Circuit quantum electrodynamics : beyond the linear dispersive regime

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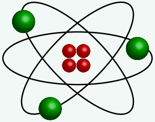
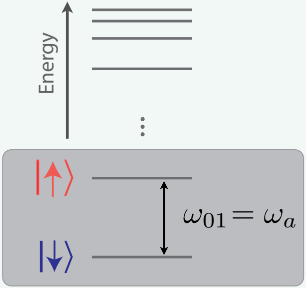
UNIVERSITÉ DE SHERBROOKE

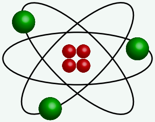
Boissonneault, Gambetta and Blais, *Phys. Rev. A* **77** 060305 (R) (2008)

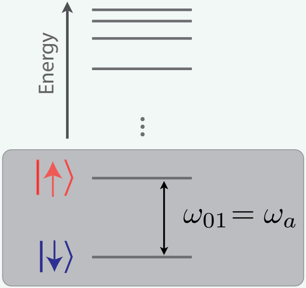


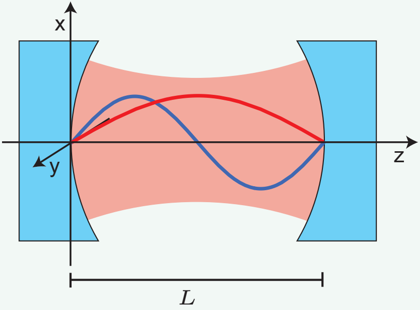


Two-levels system Hamiltonian

$$H = \frac{\omega_a}{2}\sigma_z \qquad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$







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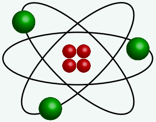
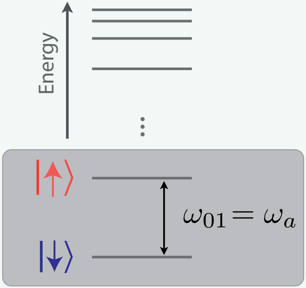
Cavity Hamiltonian

$$H = \sum_k \omega_k \left(a_k^\dagger a_k + \frac{1}{2} \right)$$

Atom and cavity





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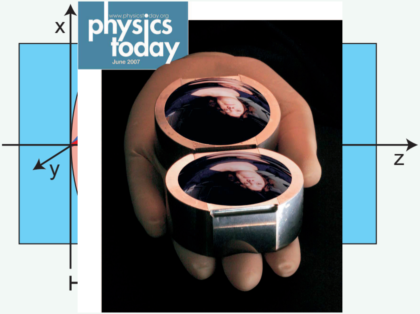

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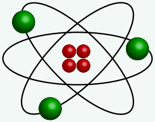

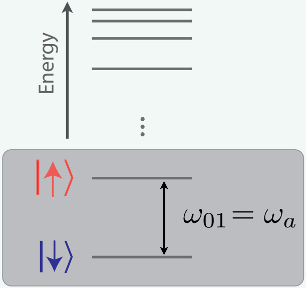

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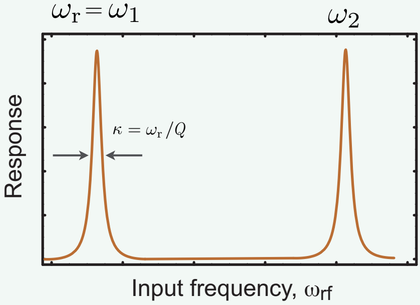


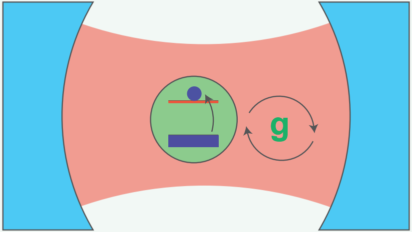
Cavity Hamiltonian

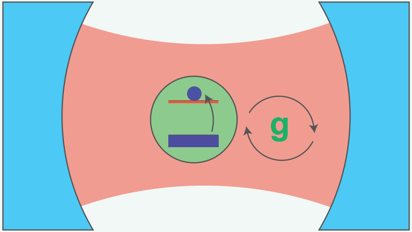
$$H = \sum_k \omega_k \left(a_k^\dagger a_k + \frac{1}{2} \right)$$

Single-mode :

$$H = \omega_r a^\dagger a$$

Cavity QED

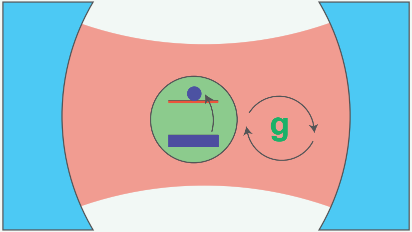




Atom-cavity interaction

$$H_I = -\vec{D}\cdot\vec{E} \approx g(a^\dagger + a)\sigma_x \approx g(a^\dagger\sigma_- + a\sigma_+)$$

$$g(z) = -d_0\sqrt{\frac{\omega}{V\epsilon_0}}\sin kz$$



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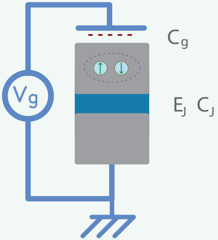
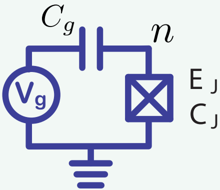
$$H = \frac{\omega_a}{2}\sigma_z + \omega_r a^\dagger a + g(a^\dagger \sigma_- + a\sigma_+)$$

