{d}}\right)$$

Direct & Dual Methods

- If the number of design variables is smaller than that of design constraints, the direct method by computing $\frac{\partial \boldsymbol{u}}{\partial \boldsymbol{d}} = \boldsymbol{K}^{-1}\left(-\frac{\partial \boldsymbol{K}}{\partial \boldsymbol{d}}\boldsymbol{u} + \frac{\partial \boldsymbol{f}}{\partial \boldsymbol{d}}\right)$ is more efficient
- On the other hand, if the number of constraints is much larger than that of design variables, then the dual method is much more efficient.

Fundamental Reference

R.L. Fox, Optimization Methods for Engineering Design, Addison-Wesley, 1971

In Practice

- In most of mechanical design problems, it is difficult to express the constraints in explicit function forms ⇒ No Analytical Sensitivity
- For example, strength of a thin walled structural component
  - yield criterion for ductile materials
  - maximum principal stress for brittle materials
  - buckling load for compressive loading

Finite Difference Method

$$\frac{D\boldsymbol{g}}{D\boldsymbol{d}} \approx \begin{cases} \dfrac{\boldsymbol{g}\left(\boldsymbol{d}+\Delta\boldsymbol{d},\,\boldsymbol{u}(\boldsymbol{d}+\Delta\boldsymbol{d})\right)-\boldsymbol{g}(\boldsymbol{d},\boldsymbol{u})}{\Delta\boldsymbol{d}} \\[2ex] \dfrac{\boldsymbol{g}\left(\boldsymbol{d}+\Delta\boldsymbol{d},\,\boldsymbol{u}(\boldsymbol{d}+\Delta\boldsymbol{d})\right)-\boldsymbol{g}\left(\boldsymbol{d}-\Delta\boldsymbol{d},\,\boldsymbol{u}(\boldsymbol{d}-\Delta\boldsymbol{d})\right)}{2\Delta\boldsymbol{d}} \\[2ex] \dfrac{\boldsymbol{g}(\boldsymbol{d},\boldsymbol{u})-\boldsymbol{g}\left(\boldsymbol{d}-\Delta\boldsymbol{d},\,\boldsymbol{u}(\boldsymbol{d}-\Delta\boldsymbol{d})\right)}{\Delta\boldsymbol{d}} \end{cases}$$

Central difference approximation is regarded as the best method to calculate the design sensitivity, even for shape design case.

Note

- Mechanical design problems are represented by rather few design variables with a lot of design constraints
  - finite difference approximation
  - dual method for analytical evaluation
- Aerospace and civil engineering structural design, we have many design variables, and then finite difference approach is not effective

Frames + Shear Panels

Exercise #7 : Sensitivity

Using the shell structure we have used in Exercise #6, find the sensitivity of the maximum Mises stress with respect to the diameter of the internal holes. Compute the design sensitivity by using the finite difference approximation.

Example of Design Sensitivity

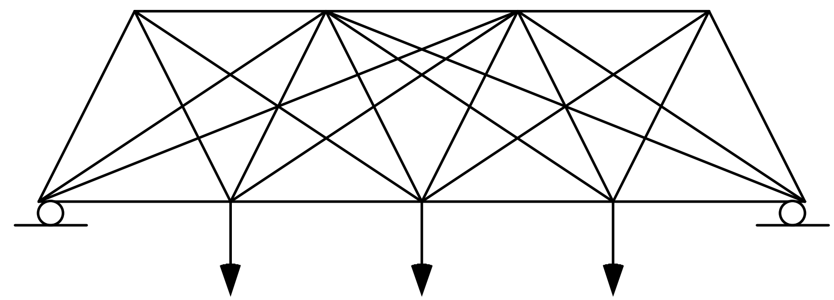
for

Truss-like Structures

No Major Problems

Weight Minimization

$$\min_{A_e,\boldsymbol{x}_i} \sum_{e=1}^{E_{\max}} \rho_e A_e L_e$$



Subject To

$$\sigma_{\bar{e}} = \max_e \left|\sigma_e\right| \le \sigma_{\max}$$ (Stress Constraint)

$$\left|\boldsymbol{u}_{\bar{i}}\right| = \max_i \left|\boldsymbol{u}_i\right| \le \Delta_{\max}$$ (Displacement Constraint)

Design Variables

Cross Sectional Area & Joint Location (Size + Shape)

Typical Performance Functions

Total Weight of a Truss/Frame Structure

$$\frac{D}{DA_e}\left(\sum_{e=1}^{E_{\max}} \rho_e A_e L_e\right) = \rho_e L_e \quad \& \quad \frac{D}{D\boldsymbol{x}_i}\left(\sum_{e=1}^{E_{\max}} \rho_e A_e L_e\right) = \frac{\partial L_e}{\partial \boldsymbol{x}_i}\rho_e A_e$$

Maximum (Axial) Stress

$$\frac{D\sigma_{\bar{e}}}{DA_e} = \frac{E_{\bar{e}}}{L_{\bar{e}}}\begin{bmatrix}-1 & 1\end{bmatrix}\frac{\partial \boldsymbol{u}_{\bar{e}}}{\partial A_e}$$

$$\frac{D\sigma_{\bar{e}}}{D\boldsymbol{x}_i} = -\frac{\sigma_{\bar{e}}}{L_{\bar{e}}}\frac{\partial L_{\bar{e}}}{\partial \boldsymbol{x}_i} + \frac{E_{\bar{e}}}{L_{\bar{e}}}\begin{bmatrix}-1 & 1\end{bmatrix}\frac{\partial \boldsymbol{u}_{\bar{e}}}{\partial \boldsymbol{x}_i}$$

Lagrangian

Lagrangian

$$L = \sum_{e=1}^{E_{\max}} \rho_e A_e L_e - \lambda_e \left(\sigma_e - \sigma_{\max} \right) - \sum_{i=1}^{n} \mu_i \left[\sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i} - \Delta_{\max} \right]$$

First Variation

$$\delta L = \sum_{e=1}^{E_{\max}} \left\{ \rho_e L_e - \lambda_e \frac{\partial \sigma_e}{\partial A_e} - \sum_{i=1}^{n} \mu_i \frac{\partial \sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i}}{\partial \boldsymbol{u}_i} \frac{\partial \boldsymbol{u}_i}{\partial A_e} \right\} \delta A_e$$

$$- \sum_{e=1}^{E_{\max}} \delta\lambda_e \left(\sigma_e - \sigma_{\max} \right) - \sum_{i=1}^{n} \delta\mu_i \left[\sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i} - \Delta_{\max} \right]$$

KKT Condition

From the variation of the Lagrange multipliers,

$$\lambda_e \left(\sigma_e - \sigma_{\max}\right) = 0 \quad , \quad \lambda_e \le 0 \quad , \quad \sigma_e - \sigma_{\max} \le 0$$

$$\mu_i \left[\sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i} - \Delta_{\max}\right] = 0 \quad , \quad \mu_i \le 0 \quad , \quad \sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i} - \Delta_{\max} \le 0$$

This implies that if the inequality constraint is not saturated, the Lagrange multiplier must be zero. Conversely, if the Lagrange multiplier is non-zero, the constraint must be saturated.

Optimality Condition

$$\lambda_e = \frac{\rho_e L_e - \sum_{i=1}^{n} \mu_i \dfrac{\partial \sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i}}{\partial \boldsymbol{u}_i} \dfrac{\partial \boldsymbol{u}_i}{\partial A_e}}{\dfrac{\partial \sigma_e}{\partial A_e}} \quad \text{if} \quad \frac{\partial \sigma_e}{\partial A_e} \neq 0 \quad \text{and} \quad \text{if} \quad \forall \delta A_e$$

$$\rho_e L_e \neq 0 \quad \& \quad \rho_e L_e \neq \sum_{i=1}^{n} \mu_i \frac{\partial \sqrt{\boldsymbol{u}_i^T \boldsymbol{u}_i}}{\partial \boldsymbol{u}_i} \frac{\partial \boldsymbol{u}_i}{\partial A_e} \Rightarrow \lambda_e \neq 0 \Rightarrow \sigma_e - \sigma_{\max} = 0$$

First Approximation : Fully Stressed Design

$$\sigma_e = \sigma_{\max} \quad \Leftrightarrow \quad \frac{\sigma_e}{\sigma_{\max}} = 1 \quad , \quad e = 1, \ldots, E_{\max}$$

Sizing can yield Fully Stressed

As a special approximation of the optimality criterion condition for a sizing problem, we can derive the concept of fully stressed design.

Interpretation

If the displacement constraint is not saturated at a node of the e-th truss member, its Lagrange multiplier must be zero. Thus, we have

$$\lambda_e = \rho_e L_e \Big/ \frac{\partial \sigma_e}{\partial A_e}$$

Since the mass density and the length of the truss element are positive, this yields

$$\lambda_e \neq 0$$

There fore, the constraint on the stress must be saturated.

Fully Stressed

Thus, if the displacement constraint is not imposed, the fully stressed state is nothing but the optimum. Therefor, even if the displacement constraint is imposed, in the most of truss members which are not related to the maximum displacement the fully stressed condition must be satisfied, and then it can be said that the fully stressed state must be a good approximation of the optimum state.

Many Design Codes in 1950s and 60s were made for Fully Stressed Design

L.Schimit disproved this need not be true.

