Comments on ultimate performance of transverse polarimeter using the rise-time calibration

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Contents

I. INTRODUCTION

There have been considerable discussions on the ultimate performance of the transverse polarimeter. It has been pointed out several times by Michael Dueren, that the absolute calibration of the transverse polarimeter can ultimately depend on a precise rise time measurement. However, no numbers have been reported yet. An important factor in the calibration of the transverse polarization using the rise time method is that the machine conditions and the polarimeter have to stable over the period of the calibration measurement. The counting speed of the transverse polarimeter defines the measurement interval to obtain the required statistical precision, even with a prefect instrument. Systematic drifts must be controlled over that interval. The key question is then, how long is the interval for a perfect instrument under perfect machine conditions. That gives an upper limit on the ultimate performance of the rise time calibration. No systematic effects have been included. The facts presented here are based solely on a statistical analysis of the rise time behaviour.

II. RISE TIME CALIBRATION

In electron storage rings the electron spin can become polarized antiparallel to the magnetic bending field as a result of synchrotron radiation emission (Sokolov-Ternov effect [1]). The polarization increases with time according to

$$P(t) = P_{max}(1 - e^{-t/\tau}), \tag{1}$$

where τ is the polarization build-up time and P_{max} is the equilibrium polarization. The maximum polarization achievable in an ideal flat machine and in absence of depolarizing effects is 92.4%. In the presence of spin rotators, however, the maximum polarization is reduced to 89.4% to first order.

In a real machine, there are also strong depolarizing effects which can counteract the Sokolov-Ternov effect. The strength of these depolarizing processes can be quantified with a time constant τ_D. This process works simultaneously with the Sokolov-Ternov build-up, and the value of the asymptotic polarization P_{max} is determined by the relative strengths of the two processes according to

$$P_{max} = P_{ST}\frac{\frac{1}{\tau_{ST}}}{\frac{1}{\tau_{ST}} + \frac{1}{\tau_D}}. \tag{2}$$

The effective build-up time τ is also reduced by spin diffusion to

$$\frac{1}{\tau} = \frac{1}{\tau_{ST}} + \frac{1}{\tau_D}. \tag{3}$$

By rearranging Eq. 2 and 3,

$$P_{max} = \tau\left(\frac{P_{ST}}{\tau_{ST}}\right), \tag{4}$$

it follows that the measurement of the build-up time can provide a separate measurement of the polarization. Ideally these two methods should give the same results. However, if the two methods show differences the spatial asymmetries can be calibrated using the characteristic rise time behaviour.

III. SIMULATION OF PERFECT INSTRUMENT

The key question of how long a time interval is needed to obtain a required statistical precision with a perfect polarimeter under perfect machine conditions is discussed in this section. This time interval depends on the counting speed of the transverse polarimeter. Two different cases are presented:

1. Do a spatial asymmetry measurement with a statistical precision of 0.02 every two minutes.
2. Do a spatial asymmetry measurement with a statistical precision of 0.01 every minute.

These two cases probably represent a reasonable boundary for a conservative and an optimistic approach.

A. Measurement Errors

The vertical component of the electron polarization P_Y is measured through the vertical asymmetry distributions and through the mean shifts [2]. The magnitude of the asymmetries occurring in the measurement of the electron polarization can be small: with a vertical electron polarization of $P_Y = 0.5$ the maximum asymmetry $A(y, E_\gamma)$ is about 0.15. Therefore a large number of photons is required for a polarization measurement with small statistical error. To quantify the problem the analysis method of the mean shift Δy is chosen. The polarization is given by

$$P_Y = \frac{\Delta y}{\Delta S_3 \Pi} \tag{5}$$

and the statistcal error δP_Y is then

$$\delta P_Y = \frac{1}{\Delta S_3 \Pi} \frac{\sigma_y}{\sqrt{N}}, \tag{6}$$

where σ_y is the rms width of the scattered distributions defined by the electron beam properties, $N = N_L + N_R$ is the total number of photons used to define the mean position and Π the anaylzing power; for more details see Ref. [2]. The error on ΔS_3, which is the degree of circular polarization in the laser light, has been neglected.

