Contracts, Biases and Consumption of Access Services
Online Appendix

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Appendix A: Proofs

In all the proofs for notational simplicity we assume V has continuous support.

Proof of Theorem 2: i)and ii) Suppose $t\bar{F}(q) < k$, then $J^d(k,t,q) = E(V|V > q)t\bar{F}(q)$ which is decreasing in q, so $q^* \leq q(k,t)$. Now consider the case $t\bar{F}(q) \geq k$. The first order derivative with respect to q is $\frac{\partial J^d(k,t,q)}{\partial q} = \frac{\partial}{\partial q}\left[k\frac{E(V-q)^+ - E(V-p)^+}{\bar{F}(q)} + kq\right] \geq 0$ which implies J^d is increasing in q. Therefore, $q^* = \min(q(k,t),p) = q(k,t,p)$.

iii) Note that $E(V|V > q) = q + \frac{E(V-q)^+}{\bar{F}(q)}$, then the optimal utility is given by $J^d(k,t) = J^d(k,t,q^*) = E(V|V > q^*)\min(t\bar{F}(q^*),k) + E(V-p)^+\frac{(t\bar{F}(q^*)-k)^+}{\bar{F}(q^*)} = [q^* + \frac{E(V-q^*)^+}{\bar{F}(q^*)}]\min(t\bar{F}(q^*),k) + E(V-p)^+\frac{(t\bar{F}(q^*)-k)^+}{\bar{F}(q^*)}$.

If $q^* = q(k,t)$ then $\bar{F}(q(k,t)) = k/t$ and $J^d(k,t,q(k,t)) = [q(k,t) + \frac{tE(V-q(k,t))^+}{k}]k + E(V-p)^+.0 = tE(V-q(k,t))^+ + kq(k,t)$. The derivative with respect to t is $E(V-q(k,t))^+ \geq 0$, and the derivative with respect to k is $q(k,t)$. On the other hand, if $q^* = p$ then $J^d(k,t) = [p + \frac{E(V-p)^+}{\bar{F}(p)}]\min(t\bar{F}(p),k) + E(V-p)^+\frac{(t\bar{F}(p)-k)^+}{\bar{F}(p)} = kp + tE(V-p)^+$. The derivative with respect to t is $E(V-p)^+$, the derivative with respect to k is p. We can summarize by writing $\frac{\partial J^d(k,t)}{\partial t} = E(V-q^*)^+$, $\frac{\partial J^d(k,t)}{\partial k} = q^*$, and $\frac{\partial J^d(k,t)}{\partial p} = -1(q(k,t) > p)pf(p)$.

The monotonicity of $J^d(k,t)$ in k and t follows from $\frac{\partial J^d(k,t)}{\partial k} = q^* \geq 0$ and $\frac{\partial J^d(k,t)}{\partial t} = E(V-q^*)^+ \geq 0$. Moreover, notice that $\frac{\partial^2 J^d(k,t)}{\partial k^2} = -\frac{1}{tf(q^*)}$, $\frac{\partial^2 J^d(k,t)}{\partial t^2} = -\frac{k}{t^2 h(q^*)}$, $\frac{\partial^2 J^d(k,t)}{\partial k \partial t} = \frac{1}{th(q^*)}$, so $\frac{\partial^2 J^d(k,t)}{\partial k^2}\frac{\partial^2 J^d(k,t)}{\partial t^2} - \frac{\partial^2 J^d(k,t)^2}{\partial k \partial t} = 0$ where $h(q)$ is the hazard rate at q. Therefore, $J^d(k,t)$ is jointly concave in (k,t) for fixed p.

iv) If $q(k,t) < p$ then $J^d(k,t)$ is independent of p. Otherwise it is given by $tE(V-p)^+$ and decreasing in p.

Proof of Theorem 3: Recall that $q_W^{DDT}(k,t,p) = q_W(k,t,p) = \min(q_W(k,t),p)$ where $q_W(k,t) = \min\{q \geq 0|P(W > q) \leq \frac{k}{t}\}$. If $V \succeq W$ then $\bar{F}_V(y) \geq \bar{F}_W(y)$ for all $y \geq 0$. Suppose that $\bar{F}_W(q_W(k,t)) = \frac{k}{t}$. By the definition of stochastic ordering $\bar{F}_V(q_W(k,t)) \geq \bar{F}_W(q_W(k,t)) = \frac{k}{t}$, therefore, $q_V(k,t)$ must be greater than or equal to $q_W(k,t)$ as $\bar{F}_V(q_V(k,t)) = \frac{k}{t}$.

