Review Problems for Midterm Examination
MEAM 501, Fall 1998

1. Formulate the following boundary value problem

$$-\frac{d}{dx}\left(a(x)\frac{du}{dx}\right)+u(x)\frac{du}{dx}+k(x)u = f(x) \quad , \quad x\in(0,L)$$
$$u(0)=u_0$$

by the weighted residual method.

2. Formulate the following boundary value problem

$$-\frac{d}{dx}\left(a(x)\frac{du}{dx}\right)+k(x)u = f(x) \quad , \quad x\in(0,b)\cup(b,L)$$
$$a(b_-)\frac{du}{dx}(b_-)-a(b_+)\frac{du}{dx}(b_+)=P$$
$$u(0)=u_0$$

by the weighted residual method and by applying the integration by parts rule, where $g(b_-)=\lim_{\varepsilon\to 0} g(b-\varepsilon)$, $g(b_+)=\lim_{\varepsilon\to 0} g(b+\varepsilon)$, b in a point inside of the interval (0,*L*), and *P* is a given number. After obtaining the weighted residual formulation, approximate it by

$$u(x)=\sum_{j=1}^{n} u_j\phi_j(x) \quad , \quad w(x)=\sum_{i=1}^{n} w_i\psi_i(x)$$

where *w* is an arbitrary weighting function, and obtain a discrete problem

$$\mathbf{Ku}=\mathbf{f}$$

in terms of *a*, *k*, *f*, *P*, ϕ_j , ψ_i, and *L*. That is, find the i-j component of **K** and i component of **f**.

3. (Continuation of Problem 2) Assume $L = a(x) = k(x) = f(x) = 1,\ b = \frac{1}{2}$, $P = 1$ and $u_0 = 0$, and

$$\phi_1(x) = \psi_1(x) = 1 - 4\left(x - \frac{1}{2}\right)^2 \quad , \quad \phi_2(x) = \psi_2(x) = 2x\left(x - \frac{1}{2}\right)$$

(1) Obtain the 2-by-2 matrix **K** and 2 component vector **f**, where $n = m = 2$ are assumed too.

(2) Find the eigenvalues λ of the matrix **K** together with the eigenvectors $\boldsymbol{x}$.

(3) Solve the matrix equation $\mathbf{Ku} = \mathbf{f}$.

(4) Represent the solution **u** as a linear combination of the two eigenvectors of the 2-by-2 matrix **K**.

(5) Find which mode (i.e. eigenvector) is dominant.

4. Answer to the following questions:

(1) For a m-by-n matrix **A**, state the definition of the singular value decomposition of **A**.

(2) Using the result of the singular value decomposition, state what is the rank of **A**, what is the range of **A**, and what is the null space of **A** ?

(3) State your idea why we are interested in knowing the range of **A** and the null space of **A**.

(4) What is an orthogonal matrix **Q** ?

(5) When two vectors **u** and **v** are orthogonal ?

(6) Show that if a matrix **Q** defined by $\mathbf{Q} = [\mathbf{q}_1, \mathbf{q}_2, \ldots\ldots, \mathbf{q}_n]$ where $\mathbf{q}_i$ are orthonormal, **Q** is an orthogonal matrix.

(7) Describe the Gram-Schmidt orthonormalization process for n number of linearly independent vectors.

(8) What is the definition of linearly independent vectors $\mathbf{v}_i$, $i = 1, \ldots, n.$?

(9) What is the Householder transformation **P** defined by a vector **v** ? State the property of **P**.

(10) What is the QR decomposition of a square matrix **A** ?

(11) State QR algorithm to find the eigenvalues and eigenvectors of a symmetric square matrix **A**.