Physics 340 **Midterm # 2 - Solutions** **16-October-00**

Name (please print):____

Student ID #: – –

This is a closed book exam. You may use a calculator.
A sheet with useful constants and equations is appended at the end.

There are 10 short problems in this exam. Each is directed at a single concept which we have covered. This is a 50 minute exam, so you have 5 minutes per problem. If you encounter one which you don't know how to approach, skip over it and make sure you have time to do all those which you know how to do.

Part I:
Questions 1 - 8 are multiple choice with no partial credit. Select the letter corresponding to the one best answer and write it under the question number in the answer table at the bottom of this page. Check the answer table carefully, as only it will be examined.

Part II:
For questions 9 and 10, partial credit will be awarded as appropriate. Show all the work needed to get your answer. Use blank areas (including backs of pages) for calculations.

	Points	**Score**
Part I	72	
Part II	28	

Total: ________________

ANSWER TABLE for Part I: Below each question number, insert the letter corresponding to the one best answer.

Question	1	2	3	4	5	6	7	8
Your answer	B	A	D	C	D	D	B	A

Part I (72 pts total) NO PARTIAL CREDIT - TRANSFER YOUR ANSWERS TO THE ANSWER TABLE

1. (9 pts) A stretched string is observed to vibrate in its fundamental mode when it is held fixed at two supports 60cm apart. If the mass of the string is 30g and the tension on the string is 70N, what is the frequency of oscillation?

(A) 62.4 Hz **(B) 31.2 Hz** (C) 6.24 Hz (D) 3.12 Hz (E) 0.624 Hz

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{70\ N}{(0.03\ kg\ /\ 0.6\ m}} = 37.4\ m/s$$

$$d = \lambda/2 = 0.6\ m \quad \Rightarrow \quad \lambda = 1.2\ m$$

$$f = \frac{v}{\lambda} = 31.2\ Hz$$

2. (9 pts) A spring force $F = -k\ x$ and a damping force $F_d = -2\delta\ m\ v = -2\delta\ m\ dx/dt$ act on a particle of mass m=0.4kg. The spring constant k=5.0N/m and δ=0.125/s. Over what time interval does the amplitude of motion of the particle decrease by a factor of ten?

(A) 18.4 s (B) 1.25 s (C) 12.5 s (D) 184 s (E) 8.0 s

$$x_2 = \frac{x_1}{10} \quad \text{so} \quad e^{-\delta t} = 0.1 \quad \text{or} \quad -\delta t = \ln 0.1 \quad \text{and} \quad t = \frac{\ln 10}{\delta} = 18.4s$$

3. (9 pts) What is the ratio of intensities that differ by 25 decibels?

(A) 25 (B) 625 (C) 5 **(D) 316** (E) 2.5

$$25\ dB = (10\ dB)\ \log\frac{I_2}{I_1} \quad \text{or} \quad \log\frac{I_2}{I_1} = 2.5 \quad \text{or} \quad \frac{I_2}{I_1} = 316$$

4. (9 pts) Consider the following two waves:

$$\begin{aligned} y_1 &= y_{m1}\ \sin(kx - \omega t) \\ y_2 &= y_{m2}\ \sin(kx - \omega t + \phi) \end{aligned}$$

where y_{m1}=3.0cm, y_{m2}=4.0cm, and ϕ=$\pi/2$ rad.

What is the amplitude of the resultant wave if these two waves are added together?

(A) 8.5 cm (B) 0 cm **(C) 5 cm** (D) 4.2 cm (E) 7 cm

$\pi/2\ rad = 90^\circ$ is a right angle

$\Rightarrow$ use Pythagorean theorem: $y = \sqrt{y_{m1}^2 + y_{m2}^2} = \sqrt{9 + 16\ cm^2} = 5\ cm$

5. (9 pts) Waves on a string with length L, mass M, under tension τ, deliver 40 Watts of power to a receiver at the end of the string. What will be the power received if the string mass is reduced by a factor of 2, leaving all other parameters (including L, τ, and ω) the same? (You may neglect any impedance mismatch between string and receiver)

(A) 40.0 W (B) 56.6 W (C) 20.0 W **(D) 28.3 W** (E) 10.0 W

$$P \propto \mu v \omega^2 A^2 \quad \text{and} \quad v = \sqrt{\frac{\tau}{\mu}}$$

$$\text{so} \quad P \propto \mu \sqrt{\frac{\tau}{\mu}}\, \omega^2 A^2 \quad \text{or} \quad P \propto \sqrt{\mu}\sqrt{\tau}\omega^2 A^2$$

$$\text{so} \quad \frac{P_2}{P_1} = \sqrt{\frac{\mu_1/2}{\mu_1}} \quad \Rightarrow P_2 = \frac{40\ W}{\sqrt{2}} = 28.3\ W$$

