Studies of the Measurement of the Transverse Polarization of the Electron/Positron Beam at DESY

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1 A Preliminary Analysis of Fourteen Risetime Curves

The transverse polarization of the electron (or positron) beam at DESY is determined by measuring the small spatial asymmetries that characterize the Compton scattering of circularly polarized light with linearly polarized electrons. The scattered photon strikes a sampling calorimeter. Wavelength shifters connected to the tops and bottoms of the scintillators in the calorimeter make it possible to compare the energies deposited in the upper and lower halves of the calorimeter (E_{up} and E_{down}) for each backscattered photon. The position of the photon on the face of the detector is derived from the value of η as defined by

$$\eta = \frac{E_{up} - E_{down}}{E_{up} + E_{down}}. \tag{1}$$

The electron polarization is derived from the so-called Π-function analysis of the η distributions, which is given by

$$\triangle\eta_v(E_\gamma) = \frac{\langle\eta\rangle_L - \langle\eta\rangle_R}{2} = \triangle S_3 P_y \Pi(E_\gamma), \tag{2}$$

where $\triangle S_3$ is the difference in the helicities of the laser beam, P_y is the transverse polarization of the electron beam, and $\langle\eta\rangle_L$ and $\langle\eta\rangle_R$ are the means of η for left and right circularly polarized laser beams, respectively. The Π-function $\Pi(E_\gamma)$ depends not only on the energy E_γ of the scattered photon. There is also a dependence on the profile of the electron beam, and the averaging over the y-values of the compton scattered photons on the detector face (the η-to-y transformation).

As with all measurements of small spatial asymmetries, the Π-function analysis is vulnerable to systematic errors. The current work studies the systematic errors present in this measurement, with particular attention paid to a risetime measurement calibration of the Π-function calculation.

1.1 Fluctuation Analysis of Constant Polarization Data

For the beam conditions during the Fall of 1994, the Π-function analysis of the transverse polarization is characterized by 1-2% statistical errors for each one-minute measurement.

The statistical errors are directly related to the intensities of the laser and positron beams, and to the luminosity of the interaction.

The analysis of constant polarization data allows us to look at the characteristic fluctuations in the measured polarizations without the added complication of time dependence in the actual polarization, to determine if the fluctuations are consistent with that expected from the uncertainty of the measurements. For this discussion, we studied the polarization data in runs 2212–2216, taken during October of 1994, starting approximately ten time constants τ into a luminosity run, shown in Fig. 1.

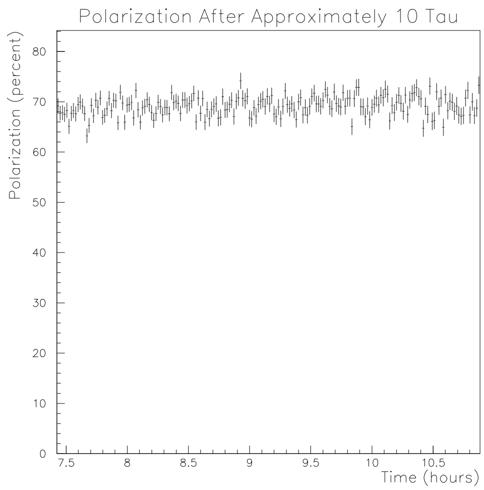

Fig. 1 Constant polarization data.

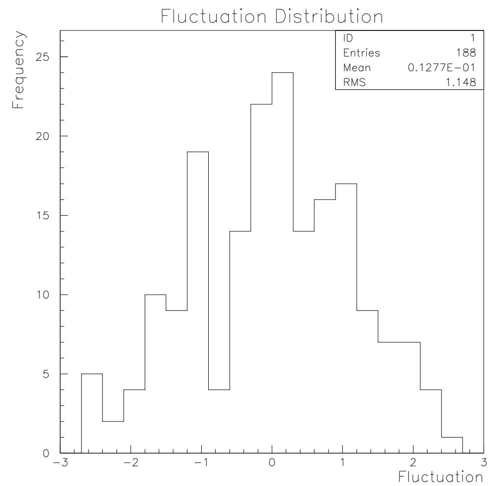

Fig. 2 Fluctuation analysis of constant polarization data.