We prove $\Delta J_V(k,t) \geq \Delta J_W(k,t)$ by induction on k and t. Notice that it is trivially true for $t=1$ for every $k>1$ as $\Delta J_X(k,1) = J_X(k,1) - J_X(k-1,1) = EX^+ - EX^+ = 0$, for $k=1$ $J_X(1,1) - J_X(0,1) = EX^+ - E[X-p]^+ = E[\min(X,p)]$, and $E[\min(V,p)] \geq E[\min(W,p)]$ if $V \succeq W$. Suppose that it is true for $k=1$ and $t-1$. Then,

$$\begin{aligned}
\Delta J_V(1,t) &= E\max\{V + J_V(0,t-1), J_V(1,t-1)\} + -J_V(0,t) \\
&= E\max\{V + J_V(0,t-1), J_V(1,t-1)\} + -J_V(0,t-1) - E[V-p]^+ \\
&= E\max\{V, \Delta J_V(1,t-1)\} - E[V-p]^+ \\
&\geq E\max\{V, \Delta J_W(1,t-1)\} - E[V-p]^+ \\
&= \Delta J_W(1,t-1) + E[V - \Delta J_W(1,t-1)]^+ - E[V-p]^+ = \Delta J_W(1,t-1) + \int_{\Delta J_W(1,t-1)}^{p} \bar{F}_V(y)dy \\
&\geq \Delta J_W(1,t-1) + \int_{\Delta J_W(1,t-1)}^{p} \bar{F}_W(y)dy = \Delta J_W(1,t)
\end{aligned}$$

the first inequality is by the induction hypothesis and the second inequality is by $V \succeq W$ and $\Delta J_X(1,t-1) \leq p$. Now, suppose that it is true for $1,...,k$ and $t-1$: $\Delta J_V(k,t-1) \geq \Delta J_W(k,t-1)$. We have

$$\begin{aligned}
\Delta J_V(k,t) &= E\max\{V + J_V(k-1,t-1), J_V(k,t-1)\} + -J_V(k-1,t) + J_V(k-1,t-1) - J_V(k-1,t-1) \\
&= E\max\{V, \Delta J_V(k,t-1)\} - J_V(k-1,t) + J_V(k-1,t-1) \\
&= \Delta J_V(k,t-1) + E[V - \Delta J_V(k,t-1)]^+ - E[V - \Delta J_V(k-1,t-1)]^+ \\
&\geq \Delta J_W(k,t-1) + E[V - \Delta J_W(k,t-1)]^+ - E[V - \Delta J_W(k-1,t-1)]^+ \\
&= \Delta J_W(k,t-1) + \int_{\Delta J_W(k,t-1)}^{\Delta J_W(k-1,t-1)} \bar{F}_V(y)dy \\
&\geq \Delta J_W(k,t-1) + \int_{\Delta J_W(k,t-1)}^{\Delta J_W(k-1,t-1)} \bar{F}_W(y)dy = \Delta J_W(k,t).
\end{aligned}$$

where the first inequality is by the induction hypothesis, the second by the assumption $\bar{F}_V(y) \geq \bar{F}_W(y)$ and $\Delta J_W(k,t-1) \leq \Delta J_W(k-1,t-1)$.

Proof of Theorem 4:

1. We assume that the decision makers have exponential utility function $U(y) = -e^{-\gamma y}$, $\gamma \geq 0$. A decision maker with γ_1 is more risk averse than a decision maker with γ_2 if $\gamma_1 > \gamma_2$. By Barz and Waldman (2007), we know that $J_\gamma(k,t) = E\max(e^{-\gamma V} J_\gamma(k-1,t-1), J_\gamma(k,t-1))$ for $k>0, t>0$ and $J_\gamma(0,t) = E\max(e^{-\gamma(V-p)} J_\gamma(0,t-1), 1)$, $t>0$ with $J(k,0) = -1$. It is more convenient to work with $G_\gamma(k,t) := -J_\gamma(k,t)$ which is the unique solution of $G_\gamma(k,t) = E\min(e^{-\gamma V} G_\gamma(k-1,t-1), G_\gamma(k,t-1))$ with initial value $G_\gamma(k,0) = 1$. With the same argument in Barz and Waldman, we can show that the optimal policy is a threshold policy: the request is accepted if $e^{-\gamma V} \leq \Delta_\gamma G(k,t) = \frac{G_\gamma(k,t)}{G_\gamma(k-1,t)}$ and rejected otherwise.

