$E_{7(7)}$ and Noether-Gaillard-Zumino current conservation

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Amplitudes 2011
University of Michigan, Ann Arbor

1. Ancient and New Story of $\mathcal{N} = 8$ Counterterms
2. Light-Cone supergraph prediction of $\mathcal{N}$=8 supergravity UV finiteness
3. Focus on the vector sector of $E_{7(7)}$ symmetry. Noether-Galliard-Zumino current conservation, NGZ identity.
4. Is it possible to construct Born-Infeld $\mathcal{N}$=8 supergravity? What does it mean for UV properties of $\mathcal{N}$=8?

Pure gravity $\mathcal{N}=0$ diverges at two loops

- Relevant counterterm is available, RK, 1974; van Nieuwenhuizen, Wu, 1977

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- Pure gravity, perturbative QFT: the book is closed, our prediction was confirmed by explicit computations, nothing new to learn about UV properties

COUNTERTERMS IN EXTENDED

SUPERGRAVITIES

R.E.Kallosh

Lebedev Physical Institute, Moscow, USSR

To the memory of Felix Berezin

The geometrical invariants, integrals over the whole supermanifold, respecting all necessary symmetries of the theory, are shown to exist starting from the 8-th (4-th) loop approximation in the N= 8 (N=4) on-shell supergravity. 3-loop counterterms, which are integrals over some subsupermanifolds, are presented on linearised level in N=4 and N=8 theories. The corresponding 3-loop non-linear invariants are discussed.

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I.E.Tamm Department of Theoretical Physics

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R.E.Kallosh

Moscow— September 1980

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HIGHER ORDER INVARIANTS IN EXTENDED SUPERGRAVITY

P. Howe
CERN -- Geneva

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U. Lindström
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ABSTRACT

On-shell linearized extended supergravity is presented in superspace for all N. The formalism is then used in the construction of higher-order invariants which may serve as counterterm Lagrangians. It is shown that three-loop counterterms exist for $N \leq 3$ and (N-1) loop counterterms for $N \geq 4$. In the full non-linear theory, the presence of a global non-compact symmetry group for $N \geq 4$ does not allow a simple extension of the (N-1) loop term, but N loop counterterms may be constructed.

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- Bern, Carrasco, Dixon, Johansson, Kosower, Roiban (2007-2009, 3 and 4 loop computations: UV finiteness. Explicit computations in $\mathcal{N}$=8 SG using the unitarity cut method are in agreement with the $\mathcal{N}$=4 Yang-Mills type formula: the theory is UV finite in dimensions

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- The possibility of UV finiteness of perturbative $\mathcal{N}$=8 SG is against the 30 years of standard wisdom on UV properties of quantum SG. A current theoretical point of view is that the 7 loop level may (or may not) be UV divergent. M. Green et al warn about L=5 : D=24/5, not 26/5.

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- Are we wasting time studying $\mathcal{N}$=8 SG ???

Old Wisdom and why we quit in 81?

- Using the existence of the covariant on-shell 4+32 dimensional superspace (Brink, Howe, 1979) and the background field method in QFT, one can use the tensor/spinor geometric calculus in the on-shell geometric superspace and construct the invariant counterterms (RK; Howe, Lindstrom, 1981). Such geometric counterterms have all known symmetries of the theory, including $E_{7(7)}$. They start at the 8-loop level. (Recent clarification of the 1/8 BPS 7-loop candidate, Bossard, Howe, Stelle, Vanhove).

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- For example one can use a superspace torsion for the 8-loop counterterm

$$S^8 \sim \kappa^{14} \int d^4x\, d^{32}\theta \,\mathrm{BerE}\, \mathrm{T_{ijk\alpha}}(\mathrm{x},\theta)\overline{\mathrm{T}}^{\mathrm{ijk}\dot\alpha}(\mathrm{x},\theta)\mathrm{T_{mnl}}^{\alpha}(\mathrm{x},\theta)\overline{\mathrm{T}}^{\mathrm{mnl}}{}_{\dot\alpha}(\mathrm{x},\theta) \ .$$

Here $T_{ijk\alpha}(x,\theta)$ is the superspace torsion superfield whose first component is a spinor field, **56** (and $\overline{\mathbf{56}}$) in $SU(8)$.

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- From this perspective to explain finiteness we have to explain an infinite number of zeros !

- Using the superampltudes, the linearized counterterms were constructed Beisert, Elvang, Freedman, Kiermaier, Morales, Stieberger, 2010. The soft scalar limit was taken into account, which is part of $E_{7(7)}$ symmetry. The table is in agreement with the Lorentz covariant superspace counterterms. There is an infinite proliferation of the higher loop candidates for UV divergences.