Some relevant parameters of the measurements are listed in Table 1.

Fig. 2 plots a frequency distribution of the deviations of the measured polarization about the mean value. The fluctuations in Fig. 2 are defined by

$$\frac{p_i - \langle p \rangle}{\sigma_i},$$

where p_i is the measured polarization at time t_i, σ_i is the statistical uncertainty of the measurement, and $\langle p \rangle$ is the weighted mean polarization as defined by

$$\langle p \rangle = \frac{\sum_{i=1}^{N} \frac{p_i}{\sigma_i^2}}{N \sum_{i=1}^{N} \frac{1}{{\sigma_i}^2}}.$$

For the data in Fig. 1, $\langle p \rangle = 68\%$. Note that the root mean square (RMS) deviation of the distribution in Fig. 2 would be unity for purely statistical fluctuations in the data. We can estimate the nonstatistical (*i.e.* systematic or random) contribution to the fluctuations in Fig. 2 assuming a gaussian distribution for the systematic error, using the formula

$$\sqrt{\sigma_{stat}^2 + \sigma_{syst}^2} = 1.148\sigma_{stat},$$

Table 1: Polarization measurement parameters.

Injection time	Beam energy (GeV)	Beam intensity (mA)	Beam lifetime (τ, hours)	Laser intensity (Watts)	Luminosity
15-10-94, 05:13	27.584	15.080	10.60	8.50	54.4%

where 1.148 is the RMS value of the distribution plotted in Fig. 2. Assuming a constant statistical error of $\sigma_{stat} \approx 1.5\%$ for every minute to minute measurement, we get

$$\sigma_{syst} = \sigma_{stat}\sqrt{(1.148)^2 - 1} \approx 0.8\%.$$

The value of σ_{syst} suggests that the polarization can be stable over an interval of approximately three and a half hours, and that the fluctuations in the measured polarization are dominated by the statistical uncertainty of the measurement. Alternatively, this result could be used to conclude that the statistical errors are being calculated correctly.

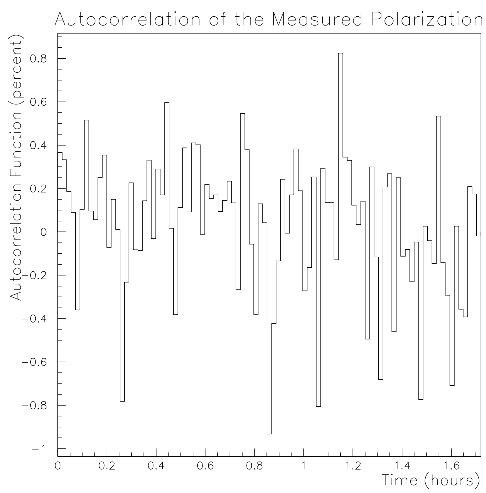

Fig. 3 Autocorrelation analysis of constant polarization data.

One possible source for a systematic contribution to the distribution in Fig. 2 is if the actual polarization changes slightly during this measurement. To look for possible time dependence in the polarization data, we use the autocorrelation function, defined for time intervals $\triangle t$,

$$A(\triangle t) = A(t_j - t_i) = \sum_{i=1}^{\frac{N}{2}} \sum_{j=i+1}^{N} \frac{(p(t_i) - \langle p \rangle)(p(t_j) - \langle p \rangle)}{\text{number of } i, j \text{ pairs}}.$$

The autocorrelation function studies the averages of the product of the deviations from the mean $\langle p \rangle$ separated by time intervals $\triangle t$. If the averaging is done over a large number

Table 2: Summary of risetime measurements.