Jaynes and Cummings, *Proc. IEEE* **51** 89-109 (1963)
Raimond, Brune and Haroche, *Rev. Mod. Phys.* **73** 565–582 (2001)
Mabuchi and Doherty, *Science* **298** 1372-1377 (2002)



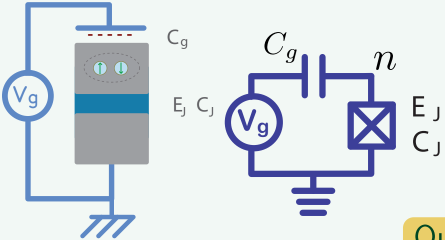


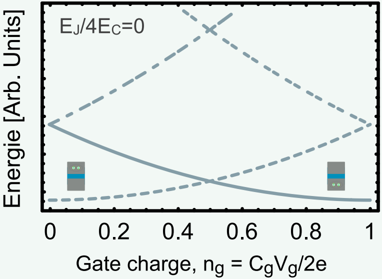
Classical Hamiltonian

$$H = 4E_C(n - n_g)^2 - E_J \cos\delta$$

$$E_C = \frac{e^2}{2(C_g + C_J)}, \qquad n_g = \frac{C_g V_g}{2e}$$

$$E_J = \frac{I_0 \Phi_0}{2\pi}$$





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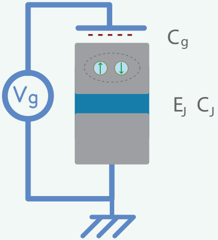
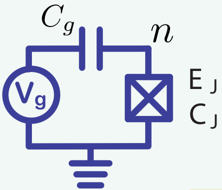
Quantum Hamiltonian

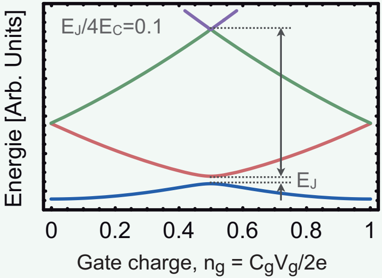
$$H = \sum_n 4E_C(n - n_g)^2 |n\rangle\langle n| - \sum_n \frac{E_J}{2}(|n\rangle\langle n+1| + \text{h.c.})$$

Restricting to $n_g \in [0,1]$: $H = \omega_a \sigma_z/2$

Shnirman, Schön and Hermon, *Phys. Rev. Lett.* **79** 2371–2374 (1997)
Bouchiat *et al.*, *Physica Scripta* **T76** 165-170 (1998)
Nakamura, Pashkin and Tsai, *Nature (London)* **398** 786 (1999)







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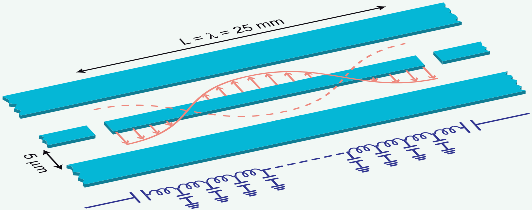
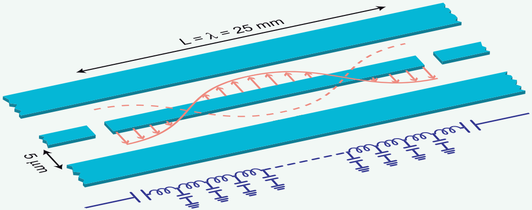
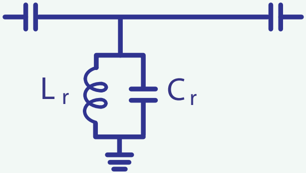
Nakamura, Pashkin and Tsai, *Nature (London)* **398** 786 (1999)