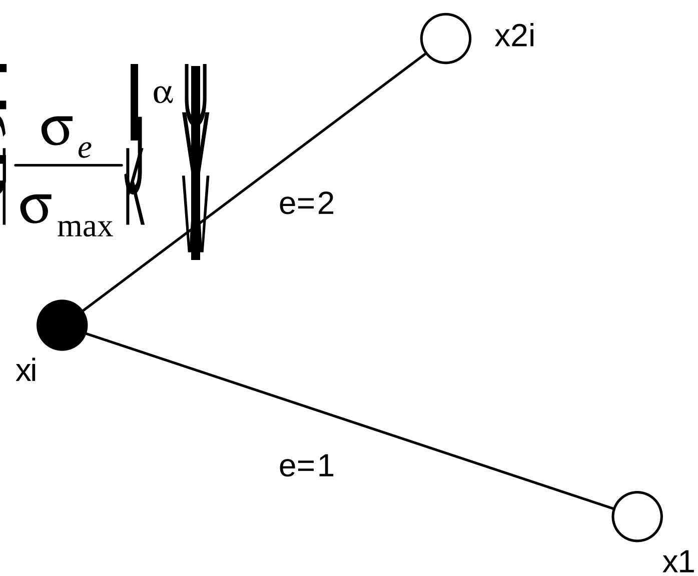
Fully Stressed Design Method

Sizing / Cross Sectional Area

$$A_e^{(k+1)} = A_e^{(k)} \left(\frac{\sigma_e}{\sigma_{max}} \right)^{\alpha} \quad , \quad k = 1,2,\ldots\ldots \quad \text{for} \quad \text{some } \alpha$$

Shape / Length & Nodal Coordinate

$$\boldsymbol{x}_i^{(k+1)} = \boldsymbol{x}_i^{(k)} + \sum_{e=1}^{i_{max}} \left(\boldsymbol{x}_{ei}^{(k)} - \boldsymbol{x}_i^{(k)} \right) \left\{ 1 - \left(\frac{\sigma_e}{\sigma_{max}} \right)^{\alpha} \right\}$$



Fully Stressed Design

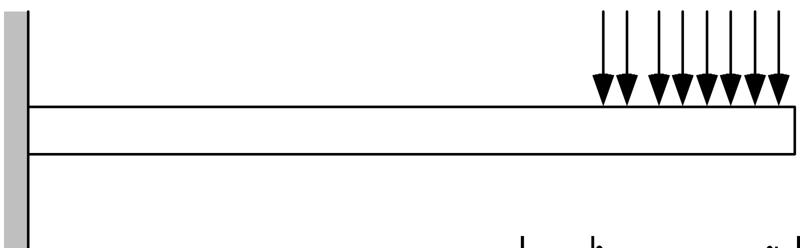
- Fully stressed design was the design method before mathematical programming method was introduced in 1960 by L. Schmit
- An effective method to find out the initial start (initial approximation) of the MPM
- This can be a Re-Design method
- This can be extended to other physical quantities and other type structures

Other Physical Quantities

- Mises equivalent stress on a boundary
- Maximum principal stress
- Maximum shear stress
- Principal Strains and/or Formed Thickness
- Strain energy density
- .... Anything Distributed along/on
- .... the Design Boundary/Domain

Fully Stresses Design

- Design variable and the quantity to be saturated must be defined in the one-to-one corresponding way

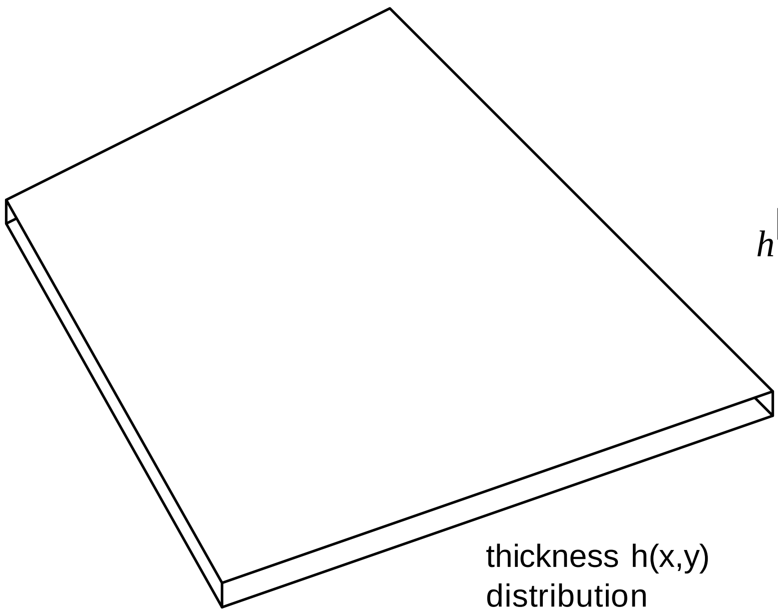


$$A^{(k+1)}(x) = A^{(k)}(x)\left(\frac{|\sigma(x,\pm h/2)|}{\sigma_{\max}}\right)$$

Axial stress is not constant in each beam element, and then the design variable Ae of the cross sectional area must be defined as the axial stress.

Natural Extension

Plate/Shell Like Structures



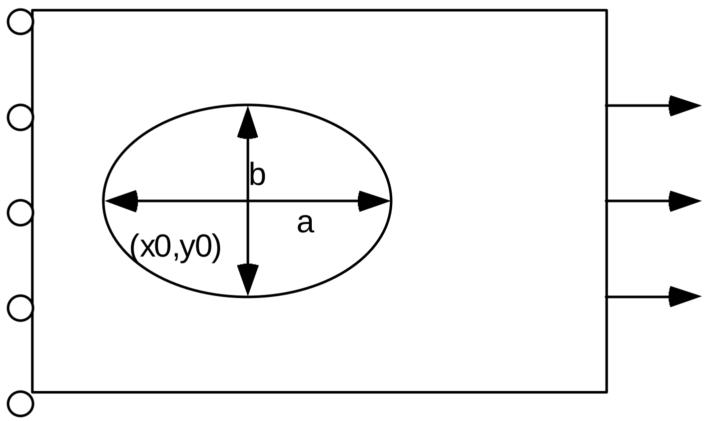
1) Mises Equivalent Stress

$$h^{(k+1)}(x,y) = h^{(k)}(x,y)\left(\frac{\max\left\{\bar{\sigma}(x,y,+h/2)\ ,\ \bar{\sigma}(x,y,-h/2)\right\}}{\sigma_{\max}}\right)^{\alpha}$$

2) Strain Energy Density

$$h^{(k+1)}(x,y) = h^{(k)}(x,y)\left(\frac{\max\left\{\frac{1}{2}\varepsilon^{T}\boldsymbol{E}\varepsilon(x,y,+h/2)\ ,\ \frac{1}{2}\varepsilon^{T}\boldsymbol{E}\varepsilon(x,y,-h/2)\right\}}{\sigma_{\max}}\right)^{\alpha}$$

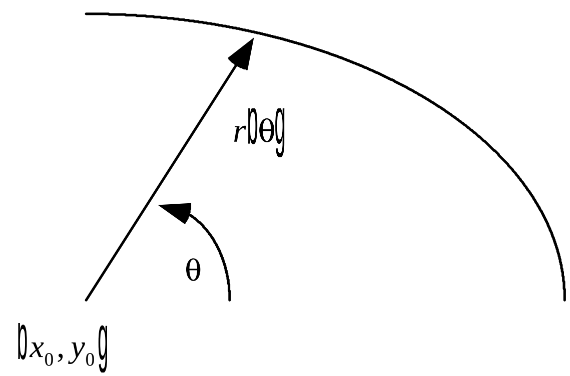
Elliptic Hole Design



Design variables are a,b, and x0, but the stresses are defined along the boundary of an elliptic hole

$$r^{(k+1)}(\theta) = r^{(k)}(\theta)\left[\frac{\bar{\sigma}\left(r^{(k)}(\theta),\theta\right)}{\sigma_{\max}}\right]^{-\alpha}$$

$r(\theta)$ = radial distance from the origin (x_0, y_0)

