

The Proton Radius Puzzle

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(23-October-2018)

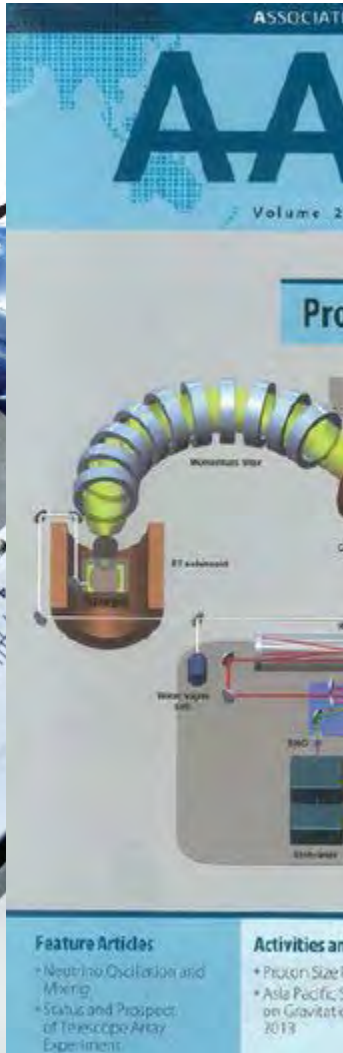
- The Proton Radius Puzzle
 - What is a radius ? How do we measure it ?
- What is the problem ?
- How do we solve it: MUSE ?

The Proton Radius Problem

July 2010



April 2013



July 2013



January 2014



The Proton Radius Problem



The Proton Radius Problem



The New York Times

- The Proton Radius Puzzle (PRP) has garnered a lot of interest!
- Not just interesting:
 - Tests our theoretical understanding of proton
 - Directly related to the strength of the Strong Interaction (QCD)
- What exactly is the puzzle ?

The Proton Radius

- Classical physics (sphere of charge density $\rho(r)$):

$$\langle r^2 \rangle = \int \rho(r) r^2 d^3 r$$

- Non-relativistic QM (w.f. of density of target $\psi(r)$):

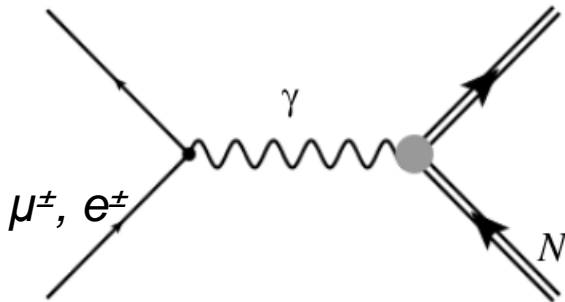
$$\langle r^2 \rangle = \int \langle \psi^*(r) | r^2 | \psi(r) \rangle d^3 r$$

- Relativistic QM (form factor $G(Q^2)$):

$$\langle r^2 \rangle = -6 \left. \frac{dG(Q^2)}{dQ^2} \right|_{Q^2=0}$$

The Proton Radius - II

- Lepton scattering



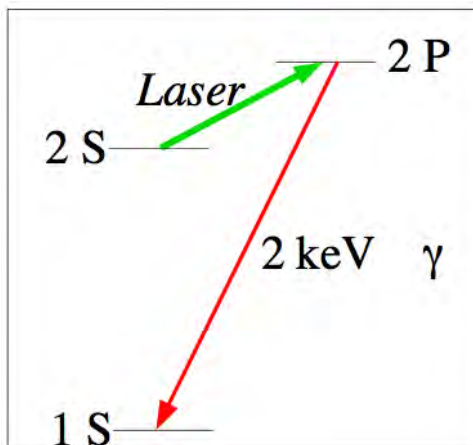
Non-relativistic scattering off extended proton:

$$\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\Big|_{point} \times (G(Q^2))^2$$

$(G(Q^2) = \int \rho(r) e^{iQ \cdot r} d^3r$ is Fourier transform of $\rho(r)$

Extract form factor from data, fit its trend with Q^2 , find slope as $Q^2 \rightarrow 0$

- Atomic Energy Levels



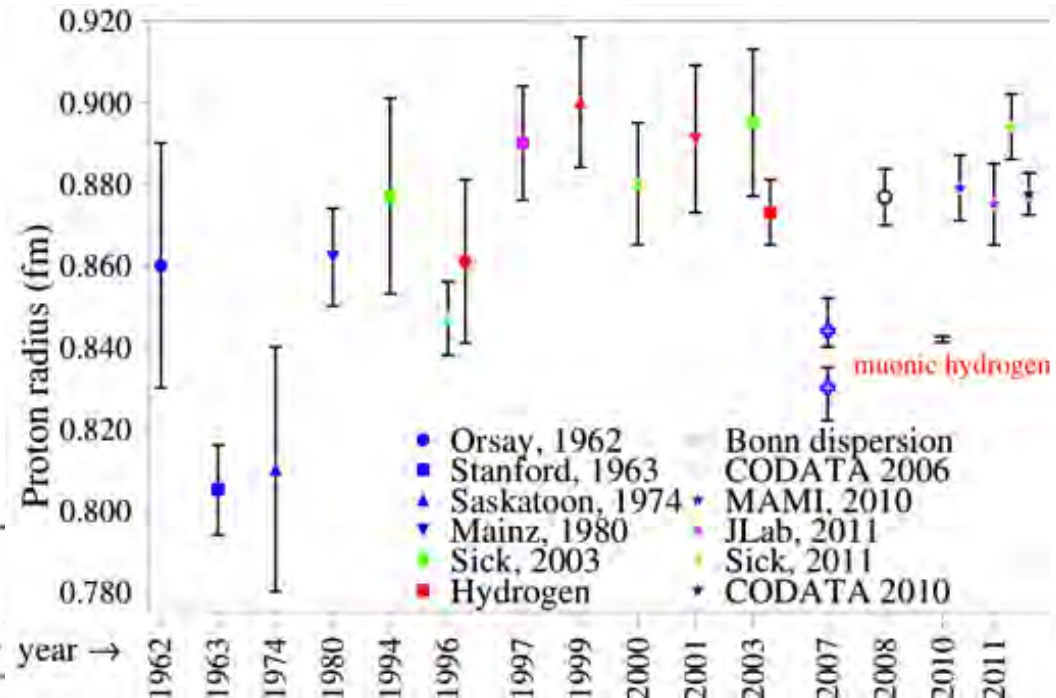
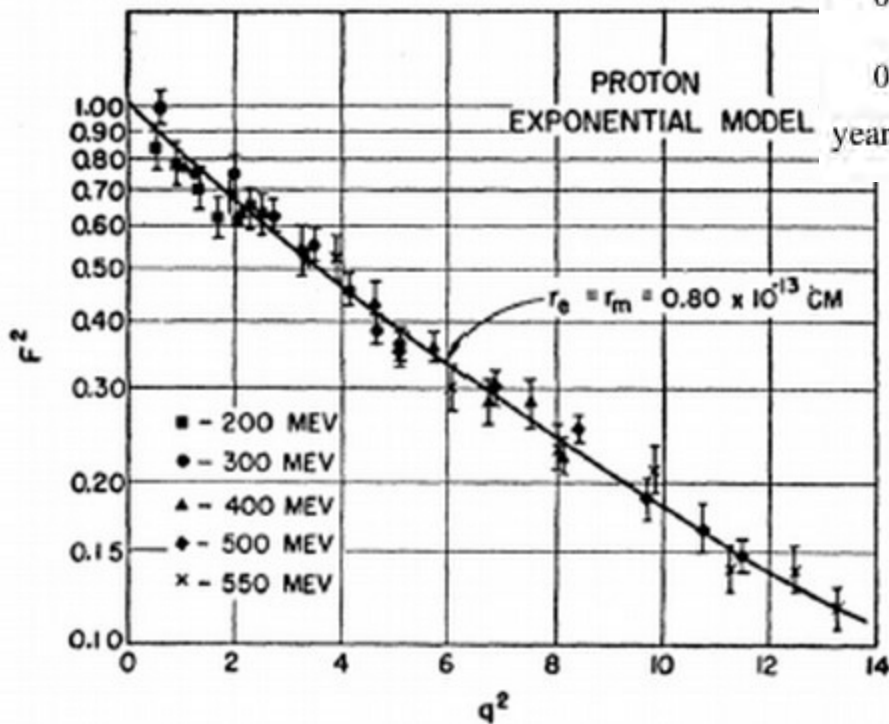
Non-relativistic (Schwinger 1952):

$$\Delta E_1 = \frac{2\pi\alpha}{3} |\phi^2(0)| \langle r_E^2 \rangle$$

Finite size of proton perturbs energies of S states – $r_p \ll r_{atomic}$, so effect proportional to electron wavefunction $\phi^2(r=0)$

The Proton Radius vs Time

Chambers and Hofstadter,
Phys Rev 103, 1454 (1956)



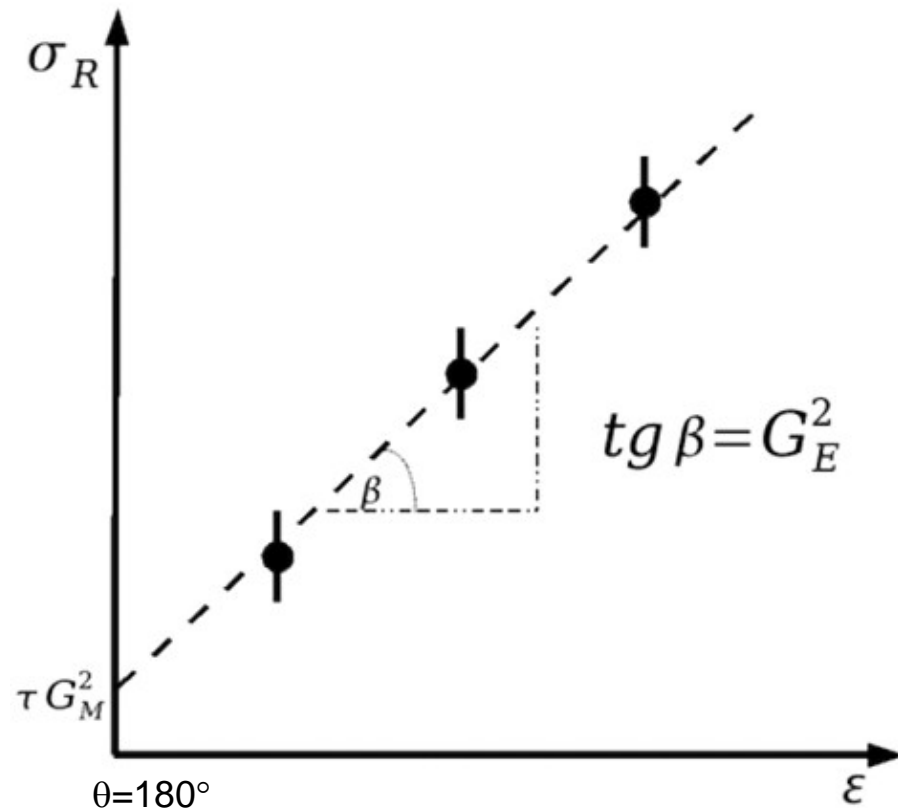
From Pohl, Gilman, Miller, Pachucki
review, arXiv:1301.0905,
Ann.Rev.NPS, modified

Electron Scattering Measurements

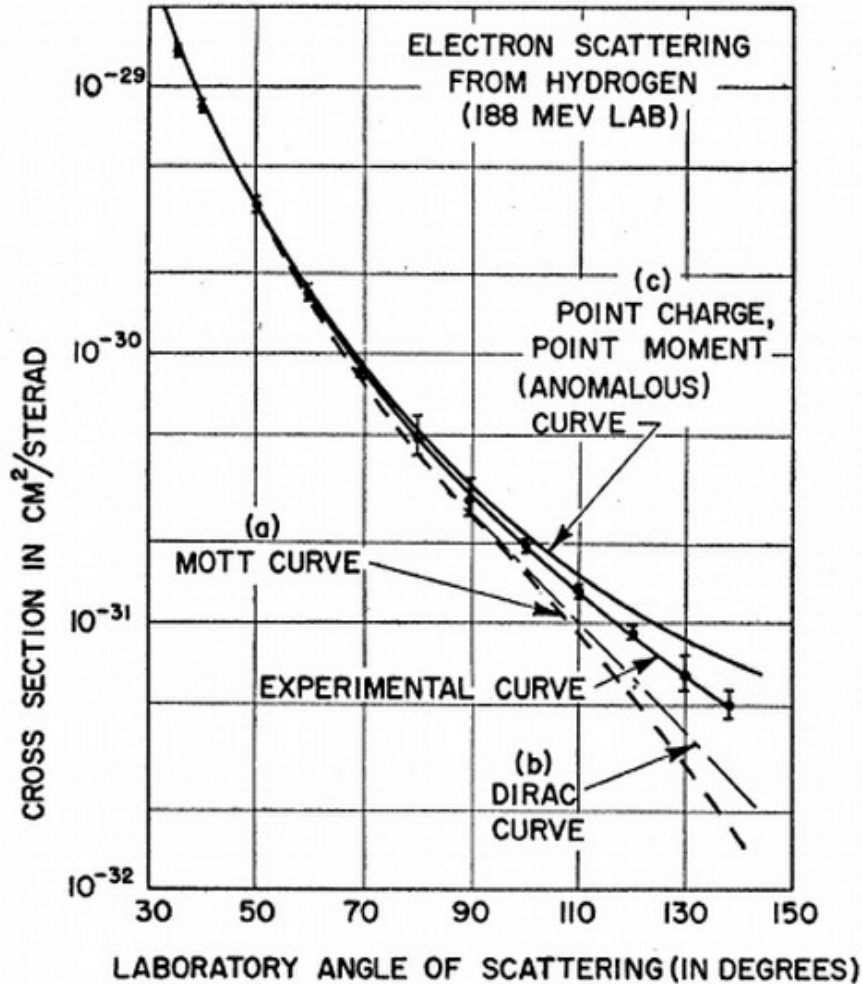
$$\sigma_R \left(\approx \frac{(d\sigma / d\Omega)}{(d\sigma / d\Omega)_{Mott}} \right) = \tau \underset{\substack{\uparrow \\ \text{current} \\ \text{distr.}}}{G_M^2} + \varepsilon \underset{\substack{\uparrow \\ \text{charge} \\ \text{distr.}}}{G_E^2} ; \quad \text{with } \tau = \frac{Q^2}{4M^2} ; \quad \varepsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2} \right]^{-1}$$

$G_E^2(0) = 1; \quad G_M^2(0) = \mu_p$

- In **one-photon** exchange (or Born approximation), form factors are related to elastic e-p scattering cross section
- Classical **Rosenbluth separation**
- Measure the reduced cross section at several values of ε (angle/beam energy combination) while keeping Q^2 fixed
- Linear fit to get intercept and slope



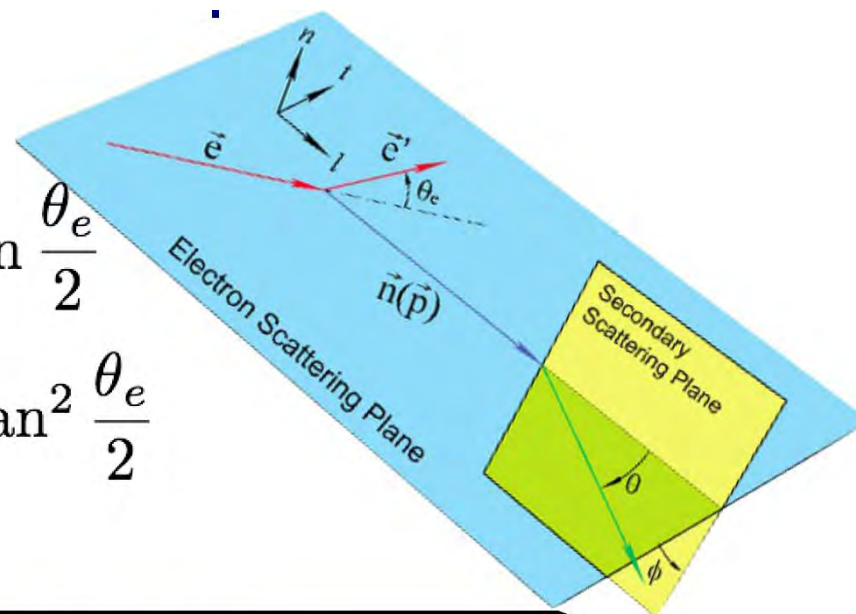
Electron Scattering Measurements (1950s)



$$\langle r_E \rangle = 0.74(24) \text{ fm}$$

- fit to RMS radius Stanford 1956
- R.W. McAllister and R. Hofstadter, Phys. Rev. **102**, 851 (1956)

Electron Scattering Measurements w/ polarization



$$I_0 P_t = -2\sqrt{\tau(1+\tau)} G_E G_M \tan \frac{\theta_e}{2}$$

$$I_0 P_l = \frac{E_e + E_{e'}}{M} \sqrt{\tau(1+\tau)} G_M^2 \tan^2 \frac{\theta_e}{2}$$

$$P_n = 0 \quad (1\gamma)$$

$$\mathcal{R} \equiv \mu_p \frac{G_E}{G_M} = -\mu_p \frac{P_t}{P_l} \frac{E_e + E_{e'}}{2M} \tan \frac{\theta_e}{2}$$

