Applied Symbology

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past and in progress work with Goncharov, Spradlin, Vergu, also Prygarin

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Plan

- Introduction
- Review: Symbols and Amplitudes

  Goncharov, Spradlin,Vergu, AV

- One-loop integrals from mixed Tate motives

  Spradlin, AV

- Extracting Motivic Content from the Symbol

  with Goncharov, Spradlin, Vergu

- Regge limit

  with Prygarin, Spradlin, Vergu

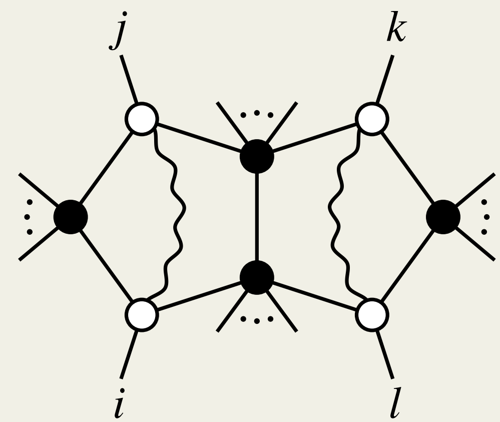
- Outlook and Conclusions

We have learned a lot about N=4 Yang-Mills scattering amplitudes.

Many have contributed to these recent exciting developments including:

Alday, Arkani-Hamed, Bargheer, Beisert, Belitski, Bern, Berkovits, Boels, Bourjaily, Brandhuber, Brodel, Bjerrum-Bohr, Bullimore, Britto, Cachazo, Caron-Huot, Carrasco, Cheung, Damgaard, Del Duca, Duhr, Dixon, Dolan, Duhr, Drummons, Eden, Elvang, Fend, Ferro, Forde, Freedman, Gaiotto, Goddard, Goncharov, Green, Henn, Heslop, Hodges, Huang, Ita, Johannson, Kallosh, Kaplan, Khoze, Kiermaier, Korchemsky, Kosower, Lipatov, Loebbert, Maitre, Mafra, Maldacena, Mason, Naculich, Nastase, Plefka, Prygarin, Roiban, Sabio Vera, Schnizer, Sever, Skinner, Smirnov, Sokatchev, Spence, Spradlin, Staudacher, Stelle, Steiberger, Svrcek, Taylor, Travaglini, Trnka, Tye, Vanhove, Vergu, Vieira, Wen, Witten

Amplitudes and Integrand



- Standard Lore

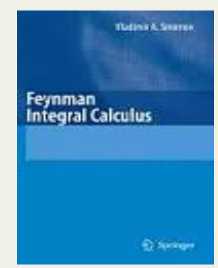
$$Amplitude = \int d(loop\ momenta)\ (Integrand)$$

- Integrand = solved for all loops and all legs in N=4

Bern, Dixon, Kosower; Arkani-Hamed, Cachazo, et al

- Integral = still very hard to evaluate! This step requires a lot of blood and tears begging for some new technology!

e.g. Smirnov's book



- Symbol = "half way between integrand and amplitude"

Goncharov, Spradlin, Vergu, AV

Symbol of an Amplitude

- Symbol of an amplitude is a new very powerful tool for analysis of multiloop amplitudes which comes from modern mathematics called theory of motives.

Goncharov, Spradlin, Vergu, AV

- Symbol of an amplitude

-- leads to the compact expression for the amplitude

-- captures the essential physical (and “motivic”) content of the amplitude

Motivations

Simplest nontrivial multi-loop amplitude in N=4 Yang-Mills is the 2-loop 6-particle MHV remainder function = what is left after subtracting IR divergences, leaving a finite and dual conformal invariant quantity

$$R(u_1, u_2, u_2)$$

$$u_1 = \frac{s_{12}s_{45}}{s_{123}s_{345}} \quad u_2 = \frac{s_{23}s_{56}}{s_{234}s_{456}} \quad u_3 = \frac{s_{34}s_{61}}{s_{345}s_{234}} \qquad s_{123} = (k_1 + k_2 + k_3)^2$$

