Midterm Examination : November 8, 1995

MEAM 501 Analytical Methods in Mechanics

1995 Fall

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1. (SVD : 35%) Let a matrix $\boldsymbol{A}$ be a m-by-n real rectangular matrix, and let $\boldsymbol{b}$ be a m-component real vector.

(1) State the singular value decomposition theorem.

(2) Obtain the pseudo-inverse (generalized inverse) $\boldsymbol{A}^{+}$ by using the singular value decomposition.

(3) Show that $\boldsymbol{x} = \boldsymbol{A}^{+} \boldsymbol{b}$ is a solution of the least squares problem of

$$\min_{\boldsymbol{x}} \frac{1}{2} \|\boldsymbol{b} - \boldsymbol{A}\boldsymbol{x}\|^2$$

where $\|\boldsymbol{g}\|$ is the natural norm of a vector $\boldsymbol{g}$ defined by $\|\boldsymbol{g}\| = \sqrt{\boldsymbol{g}^T \boldsymbol{g}}$.

2. (Householder : 35%) Answer to each question related to the Householder transformation.

(1) Define the Householder transformation $\boldsymbol{P}$.

(2) Show that it is orthogonal.

(3) Obtain its inverse.

(4) Suppose that we have a 4-by-4 matrix

$$\boldsymbol{A} = \begin{bmatrix} 1 & 5 & 9 & 13 \\ 2 & 6 & 10 & 14 \\ 3 & 7 & 11 & 15 \\ 4 & 8 & 12 & 16 \end{bmatrix}.$$

Using the Householder transformation, describe a method to form a new matrix that has the form

$$\overline{\boldsymbol{A}} = \begin{bmatrix} X & X & X & X \\ X & X & X & X \\ X & 0 & X & X \\ X & 0 & X & X \end{bmatrix},$$

that is, the (3,2) and (4,2) components are enforced to be zero. Here "X" means possibly non-zero.

(5) (Take Home Part) Check whether the algorithm stated in (4) is correct or not by using MATLAB.

3. (Eigenvalues : 30%) Let $\boldsymbol{A}$ be a n-by-n real square symmetric matrix.

(1) Show that its eigenvalues must be real.

(2) (Take Home Part) Show that all the eigenvectors are linearly independent if all the eigenvalues are distinctive, i.e., $\lambda_1 \neq \lambda_2 \neq ..... \neq \lambda_n$.

(3) (Take Home Part) Obtain the eigenvalues and eigenvectors of a matrix

$$\boldsymbol{A} = \begin{bmatrix} 5 & 2 & -1 & 0 & -1 \\ 2 & 4 & 3 & 1 & 0 \\ -1 & 3 & 5 & 2 & -1 \\ 0 & 1 & 2 & 4 & 3 \\ -1 & 0 & -1 & 3 & 5 \end{bmatrix} = \begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{a}_4 & \boldsymbol{a}_5 \end{bmatrix}$$

by applying the QR algorithm using MATLAB.

(4) (Take Home Part) Obtain the eigenvalues of the matrix A defined in (3) by solving the characteristic equation $\det(\boldsymbol{A} - \lambda \boldsymbol{I}) = 0$ by using MATLAB. Compare with the QR algorithm, and describe advantage or disadvantage of this approach.

(5) How to check whether the column vectors $\boldsymbol{a}_1$, $\boldsymbol{a}_3$, and $\boldsymbol{a}_5$ are linearly independent ?