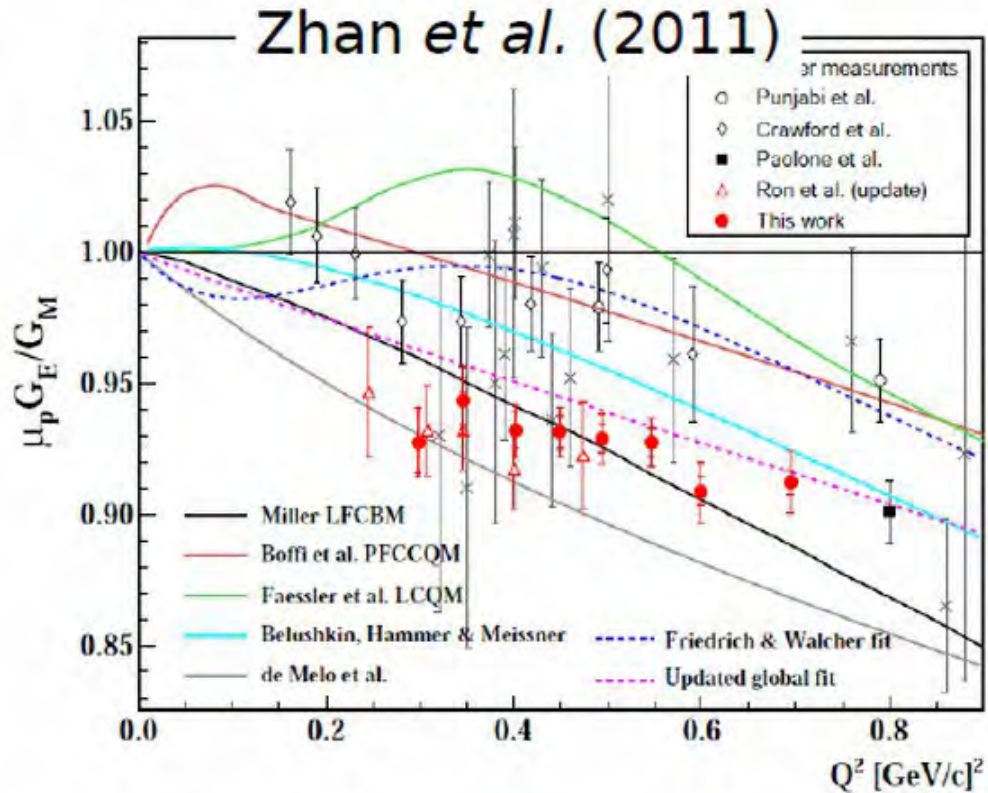
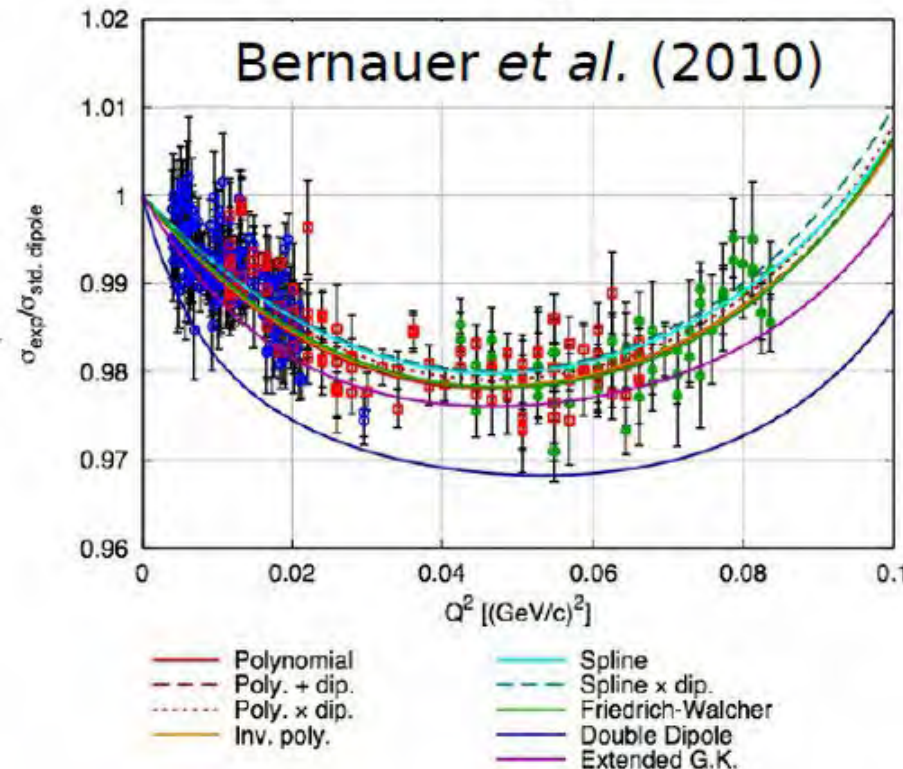
- Double polarization in elastic e-p scattering
 - measure recoil polarization or with (vector) polarized target

$${}^1H(\vec{e}, \vec{e}'\vec{p}), \quad {}^1\vec{H}(\vec{e}, \vec{e}'\vec{p})$$

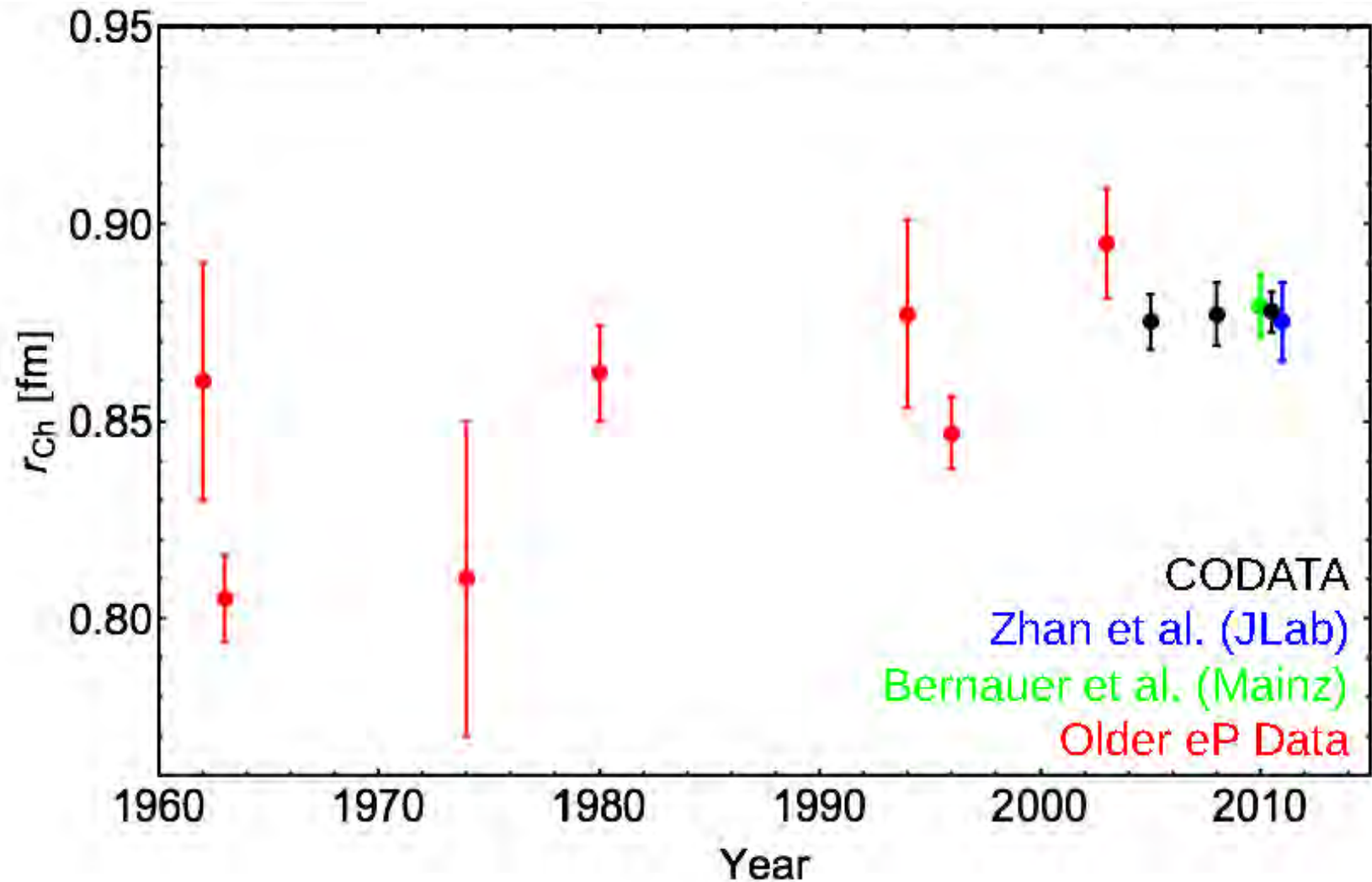
- A single measurement gives ratio of form factors

Electron Scattering Measurements (2010s)

- Bernauer et al. PRL 105, 242001: world's largest data set
 - fit functional forms to data rather than Rosenbluth separation
- Zhan *et al.* PLB 705 (2011) 59-64: Polarization measurements to get G_E/G_M , available over a large Q^2 range
 - fit(Jlab + world – Bernauer) gives radius compatible with Bernauer



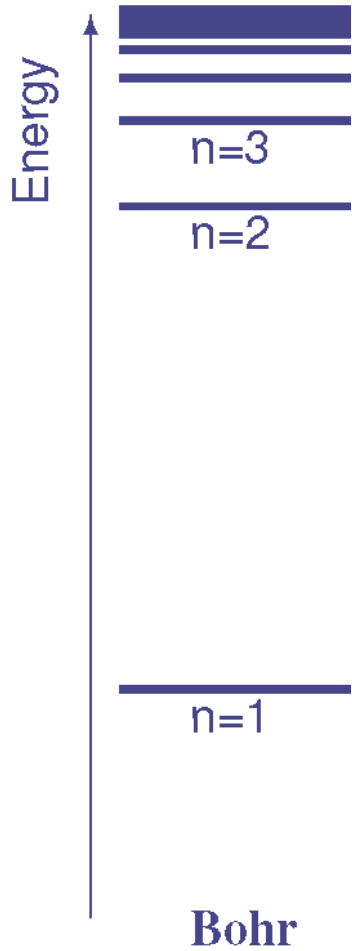
The Proton Radius vs Time from ep data



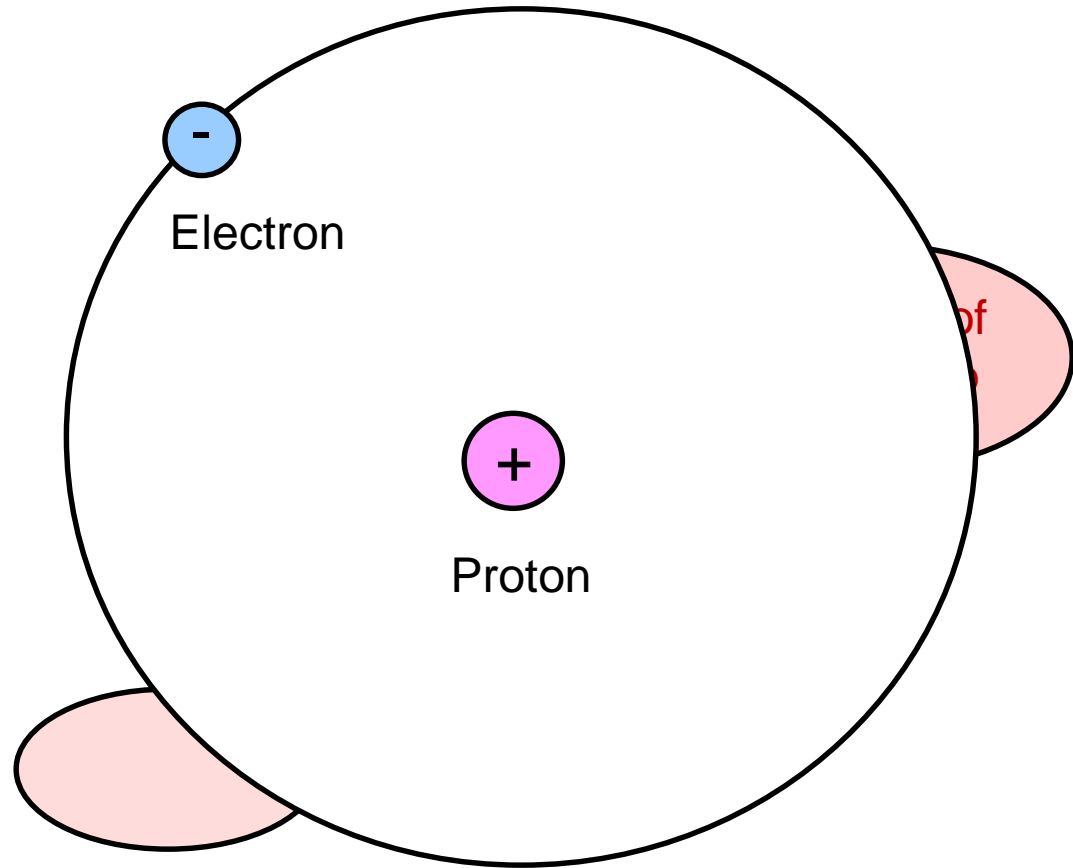
CODATA: Committee on Data for Science and Technology, the international group which publishes the recommended values for fundamental physical constants every four years.

The Proton Radius from H Lamb Shift

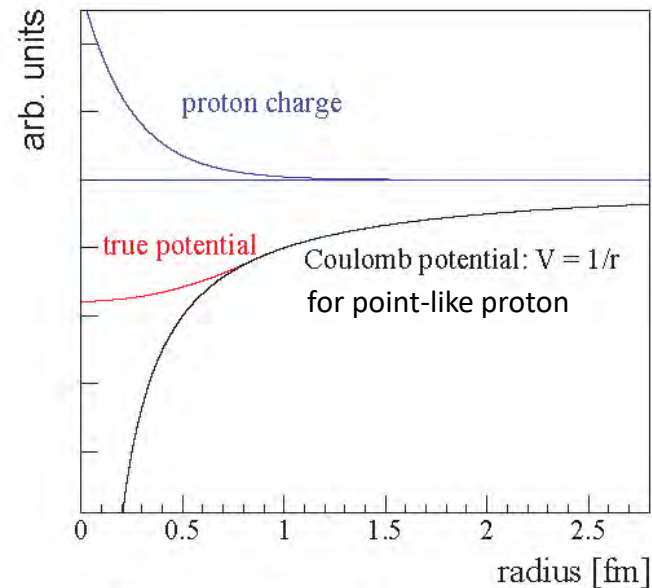
Components of a calculation



$$E = R_{\infty}/n^2$$
$$V \sim 1/r$$



Finite-size shift of atomic energy levels



S states: max. at $r=0$

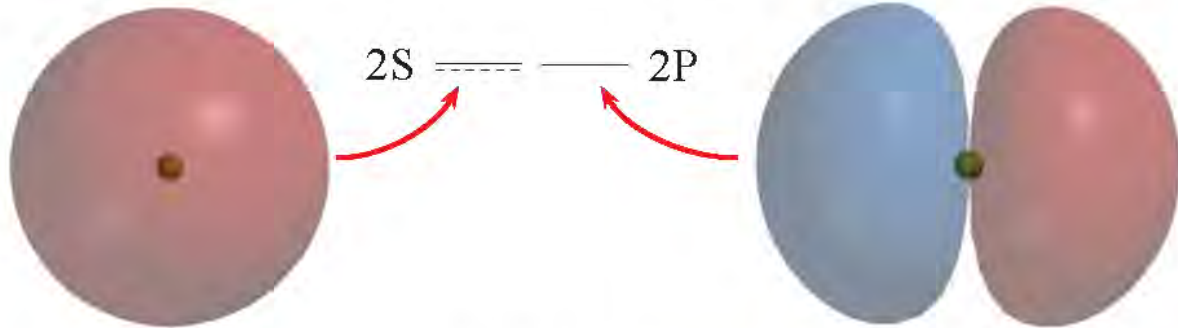
Electron sometimes **inside** the proton.

S states are shifted.

Shift is proportional to the

size of the proton

8S ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ...
4S ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ...
3S ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ≡≡≡≡ ... 3D



P states: zero at $r=0$

Electron is **not**
inside the proton.