3-loop	4-pt	5-pt	6-pt
singlet	R^4 1×MHV	~~$\varphi^2 D^2 R^3$~~	~~$\varphi^2 R^4$~~ R^4 non-linear
		↙ soft	
70		$1\times\varphi R^4$	

5-loop	4-pt	5-pt	6-pt
singlet	$D^4 R^4$ 1×MHV	~~$\varphi^2 D^6 R^3$~~	~~$\varphi^2 D^4 R^4$~~ D^4R^4 non-lin.
		↙ soft	
70		$1\times\varphi D^4 R^4$	

6-loop	4-pt	5-pt	6-pt
singlet	$D^6 R^4$ 1×MHV	~~$\varphi^2 D^8 R^3$~~	~~$\varphi^2 D^6 R^4$~~ $D^6 R^4$ non-lin.
		↙ soft	
70		$1\times\varphi D^6 R^4$	

7-loop	4-pt	5-pt	6-pt	7-pt	8-pt	9-pt	10-pt	11-pt	12-pt	13-pt	14-pt	15-pt	16-pt
singlet	D^8R^4 1×MHV	~~D^6R^5~~	D^4R^6 2×NMHV	~~D^2R^7~~	R^8 3×N^2MHV	~~$\varphi^2 D^2R^7$~~	$\varphi^2 R^8$ 4×N^3MHV	~~$\varphi^4 D^2R^7$~~	$\varphi^4 R^8$ 6×N^4MHV	~~$\varphi^6 D^2R^7$~~	$\varphi^6 R^8$ 8×N^5MHV	~~$\varphi^8 D^2R^7$~~	$\varphi^8 R^8$ 10×N^6MHV
		↙ soft		↙ soft		↙ soft		↙ soft		↙ soft		↙ soft	
70		φD^8R^4 2×		φD^4R^6 4×		φR^8 6×		$\varphi^3 R^8$ 9×		$\varphi^5 R^8$ 14×		$\varphi^7 R^8$ 19×	

8-loop	4-pt	5-pt	6-pt	7-pt	8-pt	9-pt	10-pt	11-pt	12-pt	13-pt	14-pt
singlet	$D^{10}R^4$ 1×MHV	D^8R^5 1×MHV	D^6R^6 3×NMHV	D^4R^7 3×NMHV	D^2R^8 8×N^2MHV	R^9 8×N^2MHV	$\varphi^2 D^2R^8$ 25×N^3MHV	$\varphi^2 R^9$ 22×N^3MHV	$\varphi^4 D^2R^8$ 66×N^4MHV	$\varphi^4 R^9$ 51×N^4MHV	$\varphi^6 D^2R^8$ 153×N^5MHV
		↙	↙	↙	↙	↙	↙	↙	↙	↙	
70		$\varphi D^{10}R^4$ 3×	φD^8R^5 4×	φD^6R^6 17×	φD^4R^7 16×	φD^2R^8 81×	φR^9 63×	$\varphi^3 D^2R^8$ 232×	$\varphi^3 R^9$ 211×	$\varphi^5 D^2R^8$ 1033×	

9-loop	4-pt	5-pt	6-pt	7-pt	8-pt	9-pt	10-pt	11-pt	12-pt
singlet	$D^{12}R^4$ 2×MHV	$D^{10}R^5$ 1×MHV	D^8R^6 12×NMHV 2×MHV	D^6R^7 14×NMHV	D^4R^8 117×N^2MHV 7×NMHV	D^2R^9 123×N^2MHV	R^{10} 780×N^3MHV 36×N^2MHV	$\varphi^2 D^2R^9$ 783×N^3MHV	$\varphi^2 R^{10}$ 4349×N^4MHV 169×N^3MHV
		↙	↙	↙	↙	↙	↙	↙	
70		$\varphi D^{12}R^4$ 5×$N^{0.5}$MHV	$\varphi D^{10}R^5$ 8×$N^{0.5}$MHV	φD^8R^6 122×$N^{1.5}$MHV 5×$N^{0.5}$MHV	φD^6R^7 194×$N^{1.5}$MHV	φD^4R^8 1814×$N^{2.5}$MHV 52×$N^{1.5}$MHV	φD^2R^9 2317×$N^{2.5}$MHV	φR^{10} 16485×$N^{3.5}$MHV 469×$N^{2.5}$MHV	

Various arguments were used in 2010 to explain the 3-loop finiteness via $E_{7(7)}$, Broedel, Dixon; Elvang, Kiermaier; Bossard, Howe, Stelle

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- More work has to be done, the book is far from being closed.

Unconstrained Light-Cone Superspace

- Advantages of the light-cone supergraphs: only physical degrees of freedom propagate since all gauge symmetries are fixed, unitarity is manifest, no ghosts. Some (1/2) supersymmetry is manifest. As close to the unitarity cut method as possible.

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- However, it worked well for proof of UV finiteness in the $\mathcal{N}$=4 SYM case: Mandelstam in chiral 4+8 dimensional LC superspace; Brink, Lindgren, Nilsson, in real 4+16 dimensional LC superspace, 1983

Light-by-Light Scattering Effect in Light-Cone Supergraphs

RK, Ramond, 1006.4684

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- This prediction can be deduced from the properties of light-cone supergraphs analogous to the light-by-light scattering effect in QED. A technical aspect of the argument relies on the observation that the dynamical supersymmetry action is, in fact, a compensating field-dependent gauge transformation required for the retaining the light-cone gauge condition $A_+ = 0$.