Now we will show that $\Delta_\gamma G(k,t)$ is decreasing in γ. As in Barz and Waldman (2007), we transform the problem to the following logarithmic form conditional on $V=v$, $\ln g_\gamma(k,t|v) = \min(-\gamma v + \ln G_\gamma(k-1,t-1), \ln G_\gamma(k,t-1))$ where $G_\gamma(k,t) = E[g_\gamma(k,t|V)]$. This problem has the same optimal thresholds with $G_\gamma(k,t)$. Let $\tilde{g}_\gamma(k,t|v) = -\ln g_\gamma(k,t|v)$ and $\tilde{G}_\gamma(k,t) = -\ln G_\gamma(k,t)$. Then we obtain $\tilde{g}_\gamma(k,t|v) = \max(\gamma v + \tilde{G}_\gamma(k-1,t-1), \tilde{G}_\gamma(k,t-1))$ which is similar to $J(k,t)$ and one accepts γv if $\gamma v \geq \Delta\tilde{G}_\gamma(k,t-1) = \tilde{G}_\gamma(k,t-1) - \tilde{G}_\gamma(k-1,t-1)$. By Theorem 3, we know that $\Delta\tilde{G}_{\gamma_1}(k,t) \geq \Delta\tilde{G}_{\gamma_2}(k,t)$ when $\gamma_1 \geq \gamma_2$ as one can define $V = \gamma_1 X$ and $W = \gamma_2 X$ for any random variable X. Therefore, $\Delta G_{\gamma_1}(k,t) \leq \Delta G_{\gamma_2}(k,t)$.

2. Consider a concave increasing utility function $U(y)$ for $y \geq 0$. Then the utility from the consumption of the service is given by $\min(t\bar{F}(q), k)E(U(V|V > q)) + (t - \frac{k}{\bar{F}(q)})^{+}E(U((V-p)^{+}))$. If $t\bar{F}(q) \leq k$, taking the derivative of the utility with respect to q we obtain $-tU(q)f(q)$ which is negative, therefore the optimal threshold must be less than or equal to $q(k,t)$. If $t\bar{F}(q) > k$, taking the derivative we obtain $k\frac{f(q)}{\bar{F}(q)^2}[-U(q)\bar{F}(q) + \int_q^\infty U(v)f(v)dv - \int_p^\infty U(v-p)f(v)dv]$ which can be negative or positive depending on the parameters. If it positive, then $q_{RA}(k,t) = q(k,t)$ otherwise it is less than or equal to $q(k,t)$.

References

Barz, C., and K. H. Waldmann, "Risk sensitive capacity control in revenue management", *Math. Meth. Oper. Res.* (2007) 65: 565-579.

Appendix B: Instructions

Each paragraph denotes all the instructions shown on a single screen during the experiment. Sample decision screens are also included. All instructions were also read aloud.

Experiment 1

Welcome and thank you for coming. At this point, please don't talk to any other participant, or look at their computer. If you have any questions, raise your hand and I will come help you individually. Today's session is a study of individual decision making. You will make several different choices over the course of the experiment that will affect your earnings. We will explain the details of those choices as we go along.

The first part of the experiment will involve the Cell Phone Task. For this task you will have a cell phone plan, which may include a monthly fee, some number of free calls, and the charge for answering additional calls. During the task you will receive 30 calls. The value of answering the call will be drawn randomly in each period. Call values range from \$0.15 to \$0.75. The same value distribution is used throughout the experiment, and the call value will be drawn independently in each period. In each period you choose whether to answer the call or not. If you answer the call, the value of the call will be added to your earnings for the task. You will also use one free call, or will be charged if you do not have any more free calls. If you do not answer the call your payoff will not change, nor will you use a free call. At the end of the task you will see your total payoff - including the monthly fee, the value of the answered calls, and any charges for answering calls. You will do the Cell Phone Task 4 times. One of the Cell Phone Tasks will be randomly selected for payment at the end of the experiment.