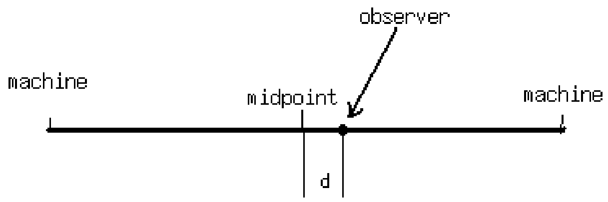
6. (9 pts) Two identical guitar strings of equal length exhibit standing waves. String #1 oscillates in its fundamental (n=1) mode, while string #2 oscillates in the second harmonic (n=2) mode. If both strings oscillate with the same frequency, what is the ratio of their tensions?

(A) 1 (B) 2 (C) 1.41 **(D) 4** (E) 1/2

$$f_1 = \frac{v_1}{2L} = f_2 = \frac{2\ v_2}{2L}$$

$$\frac{f_1}{f_2} = \frac{v_1}{2\ v_2} = \sqrt{\frac{\tau_1}{4\ \tau_2}} \quad \text{or} \quad \frac{\tau_1}{\tau_2} = 4$$

7. (9 pts) Two big identical industrial machines separated by 120m emit intense sound waves at a frequency of 20Hz. If the two machines are synchronized in their operation, how far from the midpoint on the line between the two machines should a worker stand if the worker is to be exposed to a minimum noise level?



(A) 2.5 cm **(B) 4.3 m** (C) 2.5 m (D) 8.6 m (E) 8.6 cm

$$\lambda = \frac{v}{f} = \frac{343\ m/s}{20\ Hz} = 17.15\ m$$

$$L_{left} = L + d \quad \text{and} \quad L_{right} = L - d \quad \Rightarrow \Delta L = 2\ d = \frac{1}{2}\ \lambda$$

$$\text{or}\ \ d = \frac{\lambda}{4} = \frac{17.15\ m}{4} = 4.29\ m$$

8. (9 pts) A police car traveling at 160 km/h sounds its sirens while catching up with a speeding motorist. The police siren, when standing still, emits a sound with frequency f = 500Hz, The speeding motorist, having a good musical ear, estimates the frequency to be 520Hz. Assuming that this is an accurate estimate, how fast is the speeding motorist traveling?

(A) 117 km/h (B) 154 km/h (C) 166 km/h (D) 201 km/h (E) 216 km/h

$$\frac{f'}{f} = \frac{v - v_D}{v - v_S} = \frac{v_D - v}{v_S - v}$$

$$\text{or}\ \ v_D = \frac{f'}{f}\,(v_S - v) + v = \frac{520\ Hz}{500\ Hz}\,(160\ km/h - 1234\ km/h) + 1234\ km/h = 117\ km/h$$

Part II For questions 9 and 10, partial credit will be awarded if appropriate. Show all the work needed to get your answer.

9. [14 pts total]
The oscillations of a block, sliding on a frictionless surface, are described by

$$\theta(t) = \theta_m \cos(\omega t + \phi).$$

At $t = 0$, $\theta = 0.1$ rad, $\mathrm{d}\theta/\mathrm{dt} = -0.4$ rad/s, and $\mathrm{d}^2\theta/\mathrm{dt}^2 = -0.533$ rad/s^2

$$\theta(0) = \theta_m \ \cos\phi$$
$$\dot{\theta}(0) = -\omega\ \theta_m \ \sin\phi$$
$$\ddot{\theta}(0) = -\omega^2\ \theta_m \ \cos\phi$$

(a) (5 pts) What is the angular frequency of oscillations?

$$\omega = \sqrt{-\frac{\ddot{\theta}(0)}{\theta(0)}} = \sqrt{-\frac{-0.533}{0.1}} = 2.309\ rad/s$$

Answer: $\omega =$ 2.309 rad/sec

(b) (5 pts) What is the amplitude of oscillations?

$$\theta_m = \frac{\theta(0)}{\cos\phi} = 0.2\ rad$$

Answer: $\theta_m =$ 0.2 rad

(c) (4 pts) What is the phase angle?