Number	Date	Run number	Duration (hours)
1	21-10-94	2370	1.240
2	25-10-94	2442	0.975
3	25-10-94	2444	0.975
4	27-10-94	2462	0.849
5	27-10-94	2464	0.595
6	27-10-94	2478	1.860
7	28-10-94	2482	1.137
8	28-10-94	2484	1.011
9	28-10-94	2486	0.831
10	28-10-94	2488	3.412
11	28-10-94	2492	1.192
12	29-10-94	2544	0.578
13	29-10-94	2546	0.668
14	29-10-94	2548	0.849

of pairs, the statistical fluctuations will average to zero, and therefore this function is very sensitive to changes in the actual polarization, as such changes will produce non-statistical correlations between time intervals $\triangle t$. In Fig. 3, there is no correlation between pairs of polarizations up to an hour and a half apart, which suggests the actual polarization of the beam was very stable over this time interval. The autocorrelation analysis was repeated for another time interval, and there we found evidence of a slowly varying oscillation in the measured polarization with a period of roughly 30 minutes, and an amplitude of approximately 0.7%. This analysis suggests that the Π-function calculation of the transverse polarization is very close to the level of uncertainty determined by statistical fluctuations in the measurement, and that the remaining issue is to determine if the Π-function analysis is calculating the polarization correctly. Therefore, a series of 14 risetime measurements were made to determine an absolute calibration of the Π-function analysis. For each risetime measurement, the calculated polarization at a given time interval t_i is based on the Π-function analysis, so if the Π-function analysis reproduces the correct time dependence in the risetime measurements, then the Π-function analysis is calculating the transverse polarization correctly. Some relevant information about the risetime measurements is listed in Table 2.

1.2 Fitted risetime curves and the k distribution

In this work the risetime curves, such as the ones shown in Figs. 4 and 5 are fitted to the function

$$P(t) = \frac{\tau}{k}(\frac{P_{ST}}{\tau_{ST}})(1 - e^{-(t-t_0)/\tau}), \tag{3}$$

where $\tau(\frac{P_{ST}}{\tau_{ST}}) = P(t = \infty)$. The fits were done using MINUIT. The variable τ is a parameter of the fit, as is t_0 and k. The nonimal values of P_{st} and τ_{st} during these measurements were 0.924 and 43 min, respectively (NIM **A329**, 79, 1993) [*1]. This additional parameter k provides us with a consistency check on the Π-function analysis, and the results of the values of k derived from the fits can be placed into three general categories. The first possible outcome is that a number of risetime curves yields a statistical distribution of k about unity, showing that the Π-function analysis is correct. The second possible outcome is that a number of risetime curves yields a statistical distribution of k about some number other than one, showing that the Π-function analysis is miscalibrated but otherwise sound, and the nominal polarization should be scaled by k. The third possible outcome is that a number of risetime curves yields a distribution much wider than statistical, which would mean that either the Π-function analysis or the data taking conditions requires further inspection.

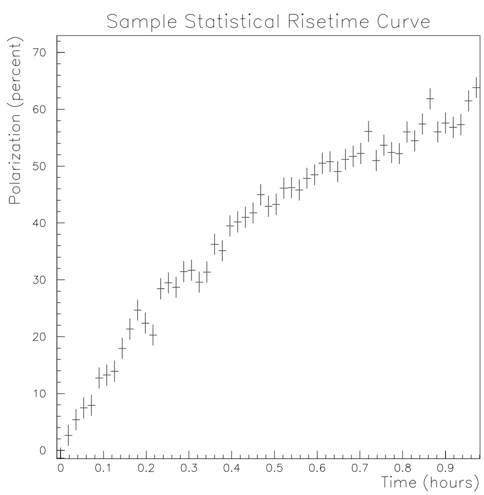

Fig. 4 Typical MC Risetime.