You can now draw sample outcomes from the distribution of call values. Each time you press the "Sample Again" button, you will see ten outcomes drawn randomly from the call value distribution. You can look at as many samples as you like. A table in the lower left will display all of the sample outcomes. Whenever you are ready, click the "Finish Sampling" button to continue the experiment.

[Call Task Decision Screen]

Please guess how many calls of value \$0.75 and \$0.15 will occur during this task. For each guess, if you are correct you will earn an additional \$0.50 in your payoff for this task. For each guess, if your guess is within 1 of the correct number, you will earn an additional \$0.25 in your payoff for this task.

You will now make a number of decisions between two lotteries. For each decision you can choose between Lottery A or Lottery B. One lottery decision will be randomly selected for payment at the end of the experiment. For that decision you will be paid the outcome for the lottery you choose.

[Lottery Task Decision Screen]

You will now play the Offer Game twice. In each game there are two players: Player A and Player B. Player A begins by deciding how to divide $5 between the two players. Player A may offer Player B any amount between $0 and $5. Player B may then accept or reject this offer. If Player B accepts than Player B gets the offered amount, and Player A gets the rest. If Player B rejects the offer, than both players get $0. You will make a decision both as Player A and Player B in each Offer Game. One of the two games will be randomly selected for payment, and you will be randomly selected to be Player A or Player B for that game. You will be randomly paired with another participant, and your payoffs will depend on each of your choices.

[Ultimatum Game Decision Screen]

Please answer the following questions. You will receive $0.25 for each question you get correct. (1) A bat and a ball cost $1.10 in total. The bat costs $1.00 more than the ball. How much does the ball cost? (answer in number of cents) (2) If it takes 5 machines 5 minutes to make 5 widgets, how long would it take 100 machines to make 100 widgets? (answer in number of minutes) (3) In a lake there is a patch of lily pads. Every day the patch doubles in size. If it takes 48 days for the patch to cover the whole lake, how long would it take for the patch to cover half of the lake? (answer in number of days)

Before we conclude, please answer the following hypothetical question. Assume that you have spent $600 on a ticket for a weekend ski trip to Wyoming. Several weeks later you buy a $300 ticket for a weekend ski trip in Colorado. You think you will enjoy the Colorado ski trip more than the Wyoming ski trip. As you are putting your just-purchased Colorado ski trip ticket in your wallet, you notice that the Wyoming ski trip and the Colorado ski trip are for the same weekend. It's too late to sell either ticket, and you cannot return either one. You must use one ticket and not the other. Which ski trip will you go on?