$$\frac{\dot{\theta}(0)}{\theta(0)} = -\ \omega\ \tan\phi \qquad \text{or} \qquad \tan\phi = \frac{-0.4}{0.1}/2.309 = 1.733 \qquad \text{or} \qquad \phi = \frac{\pi}{3}\ rad$$

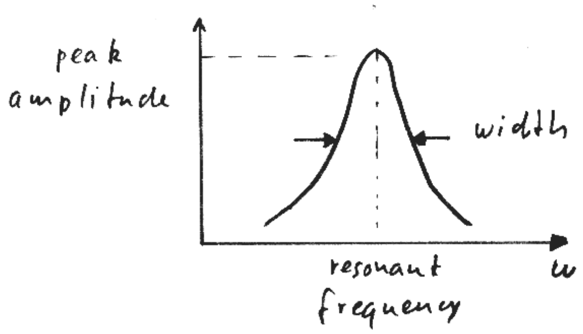
Answer: $\phi =$ $\frac{\pi}{3}$ rad

similar to Problem 16-2

10. [14 pts total]
A damped, driven, harmonic oscillator satisfies the following equation of motion:

$$m\,\ddot{x} + \delta\,\dot{x} + k\,x = F_0\cos(\omega t)$$

If the damping constant (δ) is <u>increased</u> while the parameters m, k, and F are held constant, then:



(a) (5 pts) The resonance frequency:

decreases
remains constant
increases

Answer:	decreases

(b) (5 pts) The resonance (peak) amplitude:

decreases
remains constant
increases

Answer:	decreases

(c) (4 pts) The width of the amplitude response curve (quality) as a function of ω:

decreases
remains constant
increases

Answer:	increases

Useful constants and equations

speed of sound in air: $v_s = 343$ m/s $= 1234$ km/h $= 767$ mph

Oscillations:

Simple Harmonic Motion:: $x(t) = x_m \cos(\omega t + \phi)$

$\omega = \frac{2\pi}{T} = 2\pi f = \sqrt{\frac{k}{m}}$

Pendulums: $T = 2\pi \sqrt{I/\kappa}$ torsion pendulum

$T = 2\pi \sqrt{L/g}$ simple pendulum

$T = 2\pi \sqrt{I/mgh}$ physical pendulum

Damped Harmonic Motion: $x(t) = x_m e^{-\delta t} \cos(\omega t + \phi)$

$\omega^2 = \omega_0^2 - \delta^2$ with $\omega_0^2 = \frac{k}{m}$

Forced Oscillations: $x(t) = A(\omega) \cos(\omega t + \phi)$

with $A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4\delta^2\omega^2}}$

$\omega_{res} = \sqrt{\omega_0^2 - \delta^2}$

Waves

Sinusoidal Wave: $y(x,t) = y_m \sin(kx - \omega t)$ moving in +x-direction

Wave Speed: $v = \frac{\omega}{k} = \frac{\lambda}{T} = \lambda f$

Stretched string: $v = \sqrt{\frac{\tau}{\mu}}$ wave speed

$\overline{P} = \frac{1}{2}\mu v \omega^2 y_m^2$ average power transmitted

$f = \frac{v}{\lambda} = n \frac{v}{2L}$ resonant frequency for $n = 1, 2, \ldots$

Interference: $y(x,t) = (2y_m \cos\frac{\phi}{2})\,(\sin(kx - \omega t + \frac{\phi}{2}))$

Standing Waves: $y(x,t) = [2y_m \sin kx] \cos \omega t$

Sound

Sound Waves: $v = \sqrt{\frac{B}{\rho}}$

$\Delta p = \Delta p_m \sin(kx - \omega t)$

$\Delta p_m = (v\rho\omega) s_m$ where s_m is maximum displacement

Interference: $\phi = \frac{\Delta L}{\lambda} 2\pi$ $\Delta L = m\lambda$ constructive

$\Delta L = (m + \frac{1}{2})\lambda$ destructive

Sound Intensity: $I = \frac{P}{A} = \frac{1}{2}\rho v \omega^2 s_m^2$

Sound Level: $\beta = (10 \text{ dB}) \log \frac{I}{I_0}$ where $I_0 = 10^{-12}$ W/m^2

Beats: $f_{beat} = f_1 - f_2$

Doppler Effect: $f' = f \frac{v \pm v_D}{v \mp v_S}$ for sound

$f' = f\,(1 \pm u/c)$ for light