Orbital pictures from Wikipedia

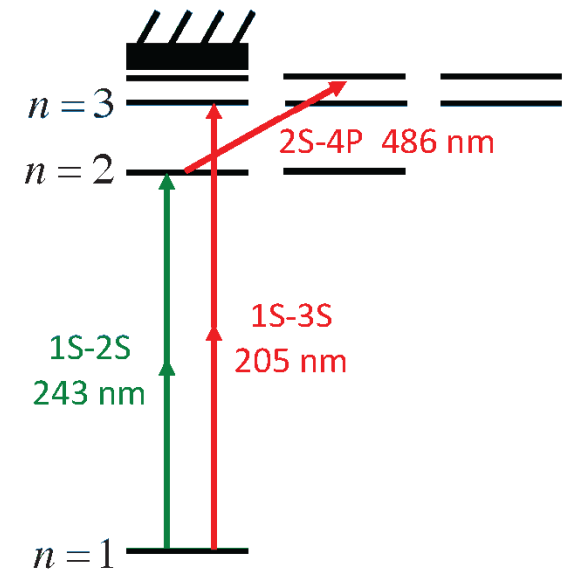
Pictures: R. Pohl

Hydrogen Atom Spectroscopy

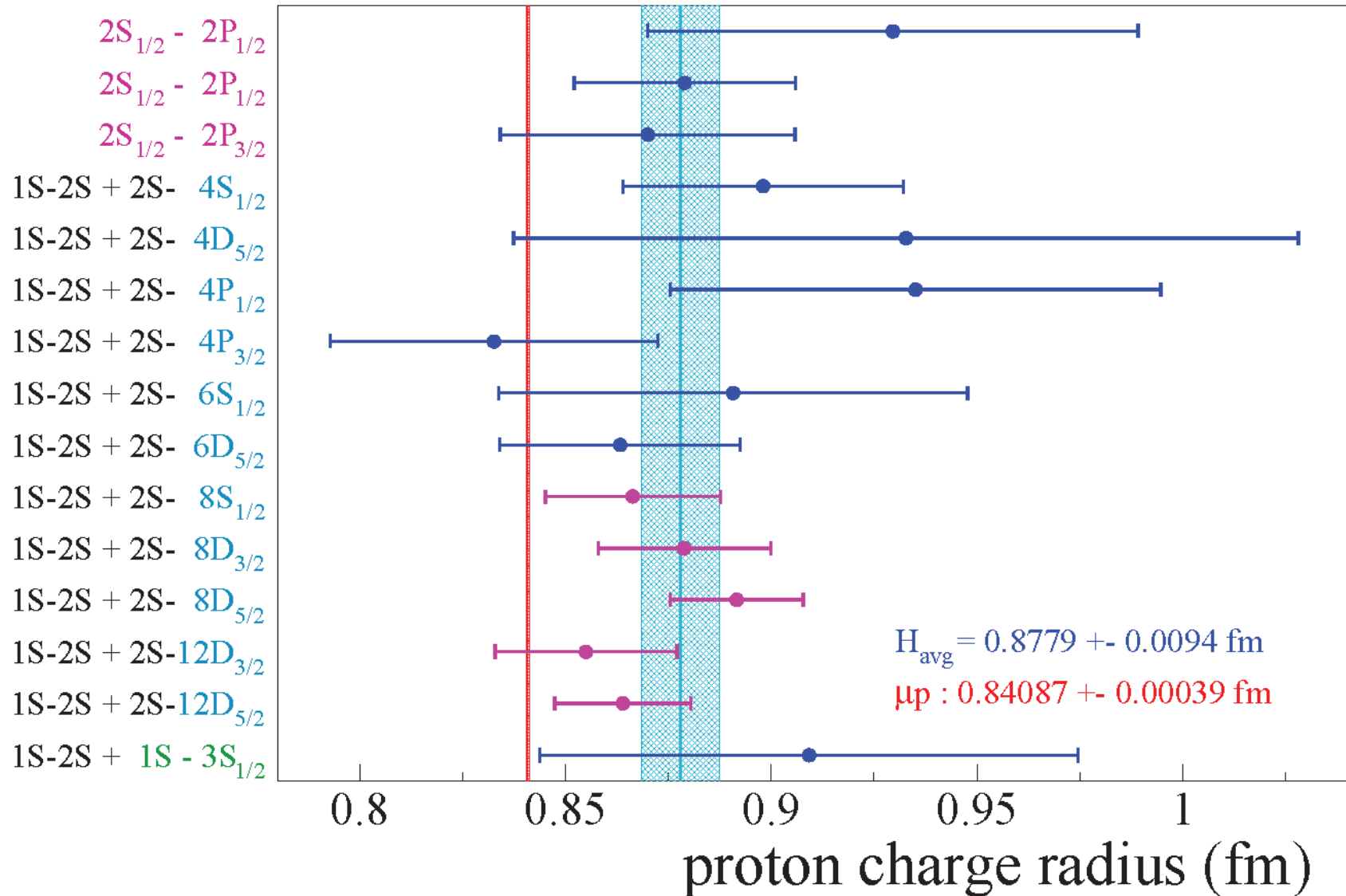
$$E_{nS} \simeq -\frac{R_\infty}{n^2} + \frac{L_{1S}}{n^3}$$

Lamb shift: $L_{1S}(r_p) = 8171.636(4) + 1.5645 \langle r_p^2 \rangle$ MHz

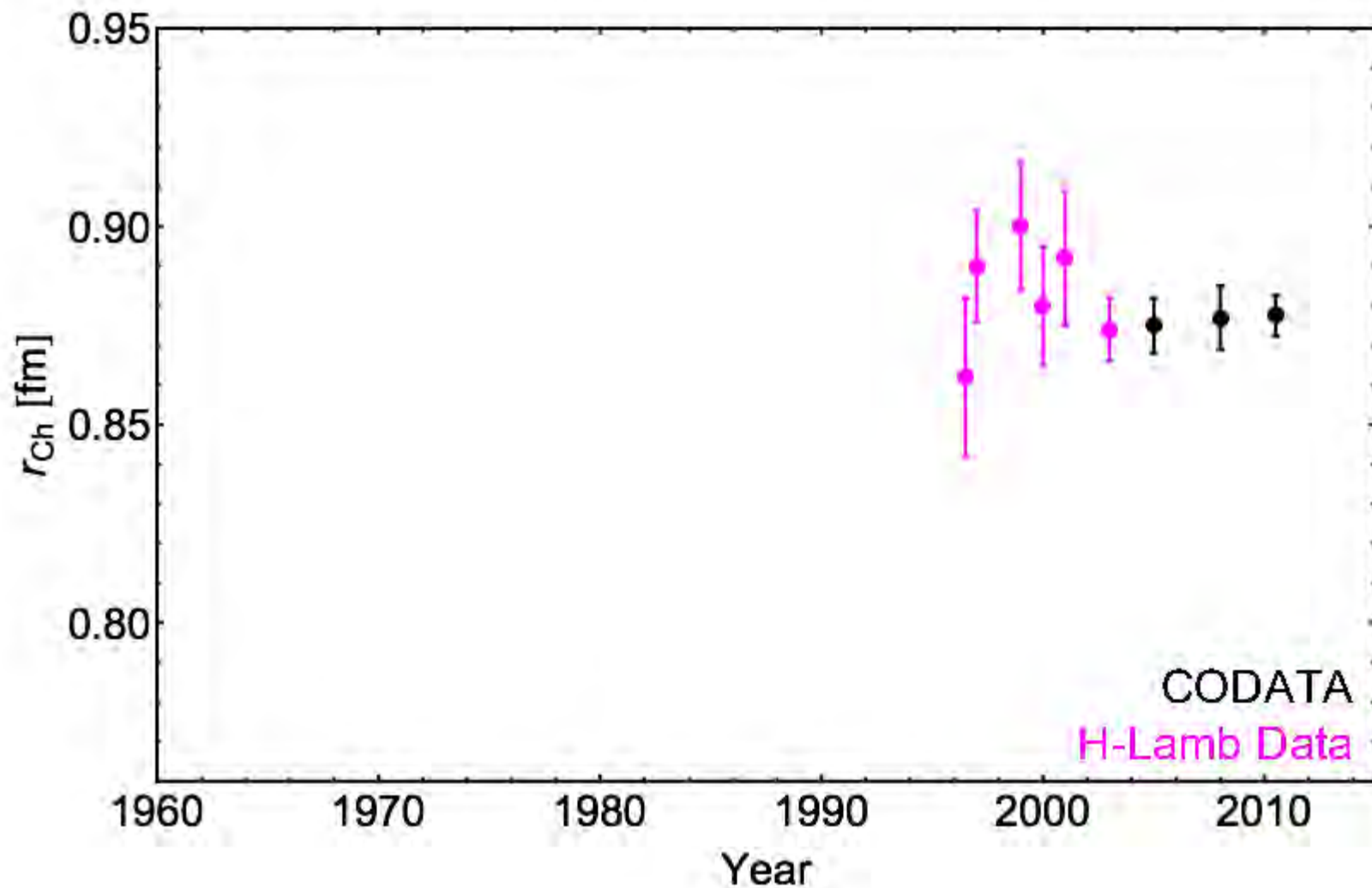
- 2 measurements required to determine R_∞ and r_p
 - A **single** narrow transition: 1S-2S ($\Delta\nu = 1.3$ Hz) measured with high accuracy.
 - Other transitions: natural width \sim MHz.
- Each measurement, combined with 1S-2S, yields a **correlated pair** (R_∞, r_p) .



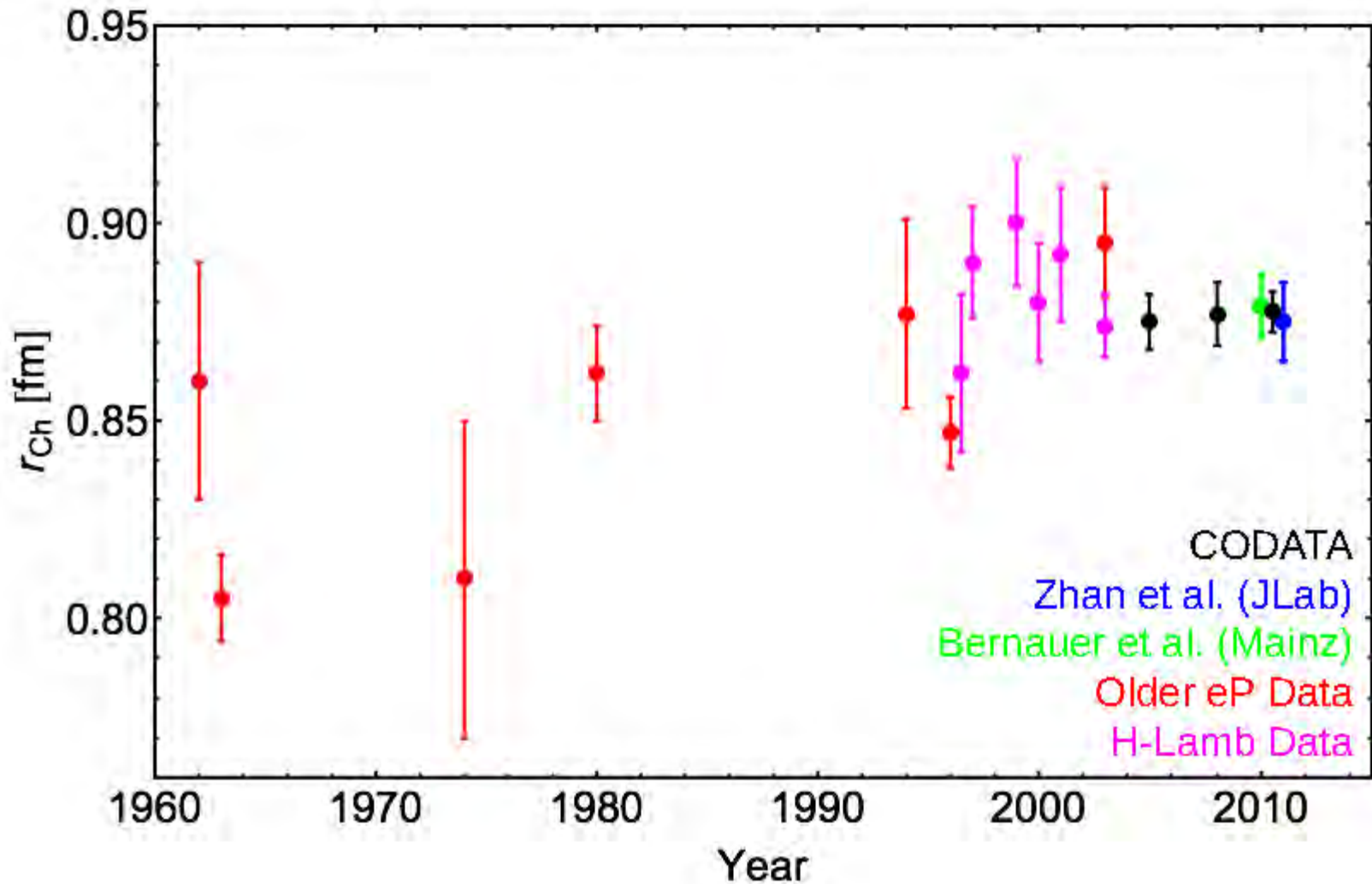
Hydrogen Atom Spectroscopy



The Proton Radius vs Time from H Lamb Shift data



The Proton Radius from H Lamb Shift and ep

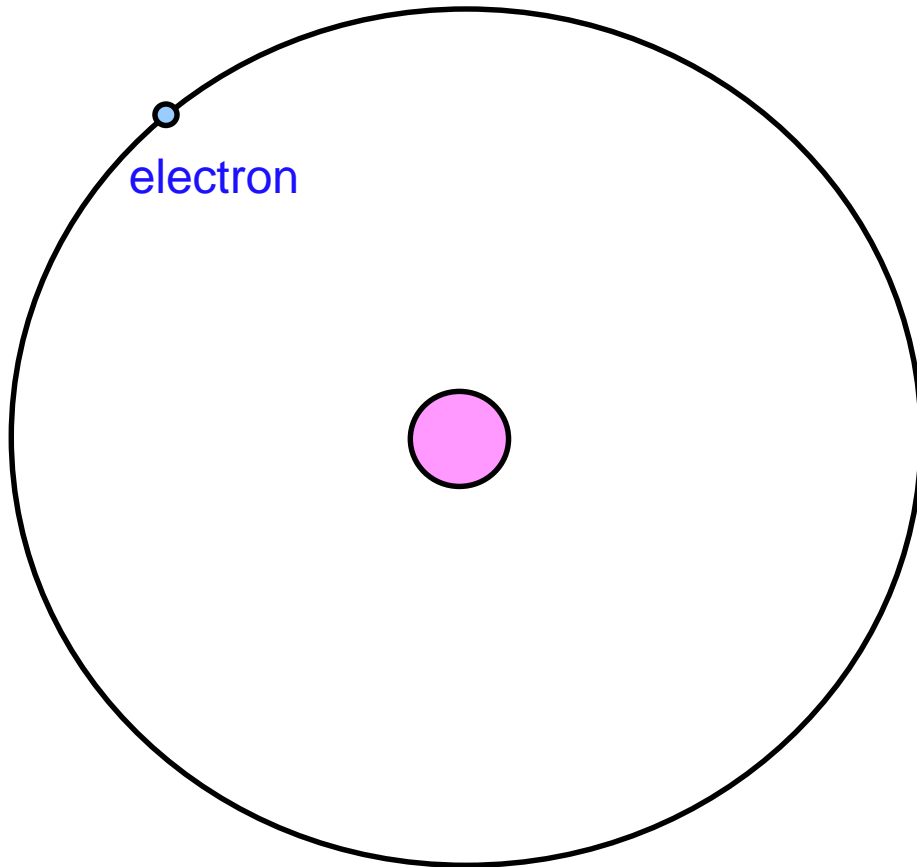


proton rms charge radius measured with electrons:
 0.8770 ± 0.0045 fm (CODATA2010+Zhan et al.)

Why Measure with μH ?

Regular hydrogen:

electron e^- + proton p



Muonic hydrogen:

muon μ^- + proton p

muon mass $m_\mu = 207 m_e$

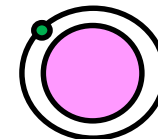
Bohr radius $a_{B,\mu} = 1/207 a_{B,e}$

Probability for μ^- to be inside proton:

$$\cong \left(\frac{r_p}{a_B} \right)^3 = (r_p \alpha)^3 m^3$$

$\rightarrow 207^3 \approx 8 \text{ million}$

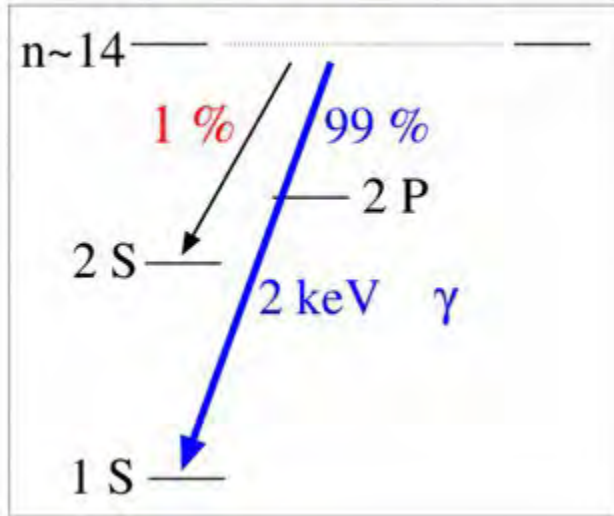
muon



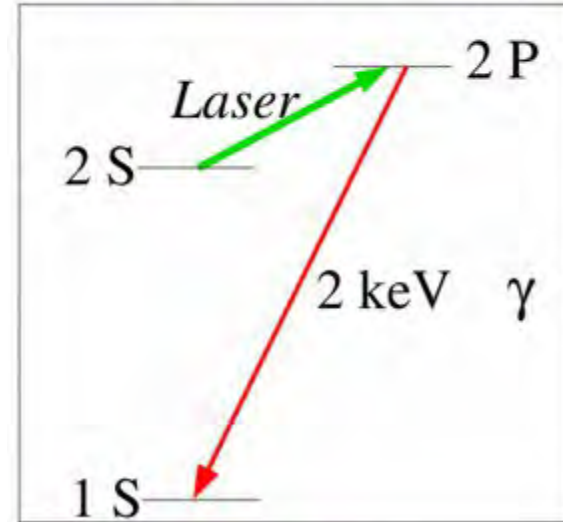
muon is **much** more sensitive to proton radius

How to Measure with μH ?

