**Physics 340** **Midterm # 3 - Solutions** **15-November-00**

Name (please print):________________________________________

Student ID #: – –

This is a closed book exam. You may use a calculator.
A sheet with useful constants and equations is appended at the end.

There are 10 short problems in this exam. Each is directed at a single concept which we have covered. This is a 50 minute exam, so you have 5 minutes per problem. If you encounter one which you don't know how to approach, skip over it and make sure you have time to do all those which you know how to do.

Part I:
Questions 1 - 8 are multiple choice with no partial credit. Select the letter corresponding to the one best answer and write it under the question number in the answer table at the bottom of this page. Check the answer table carefully, as only it will be examined.

Part II:
For questions 9 and 10, partial credit will be awarded as appropriate. Show all the work needed to get your answer. Use blank areas (including backs of pages) for calculations.

| | **Points** | **Score** |
|---|---|---|
| **Part I** | **72** | |
| **Part II** | **28** | |

**Total:** ____________________

**ANSWER TABLE for Part I:** Below each question number, insert the letter corresponding to the one best answer.

| **Question** | **1** | **2** | **3** | **4** | **5** | **6** | **7** | **8** |
|---|---|---|---|---|---|---|---|---|
| | | | | | | | | |
| **Your answer** | **C** | **D** | **B** | **E** | **C** | **D** | **A** | **C** |

Part I (72 pts total) NO PARTIAL CREDIT – TRANSFER YOUR ANSWERS TO THE ANSWER TABLE

1. (9 pts) Light with intensity $I_0$ passes through a vertical polarizing plate. It then passes through a polarizing plate which makes an angle of 35° to the vertical. Finally it passes through a second vertically polarizing plate. What is the intensity of the transmitted light in terms of $I_0$ ?

(A) 0.05 $I_0$ (B) 0.11 $I_0$ **(C) 0.23 $I_0$** (D) 0.25 $I_0$ (E) 0.5 $I_0$

(1) $I_1 = 0.5\ I_0$

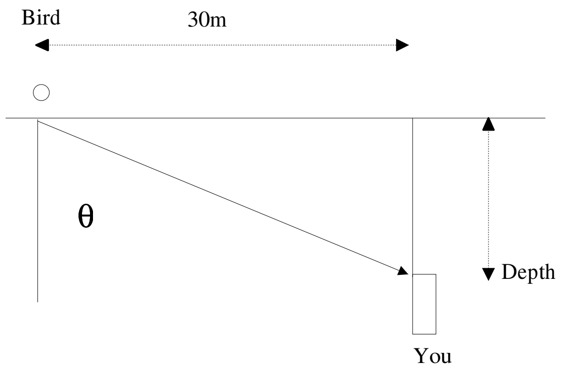
(2) $I_2 = I_1\ \cos^2 35°$

(3) $I_3 = I_2\ \cos^2 35°$

so $I_3 = 0.5\ I_0\ \cos^4 35° = 0.225\ I_0$

2. (9 pts) If you go swim underwater and look at the surface, you will find that you can only see through the water into the air for some range of angles from 0° to some maximum angle $\theta_{max}$. If you wish to be able to see a bird which is about to land on the water 30m from you, how deep must you be?

(A) 22.6 m (B) 30.0 m (C) 34.9 m **(D) 26.3 m** (E) 0.04 m



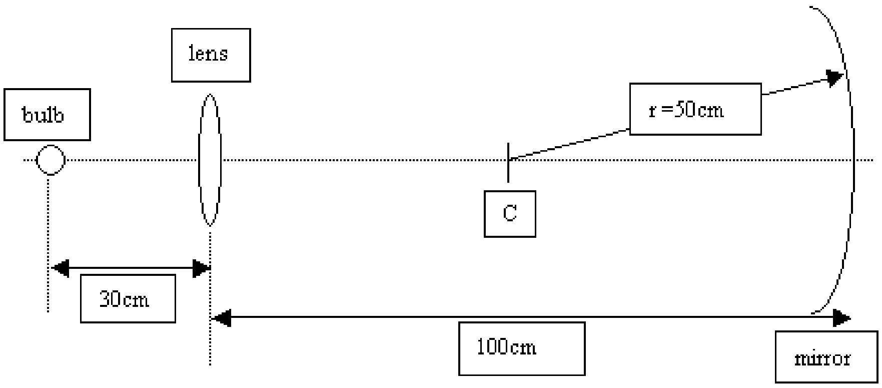

Total internal reflection in water

$$\sin\theta_{max} = \frac{1}{1.33} \quad \text{or} \quad \theta = 48.8°$$

$$D \cdot \tan\theta = 30\ m \quad \text{or} \quad D = \frac{30m}{\tan 48.8°} = 26.3\ m$$