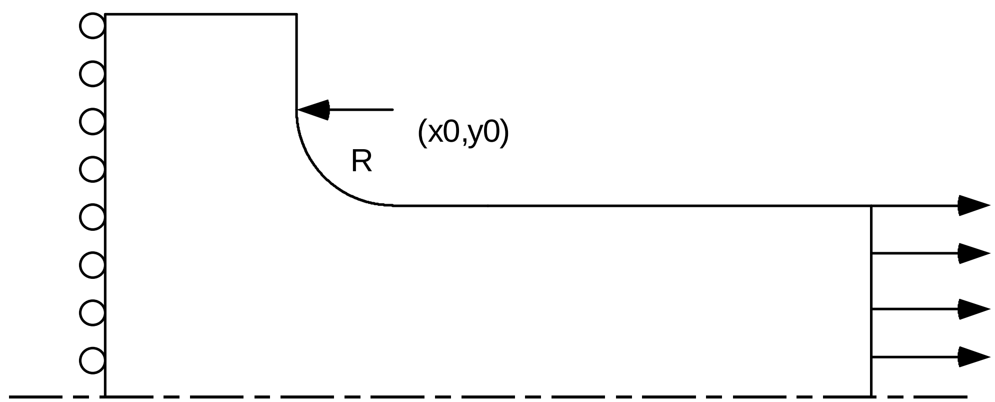


Least Squares Curve Fitting

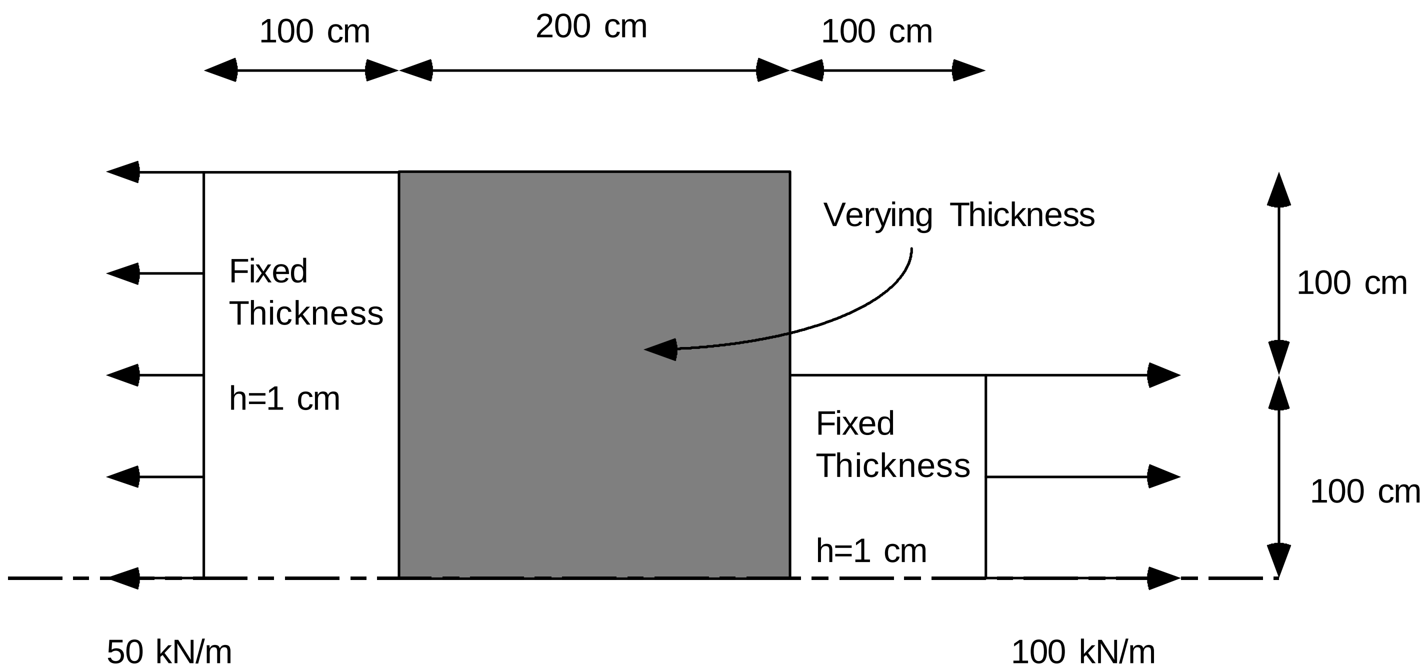
$$\min_{\substack{(x_0,y_0)\\ a\,\&\,b}} \left\| \begin{Bmatrix} x_0 + a\cos\theta \\ y_0 + b\sin\theta \end{Bmatrix} - \begin{Bmatrix} r^{(k+1)}(\theta)\cos\theta \\ r^{(k+1)}(\theta)\sin\theta \end{Bmatrix} \right\|$$

Exercise #8 : Fillet Arc Design

Find an algorithm of the fully stresses design of the location of the origin of the arc fillet shape together with the radius.



Exercise #8 : Taylor's Design



Set up a fully stressed design problem for finding the optimum thickness distribution. Also set up a shape design problem for a constant thickness, as well as a topology optimization.

$$\min_{h_{\min} \le h(x,y) \le h_{\max}} \int_{\Gamma_t} \boldsymbol{u}^T \boldsymbol{t}\, d\Gamma$$

$$\boldsymbol{u} \quad : \quad \int_{\Omega} \varepsilon(\boldsymbol{v})^T \boldsymbol{E} h(x,y) \varepsilon(\boldsymbol{v})\, d\Omega = \int_{\Gamma_t} \boldsymbol{v}^T \boldsymbol{t}\, d\Gamma \quad , \quad \forall \boldsymbol{v}$$

$$\int_{\Omega} h(x,y)\, d\Omega \le V_0$$

This formulation is identical to the homogenization design method.

J.E. Taylor in 1967 based on the work of Taylor and Prager in 1967

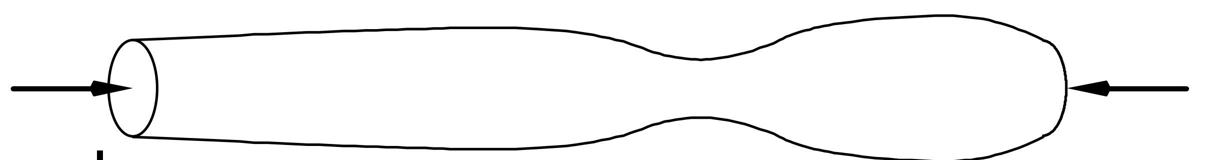
Extension

- Something is constant in the optimality condition, then we can "derive" fully stressed design formulation.

$$\phi(d) = \text{constant} \quad \Rightarrow \quad d^{(k+1)} = d^{(k)} + \beta \left(\frac{\phi\left(d^{(k)}\right) - \frac{1}{|\Omega|}\int_{\Omega} \phi\left(d^{(k)}\right) d\Omega}{\frac{1}{|\Omega|}\int_{\Omega} \phi\left(d^{(k)}\right) d\Omega} \right)^{\alpha}$$

Exercise #10 : Critical Load

$$\max_{\substack{A \\ \int_0^L A dx \le V_0}} P_{cr} = \frac{\int_0^L EI \left(w''\right)^2 dx}{\int_0^L A \left(w'\right)^2 dx} \quad , \quad I = \frac{\pi d^4}{64} \;\;\&\;\; A = \frac{\pi d^2}{4}$$



$$L = \frac{\int_0^L EI \left(w''\right)^2 dx}{\int_0^L A \left(w'\right)^2 dx} - \lambda\left(\int_0^L A dx - V_0\right)$$

$$\delta L = \frac{\int_0^L A \left(w'\right)^2 dx \, \delta \int_0^L E \frac{A^2}{4\pi} \left(w''\right)^2 dx - \int_0^L E \frac{A^2}{4\pi} \left(w''\right)^2 dx \, \delta \int_0^L A \left(w'\right)^2 dx}{\left(\int_0^L A \left(w'\right)^2 dx\right)^2} - \delta\lambda\left(\int_0^L A dx - V_0\right) - \lambda\delta\left(\int_0^L A dx - V_0\right)$$

$$= \frac{\int_0^L 2E \frac{A}{4\pi} \left(w''\right)^2 \delta A\, dx - P_{cr} \int_0^L \delta A \left(w'\right)^2 dx}{\int_0^L A \left(w'\right)^2 dx} - \lambda \int_0^L \delta A\, dx - \delta\lambda \left\{ \int_0^L A\, dx - V_0 \right\}$$

$$= \frac{\int_0^L \left[2E \frac{A}{4\pi} \left(w''\right)^2 - P_{cr} \left(w'\right)^2 - \lambda \int_0^L A \left(w'\right)^2 dx \right] \delta A\, dx}{\int_0^L A \left(w'\right)^2 dx} - \delta\lambda \left\{ \int_0^L A\, dx - V_0 \right\}$$

$$= 0 \quad \forall \delta A \;, \; \forall \delta\lambda \le 0$$

$$2E \frac{A}{4\pi} \left(w''\right)^2 - P_{cr} \left(w'\right)^2 = \lambda \int_0^L A \left(w'\right)^2 dx = \text{constant}$$

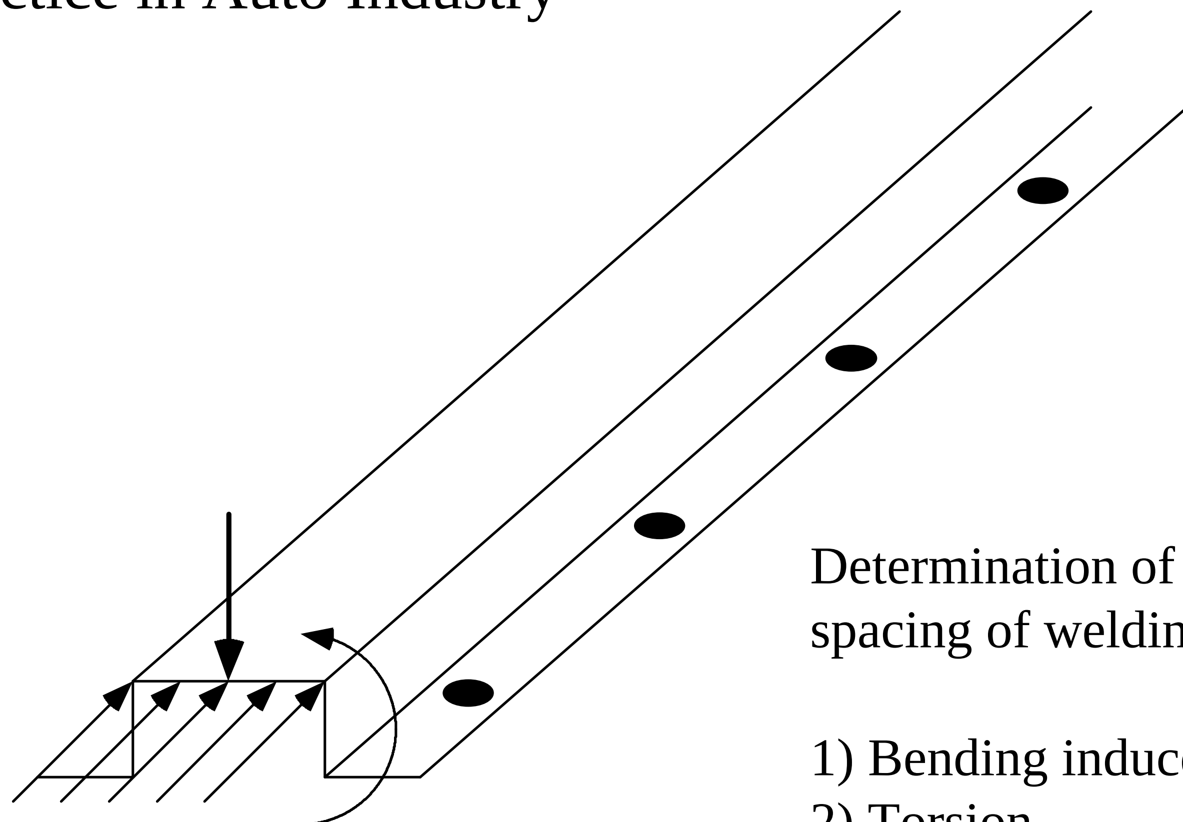
Something is Constant

“Fully Stressed Design”