From Light-Cone Superspace Action to Supersymmetric Helicity Amplitudes

RK, Broedel, 1103.0322

- The actions for $\mathcal{N}=4$ SYM and $\mathcal{N}=8$ supergravity in terms of a chiral superfield with only physical degrees of a freedom originate from the LC superspace actions. The Lorentz covariant supergravity cubic vertex is the square of the gauge theory one

$$S_3^{N=4} = f^{a_1 a_2 a_3} \int \prod \{d^8 z_i \varphi_{a_i}(z_i)\} \left[\frac{\delta^4(\sum p_i)\delta^8(\lambda^i \eta_i)}{\langle 12\rangle\langle 23\rangle\langle 31\rangle} + \frac{\delta^4(\sum_i p_i)\delta^4(\frac{1}{2}\epsilon^{ijk}[ij]\eta_k)}{[12][23][31]} \right]$$

Manifestly supersymmetric double-copy BCJ-type relation between $\mathcal{N}=4$ and $\mathcal{N}=8$

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- These first studies of the LC path integrals have not revealed any obvious deficiencies of the light-cone supergraph method of computations and we see no clear reason to distrust its predictions: NO COUNTERTERMS!

Light-Cone Superspace Counterterms

RK, 1009.1135

- The $\mathcal{N}$=8 action in real LC superspace depends on one unconstrained chiral scalar superfield.

$$S^{\rm real}[\phi,\bar\phi] = \frac{1}{2\kappa^2}\int d^4x\, d^8\theta\, d^8\bar\theta\, \mathcal{L}^{\rm real}(\phi,\bar\phi) \qquad \bar d_a\bar\phi = 0\,, \qquad d^a\phi = 0$$

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- The CT's in the chiral LC superspace are available and they are easily described using helicity amplitudes structures. They are also easily related to Lorentz covariant CT's. However, the non-vanishing on-shell CT's are not available in the real LC superspace. It is a generalization of the perturbative F-term non-renormalization theorem. LC supergraphs predict UV finiteness of $\mathcal{N}=8$ supergravity!

Current status of LC prediction of UV finiteness of $\mathcal{N}=8$

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- I have not seen any criticism so far of my more recent proof that they are not available.
- While waiting for any reaction on the LC story, I focused on the implications of $E_{7(7)}$ symmetry in covariant formalism.

Noether (1918), Gaillard and Zumino (1981)

- Emi Noether theorem “Invariante Variationsprobleme” published in Nachr. D. König. Gesellsch. D. Wiss. Zu Göttingen, Math-phys. Klasse 1918 (3): 235-257 : Any differentiable global symmetry of the action of a physical system has a corresponding conservation law

$$\mathcal{L} \to \mathcal{L} + \alpha\, \partial_\mu \mathcal{J}^\mu\,, \qquad \phi \to \phi + \alpha \Delta\phi$$

A conserved Noether current is $J_\mu^N \equiv \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)} \Delta\phi - \mathcal{J}^\mu, \quad \partial_\mu J^{\mu\, N} = 0$

and time-independent Noether charge $Q^N \equiv \int d^3x J^{0N}$

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- Duality symmetry is a differentiable global symmetry of a system, but not of the total action, as discovered by Gaillard and Zumino in studies of supergravity.
- Noether theorem in the vector sector requires a generalization, which we call NGZ current conservation or equivalent to it NGZ identity.

Exact $E_{7(7)}$ transformations acting on scalars and vectors

- Classical $\mathcal{N}$=8 supergravity, kinetic terms for vectors $\mathcal{N}_{\Lambda\Sigma}(\phi)$ depends on scalars

$$S = \frac{1}{4\kappa^2}\int d^4x\, e\Big(-\frac{1}{2}R + \mathrm{Im}\mathcal{N}_{\Lambda\Sigma}F^{\Lambda}_{\mu\nu}F^{\mu\nu\Sigma} + \mathrm{Re}\mathcal{N}_{\Lambda\Sigma}\tilde{F}^{\Lambda}_{\mu\nu}F^{\Sigma}_{\rho\sigma} + \frac{1}{2}g_{ij}(\phi)\partial_\mu\phi^i\partial^\mu\phi^j\Big)$$

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  28 $F^{\Lambda}_{\mu\nu} = \partial_\mu\mathcal{A}^{\Lambda}_{\nu} - \partial_\nu\mathcal{A}^{\Lambda}_{\mu}$, with BI $\partial_\mu\tilde{F}^{\mu\nu\,\Lambda} = 0$. The dual field strength $G^{\mu\nu}_{\Lambda}$ is defined as a derivative of the action over $F_{\mu\nu}$, namely

$$\tilde{G}^{\mu\nu}_{\Lambda} = 2\frac{\delta S(F,\phi,g)}{\delta F^{\Lambda}_{\mu\nu}} \qquad \Rightarrow \text{vector EOM} \qquad \partial_\mu\tilde{G}^{\mu\nu}_{\Lambda} = 0$$

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- Vectors in 56 transform linearly as a doublet. $E_{7(7)}$ mixes BI and EOM

$$\begin{pmatrix}F\\G\end{pmatrix}'=\mathcal{S}\begin{pmatrix}F\\G\end{pmatrix},\qquad\begin{pmatrix}\partial\tilde{F}\\\partial\tilde{G}\end{pmatrix}'=\mathcal{S}\begin{pmatrix}\partial\tilde{F}\\\partial\tilde{G}\end{pmatrix},\qquad\mathcal{S}\equiv\begin{pmatrix}\hat{A}&\hat{B}\\\hat{C}&\hat{D}\end{pmatrix},$$