Numerically known since '08 Bern, Dixon, Kosower, Roiban, Vergu, Spradlin, AV
Henn,Drummond, Korchemsky, Sokatchev

In a heroic effort, Del Duca, Duhr, Smirnov '09 found a manageable way to evaluate the relevant diagrams and obtained an analytic formula for it:

DDS Formula

$$R^{(2)}_{6,WL}(u_1,u_2,u_3) = \tag{H.1}$$

$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+$$
$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{1}{24}\pi^2 G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{1}{24}\pi^2 G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)+$$
$$\frac{3}{2}G\left(0,0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)+\frac{3}{2}G\left(0,0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},0,\frac{1}{u_3};1\right)+$$
$$G\left(0,\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_1};1\right)+$$
$$G\left(0,\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},0,\frac{1}{u_3};1\right)+G\left(0,\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_1};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},0,\frac{1}{u_2};1\right)+G\left(0,\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)+$$
$$G\left(0,\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{2}G\left(0,\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3},0;1\right)+\frac{1}{2}G\left(\frac{1}{1-u_1},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$

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$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1,0;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},0;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},1;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_2};1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_1},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,0,\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1},0;1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{1-u_2},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0,\frac{1}{1-u_2};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1,0;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},1;1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_1};1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_1+u_2};1\right)-G\left(\frac{1}{u_2},0,0,\frac{1}{u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)-\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2},0;1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{1-u_3},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0,\frac{1}{1-u_3};1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1,0;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},0;1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{79\pi^4}{360}+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)-G\left(\frac{1}{u_3},0,0,\frac{1}{u_1};1\right)-$$

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$$G\left(\frac{1}{u_3},0,0,\frac{1}{u_2};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_1+u_3};1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,0,\frac{1}{u_2+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)-\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{24}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)+\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)+$$
$$\frac{1}{8}\pi^2\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{123},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{132},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{132},\frac{1}{1-u_1};1\right)+\mathcal{G}\left(0,0,v_{213},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{213},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,0,v_{312},\frac{1}{1-u_3};1\right)+\mathcal{G}\left(0,0,v_{321},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},0,v_{132};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},0,v_{231};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},0,v_{321};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-\frac{1}{2}\mathcal{G}\left(0,\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1},0;1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{123},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},0;1\right)-\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{123};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,0,v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},0,\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{123};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},0,v_{132};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},1;1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3},0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0,\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1,\frac{1}{1-u_1};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{213};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,0,v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},0,\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{213};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},0,v_{231};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0,\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1,\frac{1}{1-u_2};1\right)+$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1,\frac{1}{1-u_2};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{312};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,0,v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},0,\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{312};1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},0,v_{321};1\right)-\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1,0;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2},0;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},1;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,0;1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0,\frac{1}{1-u_3};1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},0;1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,0;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,0;1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1,\frac{1}{1-u_3};1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},0;1\right)-$$

$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},1;1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},0,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},0,\frac{1}{1-u_1},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,0,\frac{1}{1-u_1};1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},1,1,\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1},\frac{1}{1-u_1};1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},0,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1,\frac{1}{1-u_1};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},\frac{1}{1-u_1},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,1,\frac{1}{1-u_1};1\right)-\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1,1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,1,\frac{1}{1-u_2};1\right)-\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},0,\frac{1}{1-u_2},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},1,0,\frac{1}{1-u_2};1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},1,1,\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2},\frac{1}{1-u_2};1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1,\frac{1}{1-u_2};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},\frac{1}{1-u_2},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},0,\frac{1}{1-u_3},1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},1,0,\frac{1}{1-u_3};1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},1,1,\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},0;1\right)-$$
$$\frac{5}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3},\frac{1}{1-u_3};1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},0,1;1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,0;1\right)-\frac{5}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,1;1\right)+\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1,\frac{1}{1-u_3};1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},\frac{1}{1-u_3},1;1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,1,\frac{1}{1-u_3};1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3},1;1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1,1;1\right)-\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_1},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}G\left(\frac{1}{1-u_2},1,\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$