“prompt” ($t \sim 0$)



“delayed” ($t \sim 1 \mu\text{s}$)



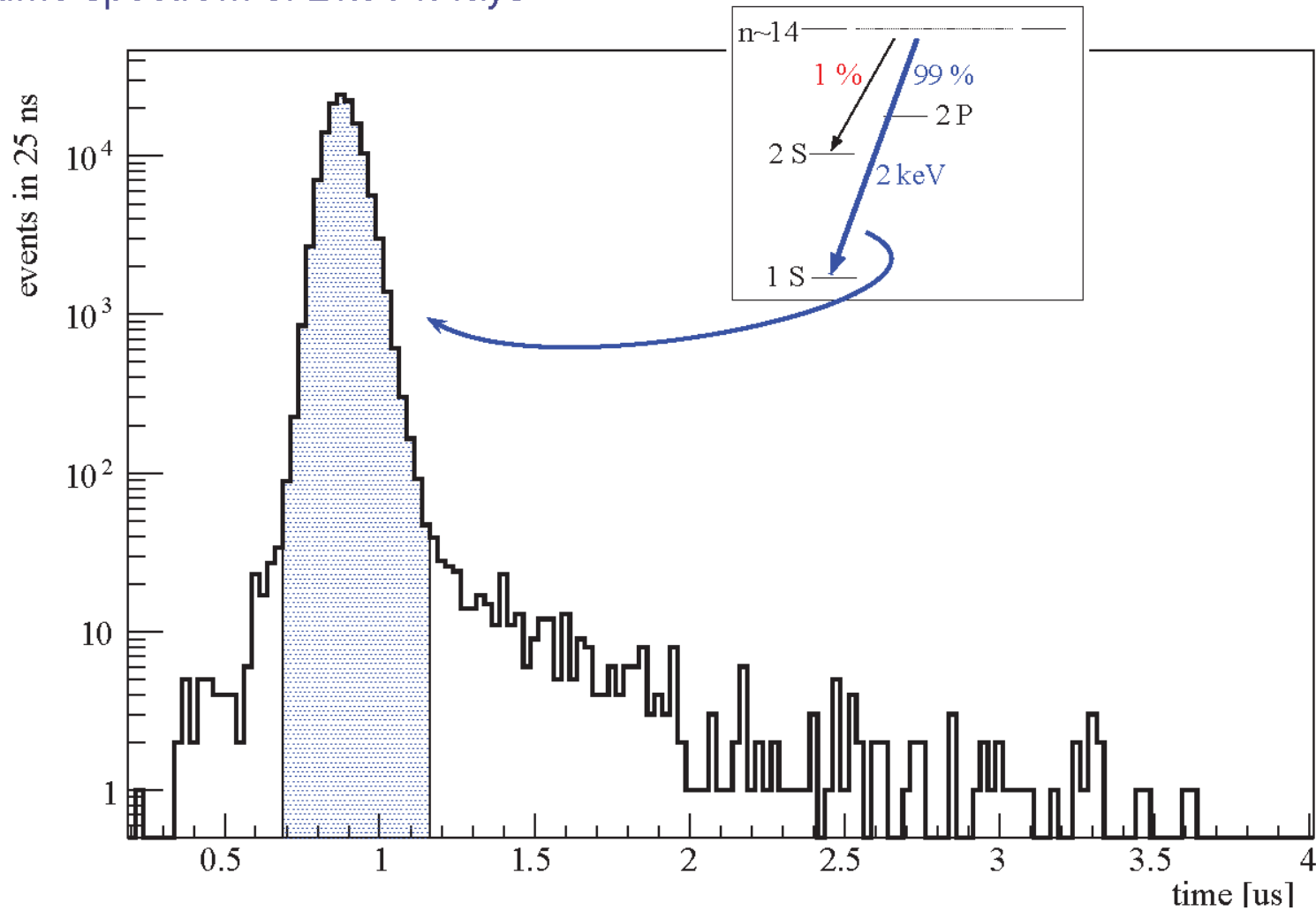
- beautifully simple, but technically challenging!
- form $\mu\text{H}^*(n \sim 14)$ by shooting μ beam on 1 mbar H_2 target
 - 99% decay to 1S, giving out fast γ pulse
 - 1% decay to longer-lived 2S state
 - S2 state excited to 2P state by tuned laser & decay with release of delayed γ
- vary laser frequency to find transition peak $\rightarrow \Delta E$ (2S to 2P) $\rightarrow r_p$

Pictures: R. Pohl

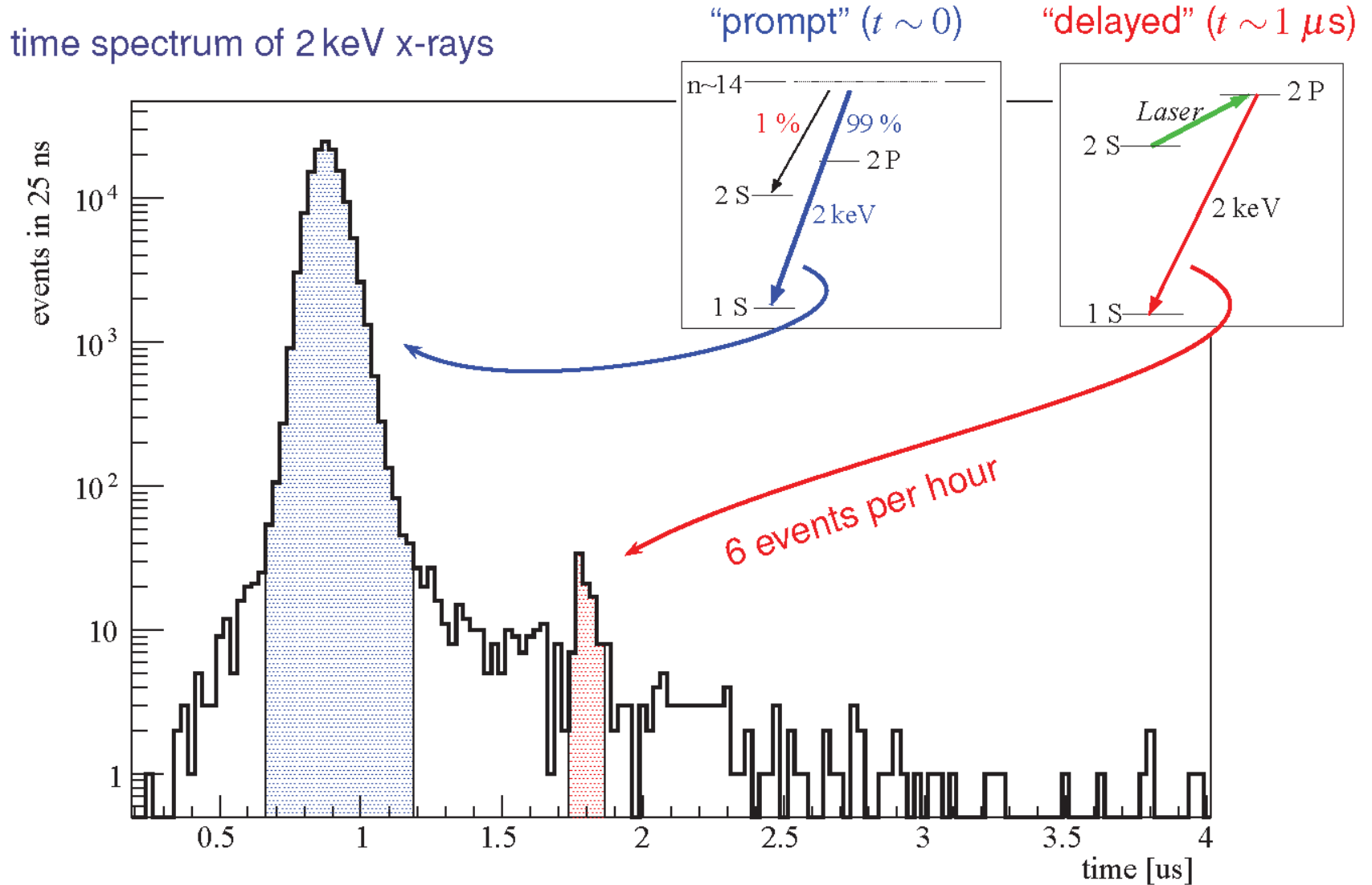
How to Measure with μH ?

time spectrum of 2 keV x-rays

"prompt" ($t \sim 0$)

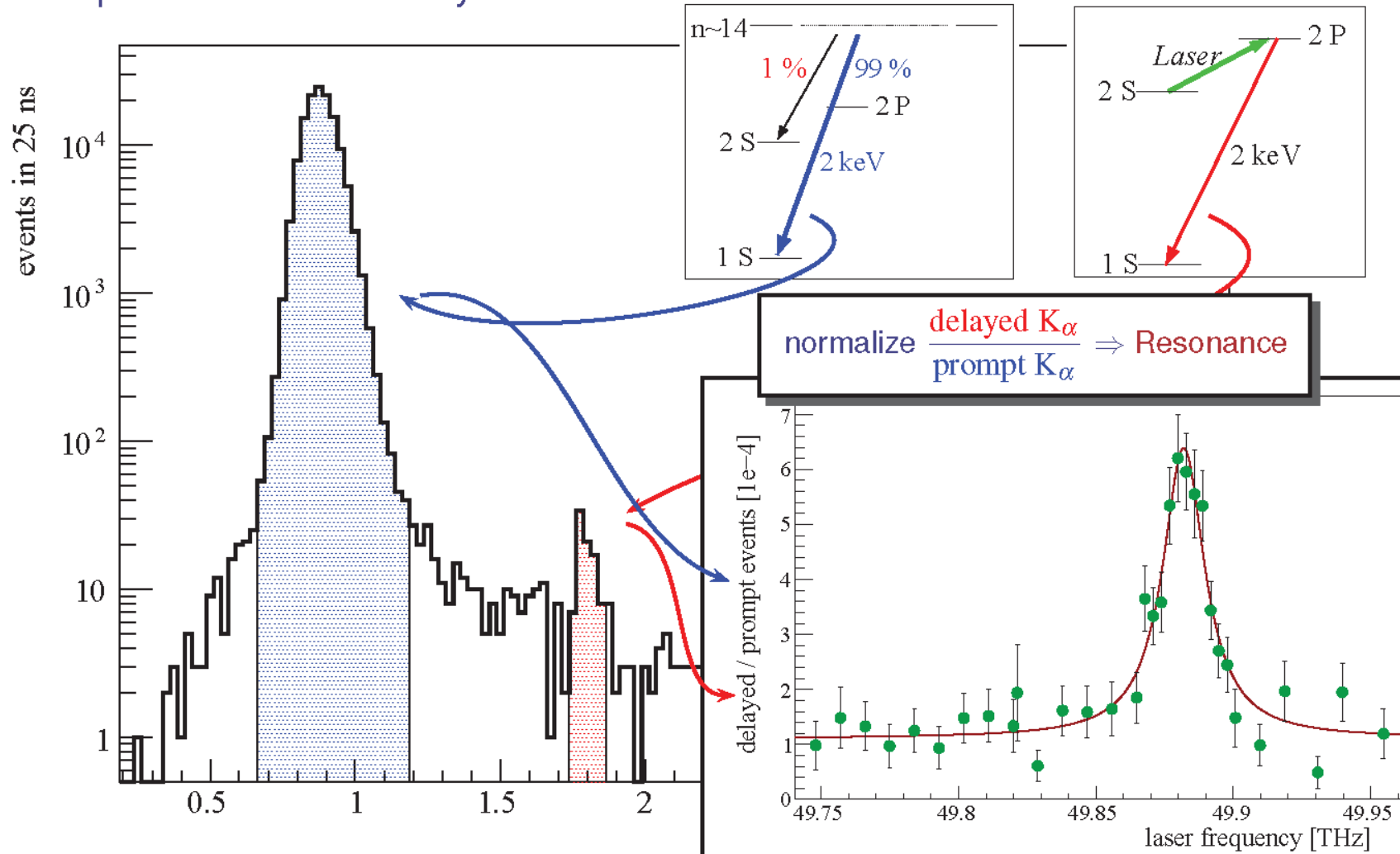


How to Measure with μH ?



How to Measure with μH ?

time spectrum of 2 keV x-rays

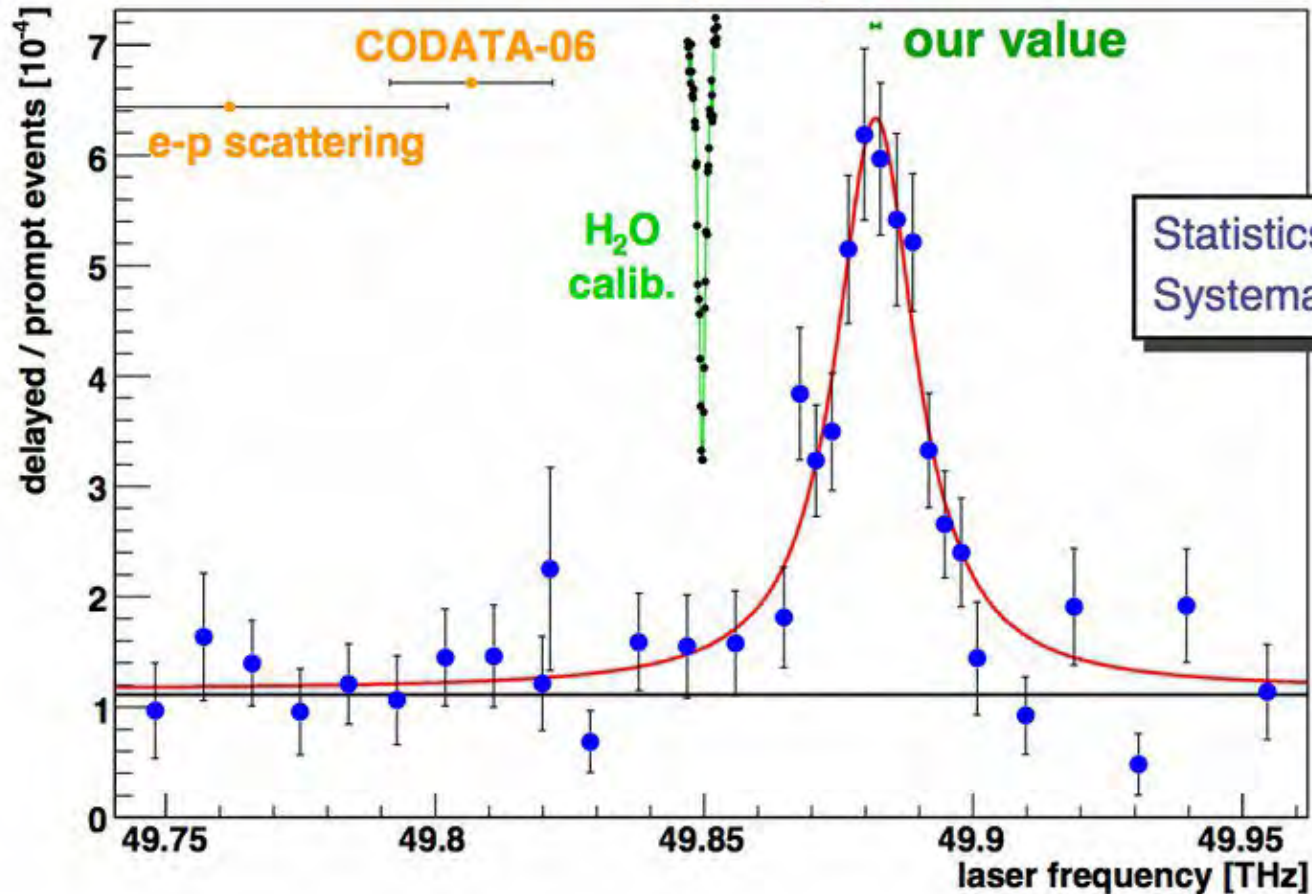


$$\Delta E(2P_{3/2}^{F=2} - 2S_{1/2}^{F=1}) = 209.9779(49) - 5.2262r_p^2 + 0.0347r_p^3 \text{ [meV]}$$

Proton Radius from μH (CREMA)