Charge qubit and coplanar resonator



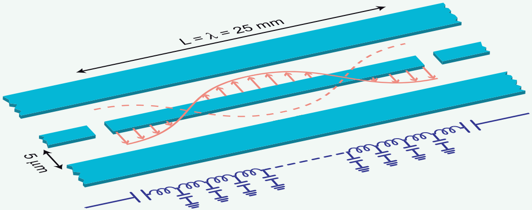


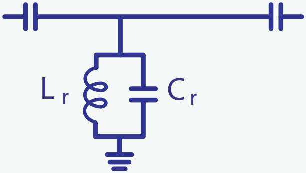


Classical Hamiltonian

$$H = \frac{\Phi^2}{2L_r} + \frac{1}{2} C_r V^2$$

$$\omega_r = \sqrt{\frac{1}{L_r C_r}}$$





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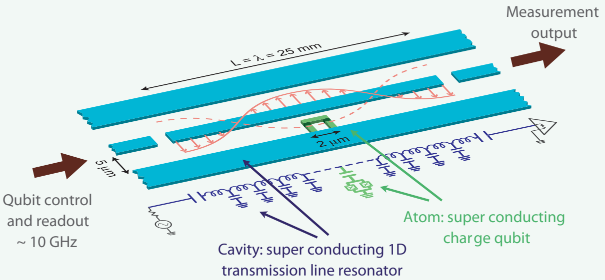
Quantum Hamiltonian

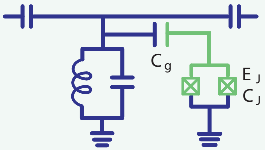
$$V = \sqrt{\frac{\omega_r}{2C_r}}(a^\dagger + a), \qquad \Phi = i\sqrt{\frac{\omega_r}{2L_r}}(a^\dagger - a)$$

$$H = \omega_r \left(a^\dagger a + \frac{1}{2} \right)$$

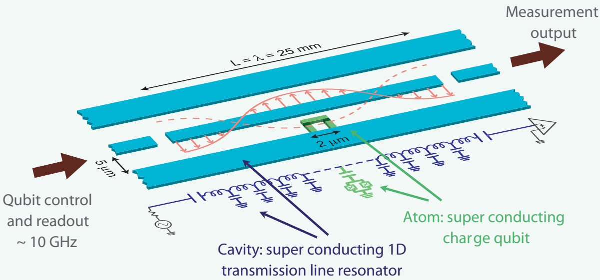
Quantum Fluctuations in Electrical Circuits, M. H. Devoret, Les Houches Session LXIII, Quantum Fluctuations p. 351-386 (1995).

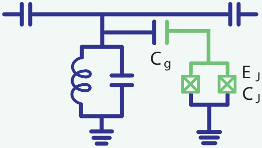
Circuit QED


Measurement
output
L = λ = 25 mm
2 µm
5 µm
Qubit control
and readout
~ 10 GHz
Cavity: super conducting 1D
transmission line resonator
Atom: super conducting
charge qubit


C g
E J
C J

Circuit QED





Parameters

- g : Qubit-cavity interaction
- ω_a : Qubit frequency
- ω_r : Resonator frequency
- $\Delta = \omega_a - \omega_r$: Detuning

$$H = \frac{\omega_a}{2}\sigma_z + \omega_r a^\dagger a + g(a^\dagger \sigma_- + a\sigma_+)$$

- Blais *et al.*, *Phys. Rev. A* **69** 062320 (2004)
- Wallraff *et al.*, *Nature* **431** 162 (2004)
- Wallraff *et al.*, *Phys. Rev. Lett.* **95** 060501 (2005)
- Leek *et al.*, *Science* **318** 1889 (2007)
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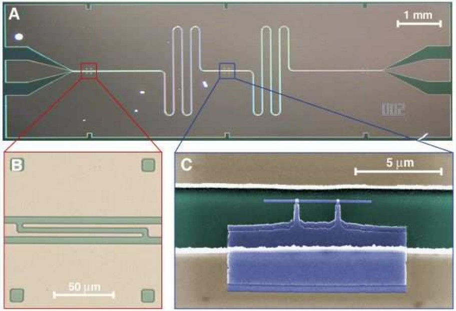
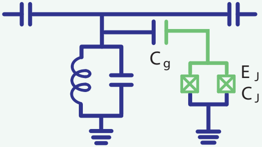
- Houck *et al.*, *Nature* **449** 328 (2007)
- Majer *et al.*, *Nature* **449** 443 (2007)





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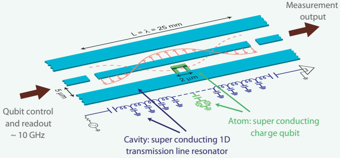
Jaynes-Cummings

$H = \omega_r a^\dagger a + \omega_a \frac{\sigma_z}{2} + g(a^\dagger \sigma_- + a\sigma_+)$

Small parameter $\lambda = g/\Delta$



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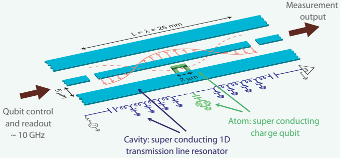
Linear dispersive

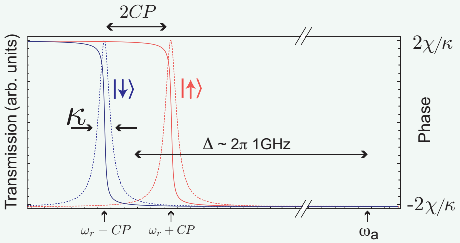
$H^D = (\omega_a + \chi)\frac{\sigma_z}{2} + (\omega_r + \chi\sigma_z)a^\dagger a$

Lamb shift ($\chi = g\lambda = g^2/\Delta$)

Stark shift or cavity pull

Valid if $\bar{n} \ll n_{\text{crit.}}$, where $n_{\text{crit.}} = 1/4\lambda^2$.





