Midterm Examination 1997 Winter
MEAM 305 Introduction to Finite Element Methods

March 14, 1997

1. Answer to the following quetions :

a) In which yaer was the finite element method introduced ? Who are the authors of the first paper ?

b) Who is the person named the finite element method ? When ?

c) Whai is the isoparametric element ?

d) State the principle of minimum potential energy. Explain this principle using a spring model as shown in Fig. 1.

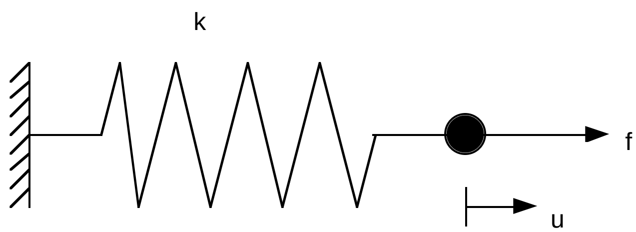

Figure 1. A Spring Model with A Single Degree of Freedom

Here k is the spring constant (stiffness), f is an applied load, and u is the amount of elongation or contraction.

e) Derive the stiffness of the bar element for axial loading and deformation. Here we assume that Young's modulus of the bar is E, cross sectional area is A, and the length of the bar is L, see Fig. 2.

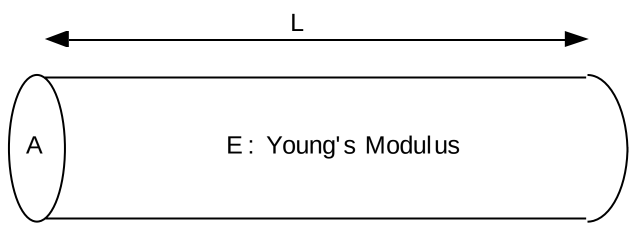

Figure 2. A Bar Element for Axial Loadings

2. Consider an 8 node hexagonal element shown in Fig. 3.

a) Define the shape functions $N_i(\xi, \eta, \zeta)$, $i = 1, 2, ..., 8$ in terms of the parametric coordinates ξ, η, and ζ.

b) Evaluate N_4 at the cetroid of the element, at the second node, and at the fourth node.

c) Verify the property $\sum_{i=1}^{8} N_i(\xi,\eta,\zeta) = 1$.

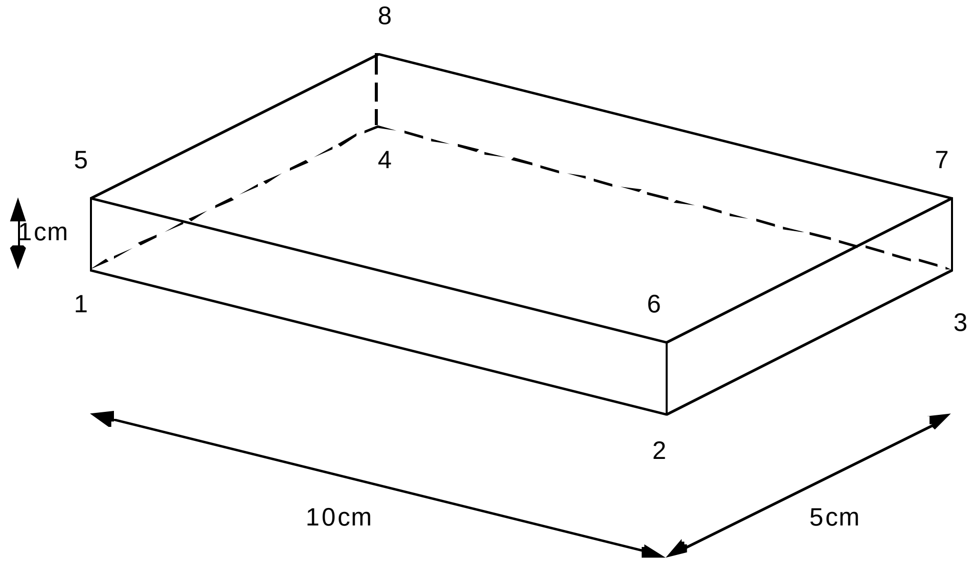

Figure 3. A 8 Node Hexagonal Element with Physical Dimensions

d) Sketch the local coordinate system (*x*,*y*,*z*). Where is the origin ?

e) Using the differential relation to an arbitrary function *g*

$$\begin{Bmatrix} \frac{\partial g}{\partial \xi} \\ \frac{\partial g}{\partial \eta} \\ \frac{\partial g}{\partial \zeta} \end{Bmatrix} = \begin{bmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} & \frac{\partial z}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} & \frac{\partial z}{\partial \eta} \\ \frac{\partial x}{\partial \zeta} & \frac{\partial y}{\partial \zeta} & \frac{\partial z}{\partial \zeta} \end{bmatrix} \begin{Bmatrix} \frac{\partial g}{\partial x} \\ \frac{\partial g}{\partial y} \\ \frac{\partial g}{\partial z} \end{Bmatrix}$$

compute

e1) the three displacement components $\{u_x \quad u_y \quad u_z\}$ in the local coordinate system

e2) the normal strain $\varepsilon_x = \frac{\partial u_x}{\partial x}$, and the shear strain $\gamma_{xy} = \frac{\partial u_x}{\partial y} + \frac{\partial u_y}{\partial x}$

when

$$\boldsymbol{d}^T = \{0 \quad 0 \quad 0 \quad 0.01 \quad 0 \quad 0 \quad 0.01 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0.01 \quad 0 \quad 0 \quad 0.01 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0\}$$

f) Find the values of ε_x at the centroid and at node 7.