Therefore, the error bar on each individual measurement does not depend on the magnitude of P_y.

B. Procedure

A random number generator has been used to generate statistical distributions of data points which fluctuate according to the assumed statistical precision. These fluctuations are overlaid on an exponential curve generated according to Eq. 1. The asymptotic value of the polarization, P_{max}, is chosen to be 0.70, and the rise time τ according to Eq. 4 is around 30 minutes. This resembles closely rise time measurements one expects for high polarization data.

C. Results

Many different sets of randomised exponential spectra have been generated and analyzed with MINUIT, by fitting the randomised exponential spectra with Eq. 1. Both, P_{max} and τ have been fitted. P_{max} has not been constrained in the fit because we assume to rely on the rise time measurement to calibrate the polarization scale.

The results are presented here for the two different cases mentioned above.

1. Asymmetry Mesurements of 0.02 every 2 minutes

The first case deals with the conservative estimate that a 0.02 measurement can be made every two minutes. Fig. 1 shows a sample of an exponential spectrum generated with the recipe discussed above.

In order to know how long a rise time measurement has to be for a certain required statistical precision in polarization, the fit is repeated again and again, each time dropping the last data point. Fig. 2 shows the results of these fits, where each point in the histograms corresponds to the time interval used for the corresponding fit. It illustrates that for short measuring intervals the fit results are far off the expected values. It also shows that for this particular set of random numbers, τ converges to within two sigmas from the expected value after a measuring interval of 200 minutes.

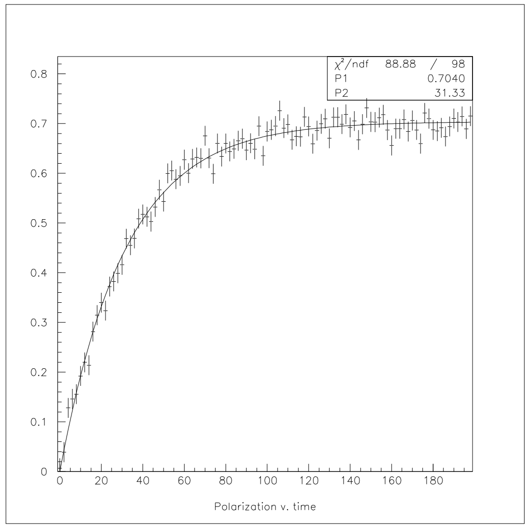

Figure 1. Simulation of a rise time measurement with a perfect polarimeter which is only influenced by statistical fluctuations. The entire measurement interval is 200 minutes long and it is assumed that a measurement with a 0.02 error is taken every 2 minutes. The spectrum was generated with $P_{max} = 0.70$ and $\tau = 30.3$. The solid line is a fit to the data.