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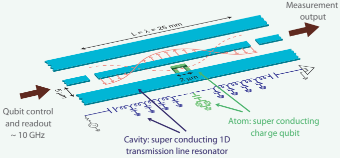

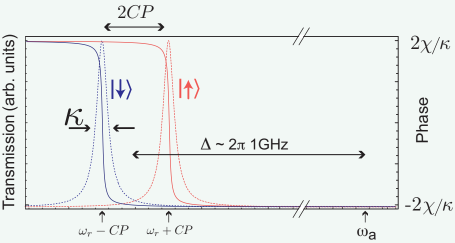

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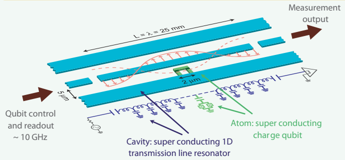

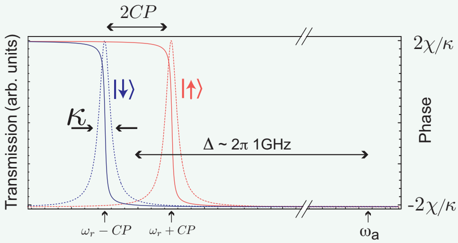

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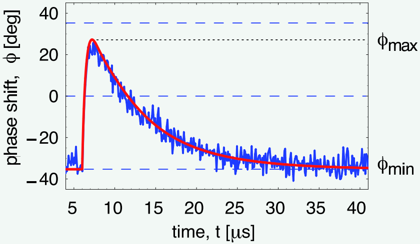
Rabi π-pulse

Wallraff *et al.*, *Phys. Rev. Lett.* **95** 060501 (2005)



Averaged 50000 times.

SNR for single-shot is 0.1.

Symbol	Optical cavity	Microwave cavity	Circuit
$\omega_r/2\pi$ or $\omega_a/2\pi$	350 THz	51 GHz	10 GHz
g/π	220 MHz	47 kHz	100 MHz
g/ω_r	3×10^{-7}	10^{-7}	5×10^{-3}

Hood *et al.*, *Science* **287** 1447 (2000)

Raimond, Brune and Haroche, *Rev. Mod. Phys.* **73** 565–582 (2001)

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Motivation

- Circuit QED is harder than cavity QED on the dispersive limit ($n_{\text{crit.}}$ is smaller)

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Motivation

- Circuit QED is harder than cavity QED on the dispersive limit ($n_{\text{crit.}}$ is smaller)
- The SNR is low, we want to measure harder... how does higher order terms affect measurement ?
- Must consider higher order corrections in perturbation theory

J-C : block diagonal

- $H = \omega_r a^\dagger a + \omega_a \sigma_z/2 + g(a^\dagger \sigma_- + a\sigma_+)$

$$H = \begin{array}{c} \\ \langle 0\downarrow| \\ \langle 0\uparrow| \\ \langle 1\downarrow| \\ \langle 1\uparrow| \\ \langle 2\downarrow| \\ \vdots \end{array} \begin{array}{cccccc} |0\downarrow\rangle & |0\uparrow\rangle & |1\downarrow\rangle & |1\uparrow\rangle & |2\downarrow\rangle & \cdots \\ -\frac{\omega_a}{2} & 0 & 0 & 0 & 0 & \cdots \\ 0 & \frac{\omega_a}{2} & g & 0 & 0 & \cdots \\ 0 & g & \omega_r - \frac{\omega_a}{2} & 0 & 0 & \cdots \\ 0 & 0 & 0 & \omega_r + \frac{\omega_a}{2} & \sqrt{2}g & \cdots \\ 0 & 0 & 0 & \sqrt{2}g & 2\omega_r - \frac{\omega_a}{2} & \cdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{array}$$

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- Total Hamiltonian
  $H = H_0 \oplus H_1 \oplus H_2 \cdots \oplus H_\infty$

$$
H = \begin{array}{c} \langle 0\downarrow| \\ \langle 0\uparrow| \\ \langle 1\downarrow| \\ \langle 1\uparrow| \\ \langle 2\downarrow| \\ \vdots \end{array}
\overset{\begin{array}{cccccc} |0\downarrow\rangle & |0\uparrow\rangle & |1\downarrow\rangle & |1\uparrow\rangle & |2\downarrow\rangle & \cdots \end{array}}{\begin{pmatrix}
-\frac{\omega_a}{2} & 0 & 0 & 0 & 0 & \cdots \\
0 & \frac{\omega_a}{2} & g & 0 & 0 & \cdots \\
0 & g & \omega_r - \frac{\omega_a}{2} & 0 & 0 & \cdots \\
0 & 0 & 0 & \omega_r + \frac{\omega_a}{2} & \sqrt{2}g & \cdots \\
0 & 0 & 0 & \sqrt{2}g & 2\omega_r - \frac{\omega_a}{2} & \cdots \\
\vdots & \vdots & \vdots & \vdots & \vdots & \ddots
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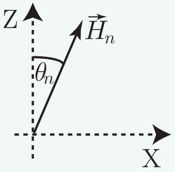