$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},0;1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(0,v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},1;1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{u_1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_1\right)-$$

$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},0;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_1\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_1\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_1\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_1\right)-\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_1\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)+\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_1\right)-\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_1\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(0,\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},0;1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)-\frac{3}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_2},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},1,\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},1;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{2}G\left(\frac{1}{u_3},0,\frac{1}{u_2};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_2\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(0,u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(0,v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},0;1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{u_2-1}{u_1+u_2-1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_2\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},0;1\right)H\left(0;u_2\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},0;1\right)H\left(0;u_2\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)+\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)-$$

$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H\left(0;u_2\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)-\frac{3}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H\left(0;u_2\right)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_1\right)H\left(0;u_2\right)+\frac{5}{24}\pi^2H\left(0;u_1\right)H\left(0;u_2\right)-$$
$$\frac{1}{4}G\left(0,\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(0,\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(0,\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}G\left(0,\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},1,\frac{1}{u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_1},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_1},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},0;1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_2},0,\frac{1}{u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}G\left(\frac{1}{u_2},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)-\frac{3}{4}G\left(\frac{1}{u_3},0,\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H\left(0;u_3\right)+\frac{1}{2}G\left(\frac{1}{u_3},\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)+$$

$$\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(0,u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(0,v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(0,v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(0,v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(0,v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{123};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},0,v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{123};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},\frac{1}{1-u_1},v_{132};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123},\frac{1}{u_3};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{123},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},0;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_1},v_{132},\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},0,v_{231};1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{213};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},\frac{1}{1-u_2},v_{231};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},0;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231},\frac{u_3-1}{u_2+u_3-1};1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},0;1\right)H\left(0;u_3\right)-\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{213},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},0;1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_2},v_{231},\frac{1}{1-u_2};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{312};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},0,v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{312};1\right)H\left(0;u_3\right)+\frac{1}{2}\mathcal{G}\left(\frac{1}{1-u_3},\frac{1}{1-u_3},v_{321};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},1;1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321},\frac{1}{1-u_3};1\right)H\left(0;u_3\right)-$$
$$\frac{3}{4}\mathcal{G}\left(v_{123},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)-\frac{3}{4}\mathcal{G}\left(v_{123},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{132},1,\frac{1}{1-u_1};1\right)H\left(0;u_3\right)+\frac{1}{4}\mathcal{G}\left(v_{132},\frac{1}{1-u_1},1;1\right)H\left(0;u_3\right)-$$
$$\frac{1}{4}\mathcal{G}\left(v_{213},1,\frac{1}{1-u_2};1\right)H\left(0;u_3\right)-\frac{1}{4}\mathcal{G}\left(v_{213},\frac{1}{1-u_2},1;1\right)H\left(0;u_3\right)+$$