Re-Design Approach

Advanced Structural Design Problem

Practice in Auto Industry

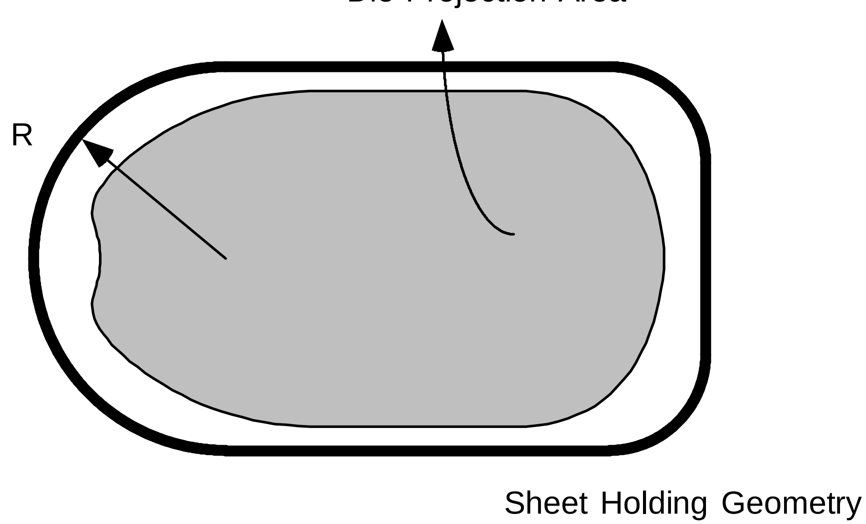


Determination of the spacing of welding spots

1) Bending induced shear
2) Torsion
3) Buckling

Exercise #11

Typical Nonlinear Mechanical Design Optimization

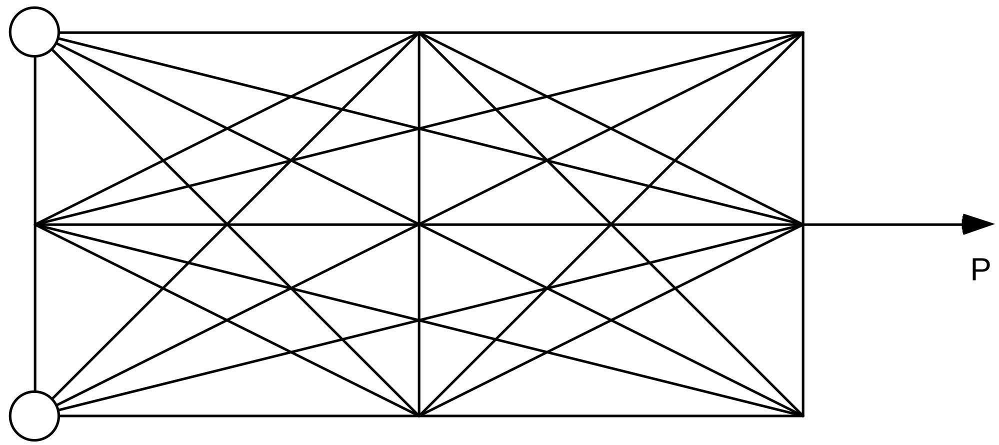


Design the radius of the left portion of the sheet holding curved line for sheet metal forming so that the thickness variation over the die projection area can be minimized after forming.

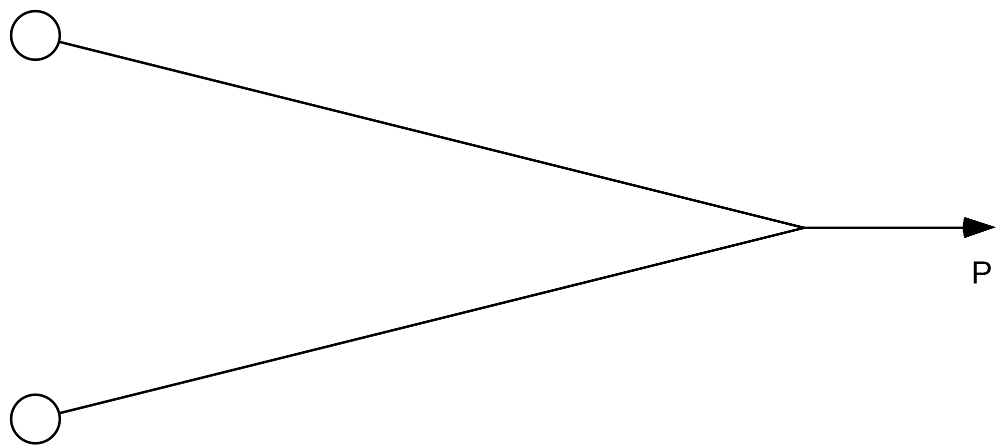
Sizing Can Yield Topology

Sizing optimization can yield topology of a structure by constructing the ground structure

Candidate truss structure



Optimum Truss



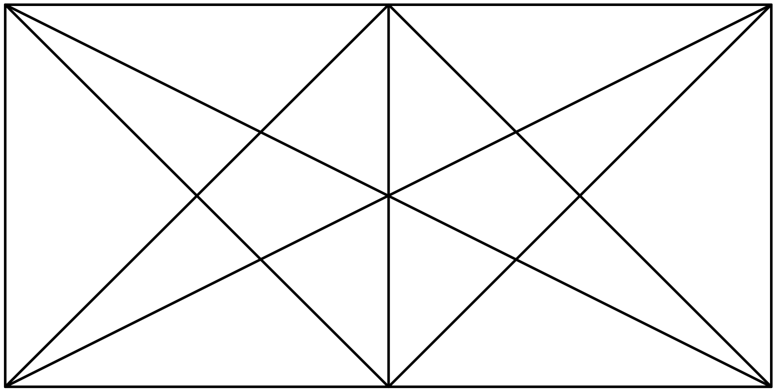
Ground Structures

Connect all the nodes

$$E_{\max} = \frac{1}{2}(n-1)n$$

n = number of nodes

$E_{\max}$ = number of elements



Sizing problems can form a topology optimization

This approach was taken in 1960s to derive the Michel truss structure

Sizing Design is Dependable

Since the sizing problem was regarded as a well behaved one, many general purpose design optimization codes were developed.

General Purpose Codes 1

- Many general purpose structural optimization codes were developed in aerospace industry in 1970s and 1980s for sizing optimization
  - ACCESS (UCLA/Schmit) ... MSC/NASTRAN
  - ELFINI (Dassault/Lucina)
  - FASTOP (Grumman)
  - LAGRANGE (MBB)

General Purpose Codes 2

- – OASIS (Stockholm/Esping)
- – OPTFORCE (Bell/Gellatly)
- – OPTI/SAMSEF (Liege/Fleury)
- – OPTIMA (Stuugart/Mlejnek)
- – OPTISYS (Saab-Scania) ... OASIS
- – ODYSSEY (General Motors/Bennett)
- – PANDA (Lockheed/Bushnell)
- – STAR (RAE/Morris)
- – TSO (General Dynamics)

SLP : most conservative

$$\min_{\substack{\boldsymbol{x} \\ \boldsymbol{Gx} \le \boldsymbol{g}}} \quad \boldsymbol{x}^{T}\boldsymbol{a} \qquad\qquad L = \boldsymbol{x}^{T}\boldsymbol{a} - \boldsymbol{\lambda}^{T}\left(\boldsymbol{Gx} - \boldsymbol{g}\right)$$

$$\delta L = \delta\boldsymbol{x}^{T}\boldsymbol{a} - \boldsymbol{\lambda}^{T}\boldsymbol{G}\delta\boldsymbol{x} - \delta\boldsymbol{\lambda}^{T}\left(\boldsymbol{Gx} - \boldsymbol{g}\right)$$

$$= \delta\boldsymbol{x}^{T}\left(\boldsymbol{a} - \boldsymbol{G}^{T}\boldsymbol{\lambda}\right) - \delta\boldsymbol{\lambda}^{T}\left(\boldsymbol{Gx} - \boldsymbol{g}\right)$$

$$\boldsymbol{x} = P_{\boldsymbol{x}}\left\{\boldsymbol{x} - \omega_{\boldsymbol{x}}\left(\boldsymbol{a} - \boldsymbol{G}^{T}\boldsymbol{\lambda}\right)\right\} \quad \text{with} \quad \boldsymbol{x}^{(0)} = P_{\boldsymbol{x}}\left(\boldsymbol{G}^{+}\boldsymbol{g}\right)$$

$$\boldsymbol{\lambda} = P_{\lambda}\left(\boldsymbol{\lambda} - \omega_{\lambda}\left(\boldsymbol{Gx} - \boldsymbol{g}\right)\right) \quad \text{with} \quad \lambda^{(0)} = P_{\lambda}\left(\boldsymbol{G}^{T+}\boldsymbol{a}\right)$$