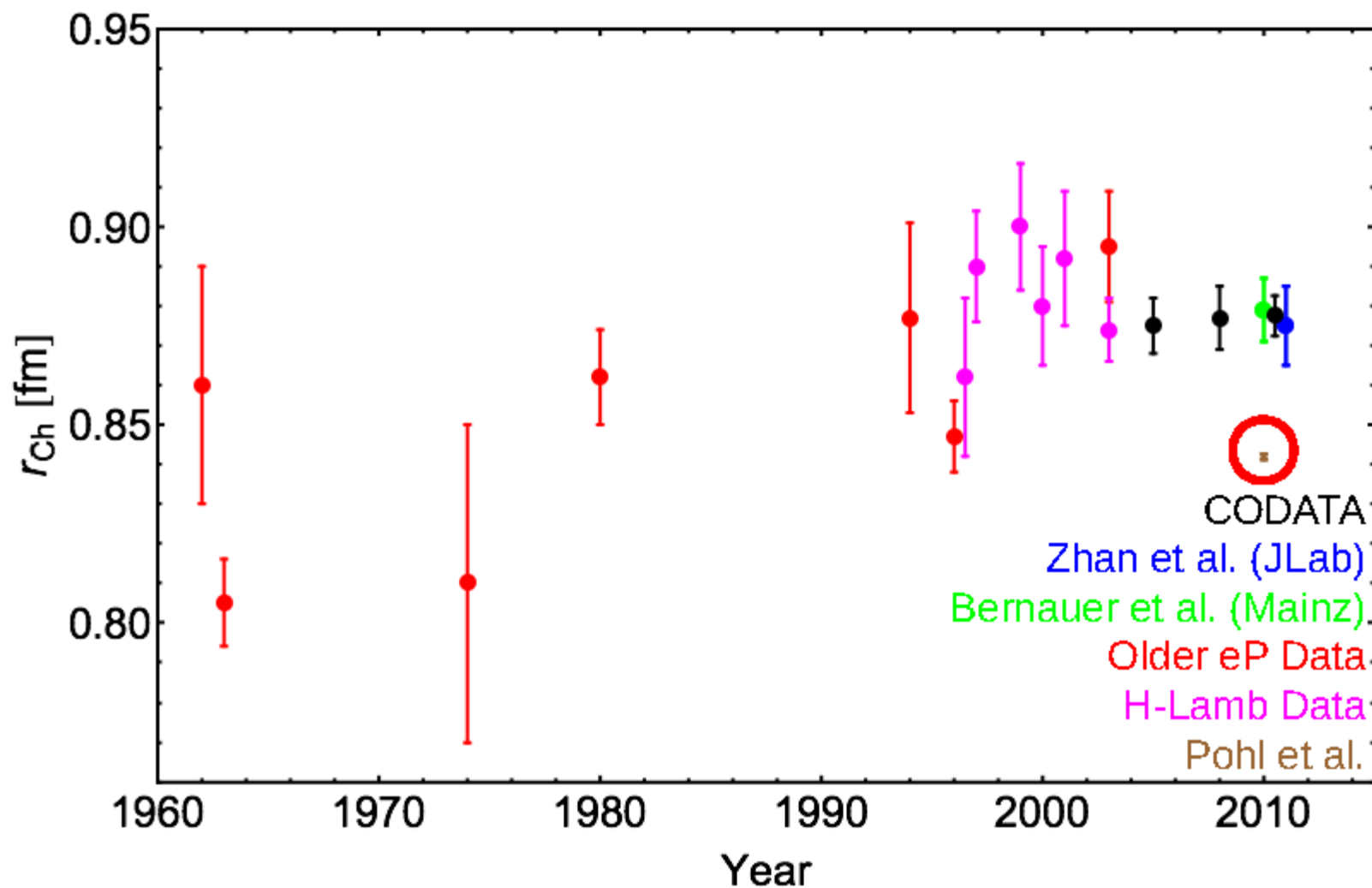
Water-line/laser wavelength:
300 MHz uncertainty

$\Delta\nu$ water-line to resonance:
200 kHz uncertainty



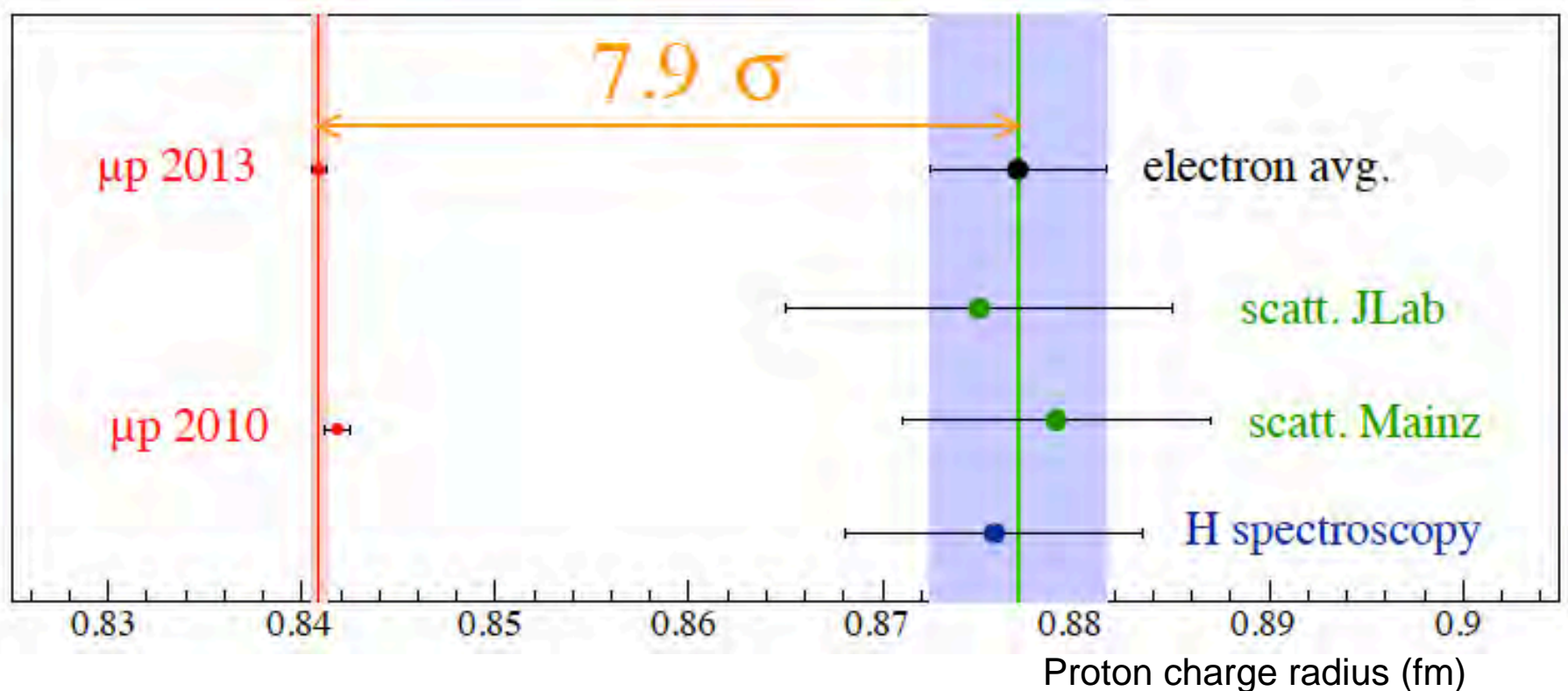
R. Pohl et al., Nature 466, 213 (2010):
 0.84184 ± 0.00067 fm: 5σ off 2006 CODATA

The Proton Radius from H & μ H Lamb Shift and ep



Puzzling & more Puzzling

- A. Antognini et al., Science **339**, 417 (2013)
- independent analysis of data of Pohl's 2010 data
 - magnetic radius agrees with e^- scattering data
 - electric radius in agreement with Pohl: **0.84087 ± 0.00039 fm**
 - 7.9σ from 2010 CODATA



Why do the muon and electron give different proton radii?

- Are there problems with the experimental results?
 - The ep (scattering) results are wrong
 - fit procedures not good enough, Q^2 not low enough
 - The ep (spectroscopy) results are wrong
 - Rydberg constant could be off by 5 sigma
 - The μp (spectroscopy) result is wrong
- Assuming the experimental results are not bad, what are viable theoretical explanations of the Radius Puzzle?
- **Beyond Standard Model Physics**
 - Pospelov, Yavin, Carlson, ...: the electron is measuring an EM radius, the muon measures an (EM+BSM) radius \rightarrow Lepton universality violation
- **Proton structure issues**
 - G. Miller: currently unconstrained correction proton polarizability affects μ , but not e (effect $\propto m_l^4$)
 - Off-shell proton in two-photon exchange leading to enhanced effects differing between μ and e
- Basically everything else suggested has been ruled out - missing atomic physics, structures in form factors, ...

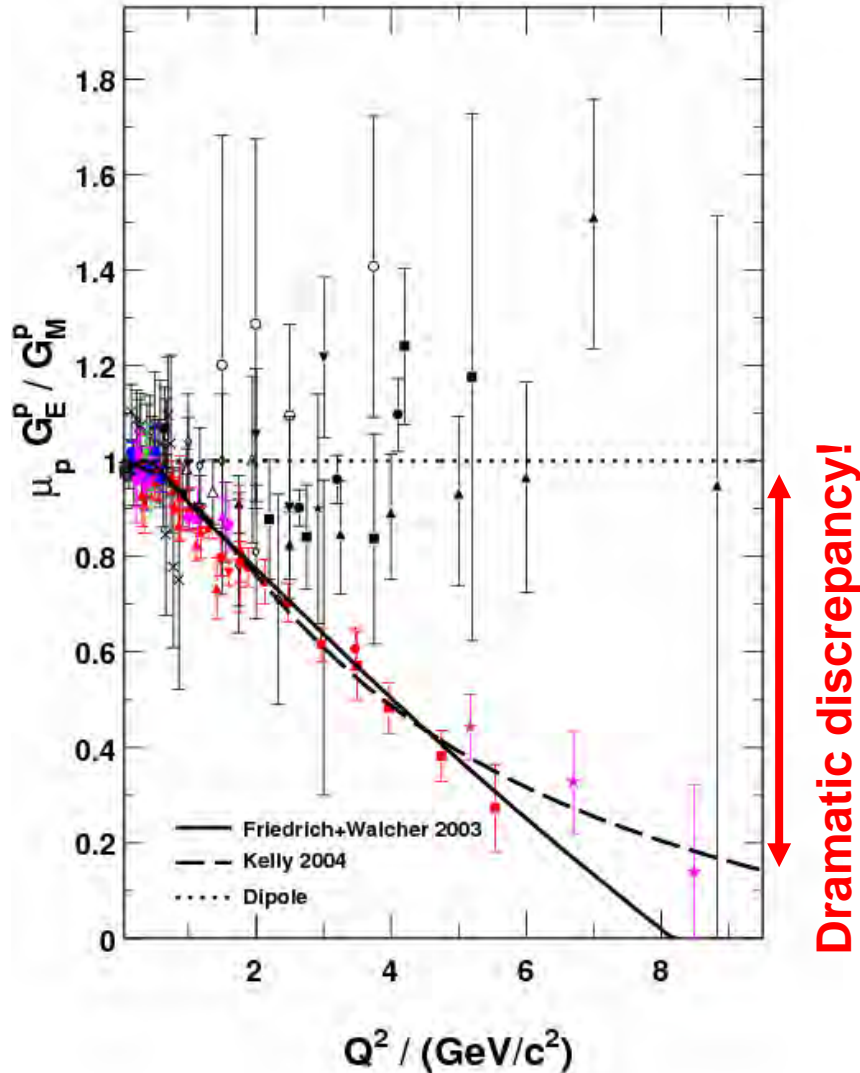
How do we Resolve the Radius Puzzle?

- New data needed to test that the e and μ are really different, and the implications of novel BSM and hadronic physics
 - **BSM**: scattering modified for Q^2 up to m_{BSM}^2 (typically expected to be MeV to 10s of MeV), enhanced parity violation
 - **Hadronic**: enhanced 2γ exchange effects
- Experiments include
 - redoing atomic hydrogen
 - light muonic atoms for radius comparison in heavier systems
 - redoing electron scattering at lower Q^2
 - Muon scattering!

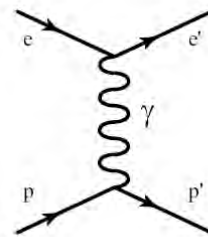
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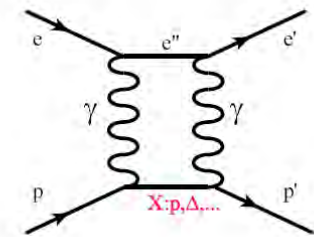
Proton Form Factor Ratio



- All Rosenbluth data from SLAC and Jlab in agreement
- Dramatic discrepancy between Rosenbluth and recoil polarization technique
- Two-photon exchange (TPE) considered best candidate
 - most prominent at high Q^2 and backward scattering angles, where cross section is suppressed



stand rad cor **independent**



TPE contributions to rad cor **not independent**

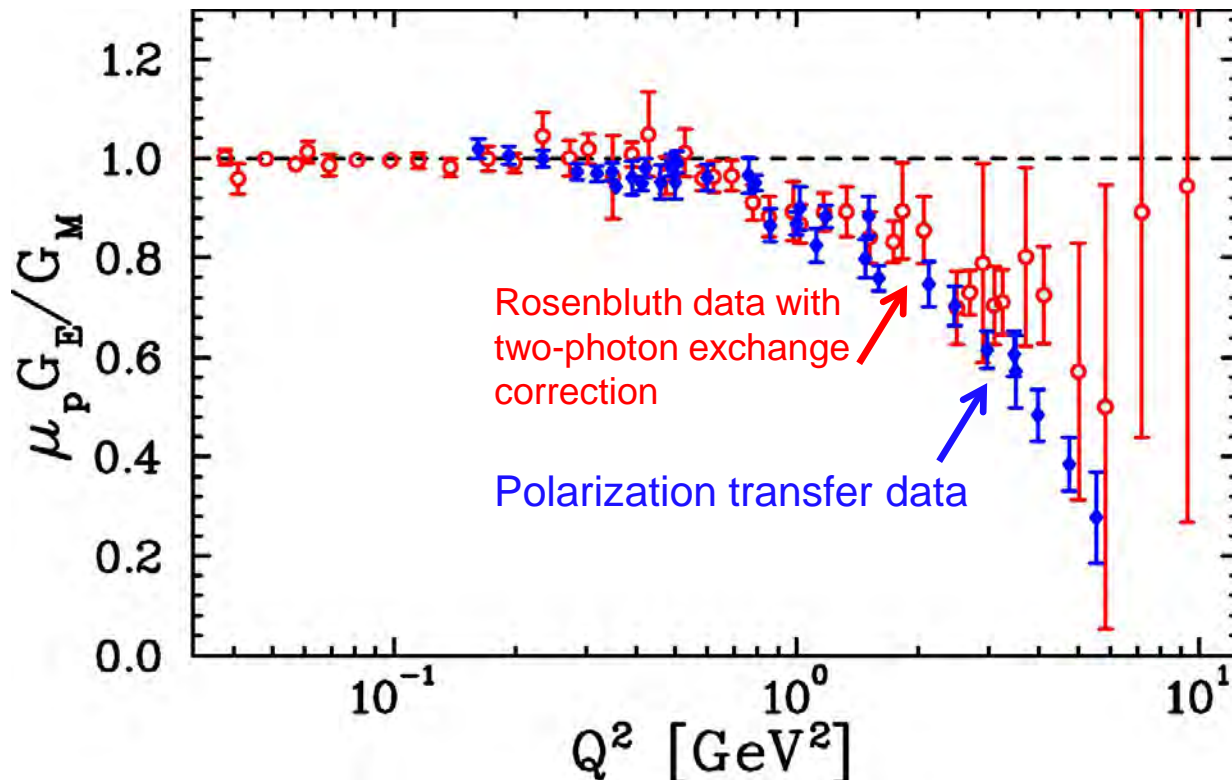
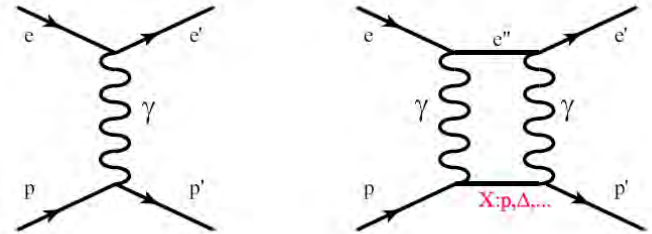
of hadronic structure

Two-photon exchange: exp. evidence

- TPE can explain form factor discrepancy

J. Arrington et al, PRC76, 035205 (2007)

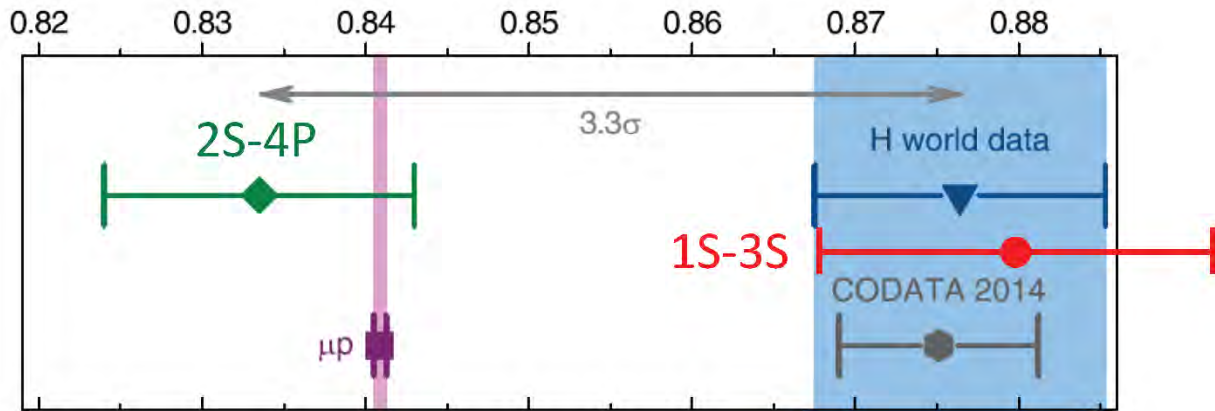
- TPE different for e^+ and e^- ?
- Are they the same for e and μ ?