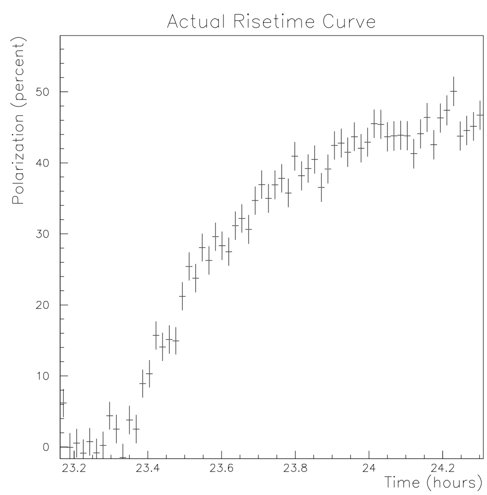

Fig. 5 Risetime curve 3.

To determine the expected range of the distribution of k, based on the statistical uncertainties of the Π-function calculations, 14,000 risetime curves (1,000 for each actual risetime measurement) with gaussian statistical errors and durations comparable to the fourteen risetime curves were generated and fitted with Eq. 3. A typical generated risetime curve is shown in Fig. 4, which corresponds in length and statistical uncertainty to the actual risetime curve in Fig. 5. The resulting frequency distribution for k obtained from fitting the 14,000 simulated risetime measurements to Eq. 2 is shown in Fig. 6. The root mean square (RMS) deviation is roughly 2%. The frequency distribution of the fourteen measured risetime curves is shown in Fig. 7, whose root mean square deviation is roughly 9%. This does not compare well with the simulated risetime curves, as it leaves a random error of $\sigma_{syst} = \sqrt{\sigma_{RMS}^2 - \sigma_{stat}^2} = \sqrt{(0.092)^2 - (0.021)^2} \approx 8.9\%$. Therefore, for a nominal transverse polarization measurement of P=0.70, the uncertainty due to this random error is ± 0.06. Given the three possible scenarios discussed previously, this result falls into the third category. In addition, the distribution of the values of k is centered at 0.96, which indicates the on-line Π-function analysis overestimates the actual polarization by roughly 4%.

We made many studies that compare the values of k obtained from the fits of Eq. 2 to the actual data with other parameters that characterize the risetime measurements. Two such plots are shown in Fig. 8. The plotted uncertainty for each value of k is the value taken from the MINUIT fits. As is shown in Fig. 7, six of the risetime measurements are clustered at $k \sim$0.925. These six risetime curves have a large range of values of τ, as is shown in Fig. 8(a), and the χ^2 of the fits for these six measurements as shown in Fig. 8(b) also varies over a large region, so the value of k does not depend strongly on either of these two parameters. We found similar relationships for all parameters we studied, such as comparisons of k with the recorded luminosity of the measurement, the electron beam intensity, the power of the laser beam, the position of mirror M3, and the voltage of the photomultipliers.