$$\frac{3}{4}\mathcal{G}\left(v_{231},1,\frac{1}{1-u_2};1\right)H(0;u_3)+\frac{3}{4}\mathcal{G}\left(v_{231},\frac{1}{1-u_2},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{312},1,\frac{1}{1-u_3};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(v_{312},\frac{1}{1-u_3},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(v_{321},1,\frac{1}{1-u_3};1\right)H(0;u_3)+\frac{1}{4}\mathcal{G}\left(v_{321},\frac{1}{1-u_3},1;1\right)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(0;u_1)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H(0;u_1)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0;u_1)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0;u_1)H(0;u_3)+$$
$$\frac{5}{24}\pi^2H(0;u_1)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H(0;u_2)H(0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(0;u_2)H(0;u_3)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(0;u_2)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H(0;u_2)H(0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0;u_2)H(0;u_3)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0;u_2)H(0;u_3)+\frac{5}{24}\pi^2H(0;u_2)H(0;u_3)+$$
$$3H(0;u_2)H(0,0;u_1)H(0;u_3)+3H(0;u_1)H(0,0;u_2)H(0;u_3)+$$
$$\frac{1}{4}H(0;u_2)H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H(0;u_3)+\frac{1}{2}H(0;u_1)H(0,1;(u_1+u_3))H(0;u_3)+$$
$$\frac{1}{4}H(0;u_1)H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H(0;u_3)+\frac{1}{2}H(0;u_2)H(0,1;(u_2+u_3))H(0;u_3)+$$
$$\frac{3}{4}H(0;u_2)H(1,0;u_1)H(0;u_3)+\frac{3}{4}H(0;u_1)H(1,0;u_2)H(0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0,0;u_1)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0,0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H(0,0;u_1)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H(0,0;u_1)-\frac{23}{24}\pi^2H(0,0;u_1)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0,0;u_2)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{312};1\right)H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},v_{321};1\right)H(0,0;u_2)-$$
$$\frac{25}{4}H(0,0;u_1)H(0,0;u_2)-\frac{23}{24}\pi^2H(0,0;u_2)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{123};1\right)H(0,0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},v_{132};1\right)H(0,0;u_3)+\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{213};1\right)H(0,0;u_3)+$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},v_{231};1\right)H(0,0;u_3)+3H(0;u_1)H(0;u_2)H(0,0;u_3)-$$
$$\frac{25}{4}H(0,0;u_1)H(0,0;u_3)-\frac{25}{4}H(0,0;u_2)H(0,0;u_3)-\frac{23}{24}\pi^2H(0,0;u_3)+\frac{1}{12}\pi^2H(0,1;u_1)+$$
$$\frac{1}{12}\pi^2H(0,1;u_2)-\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H(0;u_1)H(0;u_2)H(0,1;(u_1+u_2))+$$

$$\frac{1}{12}\pi^2H(0,1;(u_1+u_2))+\frac{1}{12}\pi^2H(0,1;u_3)+\frac{1}{4}H(0;u_1)H(0;u_2)H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{24}\pi^2H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)+\frac{1}{12}\pi^2H(0,1;(u_1+u_3))-\frac{1}{24}\pi^2H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{12}\pi^2H(0,1;(u_2+u_3))-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H(1,0;u_1)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(1,0;u_1)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(1,0;u_1)+$$
$$\frac{1}{4}G\left(\frac{1}{1-u_3},\frac{u_1-1}{u_1+u_3-1};1\right)H(1,0;u_1)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(1,0;u_1)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_3},u_{312};1\right)H(1,0;u_1)-\frac{3}{4}H(0,0;u_2)H(1,0;u_1)-\frac{3}{4}H(0,0;u_3)H(1,0;u_1)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_3-1}{u_1-1}\right)H(1,0;u_1)-\frac{1}{3}\pi^2H(1,0;u_1)-\frac{1}{2}G\left(0,\frac{1}{u_1+u_2};1\right)H(1,0;u_2)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{1-u_1},\frac{u_2-1}{u_1+u_2-1};1\right)H(1,0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_2};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_1+u_2};1\right)H(1,0;u_2)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(1,0;u_2)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(1,0;u_2)-$$
$$\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_1},u_{123};1\right)H(1,0;u_2)-\frac{3}{4}H(0,0;u_1)H(1,0;u_2)-\frac{3}{4}H(0,0;u_3)H(1,0;u_2)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_1+u_2-1}{u_2-1}\right)H(1,0;u_2)-\frac{1}{4}H(1,0;u_1)H(1,0;u_2)-\frac{1}{3}\pi^2H(1,0;u_2)-$$
$$\frac{1}{2}G\left(0,\frac{1}{u_1+u_3};1\right)H(1,0;u_3)-\frac{1}{2}G\left(0,\frac{1}{u_2+u_3};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_1},\frac{1}{u_1+u_3};1\right)H(1,0;u_3)+\frac{1}{4}G\left(\frac{1}{1-u_2},\frac{u_3-1}{u_2+u_3-1};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_2},\frac{1}{u_2+u_3};1\right)H(1,0;u_3)-\frac{1}{3}\pi^2H(1,0;u_3)+\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_1+u_3};1\right)H(1,0;u_3)+$$
$$\frac{1}{4}G\left(\frac{1}{u_3},\frac{1}{u_2+u_3};1\right)H(1,0;u_3)-\frac{1}{4}\mathcal{G}\left(\frac{1}{1-u_2},u_{231};1\right)H(1,0;u_3)+$$
$$\frac{3}{4}H(0;u_1)H(0;u_2)H(1,0;u_3)-\frac{3}{4}H(0,0;u_1)H(1,0;u_3)-\frac{3}{4}H(0,0;u_2)H(1,0;u_3)+$$
$$\frac{1}{4}H\left(0,1;\frac{u_2+u_3-1}{u_3-1}\right)H(1,0;u_3)-\frac{1}{4}H(1,0;u_1)H(1,0;u_3)-\frac{1}{4}H(1,0;u_2)H(1,0;u_3)+$$
$$\frac{1}{24}\pi^2H(1,1;u_1)+\frac{1}{24}\pi^2H(1,1;u_2)+\frac{1}{24}\pi^2H(1,1;u_3)+\frac{1}{2}H(0;u_2)H(0,0,0;u_1)+$$
$$\frac{1}{2}H(0;u_3)H(0,0,0;u_2)+\frac{1}{2}H(0;u_1)H(0,0,0;u_3)-\frac{1}{2}H(0;u_2)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{2}H(0;u_3)H\left(0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-H(0;u_1)H(0,0,1;(u_1+u_2))-$$
$$H(0;u_2)H(0,0,1;(u_1+u_2))-\frac{1}{2}H(0;u_1)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$