3. (9 pts) A thin converging lens with focal length $f = 30$cm and a spherical mirror with radius $r = 50$cm are separated by 100cm as shown in the sketch below. If a small light bulb is placed 30cm to the left of the lens, how far from the mirror surface will the image be located?

(A) 15 cm **(B) 25 cm** (C) 50 cm (D) 100 cm (E) $\infty$

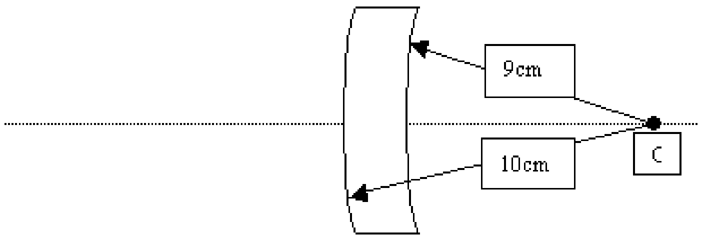



rays fall parallel on concave mirror

$$\rightarrow \quad i = f = \frac{r}{2} = 25\ cm$$

4. (9 pts) A thin glass lens (n=1.50) has two sperical surfaces with radii of 9.0cm and 10.0cm as shown in the sketch below. What is the focal length of this lens (in air)? (Give sign and magnitude)

(A) +0.4 cm (B) −45 cm (C) +45 cm (D) +180 cm **(E) −180 cm**




$$\frac{1}{f} = (n-1)\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= (1.5 - 1.0)\left(\frac{1}{+10cm} - \frac{1}{+9cm}\right)$$ (both surfaces are convex to rays from left)

or $\quad f = -180\ cm$

5. (9 pts) Light with a wavelength $\lambda = 600\text{nm}$ in glass (n=1.33) passes into air. What is the wavelength of the light in air (n=1.0)?

(A) 605 nm (B) 451 nm **(C) 798 nm** (D) 692 nm (E) 1061 nm

$$f_{air} = \frac{v_{air}}{\lambda_{air}} = f_{glass} = \frac{v_{glass}}{\lambda_{glass}}$$

$$\text{so} \quad \lambda_{air} = \lambda_{glass}\left(\frac{v_{air}}{v_{glass}}\right)\left(\frac{c}{c}\right) = \lambda_{glass}\left(\frac{n_{glass}}{n_{air}}\right) = 600\ nm\ \left(\frac{1.33}{1.0}\right) = 798\ nm$$

6. (9 pts) A thin film (n=1.4) has air on one side and glass (n=1.5) on the other. It suppresses reflection of light with wavelength $\lambda = 500\text{nm}$. How thick is the film?

(A) 175 nm (B) 350 nm (C) 700 nm **(D) 89 nm** (E) 178 nm

phase changes due to reflection on the front and back of the thin film → no contribution from reflection

$$\text{minima:} \quad 2L = \left(m + \frac{1}{2}\right)\lambda_{film} = \left(m + \frac{1}{2}\right)\frac{\lambda_{air}}{n_{film}} = \left(m + \frac{1}{2}\right)\frac{500nm}{1.4} = 179\ nm$$

$$\text{so} \quad L = 89\ nm$$

7. (9 pts) What is the maximum distance (in km) at which the Hubble Space Telescope (diameter = 2.4m) can resolve a pair of car headlamps that are separated by 1.7m and that generate light with wavelengths around 550nm?