  The kinetic function of scalars transforms under fractional transformations

$$\mathcal{N}'(\phi)=\frac{\hat{C}+\hat{D}\mathcal{N}}{\hat{A}+\hat{B}\mathcal{N}}\qquad\Rightarrow\qquad\phi'=\phi+\sigma+\ldots$$

$\mathcal{N}$=8 supergravity vectors

- For amplitude practitioners supergravity vectors originate from a double copy of $\mathcal{N}$=4 $SU(4) \times SU(4)$ SYM vectors which become $SU(8)$ states at the level of free asymptotic physical states.
- It is well known that scalars are in the coset space $G/H = E_{7(7)}/SU(8)$. However, the situation with vectors is more delicate. Namely, vectors in $\mathcal{N}$=8 supergravity Lagrangian do not transform under $SU(8)$, they transform under $E_{7(7)}$ together with their dual partners.

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- Note that in classical $\mathcal{N}$=8 supergravity the dual vectors $\mathcal{B}_{\nu\Lambda}$ are complicated non-local function of scalars and original vectors of the form

$$(\partial_\mu \mathcal{B}_\nu - \partial_\nu \mathcal{B}_\mu)_\Lambda \sim \mathcal{N}_{\Lambda\Sigma}(\phi)(\partial_\mu \mathcal{A}_\nu - \partial_\nu \mathcal{A}_\mu)^\Sigma$$

Infinitesimal form of $E_{7(7)}$ transformations acting on vectors and scalars

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- 133 of $E_{7(7)}$ include 63 Λ's in the maximal subgroup and 70 Σ's in the off-diagonal part

$$E = \exp \begin{pmatrix} 2\delta^{[I}_{[K} \Lambda^{J]}{}_{L]} & \bar{\Sigma}^{IJKL} \\ \Sigma_{IJKL} & 2\delta^{[K}_{[I} \Lambda^{L]}{}_{J]} \end{pmatrix}$$

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- But the vector part of the candidate counterterm is $E_{7(7)}$ invariant! The UV divergence breaks the $E_{7(7)}$ Noether-Gaillard-Zumino current conservation! Details in 1103.0322, 1104.5480

Reconstructive identity

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- Consider, for example, the Born-Infeld model as a deformation of the Maxwell theory.

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- Duality symmetry: the action is either quadratic in F or has infinite powers of F (We will call the case of duality with non-linear dependence on F and its derivatives a BI-type duality)

Bossard-Nicolai, 1105.1273 and Carrasco, RK, Roiban 1108.4390

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  Twisted linear self-duality constraint in $SU(8)$ covariant form is

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$$T^+_{AB} = \frac{\delta \mathcal{I}^{(1)}}{\delta \overline{T}^{+AB}}$$

  where $\mathcal{I}^{(1)}$ is a manifestly duality invariant counterterm. In $\mathcal{N}$=8 it necessarily has terms quartic in T. Therefore solving the eq. $T^+ \sim (T^-)^2 \bar{T}^+$ leads to $G(F)$ which has all powers of F^n with derivatives. After solving for $G(F)$ one has to reconstruct the action using $G \sim \frac{\delta S}{\delta F}$.

  The action must be of the BI type since it has all powers of F^n.

UV divergences in amplitude method

- For amplitude practitioners it is clear that if there is some L-loop order UV divergence, all quartic amplitudes are divergent: 4-graviton, 2-graviton-2-vector, 4-vector etc. One can't avoid having $\sim F^4$ (with derivatives) divergence since vector and graviton are in the same supermultiplet due to supersymmetry.

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- Any such CT must have F^4 term.

Thus the classical supergravity action is quadratic in F, but the CT is quartic. When it is used as a source of the deformation of the linear self-duality constraint, according to BN procedure, the action has all powers of F since we solve by iteration an eq.

$$G \sim F + g^2(G+F)^3 \sim F + g^2 F^3 + g^4 F^4 + g^6 F^7 + \ldots \Rightarrow \text{BI type}$$

this is different from the BN example where the corresponding solution for G remains linear in F

$$G \sim F + g^2 R^2 (F+G) \sim F X(R)$$

UV divergences in amplitude method

- For amplitude practitioners it is clear that if there is some L-loop order UV divergence, all quartic amplitudes are divergent: 4-graviton, 2-graviton-2-vector, 4-vector etc. One can't avoid having $\sim F^4$ (with derivatives) divergence since vector and graviton are in the same supermultiplet due to supersymmetry.

- Any such CT must have F^4 term.