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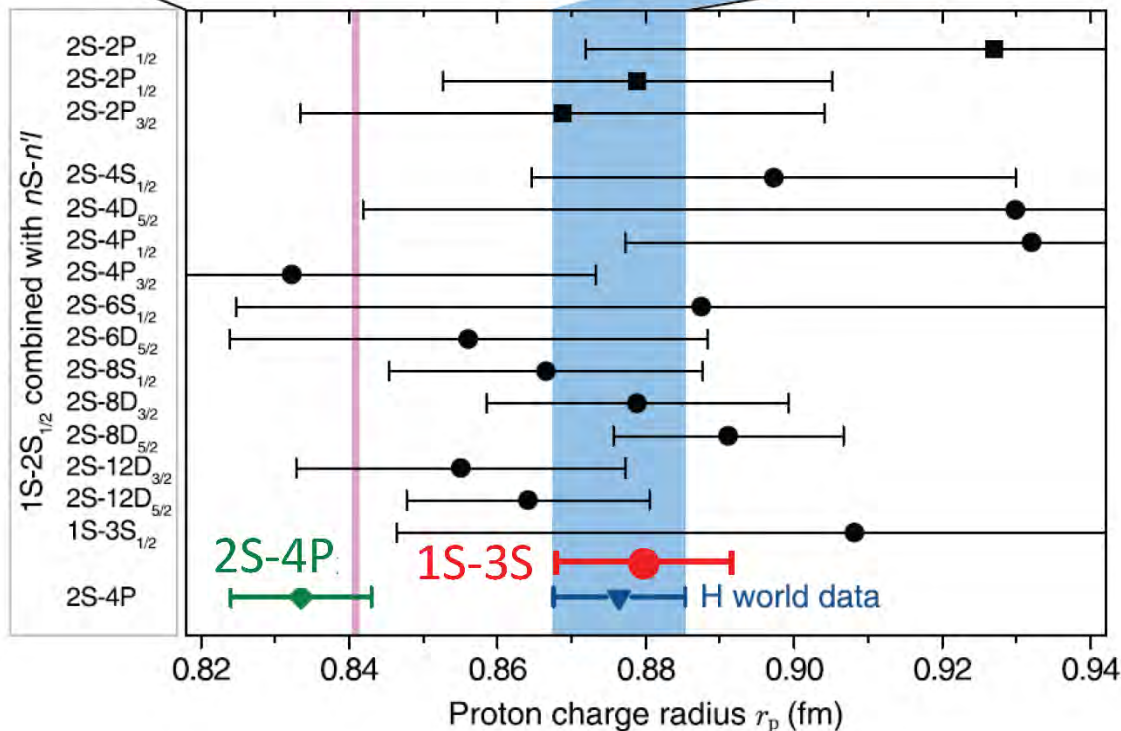
Redoing Atomic Hydrogen



MPQ (Garching): NEW proton is small in regular hydrogen, too!

LKB (Paris): Prelim.
No, it's not!

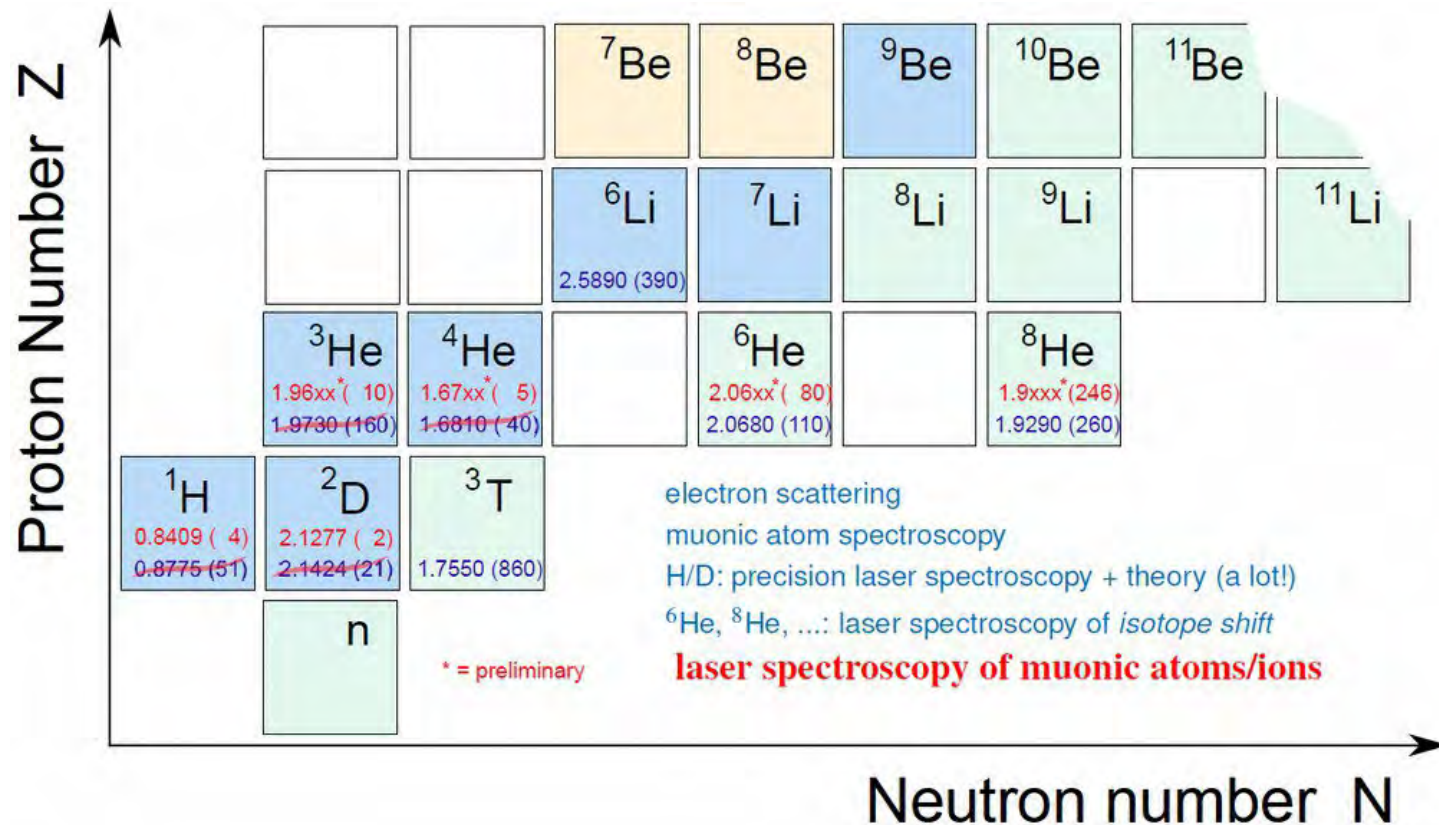
Systematics need to be carefully determined



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Light Muonic Atoms



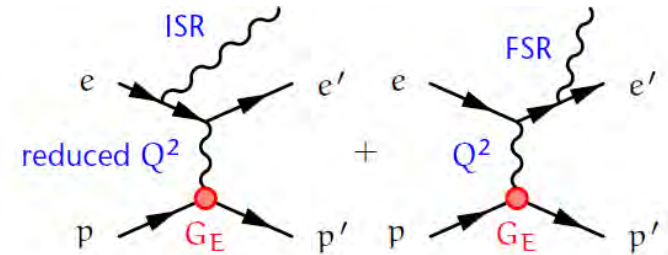
- CREMA Collaboration moved on to heavier atoms!
- Deuterium radius from μ D agrees with μ H
 - deuteron charge radius: r_d again 7σ away from CODATA
- Helium isotopes seem to agree (preliminary results)
- Puzzle seen in H & D (Z=1 radius puzzle?)

How do we Resolve the Radius Puzzle?

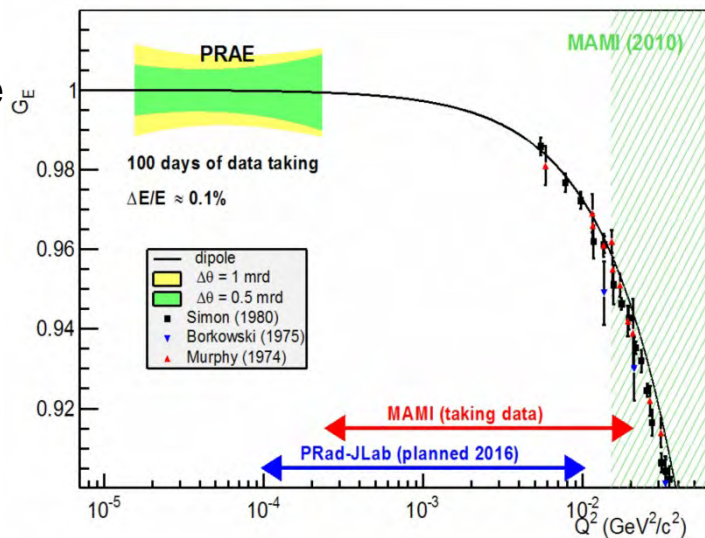
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 - **Hadronic**: enhanced 2γ exchange effects γ
- Experiments include
 - redoing atomic hydrogen
 - light muonic atoms for radius comparison in heavier systems
 - **redoing electron scattering at lower Q^2**
NB: Many efforts, not an exhaustive list!!!!
 - Muon scattering!

Redoing electron scattering at lower Q^2

- Jlab: PRad
 - low intensity beam in Hall B @ JLab into windowless gas target (1.3 billion H events)
 - Preliminary G_E slope seems to favor smaller radius (but syst errors still too large)
- Mainz: ISR
 - exploit information in radiative tail
 - dominated by coherent sum of ISR and FSR
 - investigate G_E down to $Q^2 = 10^{-4} \text{ GeV}^2/c^2$
 - results not precise enough → upgrades underway



- LPSC, Grenoble: ProRad
 - New accelerator to be built in France
 - constrain Q^2 -dependence of G_E and extrapolation to zero
 - non-magnetic spectrometer, frozen hydrogen wire / film target



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Motivation for μp scattering

Electronic hydrogen

0.8758 ± 0.0077

Spectroscopy

Muonic hydrogen

0.84087 ± 0.00039



Electron scattering

0.8770 ± 0.0060

Scattering

Muon scattering

???

How do we Resolve the Radius Puzzle?

- New data needed to test that the e and μ are really different, and the implications of novel BSM and hadronic physics

→ **BSM:** scattering modified for Q^2 up to m_{BSM}^2 (typically expected to be MeV to 10s of MeV), enhanced parity violation

→ **Hadronic:** enhanced 2γ exchange effects

MUSE will test

- Experiments include

→ redoing atomic hydrogen

→ light muonic atoms for radius comparison in heavier systems

→ redoing electron scattering at lower Q^2

→ **Muon scattering!**

Done



MUon Scattering Experiment (MUSE) at PSI



Paul Scherrer Institute
Villigen, Switzerland

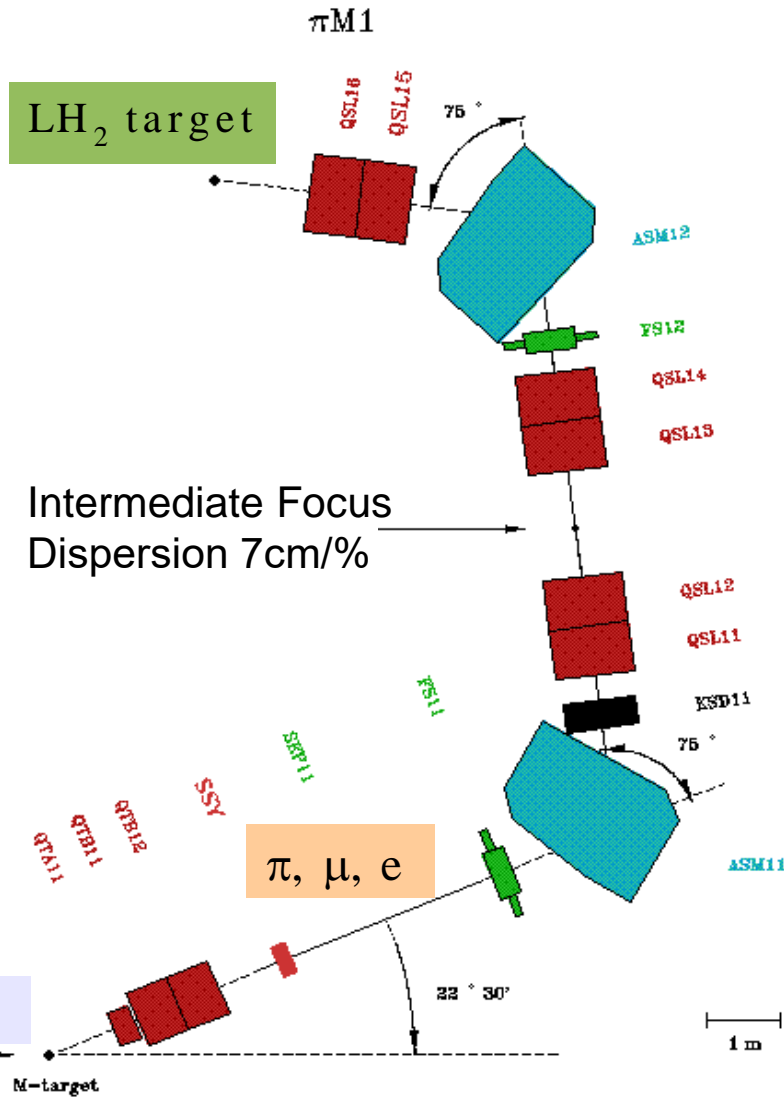
- Simultaneous measurement of $e^+/\mu^+ e^-/\mu^-$ at beam momenta of 115, 153, 210 MeV/c in $\pi M1$ channel at PSI allows:
 - Simultaneous determination of proton radius in both ep and μp scattering
 - Test of Lepton Universality
 - Determination of two photon effects
 - Separation of G_E and G_M (Rosenbluth)

Paul Scherrer Institute π M1 Beam

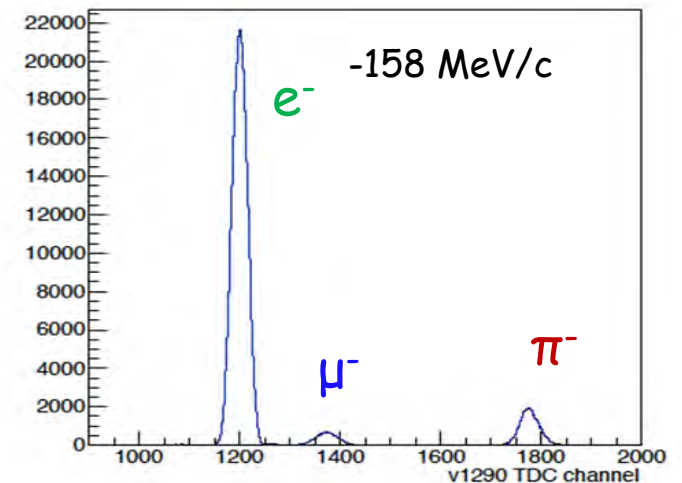
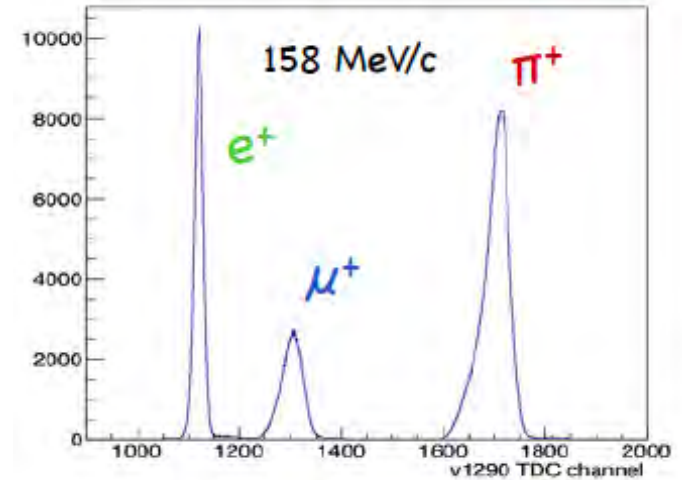


- 590 MeV proton beam, 2.2 mA, 1.3 MW beam, 50.6 MHz RF frequency
- World's most powerful proton beam
- Converted to π^\pm , μ^\pm , e^\pm in π M1 beamline
- Separate out particle species by timing relative to beam RF
- Remove as many pions as possible, trigger on e^\pm , μ^\pm