**(A) 6100 km** (B) 3900 km (C) 2300 km (D) 3100 km (E) 7400 km

$$\sin\theta_R = \theta_R = 1.22\,\frac{\lambda}{d} = 1.22\,\frac{550\times 10^{-9}m}{2.4m} = 2.8\times 10^{-7}rad$$

$$\tan\theta_R = \theta_R = \frac{1.7m}{\text{distance}}$$

or $$\text{distance} = \frac{1.7m}{\theta_R} = 6080\ km$$

8. (9 pts) Monochromatic light is normally incident on a diffraction grating with 50,000 lines in 75mm. The line is imaged at 17.5° in the first order (m=1) peak. What is the wavelength of the incident light?

(A) 300 nm (B) 200 nm **(C) 450 nm** (D) 500 nm (E) 550 nm

spacing: $$d = \frac{0.075m}{50,000} = 1.5\times 10^{-6}m$$

maxima for m=1: $d\ \sin\theta = \lambda$

or $\lambda = 1.5\times 10^{-6}m\cdot \sin 17.5^\circ = 450\ nm$

Part II For questions 9 and 10, partial credit will be awarded if appropriate. Show all the work needed to get your answer.

9. [16 pts total]
An electromagnetic plane wave propagating in free space has an electric field given by:

$$\vec{E} = E_0 \cos(kz - \omega t)\hat{\imath}$$

where

$E_0 = 100 V/m$,

$\omega = 6 \times 10^{17} s^{-1}$,

$\hat{\imath}$ = unit vector along +x direction, (we assume a right-handed Cartesian coordinate system)

a) (4 pts) What is the direction of propagation of the wave ?

$\vec{k}$ is propagation vector

| Answer = | **z** | axis |
|---|---|---|

b) (4 pts) What is the amplitude of the magnetic field ?

$$B_0 = \frac{E_0}{c} = \frac{100 \ V/m}{3 \times 10^8 \ m/s} = 3.3 \times 10^{-7} \ T$$

| Answer = | $\mathbf{3.3 \times 10^{-7}}$ | tesla |
|---|---|---|

c) (4 pts) What is the value of the wave number $k$ ?

$$k = \frac{\omega}{c} = \frac{6 \times 10^{17} \ s^{-1}}{3 \times 10^8 \ m/s} = 2 \times 10^9 \ m^{-1}$$

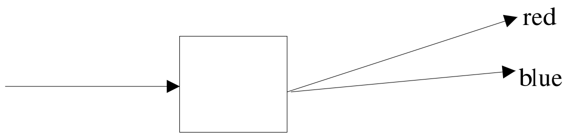
| Answer = | $\mathbf{2 \times 10^9}$ | /meter |
|---|---|---|

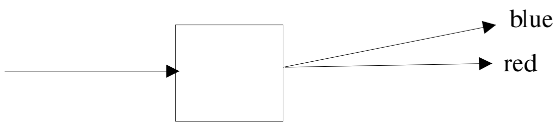
d) (4 pts) If the wave strikes a perfectly reflecting surface at normal incidence, what pressure does it exert on this surface?

$$P_r = \frac{2\ I}{c} = \frac{E_0^2}{c^2\ \mu_0} = 8.0 \times 10^{-8} \ N/m^2$$

| Answer = | $\mathbf{8.0 \times 10^{-8}}$ | newton/m$^2$ |
|---|---|---|

10. [12 pts total]
Two unknown optical devices separate light into its constituent colors. The first bends the redder light more than the bluer light, the second bends the bluer light more than the redder light.




a) (6 pts) Describe an optical device which will bend the redder light more than the bluer light.

**This optical device is consistent with a diffraction grating, where the deflection angle depends on the wavelength directly:**

$$\mathbf{d}\ \sin\theta = \mathbf{m}\ \lambda$$

b) (6 pts) Describe an optical device which will bend the bluer light more than the redder light.

**This optical device is consistent with either a prism or a diverging lens; something that disperses the light by refraction. The dispersion of most materials is such that blue light is refracted by more then red light, which we see here.**

## Useful constants and equations

Index of refraction: $n_{air} = 1.00$ $\quad$ $n_{water} = 1.33$ $\quad$ $n_{glass} = 1.50$

$\epsilon_0 = 8.85 \cdot 10^{-12} F/m$ $\quad$ $\mu_0 = 4\pi \cdot 10^{-7} H/m$

### Waves

Wave Equations (Vacuum) $\quad \dfrac{\partial^2 E}{\partial t^2} = c^2 \dfrac{\partial^2 E}{\partial x^2}$ $\quad$ and $\quad \dfrac{\partial^2 B}{\partial t^2} = c^2 \dfrac{\partial^2 B}{\partial x^2}$