Experiment 2

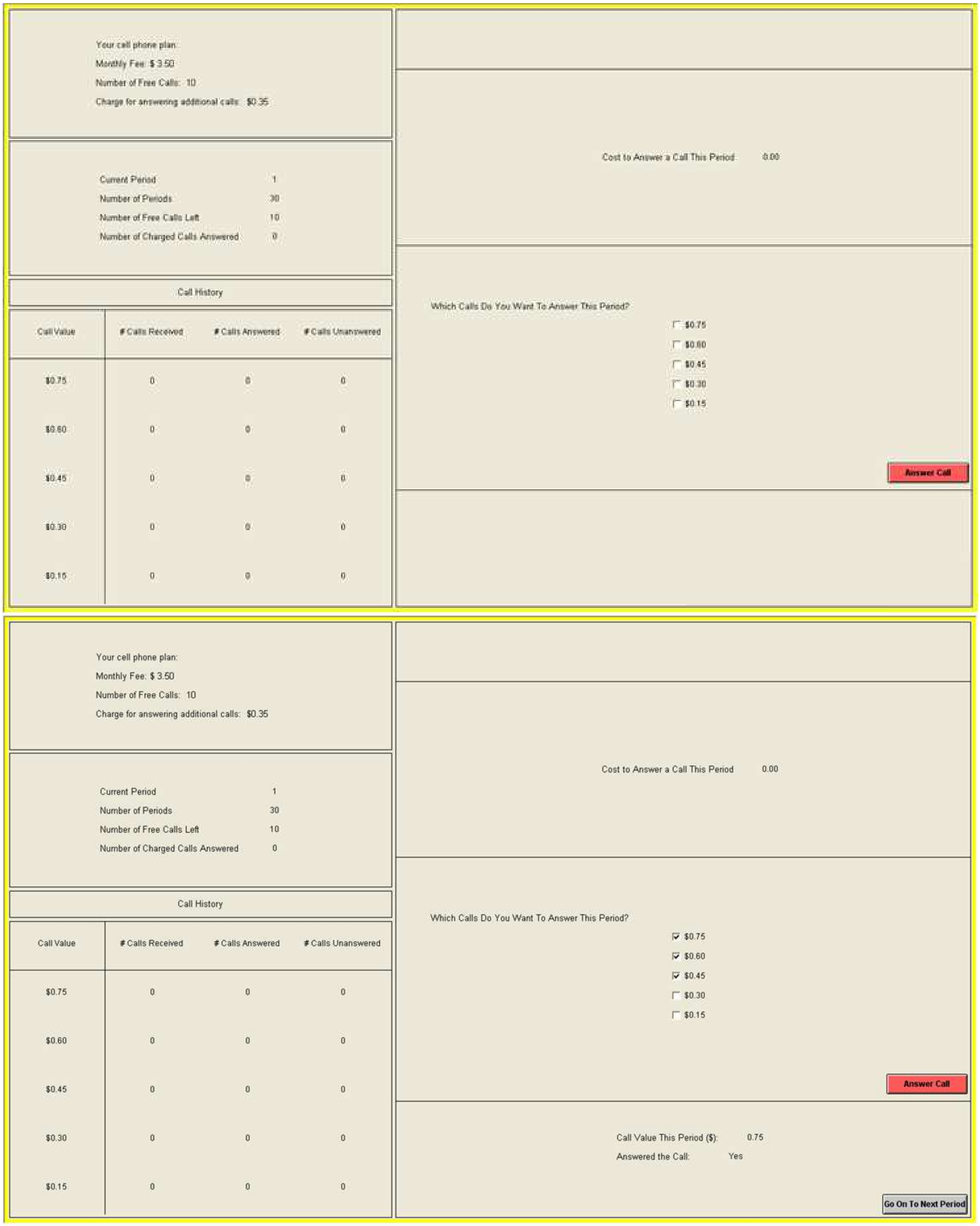
All instructions are the same except the third screen on sampling is replaced with the following instructions

Each phone call will be worth one of five values: The call will be worth $0.75 with 15% probability. The call will be worth $0.60 with 25% probability. The call will be worth $0.45 with 25% probability. The call will be worth $0.30 with 25% probability. The call will be worth $0.15 with 10% probability. Each call's value is drawn independently. To help visualize the value probabilities, imagine that there is a bucket full of 20 balls. Three balls will have $0.75 written on them, five balls have $0.60, five balls have $0.45, five balls have $0.30 and two balls have $0.15. Determining the value of a call would then be like picking one of the balls randomly, assigning that value to the call, and then putting the ball back.

Experiment 3

All instructions are the same except the regret aversion measures were omitted, and each task included the following instructions after the elicitation of the call frequency predictions (instructions for the 10 Calls contract were similar, with X ranging from $0 *to* $5*:*

You may now choose between the following two plans: (1) No Monthly Fee, No Free Calls, $0.35 to answer each call. (2) $X Monthly Fee, 20 Free Calls, $0.35 to answer each additional call. The monthly fee of $X will be set randomly between \$0 and \$10. To choose between the plans, you will enter below the largest monthly fee you would be willing to pay to have plan 2 instead of plan 1. The computer will then randomly pick X and compare it to your answer. If X is less than your answer then you will have plan 2 with a fee of X for this task. If X is more than your answer then you will have plan 1 for this task. Note that your answer will only determine which plan you will have, not what the monthly fee for plan 2 is. Therefore, it is in your best interest to say what your true value is for a plan with 20 free calls. What is the largest monthly fee you would be willing to pay for plan 2?



Call Task Decision Screen

For each Situation, please select either Lottery A, or Lottery B.

If one of these Decisions are selected for payment, you will only be informed of the outcome of the lottery you chose.

You will not be able to compare the outcome of the lottery you chose to the outcome of the other lottery.