SQP : popular method

$$\min_{\substack{\boldsymbol{x} \\ \boldsymbol{Gx} \le \boldsymbol{g}}} \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Ax} - \boldsymbol{x}^T\boldsymbol{a} \qquad L = \frac{1}{2}\boldsymbol{x}^T\boldsymbol{Ax} - \boldsymbol{x}^T\boldsymbol{a} - \lambda^T(\boldsymbol{Gx} - \boldsymbol{g})$$

$$\delta L = \delta\boldsymbol{x}^T(\boldsymbol{Ax} - \boldsymbol{a}) - \lambda^T\boldsymbol{G}\delta\boldsymbol{x} - \delta\lambda^T(\boldsymbol{Gx} - \boldsymbol{g})$$

$$= \frac{1}{\omega_x}\delta\boldsymbol{x}^T\left[\boldsymbol{x} - \boldsymbol{x} + \omega_x\left\{\boldsymbol{Ax} - \boldsymbol{a} - \boldsymbol{G}^T\lambda\right\}\right] - \frac{1}{\omega_\lambda}\delta\lambda^T\left\{\lambda - \lambda + \omega_\lambda(\boldsymbol{Gx} - \boldsymbol{g})\right\}$$

$$\boldsymbol{x} = P_x\left[\boldsymbol{x} - \omega_x\left\{\boldsymbol{Ax} - \boldsymbol{a} - \boldsymbol{G}^T\lambda\right\}\right] \quad \text{with} \quad \boldsymbol{x}^{(0)} = P_x\left\{\boldsymbol{G}^+\boldsymbol{g}\right\}$$

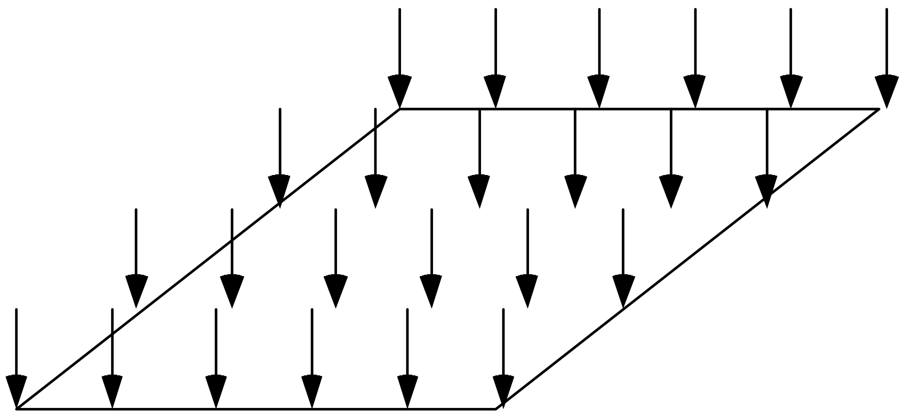
$$\lambda = P_\lambda\left\{\lambda - \omega_\lambda(\boldsymbol{Gx} - \boldsymbol{g})\right\} \quad \text{with} \quad \lambda^{(0)} = P_\lambda\left[\boldsymbol{G}^{T+}\left[\boldsymbol{Ax}^{(0)} - \boldsymbol{a}\right]\right]$$

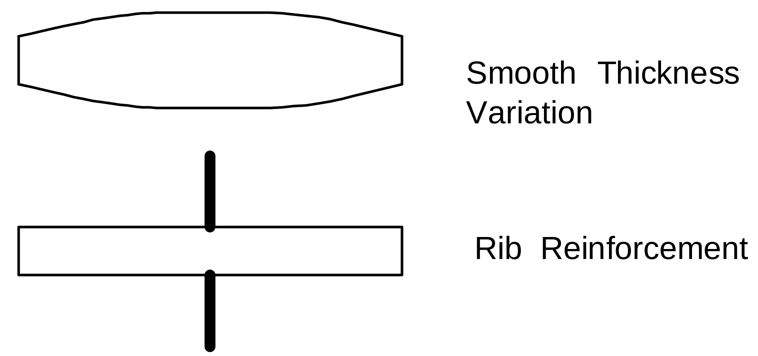
However, in 1980

Cheng and Olhoff found that the sizing problem for plate thickness distribution is not well-posed !

Exercise # 12 : Ribs ?

Which reinforcement is much more effective ?





Simply Supported
Uniformly Distributed Load

Size and Shape Design

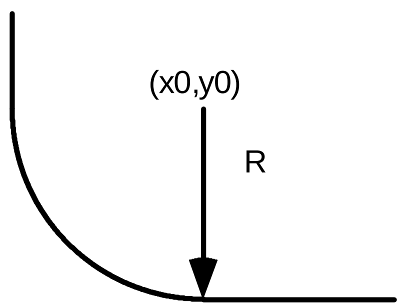
Size in MCAE means Shape

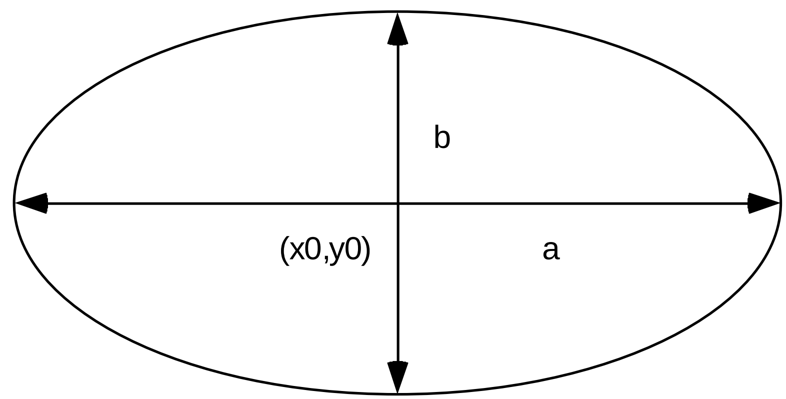
Shape Design Optimization

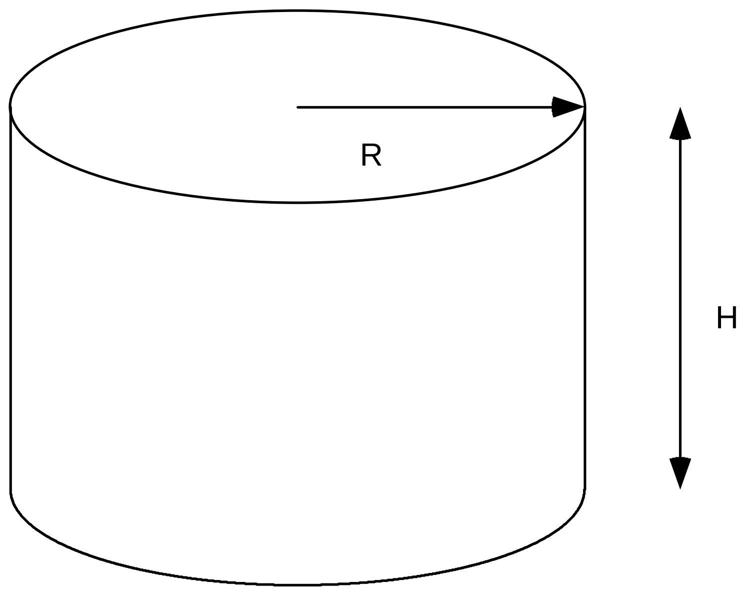
General Remarks

Toward Topology Design

Size Design in MCAE







Sizing in Mechanical Design is always related to the shape of a structure !

Shape Design Optimization

- O.C.Zienkiewicz and J.S.Campbell, Shape Optimization and Sequential Linear Programming, in an international symposium on Optimization of Structural Design, University of Wales, Swansea, January 1972
- FEM + Design Sensitivity + SLP
- Adaptation of Nodal Points on the Boundary

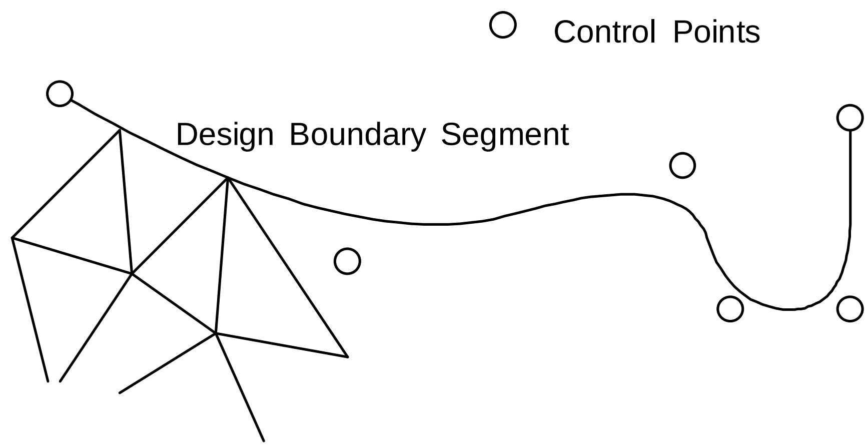
A Lot of Problems

- Without using parametric representation, they adapted the nodes of the finite element model
  - possibility of non-smoothed optimum shape due to non-smoothed stresses on the design boundary
  - possibility of excessive element distortion
  - unclear adaptation schemes

Was Not Popular

- Nodal relocation schemes were very unpopular among the researchers and engineers in practice
  - GM : Design Segment/Patch & Automatic Remeshing Scheme
  - Dassault/ELFINI : Design Segment/Patch
  - Liege/SAMSEF : Design Segment/Patch
  - SAAB-SCANIA : Design Segment/Patch

Design Segment/Patch



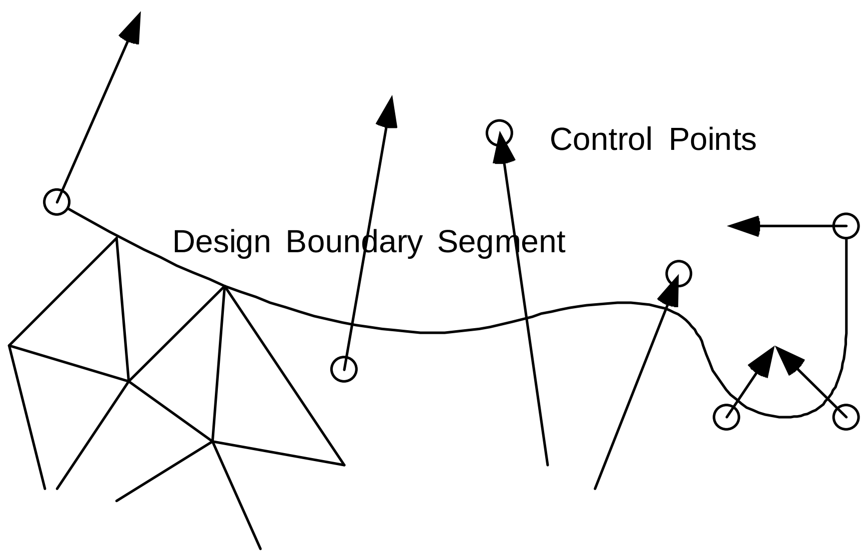
Possibility of Link with CAD Systems

but ....