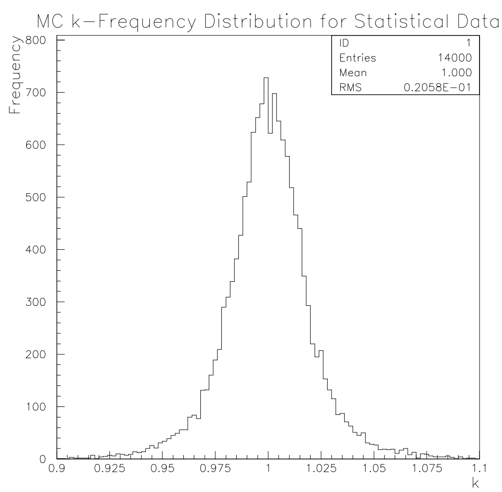

Fig. 6 MC k distribution

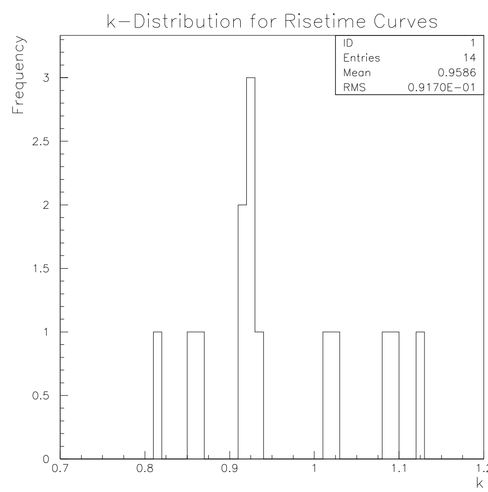

Fig. 7 Actual k distribution

It seems that the only types of effects that could produce the nearly 9% spread in the k-distribution, but not have an appreciable effect on the analysis of Section 1.1 would be a slowly varying time dependence with a time scale on the order of hours that affects the Π-function analysis, or else intermittant instabilities that only contribute to the distribution of k-values around the cluster centered at k=0.96, but were not present for the six measurements centered at k=0.925. For these types of variations, the Π-function analysis would appear to be stable for a given time interval, but a comparison of different time intervals would be inconsistent at the level of statistical fluctuations. One such type of variation, and its effect of the on-line Π-function analysis is discussed in the folllowing section.

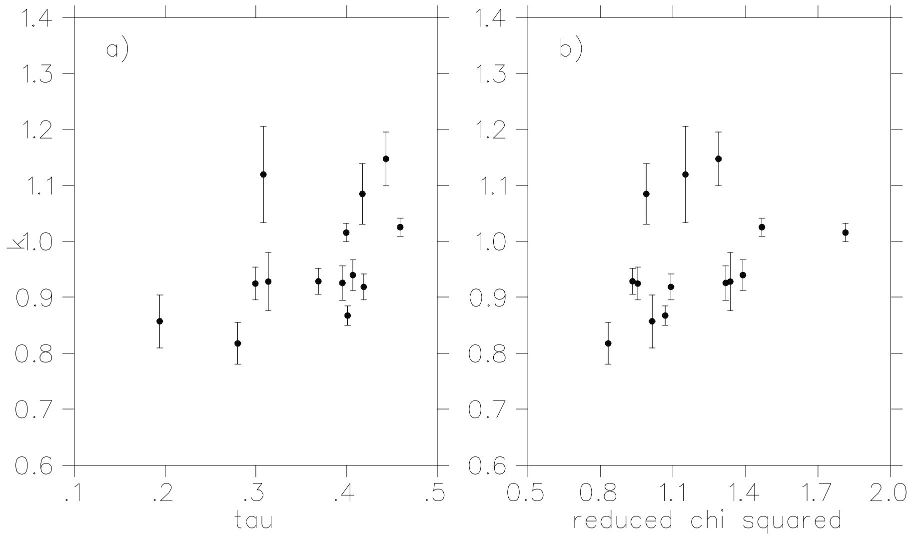

Figure 8 Studies of the k-dependence, a) plots k versus τ (the risetime constant of the measurement), b) plots k versus the χ^2 of the fit.

2 Monte Carlo studies of the Π-function analysis

In order to study the Π-function analysis, a Monte Carlo simulation `mcgeantpi` was used. A summary of the computer simulation is available. The primary objectives were to vary machine parameters that influence the measurement of the Compton scattering process, and that could in principle either change slowly over time or from fill to fill, to determine which parameters could produce systematic errors consistent with those presented in the previous section, and at the same time closely reproduce data from actual measurements, such as a y-scan of the calorimeter. For the sake of clarity, in the following discussion the treatment of the Twiss parameters will be simplified slightly. Each Twiss parameter will be treated as a variable which is completely independent of the other parameters, although this is of course not the actual situation.