  Thus the classical supergravity action is quadratic in F, but the CT is quartic. When it is used as a source of the deformation of the linear self-duality constraint, according to BN procedure, the action has all powers of F since we solve by iteration an eq.

$$G \sim F + g^2(G+F)^3 \sim F + g^2F^3 + g^4F^4 + g^6F^7 + \ldots \Rightarrow \text{BI type}$$

  this is different from the BN example where the corresponding solution for G remains linear in F

$$G \sim F + g^2R^2(F+G) \sim FX(R)$$

- Consider, for example, the 3-loop counterterm. Higher loop 4-point CT's have in addition $f(s,t,u)$

BN →

$$\frac{1}{4}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}R^{\dot\alpha\dot\beta\dot\gamma\dot\delta}R^{\alpha\beta\gamma\delta}R_{\alpha\beta\gamma\delta}$$

$$-iR_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\psi^{\dot\alpha\dot\beta\dot\gamma\,b}\partial^{\dot\delta\delta}\psi_b^{\alpha\beta\gamma}R_{\alpha\beta\gamma\delta}$$

$$-\frac{1}{2}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}F^{\dot\alpha\dot\beta\,bc}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}F^{\alpha\beta}_{bc}R_{\alpha\beta\gamma\delta} \longrightarrow (F+G)^2R^2$$

$$-\frac{1}{2}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}F^{\dot\alpha\dot\beta\,bc}\psi^{\alpha\beta}{}_{\gamma\,b}\psi_{\alpha\beta\delta\,c}$$

$$\frac{i}{3!}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\chi^{\dot\alpha\,bcd}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\chi^{\alpha}_{bcd}R_{\alpha\beta\gamma\delta}$$

$$\frac{i}{2}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\partial^{\dot\alpha\alpha}\chi^{\dot\delta\,bcd}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}F^{\delta}{}_{\alpha\,bc}\psi_{\beta\gamma\delta\,d}$$

$$\frac{1}{4!}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\phi^{bcde}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\phi_{bcde}R_{\alpha\beta\gamma\delta}$$

$$-\frac{1}{3!}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\partial^{\dot\alpha\alpha}\phi^{bcde}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\chi_{\alpha\,bcd}\psi_{\beta\gamma\delta\,e}$$

$$\frac{1}{2^3}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\phi^{bcde}F_{\alpha\beta\,bc}F_{\gamma\delta\,de}$$

$$-\frac{1}{3!^2 2^2}R_{\dot\alpha\dot\beta\dot\gamma\dot\delta}\epsilon^{bcdefghi}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\chi_{\gamma\,ghi}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\chi_{\alpha\,bcd}F_{\beta\delta\,ef}$$

$$\frac{1}{2}\psi^b_{\dot\alpha\dot\beta\dot\gamma}\psi^{\dot\alpha\dot\beta}{}_{\dot\delta}{}^{c}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}F^{\alpha\beta}_{bc}R_{\alpha\beta\gamma\delta}$$

$$-\frac{2}{2}\psi^b_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu\psi^{\dot\alpha\dot\beta\dot\gamma\,c}\partial^\mu\psi_b^{\alpha\beta\gamma}\psi_{\alpha\beta\gamma\,c}$$

$$-\frac{i}{2}\psi^b_{\dot\alpha\dot\beta\dot\gamma}F^{\dot\gamma}{}_{\dot\delta}{}^{cd}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\delta\delta}\chi^{\gamma}_{bcd}R_{\alpha\beta\gamma\delta}$$

$$2i\psi^b_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu F^{\dot\alpha\dot\beta\,cd}\partial^\mu\partial^{\dot\gamma\gamma}F^{\alpha\beta}_{bc}\psi_{\alpha\beta\gamma\,d}$$

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$$-\frac{1}{3!}\psi^b_{\dot\alpha\dot\beta\dot\gamma}\chi^{cde}_{\dot\delta}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\phi_{bcde}R_{\alpha\beta\gamma\delta}$$

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N=8 3-loop counterterm given using amplitude method in Freedman,Tonni 1101.1672

$$
\begin{aligned}
&\tfrac{2i}{3!2}\psi^{b}_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu\phi^{cdef}\,\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^\mu\phi_{bcde}\psi_{\alpha\beta\gamma\, f}\\
&-\tfrac{2i}{4!2}\psi^{b}_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu\phi^{cdef}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\phi_{cdef}\partial^\mu\psi_{\alpha\beta\gamma\, b}\\
&-\tfrac{2i}{3!}\psi^{b}_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu\partial^{\dot\alpha\alpha}\phi^{cdef}\,\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\chi_{\alpha\, cde}\partial^\mu F_{\beta\gamma\, bf}\\
&\tfrac{2i}{2^2}\psi^{b}_{\dot\alpha\dot\beta\dot\gamma}\partial_\mu\partial^{\dot\alpha\alpha}\phi^{cdef}\,\partial^\mu\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\chi_{\alpha\, bcd}F_{\beta\gamma\, ef}\\
&\tfrac{2i}{3!^2 6}\psi^{b}_{\dot\alpha\dot\beta\dot\gamma}\epsilon^{cdefghij}\partial_\mu\partial^{\dot\alpha\alpha}\chi_{\gamma\, hij}\partial^\mu\partial^{\dot\beta\beta}\chi_{\alpha\, bcd}\partial^{\dot\gamma\gamma}\chi_{\beta\, efg}\\
&\tfrac{1}{2^3}F^{bc}_{\dot\alpha\dot\beta}F^{de}_{\dot\gamma\dot\delta}\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\partial^{\dot\delta\delta}\phi_{bcde}R_{\alpha\beta\gamma\delta}\\
&\tfrac{2}{2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu F^{\dot\alpha}{}_{\dot\gamma}{}^{de}\partial^\mu\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\chi^{\alpha}_{bcd}\psi_{\alpha\beta\gamma\, e}\\
&\tfrac{2^2}{2^3}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\partial_\nu F^{\dot\alpha\dot\beta\, de}\partial^\mu\partial^\nu F^{\alpha\beta}_{bc}F_{\alpha\beta\, de}
\end{aligned}
$$

