

References on Translation Surfaces

1 Three introductions, in increasing order of technical detail

1. Wright, Alex. *Translation surfaces and their orbit closures: An introduction for a broad audience.* arXiv:1411.1827.

The beginning spends several pages discussing the three equivalent definitions of a translation surface. These notes include examples of important constructions: unfolding billiards, translation covers, and slit constructions (i.e. the “flat” version of connected sums). The second and fifth sections go into some depth discussing affine invariant submanifolds, and the third and fourth sections give a not-too-technical introduction to the dynamics of translation surfaces.

2. Zorich, Anton. *Flat Surfaces.* arXiv:math/0609392.

A beautifully-written casual exposition of the subject. He focuses first on some concrete relationships between translation surfaces and interval exchange transformations, giving a lot of visual insight into the dynamics of flat surfaces. He also includes a crash-course in Teichmüller theory, as well as an entire section discussing hopes for a “Magic Wand” theorem (these notes were written before the theorem was proved!). The appendix also serves as a cheat-sheet for the basics of ergodic theory.

3. Forni, G. & Matheus, C. *Introduction to Teichmüller theory and its applications to dynamics of interval exchange transformations, flows on surfaces and billiards.* arXiv:1311.2758.

The first three sections give a more detail-oriented introduction to the basics of translation surfaces than the other two references listed here. The further sections constitute a full course on translation surfaces, with an emphasis on dynamics.

2 Additional references

- Chen, Dawei. *Teichmüller dynamics in the eyes of an algebraic geometer.* arXiv:1602.02260.

The “Preliminaries” section is a good cheat-sheet for all the basic definitions in this subject. The rest of the paper is a concise summary of contemporary algebro-geometric results on translation surfaces.

- Masur, H. & Tabachnikov, S. *Rational Billiards and Flat Structures.* Handbook of Dynamical Systems, vol. 1A, ch. 13. Edited by B. Hasselblatt and A. Katok. Elsevier Science B.V., 2002.

If you’re looking for a fully-detailed, textbook-style treatment of Teichmüller dynamics, complete with proofs, then this is your go-to.

- Zorich, Anton. *The Magic Wand Theorem of A. Eskin and M. Mirzakhani.* arXiv: 1502.05654.

A storytelling-style overview of what the Magic Wand theorem is and why it’s important. Includes a brief biography of Maryam Mirzakhani.

- URLs for videos of research talks on translation surfaces:

- From the Fields Institute (October 2018): <http://www.fields.utoronto.ca/video-archive/event/2359>
- Carlos Matheus’ lectures at the Simons Center (April 2019): http://scgp.stonybrook.edu/video_portal/results.php?event_id=266
- From CMO (May 2019): <https://www.birs.ca/events/2019/5-day-workshops/19w5078/videos>