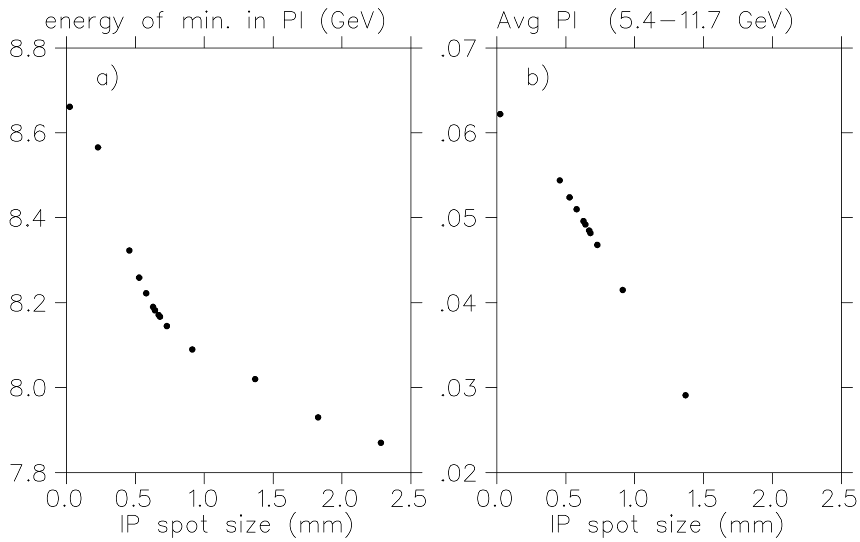

Figure 9 Studies of the Π-function, a) plots the value of the energy of the Compton scattered photon at the minimum of the Π-function, b) plots the average of the Π-function over the energy interval from 5.4 to 11.7 GeV.

Preliminary analysis concentrated on variations in the size of the laser beam spot at the interaction point. These studies revealed that changing the laser beam width over the interval from 0 mm to 4 mm had a negligible effect on the Π-function. Therefore, in the following discussion the laser beam width was set to 0 mm. The variable which was then varied was the vertical emittance ϵ_y of the positron beam. The spot size at both the interaction point and at the detector face is proportional to $\sqrt{\epsilon_y}$, as is the distribution of the directions of the electrons or positrons at the interaction point. Therefore the Π-function is much more sensitive to the changes in ϵ_y than it is to changes in the spot size of the laser beam since ϵ_y influences so many properties of the scattering distributions. The value of ϵ_y was varied in order to match the on-line Π-function and the experimental data for the y-scan runs. As of this writing we have not studied the dependence of the Π-function on other Twiss parameters such as β_y, but we hope to make such studies shortly. In the discussion that follows all Twiss parameters other than ϵ_y are kept at the values listed in Barber *et al.* (NIM **A329**, 79, 1993).

The results of the monte carlo simulations for different values of ϵ_y are shown in Fig. 9, which reveal that as ϵ_y is varied, the energy of the Compton scattered photon that corresponds to a minimum in the Π-function changes, as does the magnitude of the Π-function over the entire energy interval. The on-line Π-function has a minimum at $E_\gamma = 8.230$ GeV and an average value over the range from 5.4-11.7 GeV of -0.0495 [*2]. For an electron beam with negligible width, $\epsilon_y \sim 0$, and the results of the `mcgeantpi` simulated Π-function has a minimum at $E_\gamma = 8.661$ GeV and an average value of -0.0622. For an electron beam with a width at the interaction point of σ_y=0.228 mm, which is the nominal value, the minimum energy $E_\gamma = 8.566$ GeV and the average value is -0.0600. For an electron beam with a width σ_y at the IP of 0.628 mm, the minimum energy $E_\gamma =$

8.190 GeV and the average value is -0.0496, which is very close to the values of the on-line Π-function. This behavior is summarized in Fig. 10. For the beam width σ_y=0.628 mm, the shape of this Π-function closely matches the shape of the on-line Π-function. We therefore used this Π-function as the nominal Π-function in our calculations, even though it overpredicts the electron spot size σ_y by roughly a factor of 3 [*3].