$$\frac{1}{2}H\left(0;u_2\right)H\left(0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-H\left(0;u_1\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_1+u_3\right)\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$\frac{1}{2}H\left(0;u_3\right)H\left(0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-H\left(0;u_2\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-$$
$$H\left(0;u_3\right)H\left(0,0,1;\left(u_2+u_3\right)\right)-\frac{1}{2}H\left(0;u_2\right)H\left(0,1,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(0,1,0;u_2\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(0,1,0;u_3\right)+\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+$$
$$\frac{1}{4}H\left(0;u_3\right)H\left(0,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_1\right)-\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_1\right)-$$
$$\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_3\right)H\left(1,0,0;u_2\right)+\frac{1}{2}H\left(0;u_1\right)H\left(1,0,0;u_3\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(1,0,0;u_3\right)-\frac{1}{4}H\left(0;u_3\right)H\left(1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)-$$
$$7H\left(0,0,0,0;u_1\right)-7H\left(0,0,0,0;u_2\right)-7H\left(0,0,0,0;u_3\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+$$
$$3H\left(0,0,0,1;\left(u_1+u_2\right)\right)+\frac{3}{2}H\left(0,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+3H\left(0,0,0,1;\left(u_1+u_3\right)\right)+$$
$$\frac{3}{2}H\left(0,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+3H\left(0,0,0,1;\left(u_2+u_3\right)\right)+\frac{9}{4}H\left(0,0,1,0;u_1\right)+$$
$$\frac{9}{4}H\left(0,0,1,0;u_2\right)+\frac{9}{4}H\left(0,0,1,0;u_3\right)-\frac{1}{2}H\left(0,1,0,0;u_1\right)-\frac{1}{2}H\left(0,1,0,0;u_2\right)-$$
$$\frac{1}{2}H\left(0,1,0,0;u_3\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{2}H\left(0,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{2}H\left(0,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(0,1,1,0;u_1\right)+H\left(0,1,1,0;u_2\right)+H\left(0,1,1,0;u_3\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_2-1}{u_2-1}\right)-\frac{1}{4}H\left(0,1,1,1;\frac{u_1+u_3-1}{u_1-1}\right)-$$
$$\frac{1}{4}H\left(0,1,1,1;\frac{u_2+u_3-1}{u_3-1}\right)+H\left(1,0,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+H\left(1,0,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$H\left(1,0,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+2H\left(1,0,1,0;u_1\right)+2H\left(1,0,1,0;u_2\right)+2H\left(1,0,1,0;u_3\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_2-1}{u_2-1}\right)+\frac{1}{4}H\left(1,1,0,1;\frac{u_1+u_3-1}{u_1-1}\right)+$$
$$\frac{1}{4}H\left(1,1,0,1;\frac{u_2+u_3-1}{u_3-1}\right)+\frac{1}{2}H\left(1,1,1,0;u_1\right)+\frac{1}{2}H\left(1,1,1,0;u_2\right)+\frac{1}{2}H\left(1,1,1,0;u_3\right)-$$
$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{u_{123}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{u_{231}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)-\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-$$