→ $(F+G)^4$

$$
\tfrac{2^2}{2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\partial_\nu F^{\dot\alpha\dot\beta\, de}\partial^\mu F^{\alpha\beta}_{bd}\partial^\nu F_{\alpha\beta\, ce}
$$

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$$
\begin{aligned}
&\tfrac{1}{3!^2 2^2}\epsilon_{bcdefghi}F^{bc}_{\dot\alpha\dot\beta}\partial^{\dot\delta\delta}\chi^{def}_{\dot\gamma}\,\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\chi^{ghi}_{\dot\delta}R_{\alpha\beta\gamma\delta}\\
&\tfrac{2i}{3!}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\chi^{def}_{\dot\gamma}\,\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\phi_{bdef}\,\partial^\mu\psi_{\alpha\beta\gamma\, c}\\
&-\tfrac{2i}{2^2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\chi^{def}_{\dot\gamma}\,\partial^\mu\partial^{\dot\alpha\alpha}\partial^{\dot\beta\beta}\partial^{\dot\gamma\gamma}\phi_{bcde}\psi_{\alpha\beta\gamma\, f}\\
&\tfrac{2^2 i}{2^2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\partial_\nu\chi^{\dot\alpha\, def}\partial^\mu\partial^\nu\partial^{\dot\beta\beta}\chi^{\alpha}_{bcd}F_{\alpha\beta\, ef}\\
&-\tfrac{2^2 i}{2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\partial_\nu\chi^{\dot\alpha\, def}\partial^\mu\partial^{\dot\beta\beta}\partial^\nu\chi^{\alpha}_{bde}F_{\alpha\beta\, cf}\\
&-\tfrac{2^2 i}{3!2}F^{bc}_{\dot\alpha\dot\beta}\partial_\mu\partial_\nu\chi^{\dot\alpha\, def}\partial^{\dot\beta\beta}\chi^{\alpha}_{def}\partial^\mu\partial^\nu F_{\alpha\beta\, bc}
\end{aligned}
\qquad (6.8)
$$

$\mathcal{N}$=8 is complicated, let us look at $U(1)$ $\mathcal{N}$=0 duality, Maxwell and Born-Infeld, CKR

- $$T = F - iG\,, \qquad T^* = F + iG\,, \qquad T^\pm = \tfrac{1}{2}(T \pm i\tilde{T})$$
  and the $U(1)$ duality symmetry is
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  $$\delta\mathcal{L} = \frac{1}{4}(\tilde{G}BG - \tilde{F}BF)$$
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  $$F\tilde{F} + G\tilde{G} = 0\,,$$
- In the Maxwell case with $g = 0$ there is a simple duality covariant linear twisted self-duality constraint $G = \tilde{F}$ and $F = -\tilde{G}$, which in self-dual notation is
  $$T^+ = F^+ - iG^+ = 0$$

$U(1)$ $\mathcal{N}$=0 duality, general solution of NGZ identity and Born-Infeld

NGZ identity has many solutions, one of them is BI but there are more (there is an arbitrary function of one real variable available)

- BN original proposal for $U(1)$ $\mathcal{N}$=0 duality is to deform the Maxwell constraint $T^+ = 0$ by the variation of the initial source of deformation, in this case $\mathcal{I}^{(1)} = \frac{g^2}{32}(T^-)^2(T^{*+})^2$

$$T^+_{\mu\nu} = \frac{\delta \mathcal{I}^{(1)}}{\delta \bar{T}^{*+}_{\mu\nu}} = \frac{g^2}{16} T^{*+}_{\mu\nu}(T^-)^2$$

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- We have found a generalized procedure which requires a more general source of the deformation of the linear twisted self-duality constraint and allows to reproduce all solutions of the NGZ identity. In particular, we can get the BI model.

Where is BI? Our generalized procedure.

- Our ansatz for the action and for the deformation of the linear twisted self-duality

$$\mathcal{L} = \Big(g^{-2} \sum_{m=0,p=0} g^{2(p+2m)} c_{(p,2m)} t^p z^{2m} \Big) - c_{(0,0)} g^{-2}$$

where $t = F^2/4$, and $z = F\tilde{F}/4$ and we recover the Gibbons-Rasheed "function of one variable' s worth of Lagrangians"

$$T^+_{\mu\nu} = \frac{g^2}{16} T^{*\,+}_{\mu\nu} (T^-)^2 \Big[1 + \sum_{n=0} d_n \Big(\tfrac{1}{4}\, g^4 (T^{*\,+})^2 (T^-)^2 \Big)^n \Big],$$

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- Born-Infeld
We have found that to reconstruct the BI action $\mathcal{L}_{BI} = \frac{1}{g^2}(1 - \sqrt{-\det(\eta_{\mu\nu} + gF_{\mu\nu})}$ using the generalized procedure order by order we had to define the deformation of the linear twisted self-duality as follows