Design boundary segments/patches are defined independently of the finite element model, using splines and control points.

Design variables are the location of control points, not the nodes of the FE model on the design boundary

Adaptation Scheme



Schnuck's Method

Control points as well as finite element nodes are adapted in the normal direction to the design boundary

Possibility of crashing

Must be combined with automatic mesh generation

GM's Success

Drs. Bennett and Botkin made a great success in shape optimization by applying

1. Design Segment/Patch Approach
2. Adapting the control points to the normal direction
3. Applying a full automatic mesh generation scheme developed by M. Shepherd in RPI
4. Applying the adaptive finite element method to control FE approximation error, especially the error of the stress

However, this could not become a successful product

GM : Mathematically Right

GM's success was great, and the best possible shape design optimization program we could have, even in mathematics.

Mathematical theory of shape optimization by Dal Maso and Buttazzo says that FEM models must be independent of a parametric representation of the boundary shape, and it the number of parameters are finite, then there exists at least one optimum shape.

If the number of parameters is increasing, then the optimum solution need not converge to a unique one.

What this means ?

Shape should be represented by less number of parameters, that is, each design segment should have simple geometry without using sophisticated higher order splines.

More number of parametric design variables need not be effective, and we may need to expect quite different results from the case of less number of parameters.

More flexibility by more parameters makes easy crash of multiple design segments, and it becomes difficult to control

GM's Success

was too great !

Very Few Could Follow
What They have Done.

Stacked !

Practical Approach

Full integration of

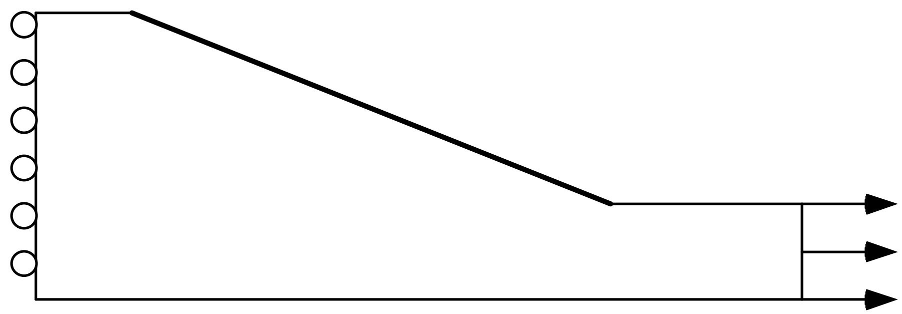
1) CAD like representation of Design Segments
2) Control Point Adaptation
3) Adaptive Finite Element Method
4) Full Automatic Mesh Generation Method

is not realistic in practice.

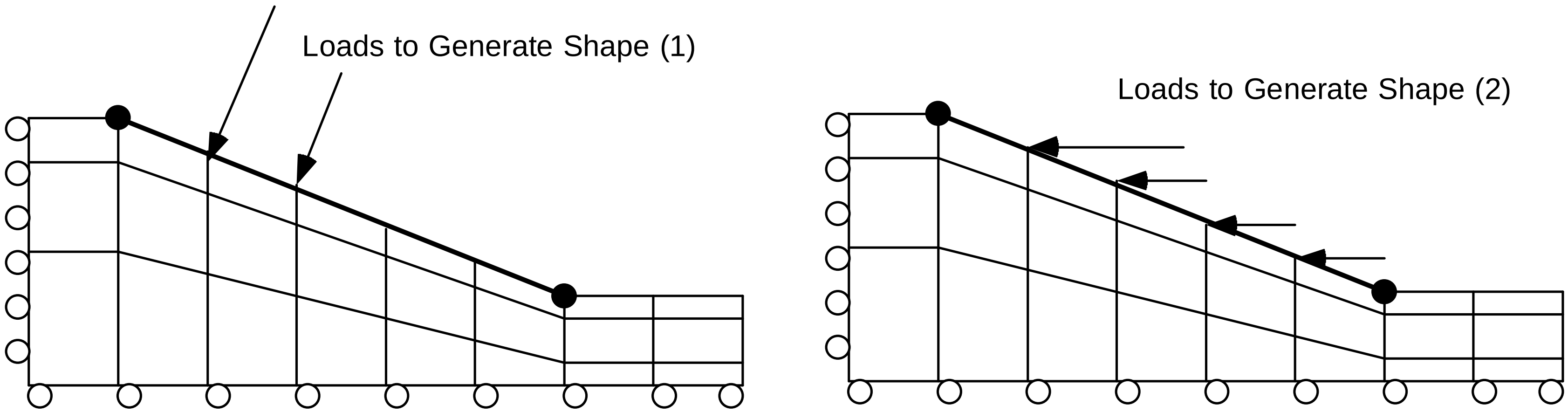
What is a possible alternate ?

a) GENESYS Approach
b) Bio-mechanical Growth Approach

GENESYS Approach



Linear Combination of Base Shapes generated by FE deformation by artificial loads



Advantage

- Design boundary change is smooth and can be controlled, since elastic deformation due to fictitious loads is regarded as a base design change
- Finite element distortion is minimized
- Remeshing methods need not be integrated, since the initial finite element connectivity is maintained during optimization

Success of GENESYS

- By creating interactive preprocessor to define the base shapes for the design change, but it is independent of CAD soft
- Three-dimensional curved design segments and are treated by the same way
- FORD extensively uses this after Topology Design results to make detailed design

Bio-mechanical Growth

- Similarity with Thermal Deformation
  - increasing temperature results expansion
  - cooling results shrinkage of a structure
- Temperature Change = Difference between the Current Stress and the Targeted One in the optimality criteria method
- Azegami @Toyohashi Technical University
- Sauter (Germany) etc

Approach

For the Fully Stressed Design

thermal loading

$$\boldsymbol{f}^{e}_{thermal} = \int_{\Omega_e} \boldsymbol{B}^T \boldsymbol{D} \Delta T a \, d\Omega$$

fictitious loading

$$\boldsymbol{f}^{e}_{fictitious} = \int_{\Omega_e} \boldsymbol{B}^T \boldsymbol{D} \left(\frac{\sigma - \sigma_{\text{target}}}{\sigma_{\text{target}}} \right)^{\alpha} a \, d\Omega$$

Shrink if stress is too low, enlarge if stress is too high

Characteristics

- Bio-mechanical growth approach is quite powerful for the fully stressed design approach and also for the optimality criteria method for design optimization
- It is similar to GENESYS approach in the sense that fictitious loadings are considered to adapt the design shape
- and no need to make remeshing schemes

Nature of Shape Change

- If dramatic shape change is not required, CAD linked remeshing scheme with full automatic mesh generation methods is not quite essential.
- Thus, if shape design is considered after topology optimization, then both GENESYS and Bio-mechanical growth approaches are sufficiently powerful.

Shape Design Optimization

- Since Topology Design Optimization does not include many design constraints, Shape Design stage should involve all kind of design restriction not only for
  - stiffness, strength, local buckling
- but also
  - manufacturability
  - geometric constrains