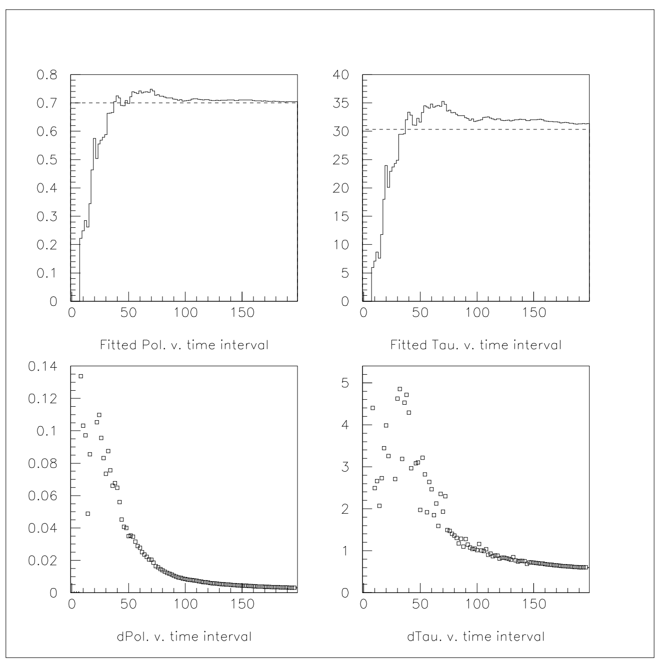

Figure 2. Results of fitting the data which are displayed in Fig. 1. The time interval is displayed on the absissa. Each point in the spectrum corresponds to a specific time interval. The top row shows the fitted values and the bottom the errors in the fitted values. This illustrates that for short time intervals the fitted results are far off the expected values, indicated with dashed lines.

Fig. 3 shows fitted results for three different samples of randomised exponential spectra. It also displays the errors in the fitted rise time versus time interval averaged over many different sets of randomised exponential spectra. This shows clearly that the system has to be stable for 180 min (110 min) to get a precision of 2% (3%) in τ.

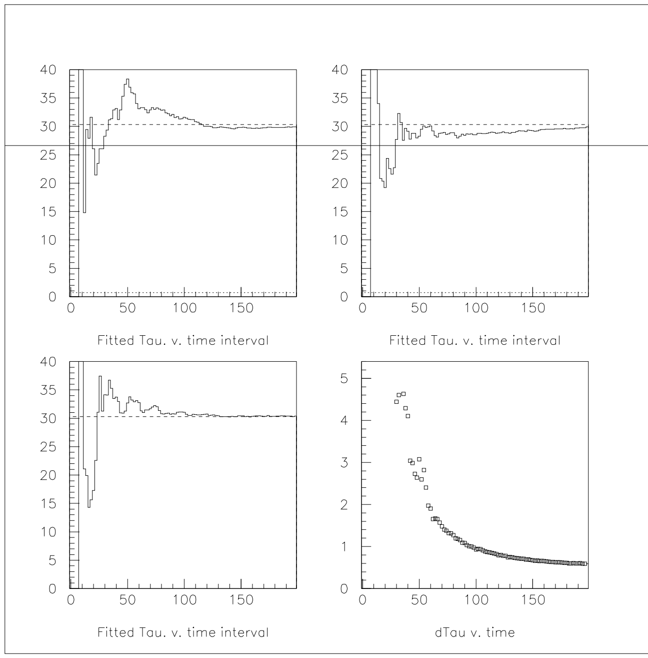

Figure 3. Fitted results for three different samples of exponential curves are shown. Also shown is a histogram with errors in the fitted rise time versus time interval averaged over many different sets of exponential curves.

2. Asymmetry Mesurements of 0.01 every 1 minute

The second case deals with the more optimistic estimate that a 0.01 measurement in τ can be made every minute. Fig. 4 shows an example of a spectrum generated with these conditions.

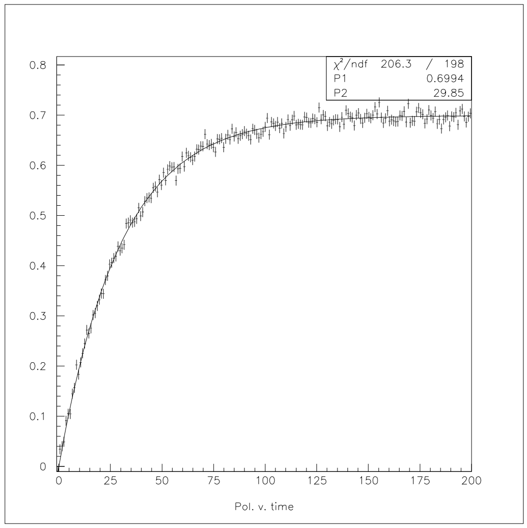

Figure 4. Simulation of a rise time measurement with a perfect polarimeter which is only influenced by statistical fluctuations. The entire measurement interval is 200 minutes long and it is assumed that a measurement with a 0.01 error is taken every minute. The spectrum was generated with $P_{max} = 0.70$ and $\tau = 30.3$, indicated with the dashed lines.