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- Writing this as

$$T^+_{\mu\nu} = \frac{\delta\mathcal{I}(T^-, \bar{T}^+, g)}{\delta \bar{T}^+_{\mu\nu}}$$

Hypergeometric PFQ

We have found that the required deformation source takes the following form

$$\mathcal{I}(T^-, \overline{T}^+, g) = \frac{6}{g^2}\Big(1 - {}_3F_2(-\tfrac{1}{2}, -\tfrac{1}{4}, \tfrac{1}{4}; \tfrac{1}{3}, \tfrac{2}{3}; -\tfrac{1}{27}\, g^4\, (\overline{T}^+)^2\, (T^-)^2)\Big)$$

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- Is there a deep meaning here? A principle?
- Here we knew the BI action and therefore we were able to reconstruct the deformation even in the case when the initial source did not work.

Covariant procedures for perturbative non-linear deformation of duality-invariant theories

We have found analogous results for $\mathcal{N}$=1 supersymmetric BI-type models and we are working on $\mathcal{N}$=2 case. We find that the choice of manifestly duality invariant $\mathcal{I}$ for known models with $\mathcal{N}$=2 + extra $\mathcal{N}$=2 spontaneously broken is even more involved.

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- For $\mathcal{N}$=1 there are some papers on BI-type supergravity, but vectors are not part of the supergravity multiplet, and there is no duality.
- The $\mathcal{N}\geq$ 2 BI supergravity is not, yet, available. We will try to construct it.

Covariant procedures for perturbative non-linear deformation of duality-invariant theories

We have found analogous results for $\mathcal{N}$=1 supersymmetric BI-type models and we are working on $\mathcal{N}$=2 case. We find that the choice of manifestly duality invariant $\mathcal{I}$ for known models with $\mathcal{N}$=2 + extra $\mathcal{N}$=2 spontaneously broken is even more involved.

- Thus, on one hand, we were able to generalize the original BN procedure and produce the BI action. On the other hand we do not see a clear underlying principle for making a decision about the source of deformation of the twisted self-duality.
- Our choice of a function $\mathcal{I}(T^-, \overline{T}^+, g) = \frac{6}{g^2}\left(1 - {}_3F_2(-\frac{1}{2}, -\frac{1}{4}, \frac{1}{4}; \frac{1}{3}, \frac{2}{3}; -\frac{1}{27}\, g^4\, (\overline{T}^+)^2\, (T^-)^2)\right)$ enabled us to reconstruct the geometric action $\frac{1}{g^2}(1 - \sqrt{-\det(\eta_{\mu\nu} + gF_{\mu\nu})}$.
- So, what is the lesson for $\mathcal{N}$=8 BI-type supergravity ?
- For $\mathcal{N}$=1 there are some papers on BI-type supergravity, but vectors are not part of the supergravity multiplet, and there is no duality.
- The $\mathcal{N}\geq 2$ BI supergravity is not, yet, available. We will try to construct it.
- Crucial point: will it be possible to discover $\mathcal{N}$=8 BI supergravity with F^n terms (with derivatives and all n) in the action using the generalization of the BN deformation of the linear twisted self-duality?

BI $\mathcal{N}$=8???

- The classical $\mathcal{N}$=8 theory has one gravitational coupling and it is strictly quadratic in vector fields F

$$S_{N=8}(\kappa^2) = \frac{1}{2\kappa^2}(R - F\mathcal{N}(\phi)F + ...)$$

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- We will find that there is no consistent N=8 Born-Infeld-type supergravity. In such case an unbroken $E_{7(7)}$ would predict UV finiteness.
- We will be able to construct N=8 Born-Infeld-type supergravity.

$$S^{BI}_{N=8}(\kappa^2, g^2) = \frac{1}{2\kappa^2}(R - F\mathcal{N}(\phi)F + ...) + g^2 F^4 f(s,t,u) + ...g^{2m} F^n f(s,t,u,...) + ...$$

In such case it is not even clear how the existence of such $\mathcal{N}$=8 BI supergravity affects the predictions for the UV properties of the original $\mathcal{N}$=8 supergravity theory. We will deal with this issue if we find BI $\mathcal{N}$=8

What are chances to find $\mathcal{N}=8$ BI supergravity and use it to disprove the $E_{7(7)}$ UV finiteness argument???

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- A wise person once said: "Born-Infeld $\mathcal{N}$=8 supergravity does sound like a tall order!"