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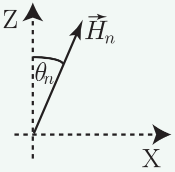

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Diagonalization

- Rotation around Y axis
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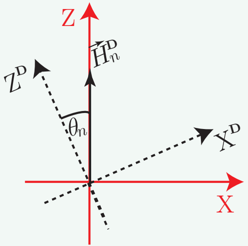

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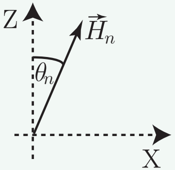

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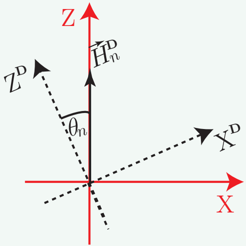

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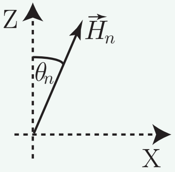
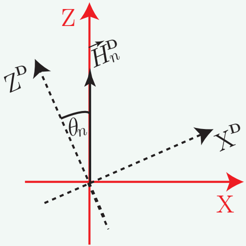


$\theta_n = \arctan(2g\sqrt{n}/\Delta) \approx 2g\sqrt{n}/\Delta$



Dispersive limit

- Jaynes-Cummings hamiltonian

$$H = \omega_r a^\dagger a + \omega_a \frac{\sigma_z}{2} + g(a^\dagger \sigma_- + a\sigma_+)$$

- Exact transformation : D
- Small parameter $\lambda = g/\Delta$
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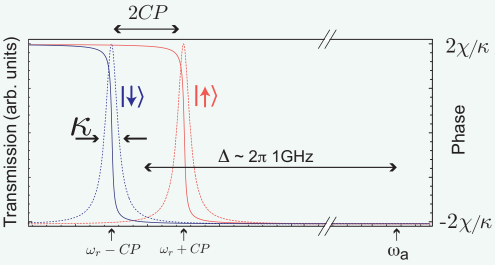
Result at order λ

$$H^D = (\omega_a + \chi)\frac{\sigma_z}{2} + (\omega_r + \chi\sigma_z)a^\dagger a$$

Lamb shift ($\chi = g\lambda = g^2/\Delta$)

Stark shift or cavity pull



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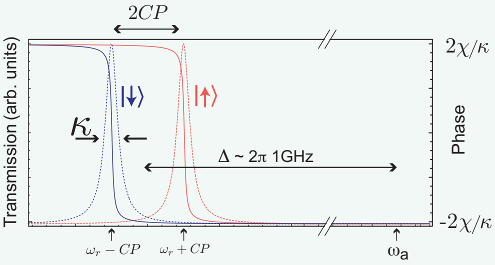
Stark shift or cavity pull

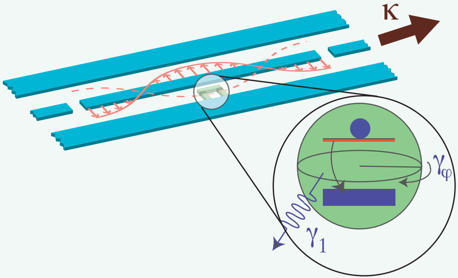
Result at order λ^2

$$H^D = (\omega_a + \chi')\frac{\sigma_z}{2} + [\omega_r + (\chi' - \zeta a^\dagger a)\sigma_z]a^\dagger a$$

$\chi' = \chi(1-\lambda^2)$, $\zeta = \lambda^2\chi$

The cavity pull decrease : $\langle CP\rangle = \chi' - \zeta\langle a^\dagger a\rangle$



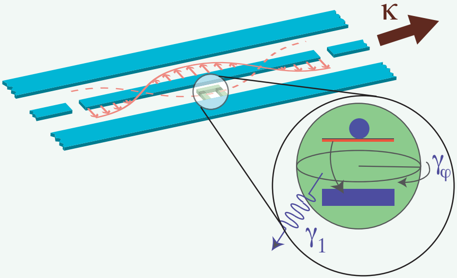


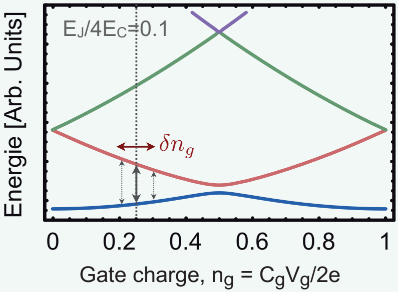
Model for dissipation

- Coupling to a bath
  $H_\kappa = \int_0^\infty \sqrt{g_\kappa(\omega)}[b_\kappa^\dagger(\omega) + b_\kappa(\omega)][a^\dagger + a]d\omega$
  $H_\gamma = \int_0^\infty \sqrt{g_\gamma(\omega)}[b_\gamma^\dagger(\omega) + b_\gamma(\omega)]\sigma_x d\omega$