Decision	Lottery A	Lottery B	I choose
1	10% chance of $2.00 and 90% chance of $1.60	10% chance of $3.85 and 90% chance of $0.10	Lottery A Lottery B
2	20% chance of $2.00 and 80% chance of $1.60	20% chance of $3.85 and 80% chance of $0.10	Lottery A Lottery B
3	30% chance of $2.00 and 70% chance of $1.60	30% chance of $3.85 and 70% chance of $0.10	Lottery A Lottery B
4	40% chance of $2.00 and 60% chance of $1.60	40% chance of $3.85 and 60% chance of $0.10	Lottery A Lottery B
5	50% chance of $2.00 and 50% chance of $1.60	50% chance of $3.85 and 50% chance of $0.10	Lottery A Lottery B
6	60% chance of $2.00 and 40% chance of $1.60	60% chance of $3.85 and 40% chance of $0.10	Lottery A Lottery B
7	70% chance of $2.00 and 30% chance of $1.60	70% chance of $3.85 and 30% chance of $0.10	Lottery A Lottery B
8	80% chance of $2.00 and 20% chance of $1.60	80% chance of $3.85 and 20% chance of $0.10	Lottery A Lottery B
9	90% chance of $2.00 and 10% chance of $1.60	90% chance of $3.85 and 10% chance of $0.10	Lottery A Lottery B
10	100% chance of $2.00 and 0% chance of $1.60	100% chance of $2.00 and 0% chance of $1.60	Lottery A Lottery B

Select either Lottery A or Lottery B for each Decision before clicking OK.

OK

Decision 11

If this Decision is selected for payment, you will be informed of the outcome of Lottery A, as well as the lottery you chose (if it is different).

You will only be able to compare the outcome of the lottery you chose to the outcome of the other lottery if you choose Lottery B.

Lottery A: 55% chance of $2.00 and 45% chance of $1.60

Lottery B: 55% chance of $3.85 and 45% chance of $0.10

Lottery A Lottery B

Decision 12

If this Decision is selected for payment, you will be informed of the outcome of Lottery B, as well as the lottery you chose (if it is different).

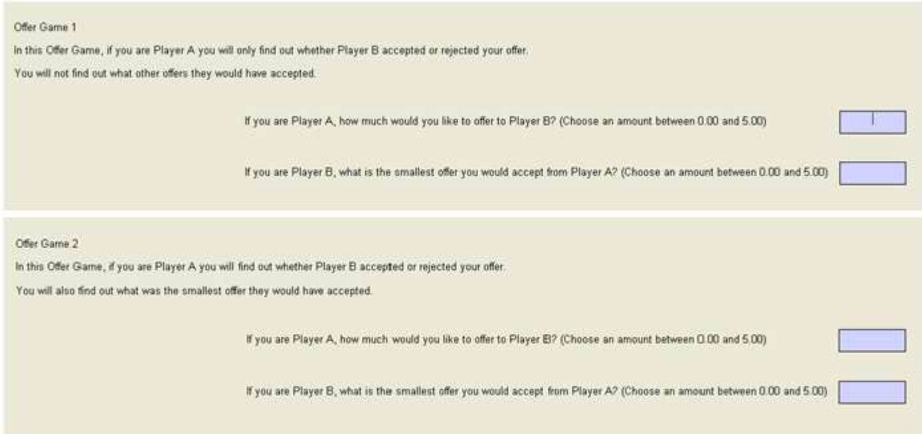
You will only be able to compare the outcome of the lottery you chose to the outcome of the other lottery if you choose Lottery A.

Lottery A: 45% chance of $2.00 and 55% chance of $1.60

Lottery B: 45% chance of $3.85 and 55% chance of $0.10

Lottery A Lottery B

Lottery Task Decision Screen



Offer Game 1

In this Offer Game, if you are Player A you will only find out whether Player B accepted or rejected your offer.

You will not find out what other offers they would have accepted.

If you are Player A, how much would you like to offer to Player B? (Choose an amount between 0.00 and 5.00)

If you are Player B, what is the smallest offer you would accept from Player A? (Choose an amount between 0.00 and 5.00)

Offer Game 2

In this Offer Game, if you are Player A you will find out whether Player B accepted or rejected your offer.

You will also find out what was the smallest offer they would have accepted.

If you are Player A, how much would you like to offer to Player B? (Choose an amount between 0.00 and 5.00)

If you are Player B, what is the smallest offer you would accept from Player A? (Choose an amount between 0.00 and 5.00)

Ultimatum Game Decision Screen