GENESYS Approach

New Version of OPTISHAPE

Topology Design

+

SHAPE DESIGN with Modified Azegami's Approach

Shape : Formulation

Typical Setting of Optimization

$$\min_{\text{design}} \int_\Omega \rho \, d\Omega$$

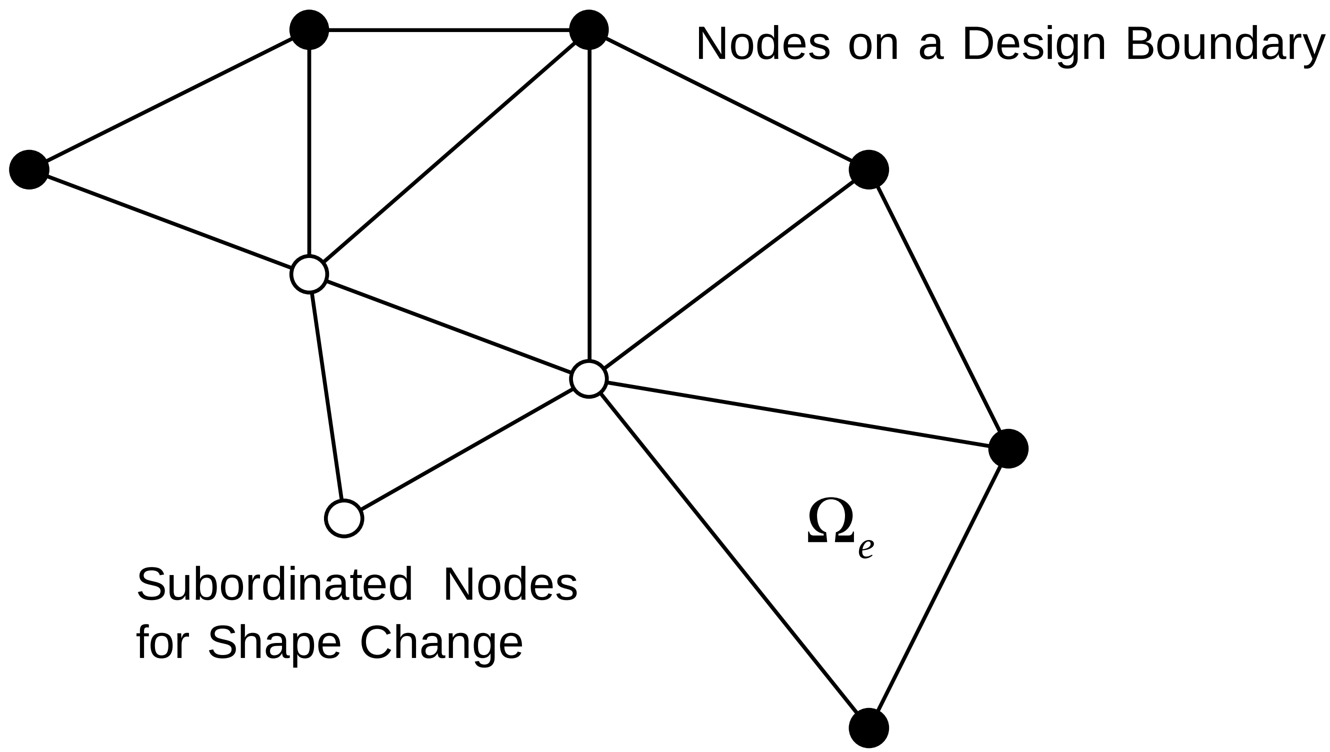
subject to

$$a(\boldsymbol{u},\boldsymbol{v}) = f(\boldsymbol{v}) \quad \forall \boldsymbol{v}$$

$$\bar{\sigma} \le \sigma_{\max}$$

$$|\boldsymbol{u}| \le u_{\max}$$

Ω = variable unknown domain


Nodes on a Design Boundary
Ω_e
Subordinated Nodes for Shape Change

Finite Element Representation

$$\min_{\text{design}} \sum_{e=1}^{E_{\max}} \rho_e \Omega_e$$

Varying in Shape Design

subject to

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$$

$$\bar{\sigma}_e \leq \sigma_{\max} \;, e = 1, \ldots, E_{\max}$$

$$\left|\boldsymbol{u}_i\right| \leq u_{\max} \;, i = 1, \ldots, I_{\max}$$

Ω_e = area / volume of finite elements

Virtual Work Principle

$$a(\boldsymbol{u},\boldsymbol{v}) = f(\boldsymbol{v}) \quad \forall \boldsymbol{v}$$

$\Leftrightarrow$

thermal load — applied traction

$$\int_\Omega \varepsilon(\boldsymbol{v})^T \boldsymbol{E}\varepsilon(\boldsymbol{u})\,d\Omega = \int_\Omega \varepsilon(\boldsymbol{v})^T \boldsymbol{E}\alpha\,d\Omega + \int_\Omega \boldsymbol{v}^T \rho \boldsymbol{b}\,d\Omega + \int_{\Gamma_t} \boldsymbol{v}^T \boldsymbol{t}\,d\Omega \quad \forall \boldsymbol{v}$$

internal virtual work — body force

Finite Element Approximation

$$\boldsymbol{K}\boldsymbol{u} = \boldsymbol{f}$$

Ω = variable unknown domain

Too Complex Requirement

A lot of mathematical evaluation is necessary to compute required design sensitivity for shape design

Standard Procedure

Ω = variable unknown domain

Design Variable = Control Points $\boldsymbol{x}_{cp}$

Step 1 : Relation between FE nodes and Design control points

$$\boldsymbol{x} = \boldsymbol{T}\boldsymbol{x}_{cp}$$

Step 2 : Design Sensitivity w.r.t. control points

$$\frac{D}{D\boldsymbol{x}_{cp}}\left(\boldsymbol{K}\boldsymbol{u}\right) = \frac{D\boldsymbol{K}}{D\boldsymbol{x}_{cp}}\boldsymbol{u} + \boldsymbol{K}\frac{D\boldsymbol{u}}{D\boldsymbol{x}_{cp}}$$

Design Sensitivity

$$\frac{D\boldsymbol{K}}{D\boldsymbol{x}_{cp}} = \frac{D}{D\boldsymbol{x}_{cp}} \sum_{e=1}^{E_{\max}} \boldsymbol{K}_e = \frac{D}{D\boldsymbol{x}_{cp}} \sum_{e=1}^{E_{\max}} \int_{\Omega_e} \boldsymbol{B}^T \boldsymbol{E} \boldsymbol{B} d\Omega = \frac{D}{D\boldsymbol{x}_{cp}} \sum_{e=1}^{E_{\max}} \int_{\Omega_R} \boldsymbol{B}^T \boldsymbol{E} \boldsymbol{B} J d\Omega_R$$

$$= \sum_{e=1}^{E_{\max}} \int_{\Omega_R} \left(\frac{D\boldsymbol{B}}{D\boldsymbol{x}_{cp}} \right)^T \boldsymbol{E} \boldsymbol{B} J d\Omega_R + \sum_{e=1}^{E_{\max}} \int_{\Omega_R} \boldsymbol{B}^T \boldsymbol{E} \left(\frac{D\boldsymbol{B}}{D\boldsymbol{x}_{cp}} \right) J d\Omega_R + \sum_{e=1}^{E_{\max}} \int_{\Omega_R} \boldsymbol{B}^T \boldsymbol{E} \boldsymbol{B} \left(\frac{DJ}{D\boldsymbol{x}_{cp}} \right) d\Omega_R$$

Design Sensitivity Analysis must be in FEA codes

$$\frac{D\boldsymbol{B}}{D\boldsymbol{x}_{cp}} = \frac{D\boldsymbol{B}}{D\boldsymbol{x}} \frac{\partial \boldsymbol{x}}{\partial \boldsymbol{x}_{cp}} = \frac{D\boldsymbol{B}}{D\boldsymbol{x}} \boldsymbol{T}$$

Design Sensitivity Analysis must be linked with spline representation of design segments/surfaces

Difficulty : Too Much

- Every FEA code does have their own special finite elements, and then design sensitivity must be performed in such a FEA code
- Geometric representation of the control points and the FE nodes must be related, and then this requires full link with CAD representation and mesh generation scheme

PARADIGM Change

is required to do shape design

How ?

Mathematicians Are Fantastic !

Murat and Tartar (France) in 1983

Kohn and Strang (USA) in 1984

Lurier, Cherkaev, and Fedrov (Russia) in 1981

Characteristic Function

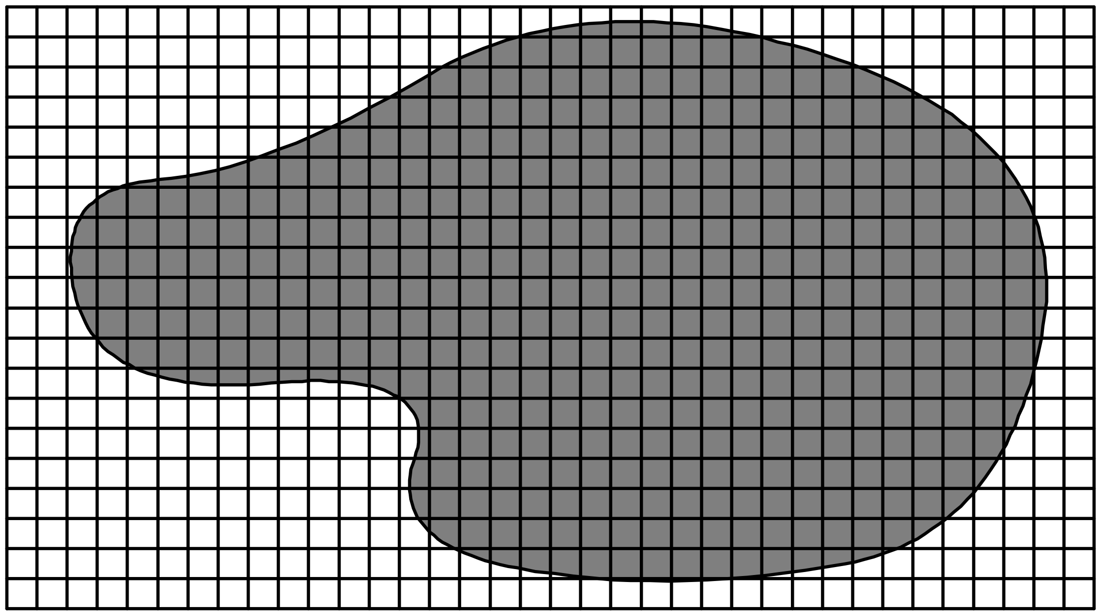
On-Off condition for the unknown domain

$$\chi_\Omega(\boldsymbol{x}) = \begin{cases} 1 & if \quad \boldsymbol{x} \in \Omega \\ 0 & if \quad \boldsymbol{x} \notin \Omega \end{cases}$$

Extended Formulation

$$\int_\Omega \varepsilon(\boldsymbol{v})^T \boldsymbol{E} \varepsilon(\boldsymbol{u}) d\Omega = \int_D \varepsilon(\boldsymbol{v})^T \chi_\Omega \boldsymbol{E} \varepsilon(\boldsymbol{u}) dD$$

$D \subset \Omega$ is the extended design domain that is fixed and known *a priori*



On/Off Switch Condition

$$\chi_e = \begin{cases} 0 \,/\, \text{off} & \text{if outside} \\ 1 \,/\, \text{on} & \text{if inside} \end{cases}$$

What this means ?