Parameters

- κ : Rate of photon loss
- γ_1 : Transverse decay rate





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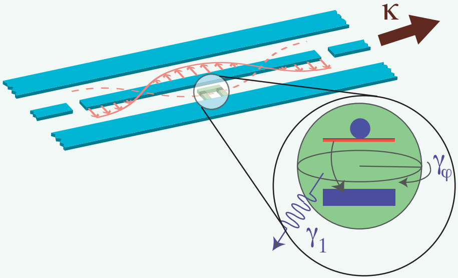

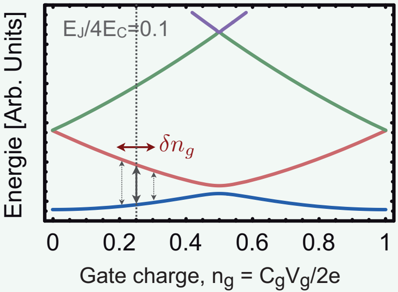

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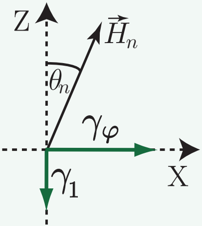
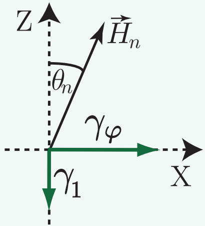
- Dephasing
  $H_\varphi = \eta f(t)\sigma_z$

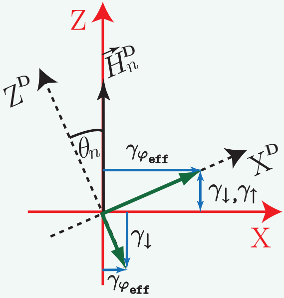
Parameters

- κ : Rate of photon loss
- γ_1 : Transverse decay rate
- γ_φ : Pure dephasing rate

Dissipation in the transformed basis







Transformation of system-bath hamiltonian

- $a \xrightarrow{D} a + \lambda\sigma_- + \mathcal{O}\left(\lambda^2\right)$
- $\sigma_- \xrightarrow{D} \sigma_- + \lambda a \sigma_z + \mathcal{O}\left(\lambda^2\right)$
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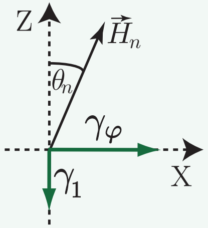

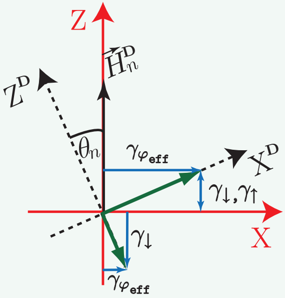

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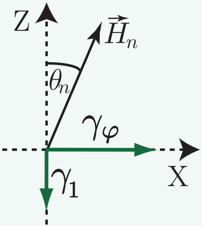
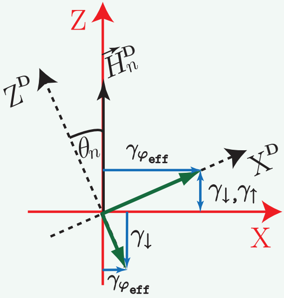
Method

- Transform the system-bath hamiltonian
- Trace out heat bath and cavity degrees of freedom (Gambetta *et al.*, *Phys. Rev. A* **77** 012112 (2008))





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New rates (assuming white noises)

- $\gamma_\downarrow = \gamma_1\left[1 - 2\lambda^2\left(\bar{n} + \frac{1}{2}\right)\right] + \gamma_\kappa + 2\lambda^2\gamma_\varphi\bar{n}$
- $\gamma_\uparrow = 2\lambda^2\gamma_\varphi\bar{n}$
- $\gamma_\kappa = \lambda^2\kappa$

Parameters for the SNR

- Number of measurement photons

$$\text{SNR} \sim \frac{\text{Num. phot.}}{}$$

Parameters for the SNR

- Number of measurement photons
- Output rate : κ

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Parameters for the SNR

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- Output rate : κ
- Fraction of photons detected : η

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Parameters for the SNR

- Number of measurement photons
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- Fraction of photons detected : η
- Info per photon : cavity pull