The same recipe for analyzing the spectra is used as in the section above.

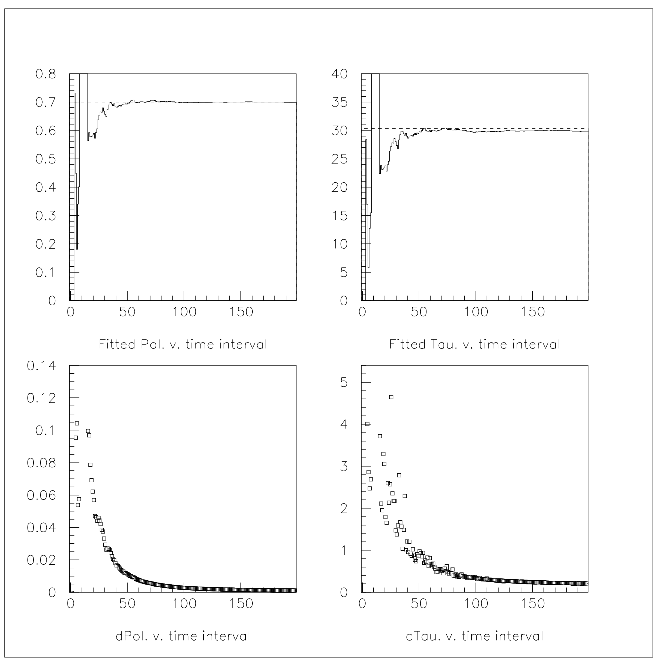

Figure 5. Results of fitting the data which are displayed in Fig. 4. The measuring time interval is displayed on the absissa. Each point in the spectrum corresponds to a specific time interval. It illustrates that for short time intervals the fit results are far off the expected values, indicated with dashed lines.

Fig. 6 shows the errors in the fitted rise time versus time interval averaged over many different sets of exponential curves. This shows clearly that the system has to be stable for 120 min (60 min) to get a precision of 1% (2%) in τ.

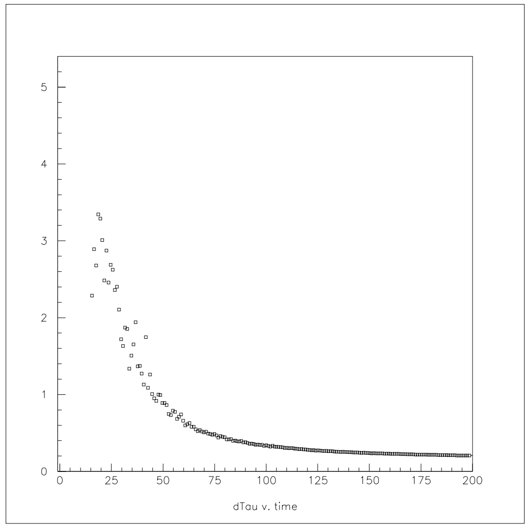

Figure 6. Errors in the fitted rise time versus time interval averaged over many different sets of exponential curves are shown

IV. CONCLUSIONS

It has been investigated how long a perfect polarimeter, which produces measurements every one or two minutes with a statistical precision of 0.01 to 0.02, has to take data to achieve a precision of 1% to 3% in the rise time calibration. Depending on the frequency and precision of each individual asymmetry measurement, one needs the entire system (electron beam and calorimeter) to remain stable for typically one to two hours.

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[1] A.A. Sokolov and I.M. Ternov, Sov. Phys. Doklady 8 (1964) 1203.
[2] D.P. Barber et al., NIM **A329** (1993) 79.