π M1 / MUSE beamline



π M1: 100-500 MeV/c RF+TOF separated π, μ, e



MUSE experiment layout

- Beam particle tracking
- Liquid hydrogen target
- Scattered lepton detection

Measure $e^\pm p$ and $\mu^\pm p$
elastic scattering

$$p \approx 115, 153, 210 \text{ MeV}/c$$

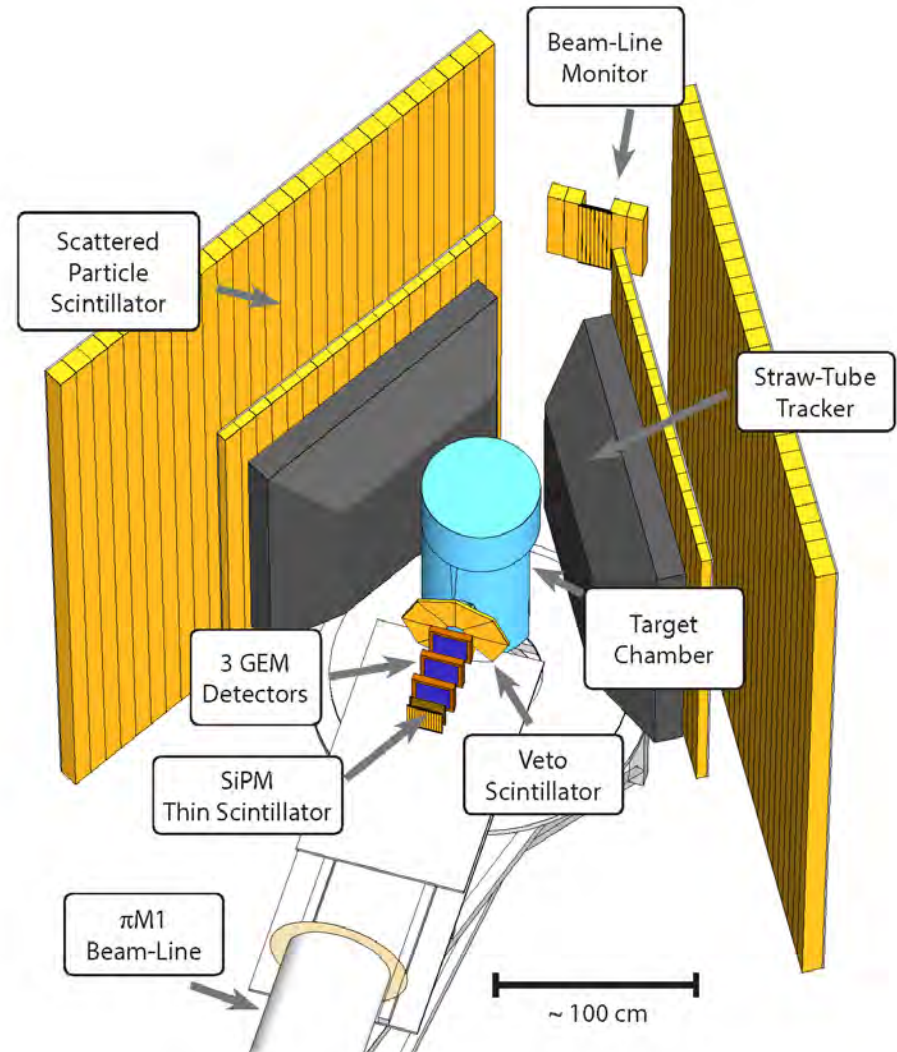
$$\theta \approx 20^\circ - 100^\circ$$

$$Q^2 \approx 0.002 - 0.07 \text{ (GeV}/c)^2$$

$$\epsilon \approx 0.256 - 0.94$$

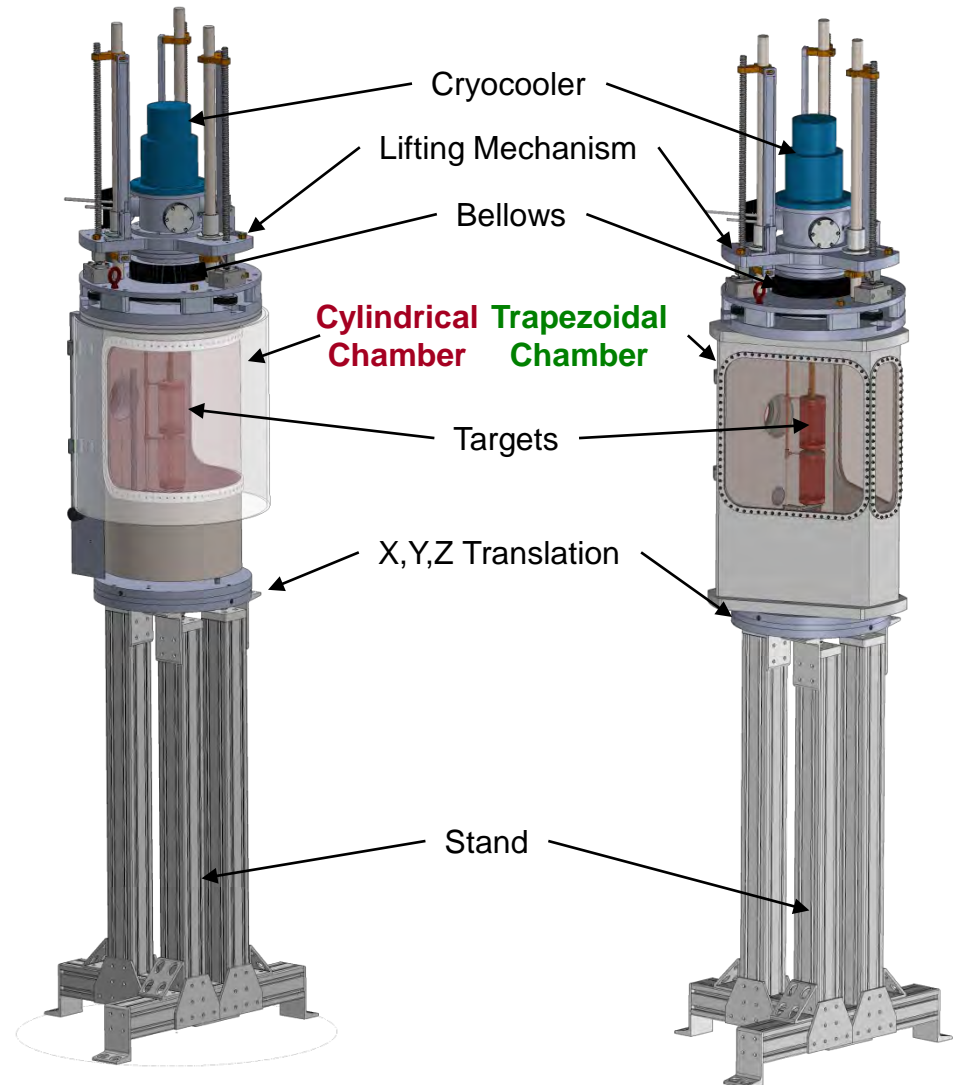
Challenges

- Secondary beam with π background
- Non-magnetic spectrometer
- Background from Møller scattering and muon decay in flight



MUSE Target Design

- Two chamber designs have been considered
 - **Cylindrical chamber** with a single wrap-around exit window
 - **Trapezoidal chamber** with three discrete exit windows
- Both designs use similar stands, target assemblies, and lifting lid assemblies
- Physicists prefer cylindrical chamber
- Engineers prefer trapezoidal chamber



Unsupported Windows form Pleats

- 127 μm Kapton window deflecting inward about 2.5" (6.35 cm) at about 0.5 atm
- C785 sailcloth (258 μm Kapton equivalent) at 1 atm still forms pleats



Does not work

Window Burst Shortly after Photo

Flat Windows don't form Pleats



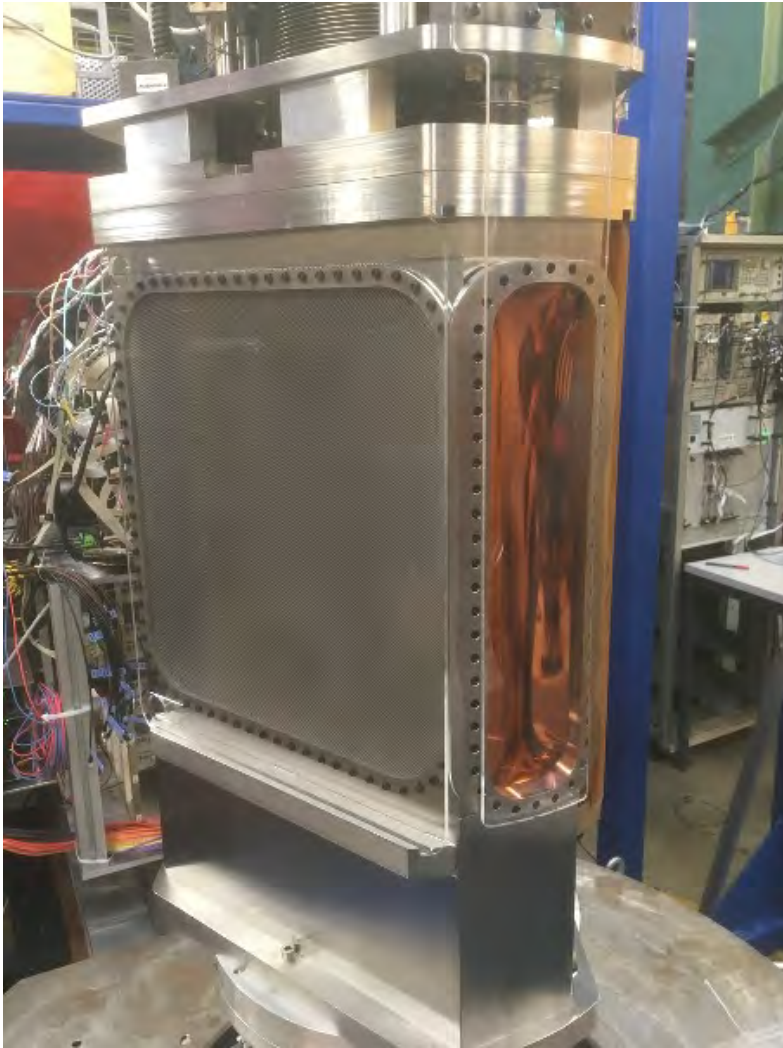
window deforms 68 mm at 1 atm



Mylar laminated on aramid fabric
window deforms 27 mm at 1 atm

Hydrogen Target

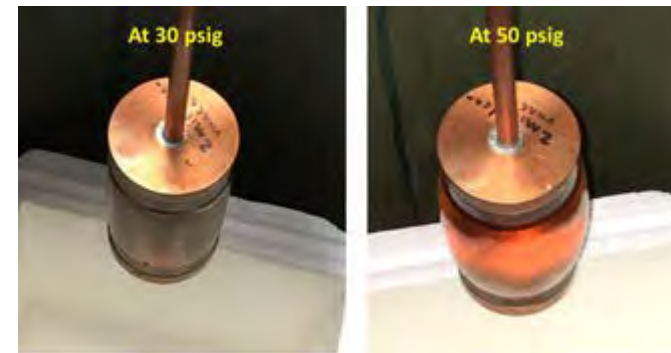
Target chamber



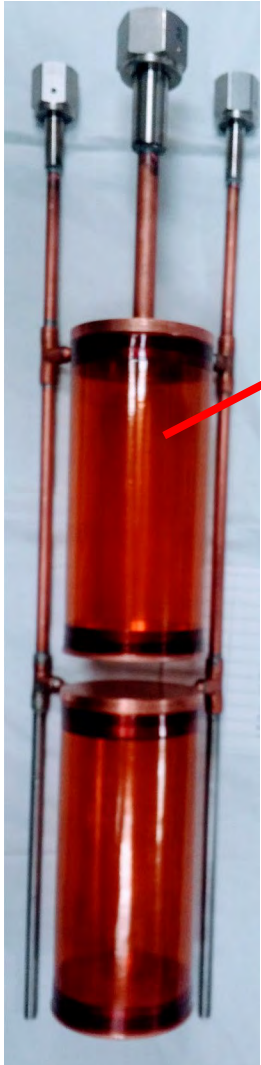
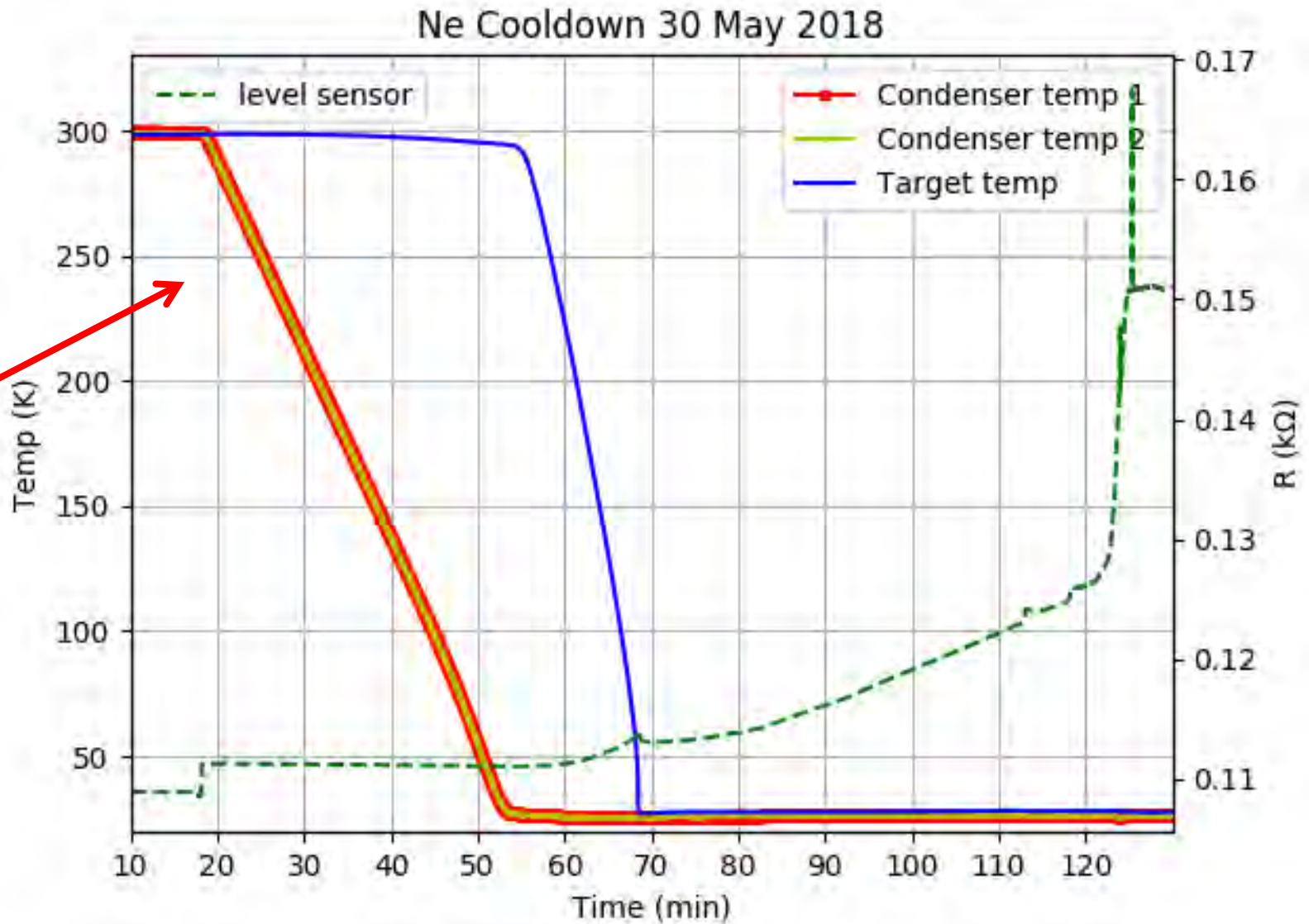
New target cells



Target cell destruction tests

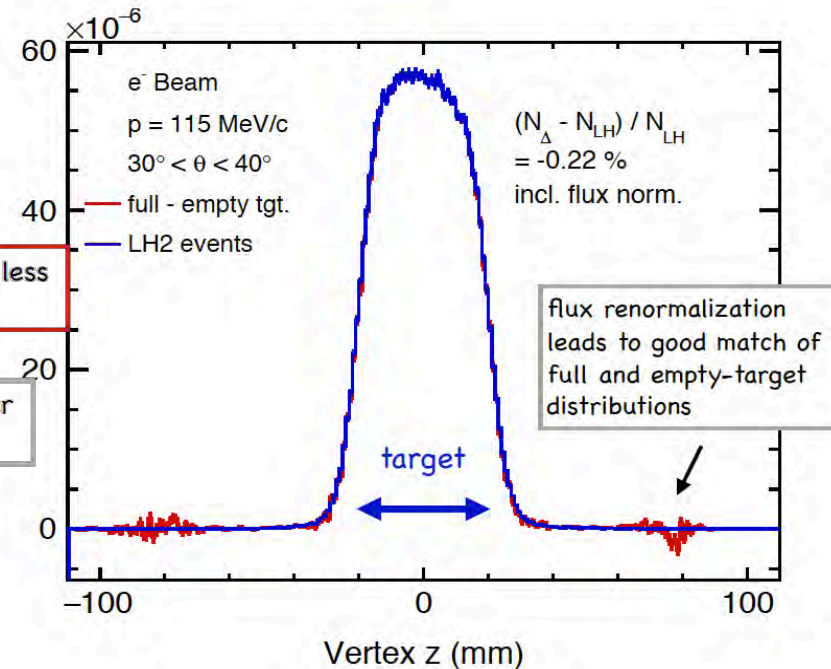
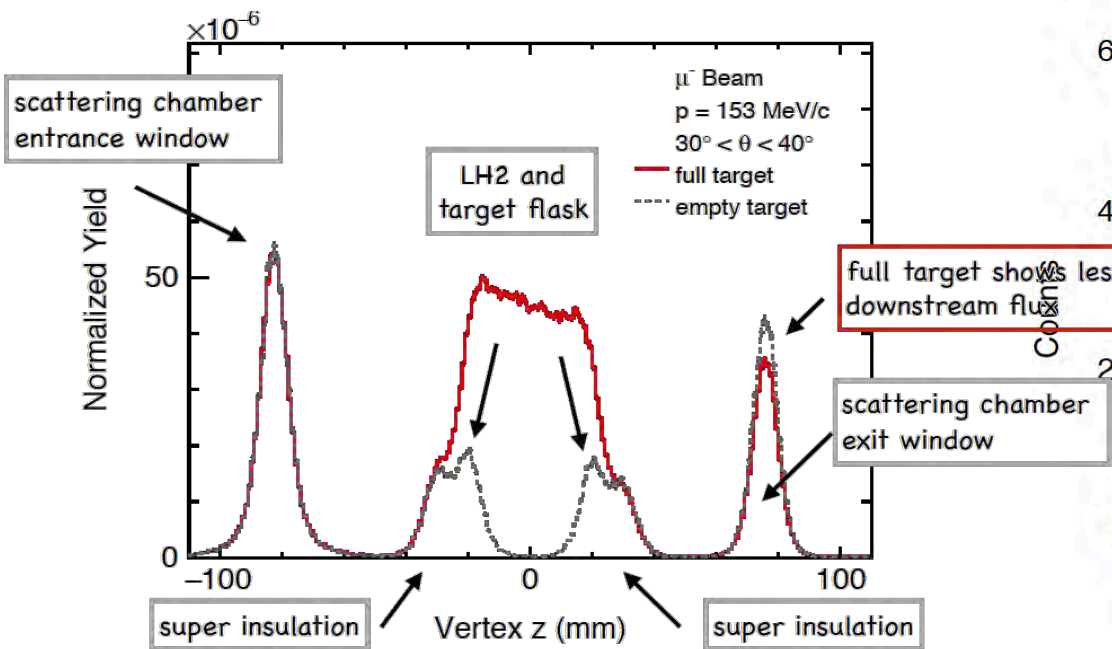
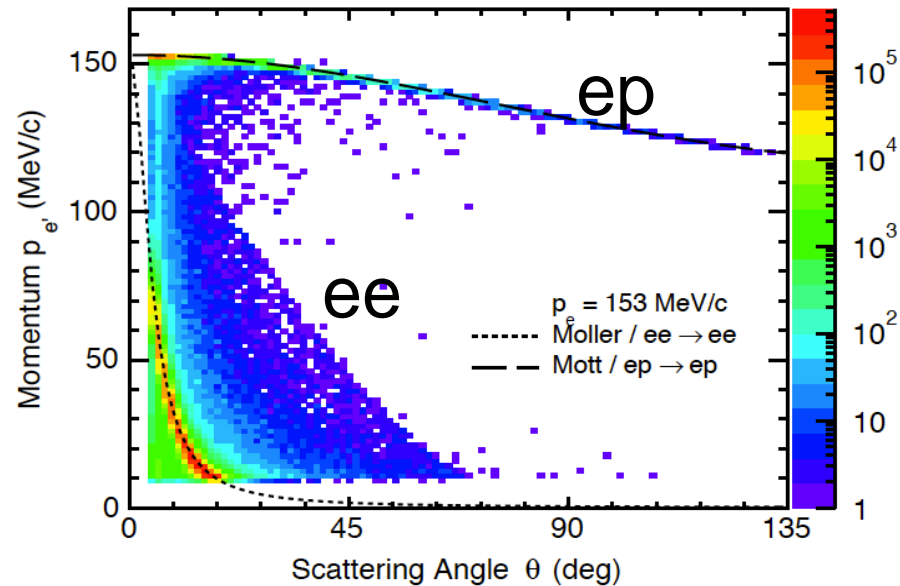