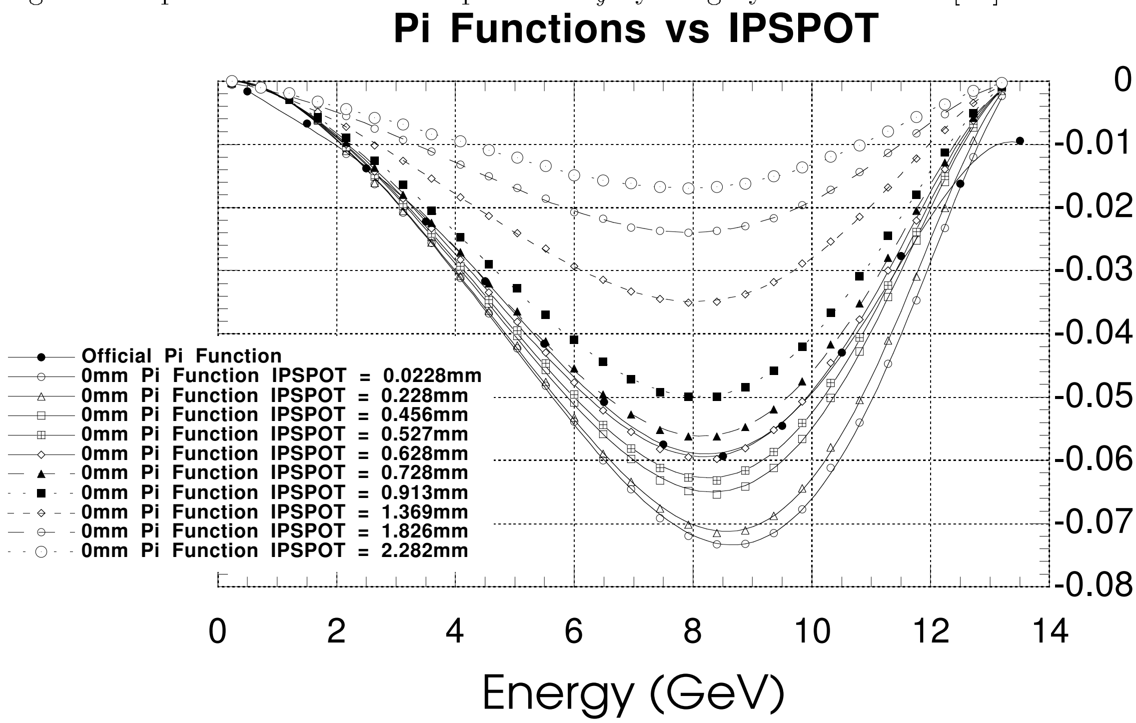

Figure 10 The calculated Π-functions for different values of ϵ_y.

A y-scan measurement from y=0 ±0.45 mm was made to study the sensistivity of the on-line Π-function analysis to small offsets in the polarimeter. As the absolute value of the y-offset increases, the calculated polarization using the on-line analysis decreases, since the on-line analysis does not include the effects of the y-offset. The results of the measured polarization versus the value of the y-offset were fitted to a gaussian function, and from this fit, the estimated change in the average of the Π-function over the energy region from 5.4 to 11.7 GeV is shown in Fig. 11. Over the range of measured offsets from ±0.45 mm, the variation in the average of the Π-function over the energy interval from 5.4 to 11.7 GeV using σ_y=0.628 mm closely matches the fit to the data. At this point, the Π-function based on the monte carlo simulation reproduced all aspects of the actual on-line Π-function. The Compton energy of the minimum in the Π-function, the average of the Π-function over the interval from 5.4 to 11.7 Gev, and the y-dependence of the measured polarization are all reproduced very well with the same value of ϵ_y. Of course, as was mentioned above, the value of σ_y=0.628 mm is larger than the expected value, and we do not mean to suggest that the nominal value of σ_y = 0.228 mm is incorrect.