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Parameters for the SNR

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  $\gamma_\downarrow + \gamma_\uparrow = \gamma_1 \left[1 - 2\lambda^2 \left(\bar{n} + \frac{1}{2}\right)\right] + \gamma_\kappa + 4\lambda^2 \gamma_\varphi \bar{n}$

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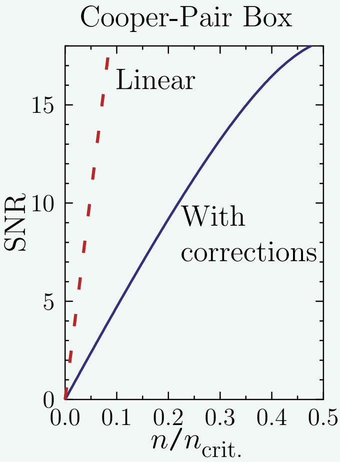
Conclusion

- SNR levels off with non-linear effects !
- Explains low experimental SNR
- Applies to all dispersive homodyne measurement



$\Delta/2\pi = 1.7$ GHz, $g/2\pi = 170$ MHz
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$n_{\text{crit.}} = 1/4\lambda^2 = 25$

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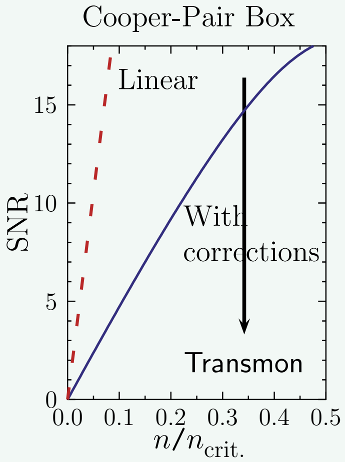

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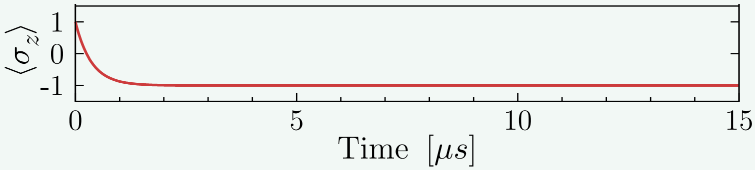
Mixing rates

- Downward rate : $\gamma_\downarrow(\bar{n})$
- Upward rate : $\gamma_\uparrow(\bar{n})$
- Heat bath with temperature $T(\bar{n}) = (\hbar\omega_r/k_B)/\log(1+1/\bar{n})$



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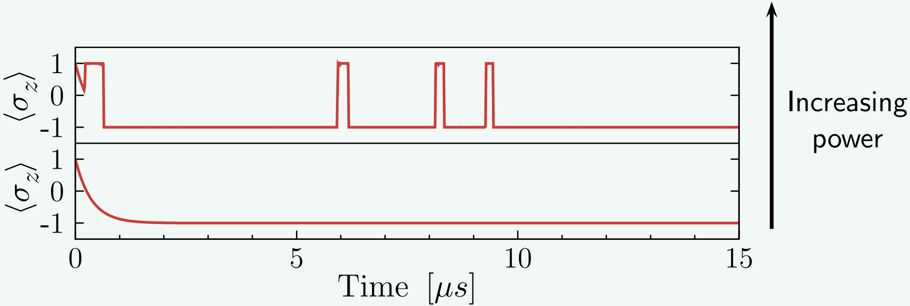

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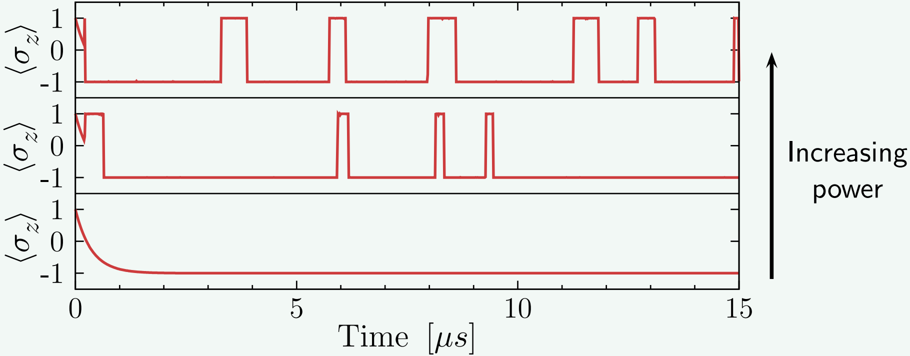
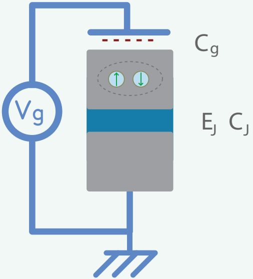
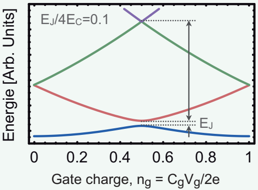