Shape design can be transformed into design of material constants (material distribution over a fixed design domain)

$$\int_D \left\{ \boldsymbol{v} \right\}^T \chi_\Omega \boldsymbol{E} \left\{ \boldsymbol{u} \right\} dD = \int_D \left\{ \boldsymbol{v} \right\}^T \boldsymbol{E}_\Omega \left\{ \boldsymbol{u} \right\} dD$$

$$\boldsymbol{E}_\Omega = \chi_\Omega \boldsymbol{E} = \text{new material constants}$$

No mesh adaptation is required !

Look at Taylor's Approach

Plate Thickness Optimization for Plane Elasticity
by John E. Taylor in 1967

$$\int_D \varepsilon(\boldsymbol{v})^T \; h(\boldsymbol{x}) \; \boldsymbol{E}\varepsilon(\boldsymbol{u})\, dD$$

Plate Thickness Designed

2D

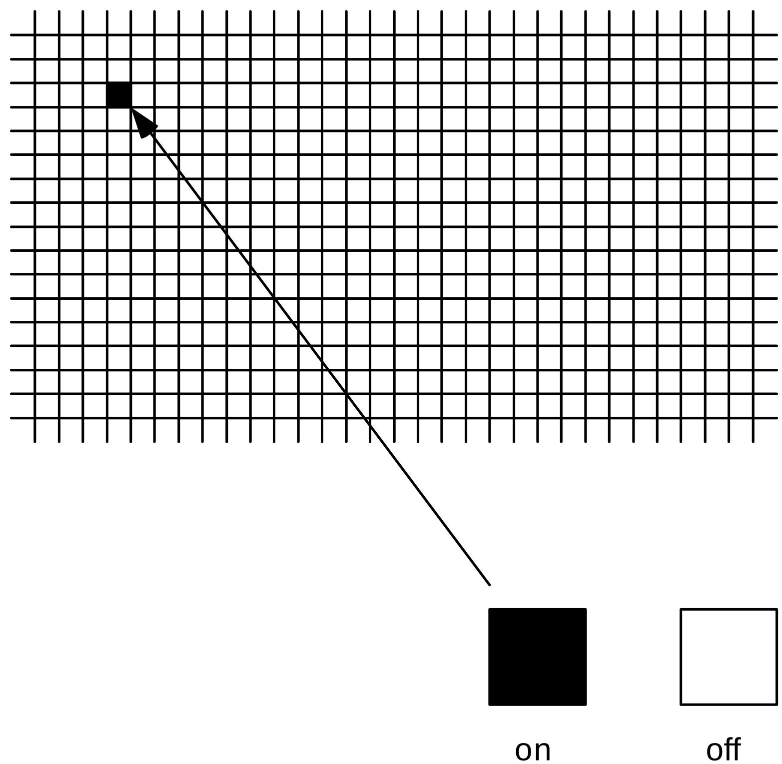
2D & 3D

$$\int_D \varepsilon(\boldsymbol{v})^T \; \chi_\Omega \; \boldsymbol{E}\varepsilon(\boldsymbol{u})\, dD$$

Extended Domain Approach

Pixel/Voxel Representation

Shape is represented by a collection of pixels/voxels as in monitors of computer graphics



OPTISHAPE

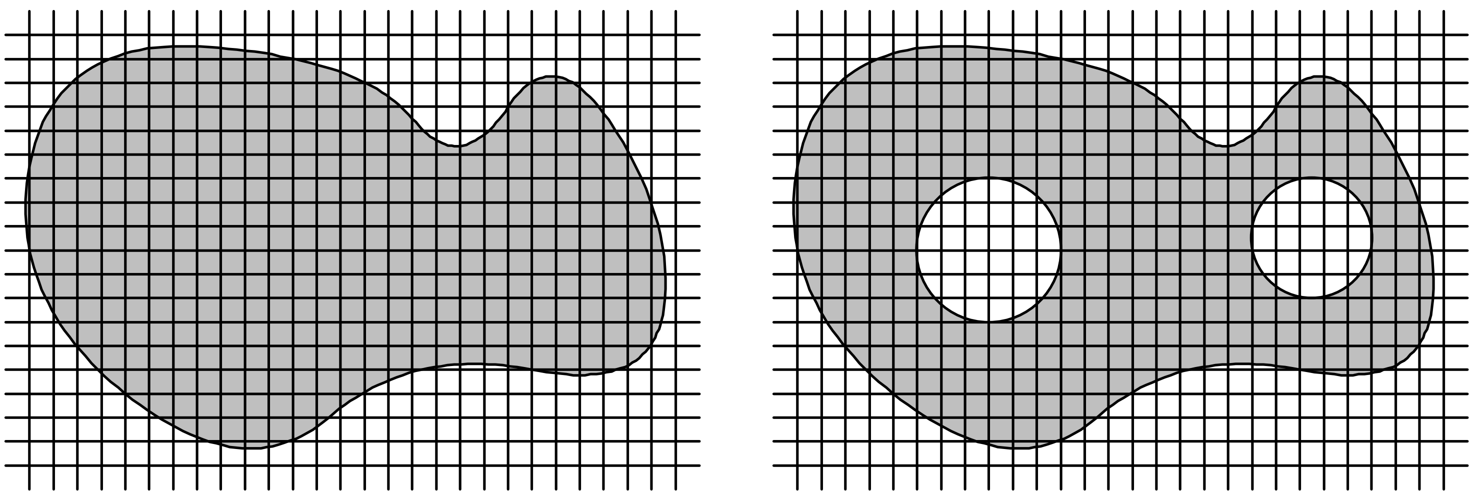
is a program based on this idea

image (pixel/voxel) based

representation of the shape

Very Flexible & Simple

Generating holes inside is not a problem !



that is not only SHAPE but also TOPOLOGY
of a structure can be designed in this approach

OPTISHAPE

for

Shape Optimization

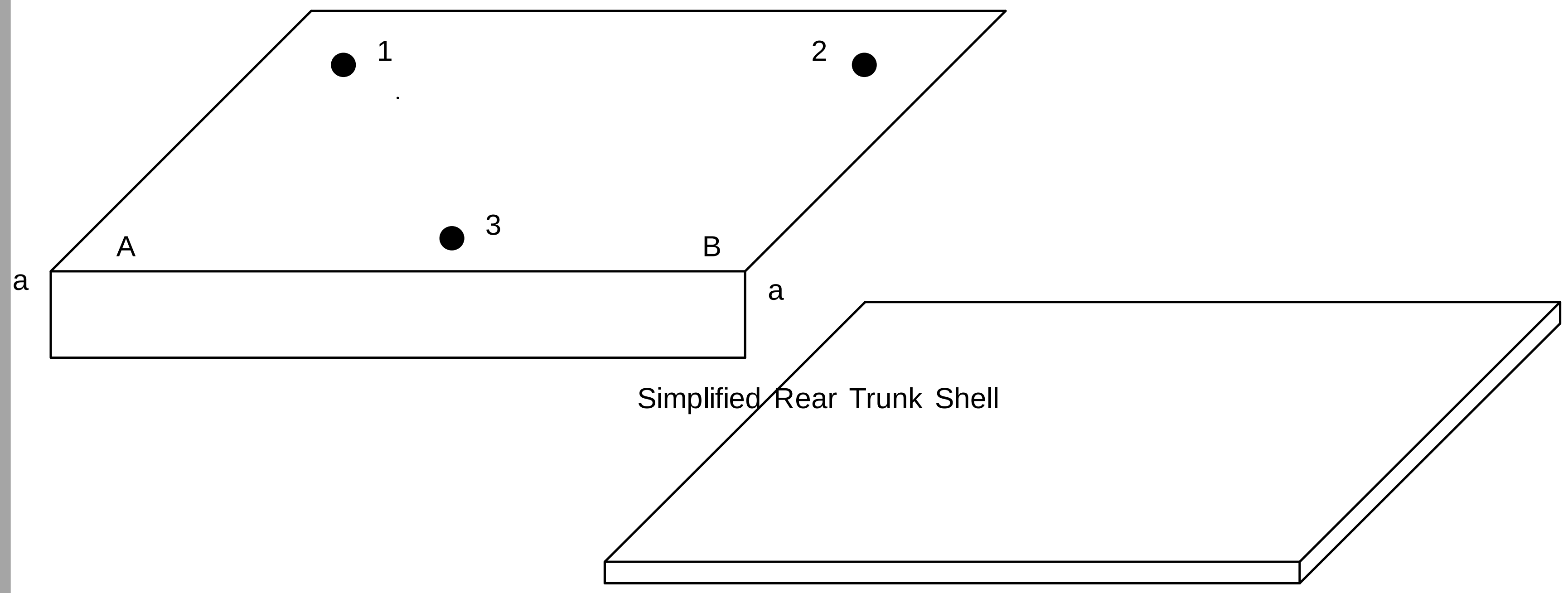
and

Topology Optimization

Exercise #12 : OPTISHAPE

- Three Point Supports

Load Cases
1) Uniform Pressure on the Upper Plate
2) Point Loads at A and B independently when support 3 fails
3) Distributed Edge Load on Line a-a when support 3 fails



Simplified Rear Trunk Shell

Three Dimensional Design Domain Under the Upper Plate (Discretized by 50x30x4 Mesh)