Hydrogen Target Cooldown

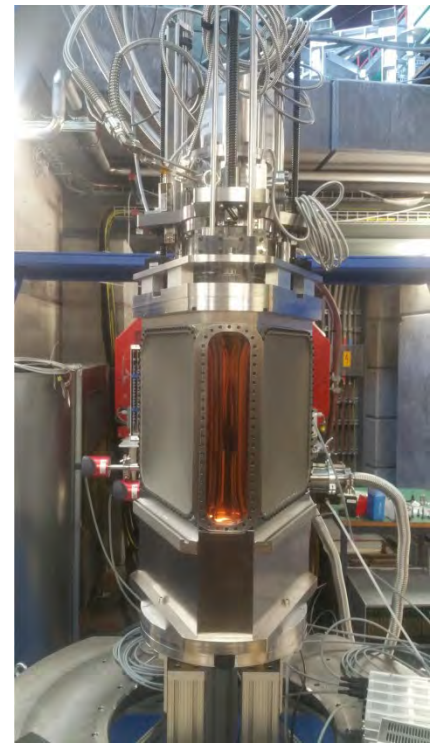
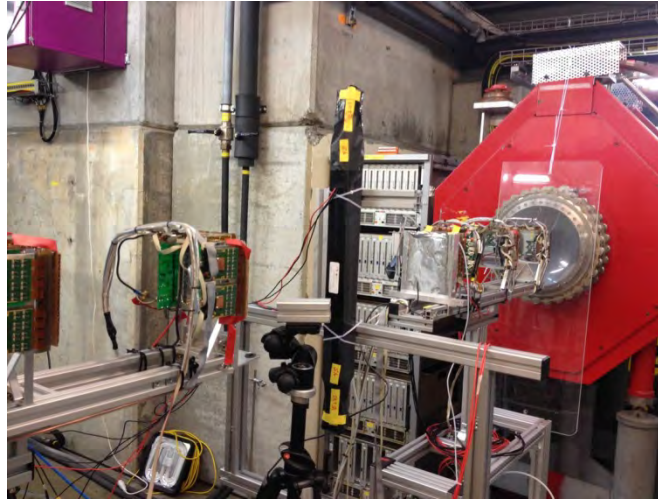
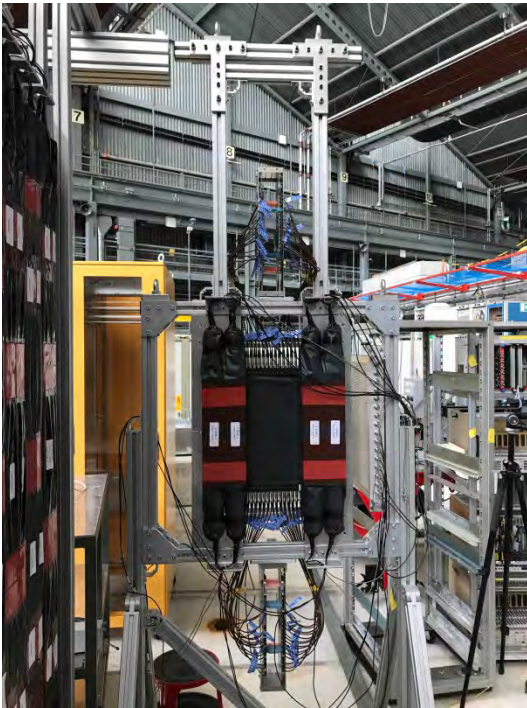


Target Simulations

- Particle vertex and scattering-angle reconstruction meet MUSE requirements
- Background from target walls and windows can be cleanly eliminated or subtracted



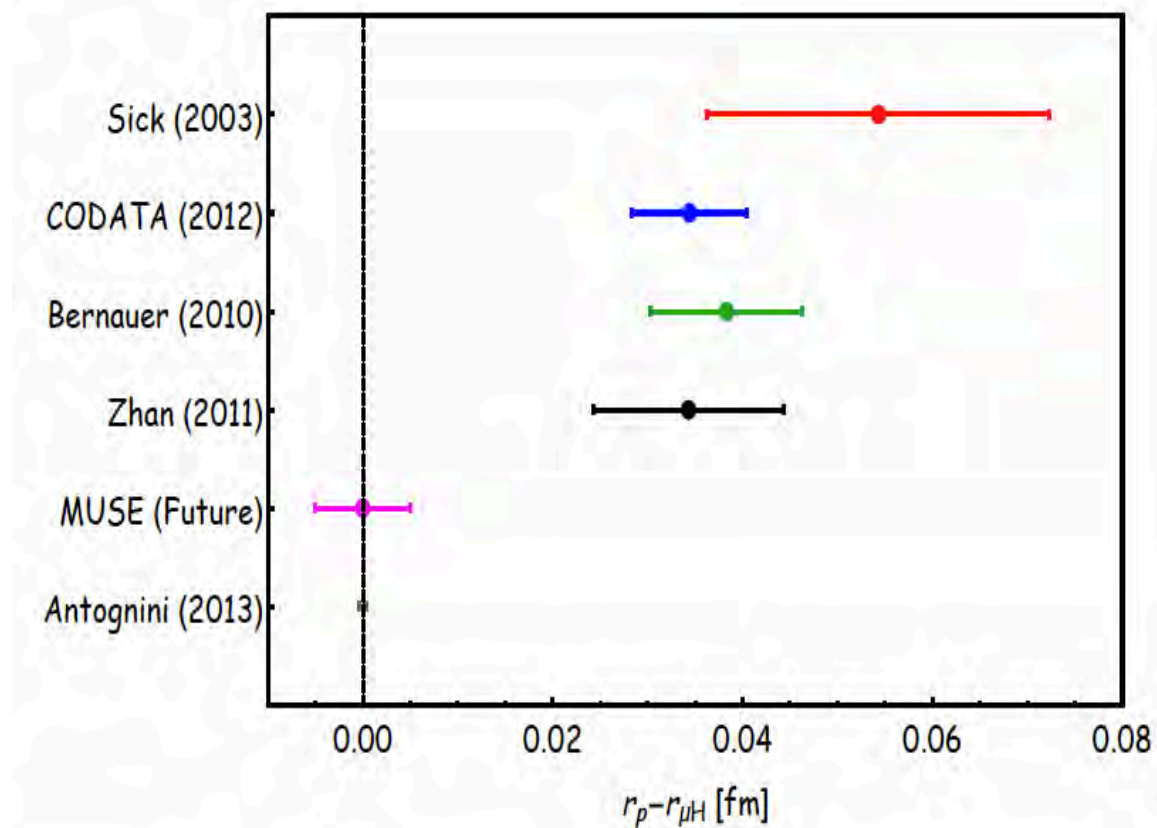
MUSE status



- 16 test runs (2012 – 2018) demonstrate simulation agreement & reliable performance
- Construction almost completed
 - Two six-month data-taking runs in 2019/20

Projected sensitivity for MUSE

- Extract radius from ep and μp form factors
- Error on radius difference ~ 0.009 fm
- MUSE will
 - verify the effect
 - compare form factors
 - compare cross sections
 - test two photon effect
 - solve the PRP?



MUon Scattering Experiment (MUSE) at PSI

58 MUSE collaborators from 25 institutions in 5 countries:

A. Afanasev, A. Akmal, J. Arrington, H. Atac, C. Ayerbe-Gayoso, F. Benmokhtar, N. Benmouna, J. Bernauer, A. Blomberg, E. Brash, W.J. Briscoe, E. Cline, D. Cohen, E.O. Cohen, K. Deiters, J. Diefenbach, B. Dongwi, E.J. Downie, L. El Fassi, S. Gilad, R. Gilman, K. Gnanvo, R. Gothe, D. Higinbotham, Y. Ilieva, L. Li, M. Jones, N. Kalantarians, M. Kohl, G. Kumbartzki, J. Lichtenstadt, W. Lin, A. Liyanage, N. Liyanage, W. Lorenzon, Z.-E. Meziani, P. Monaghan, K.E. Mesick, P. Moran, J. Nazeer, C. Perdrisat, E. Piasetzsky, V. Punjabi, R. Ransome, R. Raymond, D. Reggiani, P.E. Reimer, A. Richter, G. Ron, T. Rostomyan, A. Sarty, Y. Shamai, N. Sparveris, S. Strauch, N. Steinberg, V. Sulkosky, A.S. Tadepalli, M. Taragin, and L. Weinstein



George Washington University, Montgomery College, Argonne National Lab, Temple University, College of William & Mary, Duquesne University, Massachusetts Institute of Technology, Christopher Newport University, Rutgers University, Hebrew University of Jerusalem, Tel Aviv University, Paul Scherrer Institut, Johannes Gutenberg-Universität, Hampton University, University of Michigan, University of Virginia, University of South Carolina, Jefferson Lab, Los Alamos National Laboratory, Norfolk State University, Technical University of Darmstadt, St. Mary's University, Soreq Nuclear Research Center, IIZMANN Institute, Old Dominion University

Conclusion

“It tells us that there’s still a puzzle,” Evangeline Downie from the George Washington University in Washington D.C., who was not involved in the study, told New Scientist. “It’s still very open, and the only thing that’s going to allow us to solve it is new data.”

- **Spectroscopy**
 - CODATA 2014 5.6σ from μH
 - μH disagrees with (almost) all atomic H
 - μD disagrees with atomic D (3.5σ disagreement)
 - ^xHe results seem to agree (preliminary)
- **Elastic scattering**
 - Depending on extraction agrees with / disagrees strongly with μH
 - More low Q^2 measurements in preparation / analysis / underway
 - MUSE under construction to give first precise muon scattering results
- We are **still** (possibly more) **puzzled!**

Outlook

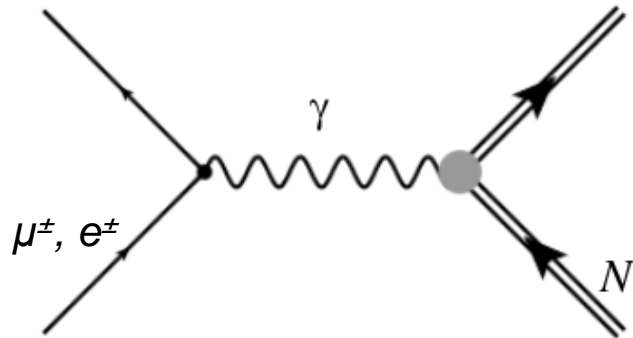
- The proton radius puzzle is a high-profile issue
 - Explanation unclear
 - PSI MUSE tests interesting possibilities: Are μp and $e p$ interactions different? If so, does it arise from 2γ exchange effects ($\mu^+ \neq \mu^-$) or BSM physics ($\mu^+ \approx \mu^- \neq e^-$)?
- Within 2-3 years we should start to see the muon scattering results, and possibly start to resolve the puzzle, perhaps seeing new physics!

Thank you

Backup slides

Lepton scattering and charge radius

Lepton scattering from a nucleon:



Vertex currents:

$$J_e^\mu = -e\bar{u}_e\gamma^\mu u_e$$

$$J_N^\mu = \bar{\psi}_N \left[F_1(Q^2)\gamma^\mu + F_2(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2M_N} \right] \psi_N$$

F_1, F_2 are the Dirac and Pauli form factors

Sachs form factors:

$$G_E(Q^2) = F_1(Q^2) - \tau F_2(Q^2)$$

$$G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

Fourier transform (in the Breit frame) gives spatial charge and magnetization distributions

Derivative in $Q^2 \rightarrow 0$ limit:

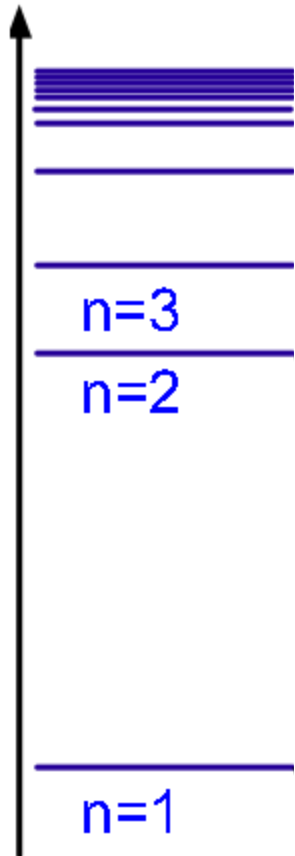
$$\langle r_E^2 \rangle = -6 \frac{dG_E^p(Q^2)}{dQ^2} \Big|_{Q^2 \rightarrow 0}$$

$$\langle r_M^2 \rangle = -6 \frac{dG_M^p(Q^2)/\mu_p}{dQ^2} \Big|_{Q^2 \rightarrow 0}$$

Expect identical result for ep and μp scattering

The Proton Radius from H Lamb Shift

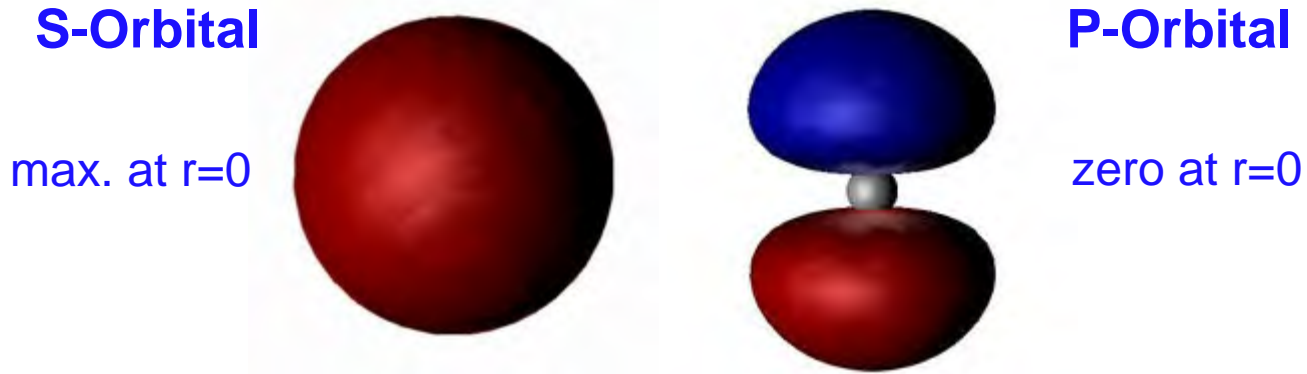
Components of the Hydrogen Energy Levels



Bohr

0.014% of
the Lamb
Shift!

Why Measure with μH ?



- While lepton is inside proton, attractive potential is lower
- Average potential reduced the longer lepton spends inside proton
- Strongly affects S orbitals, much less so P, so S-P transitions change
- Probability for lepton to be inside proton = volume of p / volume of atom:

$$\cong \left(\frac{r_p}{a_B} \right)^3 = (r_p \alpha)^3 m^3$$

- $m_\mu = 205 m_e$: so μH is $205^3 \approx 8$ million times more sensitive to r_p