Conclusions

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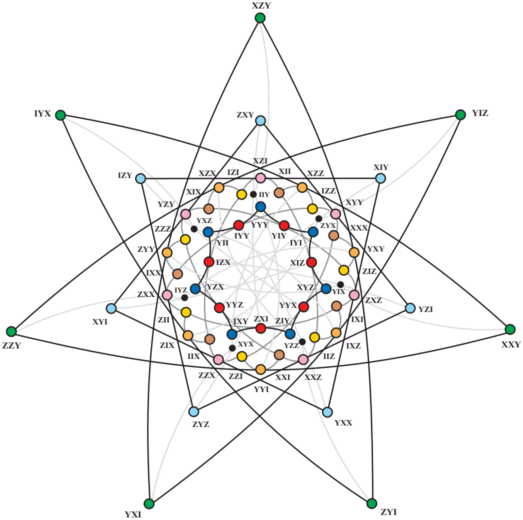
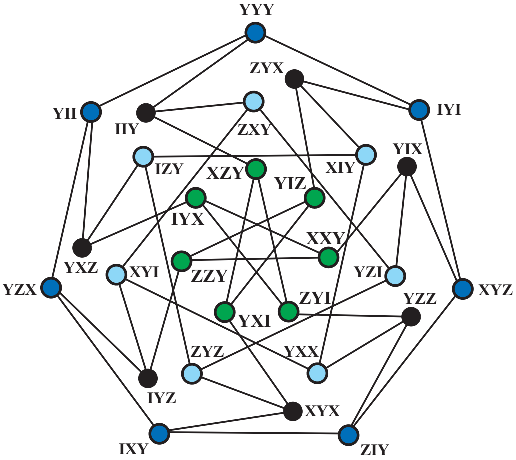
- $E_{7(7)}$ duality supports this conclusion.
- $E_{7(7)}$ duality may be insufficient to prove the UV finiteness of the conjectured Born-Infeld $\mathcal{N}$=8 supergravity, which we are trying to construct. Even if we succeed, we do not know yet whether this would affect our earlier results related to the UV properties of the original $\mathcal{N}$=8 supergravity.

Back up slides

$E_{7(7)}$ symmetric N=8 black hole entropy, 3-cubit operators and the Split Cayley Hexagon (by P. Levay et al)



The Coxeter graph, in a form showing its automorphism of order seven, as a subgraph/subgeometry of the Hexagon



Dilaton–Axion Symmetry

John H. Schwarz, hep-th 9209125

The heterotic string compactified on a six-torus is described by a low-energy effective action consisting of N=4 supergravity coupled to N=4 super Yang-Mills, a theory that was studied in detail many years ago. By explicitly carrying out the dimensional reduction of the massless fields, we obtain the bosonic sector of this theory. In the Abelian case the action is written with manifest global $O(6, 6+n)$ symmetry. A duality transformation that replaces the antisymmetric tensor field by an axion brings it to a form in which the axion and dilaton parametrize an $SL(2, R)/SO(2)$ coset, and the equations of motion have $SL(2, R)$ symmetry. This symmetry, which

$SL(2,R)$ Symmetry in Four Dimensions

Another way of describing the $SL(2,R)$ symmetry of the dilaton and axion kinetic terms is to introduce a complex modular parameter

$$\tau = \chi + ie^{-\phi}\ , \tag{41}$$

which has the nice property that under a linear fractional transformation

$$\tau \to \frac{a\tau + b}{c\tau + d} \tag{42}$$

the combination

$$\frac{g^{\mu\nu}\partial_\mu \tau \partial_\nu \bar{\tau}}{(\text{Im}\ \tau)^2} = g^{\mu\nu}(\partial_\mu \phi \partial_\nu \phi + e^{2\phi}\partial_\mu \chi \partial_\nu \chi) \tag{43}$$

is invariant.

To see how the $SL(2,R)$ symmetry works for S_F, we define

$$\mathcal{F}^{\pm}_{\mu\nu} = M\eta\mathcal{F}_{\mu\nu} \pm i\tilde{\mathcal{F}}_{\mu\nu} \,. \tag{45}$$

Then, using the identity $\mathcal{F}^{+\mu\nu}M^{-1}\mathcal{F}^{-}_{\mu\nu} = 0$, we can rewrite S_F in the form

$$S_F = -\frac{1}{16i}\int_M dx\sqrt{-g}\Big(\tau\mathcal{F}^{+\mu\nu}M^{-1}\mathcal{F}^{+}_{\mu\nu} - \bar{\tau}\mathcal{F}^{-\mu\nu}M^{-1}\mathcal{F}^{-}_{\mu\nu}\Big) \,. \tag{46}$$

To exhibit $SL(2,R)$ symmetry it is necessary to have $\mathcal{A}_\mu$ transform at the same time as τ. The appropriate choice is to require that $\mathcal{F}^{\pm}_{\mu\nu}$ transform as modular forms as follows

$$\mathcal{F}^{+}_{\mu\nu} \to (c\tau + d)\mathcal{F}^{+}_{\mu\nu} \,, \quad \mathcal{F}^{-}_{\mu\nu} \to (c\bar{\tau} + d)\mathcal{F}^{-}_{\mu\nu} \,. \tag{49}$$

This implies that

$$\tau\mathcal{F}^{+}_{\mu\nu} \to (a\tau + b)\mathcal{F}^{+}_{\mu\nu} \,, \quad \bar{\tau}\mathcal{F}^{-}_{\mu\nu} \to (a\bar{\tau} + b)\mathcal{F}^{-}_{\mu\nu} \,. \tag{50}$$

Thus the equation of motion (47) and the Bianchi identity (48) transform into linear combinations of one another and are preserved. In particular, the negative of the unit matrix sends $\mathcal{F}^{\pm}_{\mu\nu} \to -\mathcal{F}^{\pm}_{\mu\nu}$. This result is acceptable if we identify the symmetry as $SL(2,R)$, not just $PSL(2,R) = SL(2,R)/Z_2$. Note that $SL(2,R)$ is not a symmetry of the action.