The case of the transmon





Koch *et al.*, *Phys. Rev. A* **76** 042319 (2007)

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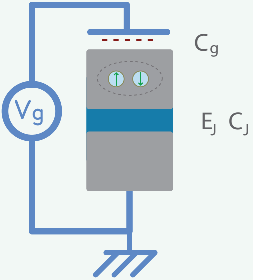

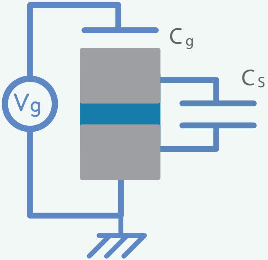

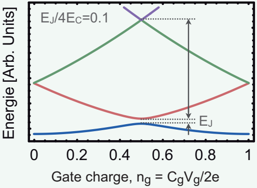

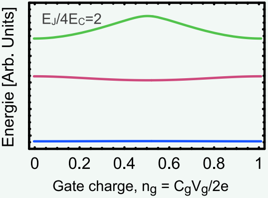

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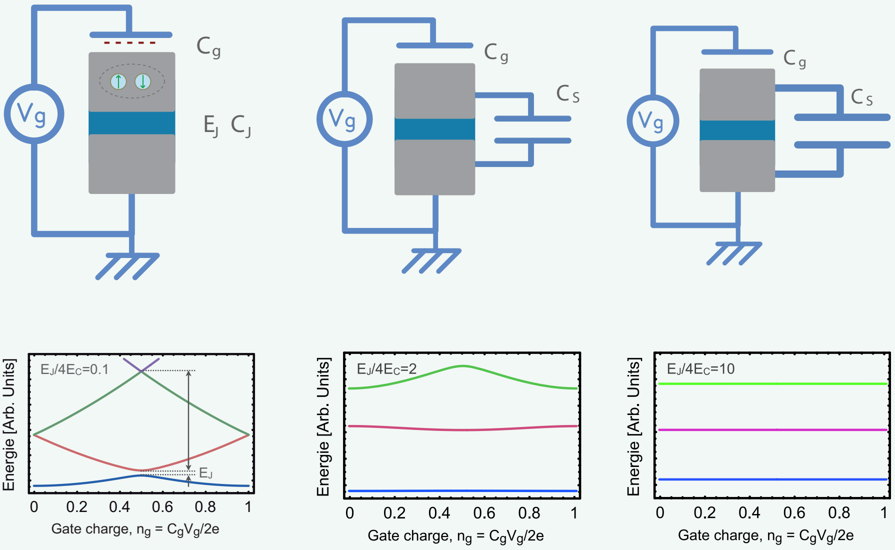

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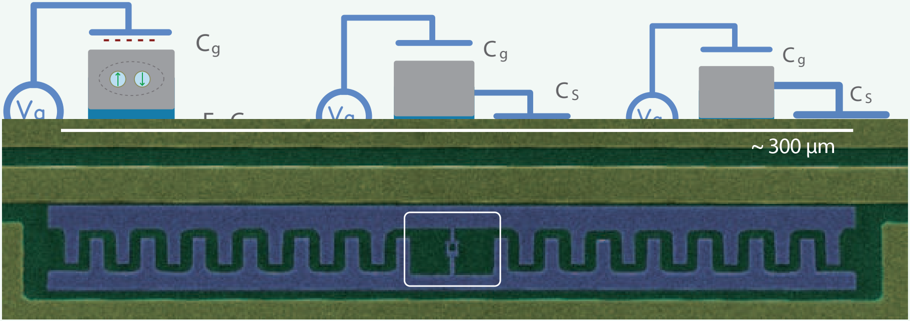

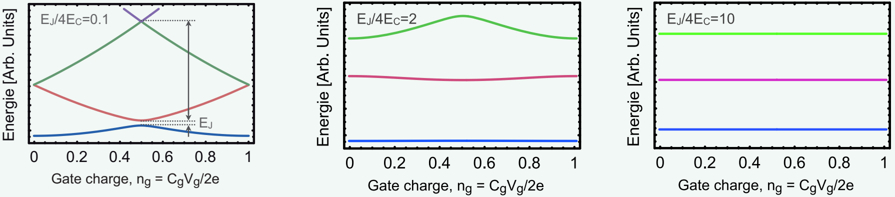

Koch *et al.*, *Phys. Rev. A* **76** 042319 (2007)

Main results

- Simple model describing the physics of measurement
- Side-effect of measuring harder : you heat your qubit (even with photons that can't be directly absorbed)
- Side-effect of measuring harder : each photon you add carries less information than the previous one
- Measuring harder $\neq$ bigger SNR

Coming soon

- The transmon (3 level system) (Koch *et al.*, *Phys. Rev. A* **76** 042319 (2007))
- Taking advantage of the non-linearity
- Comparison with experiments

More information : Boissonneault, Gambetta and Blais, *Phys. Rev. A* **77** 060305 (R) (2008)

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