Solution e.g. $E = E_0 \sin(kx \pm \omega t)$ $\quad$ with $\quad c = \frac{1}{\sqrt{\epsilon_0 \mu_0}}$ $\quad k = \frac{2\pi}{\lambda}$ $\quad \omega = 2\pi f = c\,k$

$E_0 = c\,B_0$ $\quad c = f\lambda = 3.0 \cdot 10^8\ m/s$

For e-m wave in medium: $v = \frac{c}{n}$, $\quad$ where $n = \sqrt{\frac{\epsilon\,\mu}{\epsilon_0\,\mu_0}}$

Snell's Law: $\quad n_1 \sin\theta_1 = n_2 \sin\theta_2$ $\quad$ Brewster angle: $\quad \theta_b = \tan^{-1} \frac{n_2}{n_1}$

Pointing vector: $\quad \vec{S} = \frac{1}{\mu_0}\,\vec{E} \times \vec{B}$ $\quad$ Intensity: $\quad I = \frac{1}{c\,\mu_0}\,\frac{E_0^2}{2} = \frac{1}{c\,\mu_0}\,E_{rms}^2$

Radiation Pressure: $\quad P_r = \frac{I}{c}$ (total absorption) $\quad P_r = \frac{2I}{c}$ (total reflection)

### Mirrors and Lenses

Spherical mirrors and lenses: $\quad \dfrac{1}{d_o} + \dfrac{1}{d_i} = \dfrac{1}{f}$ $\quad$ lateral magnification: $\quad m = -\dfrac{d_i}{d_o}$

For mirrors: $\quad f = \dfrac{r}{2} \begin{Bmatrix} \text{concave} = + \\ \text{convex} = - \end{Bmatrix}$

For thin lenses in air: $\quad \dfrac{1}{f} = (n-1)\left[\dfrac{1}{r_1} - \dfrac{1}{r_2}\right]$ $\quad$ where $\quad \begin{Bmatrix} \text{convex} = + \\ \text{concave} = - \end{Bmatrix}$

Magnifier: $\quad m_\theta = \dfrac{25\ cm}{f}$ $\quad$ Microscope: $\quad M = m\,m_\theta = -\dfrac{s}{f_{objective}}\,\dfrac{25\ cm}{f_{eye}}$

Telescope: $\quad m_\theta = -\dfrac{f_{objective}}{f_{eye}}$

### Interference

Double Slit experiment: Maxima when $d \sin\theta = m\,\lambda$ $\quad m = 0, 1, 2, 3, \ldots$

Minima when $d \sin\theta = (m + \frac{1}{2})\,\lambda$ $\quad m = 0, 1, 2, 3, \ldots$

Intensity $I = I_m \cos^2 \frac{\phi}{2}$, $\quad$ where $\phi = \frac{2\pi d}{\lambda} \sin\theta$

Thin film (in air): Maxima when $2L = (m + \frac{1}{2})\,\frac{\lambda}{n_2}$ $\quad m = 0, 1, 2, 3, \ldots$

Minima when $2L = m\,\frac{\lambda}{n_2}$ $\quad m = 0, 1, 2, 3, \ldots$

Reflected Wave: $\Delta\phi = \pi$ $\quad$ if $n_1 < n_2$

$\Delta\phi = 0$ $\quad$ if $n_1 > n_2$

### Diffraction

Single Slit diffraction: Minima when $a \sin\theta = m\lambda$ $\quad m = 1, 2, 3, \ldots$

Intensity $I = I_m \left(\frac{\sin\alpha}{\alpha}\right)^2$ $\quad$ where $\alpha = \frac{\pi a}{\lambda} \sin\theta$

Two objects can be resolved if: $\theta > \theta_R$ $\quad$ where $\theta_R = 1.22\,\frac{\lambda}{d}$

For diffraction grating: Maxima when $d \sin\theta = m\lambda$ $\quad m = 0, 1, 2, 3, \ldots$

X-ray diffraction from crystal with interplanar spacing $d$:

Maxima when $2 \sin\theta = m\lambda$ $\quad m = 1, 2, 3, \ldots$