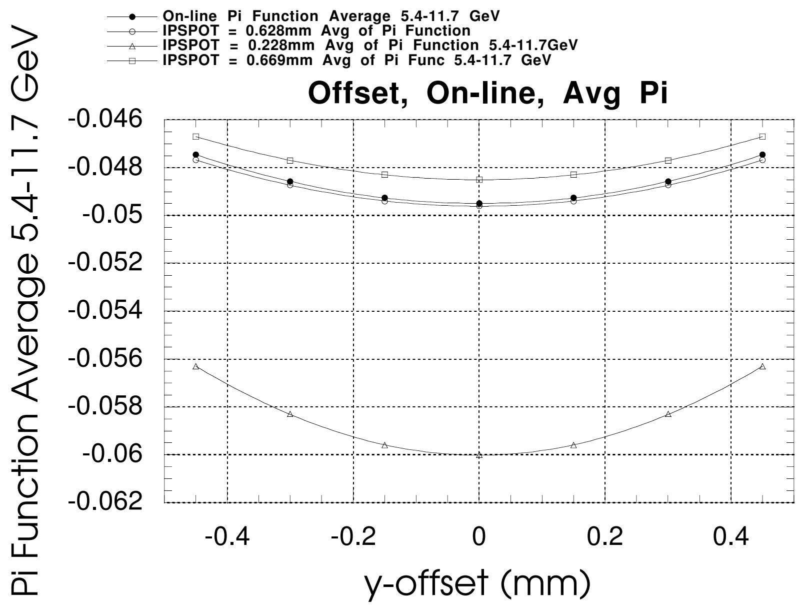

Figure 11 A comparison of the actual y-dependence of the on-line Π-function analysis to the y-dependence of the monte carlo simulation for σ_y=0.628 [*4].

These results can be used to estimate the effect that small changes in the size of the electron beam introduce to the Π-function analysis. For an electron beam width σ_y = 0.628 mm, a change in the spot size of 0.014 mm (a 2.2% relative change) yields a change in the average value of the Π-function of 0.8%. For a change in the spot size of 0.050 mm (a 7.9% relative change), the average value of the Π-function changes by 2.8%. For a variation of 0.100 mm (a 16% relative change), the average value changes by 5.6%. These results suggest that small variations in the electron beam spot size can have a significant effect on the Π-function analysis, and therefore on the extracted polarization of the beam. Another consideration is the orientation of the beam ellipse. There are machine conditions which yield a beam ellipse that is not aligned in the horizontal direction. In principle this could introduce a spread in the effective beam spot size projected along the y-axis, if the angle of the ellipse were to vary.

3 Conclusion

We have shown that there is a random error in the Π-function measurements with a standard deviation $\sigma_{syst} \sim 8.9\%$, and that the on-line Π-function analysis overestimates the polarization by 4%. Our results suggest that the systematic error modifies the Π-function analysis in a way that slowly varies with time, on the order of hours, or else is intermittant, and that for a restricted time interval it is not present at the same level. There are very few sources of systematic errors that could produce such behavior. We have considered one such type: a slowly varying electron beam profile. Clearly there are other possibilities that need to be considered. Currently we have not addressed the fact that the Twiss parameters near the interaction point are not constants, and if the laser beam spot changes, the interaction length along the path of the electron beam will change, which will alter the average values of the Twiss parameters seen by the laser beam. This could have a significant effect on the distribution of the backscattered photons on the face of the detector.

4 Addendum: (January 21, 1996)

[*1] The absolute value of k is not correct because the wrong values for P_{st} and τ_{st} were used. The beam energy was not the same as the one described in NIM **A329** (1993). The correct values for P_{st} and τ_{st} are 0.915 and 36.7 min, respectively.

[*2] The actual value used in the ONLINE programs is -0.0456.

[*3] The large value of σ_y is still not understood, however, new efforts are underway to write a new and independent Monte Carlo program.

[*4] A reanalysis of the fourteen rise time curves collected in 1994 can be found in the "Interim Report on the Measurement of the Positron Beam Polarization" by S. Barrow *et al.*. There, all the correct parameters have been used.