

Some relationships from vector analysis

$$\nabla = \left(\hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \right) : \quad \text{nabla operator}$$

$$\begin{aligned} \nabla \cdot \phi &= \text{grad } \phi: & \phi & \text{ scalar field} & \rightarrow & \text{vector field} \\ &= \left(\hat{i} \frac{\partial}{\partial x} \phi + \hat{j} \frac{\partial}{\partial y} \phi + \hat{k} \frac{\partial}{\partial z} \phi \right) \\ \nabla \cdot \vec{A} &= \text{div } \vec{A}: & \vec{A} & \text{ vector field} & \rightarrow & \text{scalar field} \\ &= \left(\frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \right) \\ \nabla \times \vec{A} &= \text{curl } \vec{A}: & \vec{A} & \text{ vector field} & \rightarrow & \text{vector field} \\ &= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \left(\frac{\partial}{\partial y} A_z - \frac{\partial}{\partial z} A_y \right) \hat{i} + \left(\frac{\partial}{\partial z} A_x - \frac{\partial}{\partial x} A_z \right) \hat{j} + \left(\frac{\partial}{\partial x} A_y - \frac{\partial}{\partial y} A_x \right) \hat{k} \end{aligned}$$

$$\nabla^2 = \nabla \cdot \nabla = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) : \quad \text{laplacian operator}$$

applied on electric potential U :

$$\nabla^2 U = \text{div grad } U = 0 \quad \text{“Laplace equation”}$$

$$\nabla^2 U = \left(\frac{\partial^2}{\partial x^2} U + \frac{\partial^2}{\partial y^2} U + \frac{\partial^2}{\partial z^2} U \right) = 0$$

Next: $\nabla \times (\nabla \times \vec{A}) = \nabla (\nabla \cdot \vec{A}) - \nabla^2 \vec{A}$

where: $\nabla \cdot \vec{A} = \frac{\partial}{\partial x} A_x + \frac{\partial}{\partial y} A_y + \frac{\partial}{\partial z} A_z \equiv S$

so that $\nabla (S) = \hat{i} \frac{\partial}{\partial x} S + \hat{j} \frac{\partial}{\partial y} S + \hat{k} \frac{\partial}{\partial z} S$

and

$$\begin{aligned} \nabla^2 \vec{A} &= \left(\frac{\partial^2}{\partial x^2} A_x + \frac{\partial^2}{\partial y^2} A_x + \frac{\partial^2}{\partial z^2} A_x \right) \hat{i} \\ &+ \left(\frac{\partial^2}{\partial x^2} A_y + \frac{\partial^2}{\partial y^2} A_y + \frac{\partial^2}{\partial z^2} A_y \right) \hat{j} \\ &+ \left(\frac{\partial^2}{\partial x^2} A_z + \frac{\partial^2}{\partial y^2} A_z + \frac{\partial^2}{\partial z^2} A_z \right) \hat{k} \end{aligned}$$