$$\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{132}}\right)-\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{24}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{213}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_3\right)\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{8}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{8}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\frac{1}{24}\pi^2H\left(0;u_1\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-\frac{1}{24}\pi^2H\left(0;u_2\right)\mathcal{H}\left(1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)-\frac{1}{4}H\left(1,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-\frac{1}{4}H\left(1,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{u_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-\frac{1}{4}H\left(1,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)+\frac{1}{24}\pi^2\mathcal{H}\left(0,1;\frac{1}{u_{312}}\right)-$$
$$\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0,0;u_3\right)\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{231}}\right)-$$
$$\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+$$
$$\frac{1}{4}H\left(0,0;u_1\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0,0;u_2\right)\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)+\frac{1}{6}\pi^2\mathcal{H}\left(0,1;\frac{1}{v_{321}}\right)-$$
$$\frac{1}{2}H\left(0;u_2\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{123}}\right)-\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{132}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{213}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_3\right)\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{231}}\right)-\frac{1}{2}H\left(0;u_1\right)H\left(0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+$$
$$\frac{1}{2}H\left(0,0;u_1\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{1}{2}H\left(0,0;u_2\right)\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)+\frac{11}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{312}}\right)-$$
$$\frac{1}{24}\pi^2\mathcal{H}\left(1,1;\frac{1}{v_{321}}\right)+\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{123}}\right)+$$
$$\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_3\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{231}}\right)+\frac{1}{2}H\left(0;u_1\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+$$
$$\frac{1}{2}H\left(0;u_2\right)\mathcal{H}\left(0,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{231}}\right)+$$

$$
\begin{aligned}
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{123}}\right)-\\
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\\
&\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{231}}\right)+\\
&\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\\
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{123}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{231}}\right)+\\
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{u_{312}}\right)+\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{123}}\right)-\\
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\\
&\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{213}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}H\left(0;u_3\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{231}}\right)+\\
&\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{312}}\right)-\frac{1}{4}H\left(0;u_1\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+\\
&\frac{1}{4}H\left(0;u_2\right)\mathcal{H}\left(1,0,1;\frac{1}{v_{321}}\right)+H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{123}}\right)-\\
&H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_3\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{231}}\right)+H\left(0;u_1\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\\
&H\left(0;u_2\right)\mathcal{H}\left(1,1,1;\frac{1}{v_{312}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{123}}\right)-\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{231}}\right)-\\
&\frac{3}{2}\mathcal{H}\left(0,0,0,1;\frac{1}{u_{312}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{132}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{213}}\right)-3\mathcal{H}\left(0,0,0,1;\frac{1}{v_{321}}\right)-\\
&\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,0,1,1;\frac{1}{u_{312}}\right)-\\
&\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(0,1,0,1;\frac{1}{u_{312}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{132}}\right)+\zeta_3H\left(0;u_1\right)+\zeta_3H\left(0;u_2\right)+\zeta_3H\left(0;u_3\right)+\\
&\frac{5}{2}\zeta_3H\left(1;u_1\right)+\frac{5}{2}\zeta_3H\left(1;u_2\right)+\frac{5}{2}\zeta_3H\left(1;u_3\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{123}}\right)+\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{231}}\right)+\\
&\frac{1}{2}\zeta_3\mathcal{H}\left(1;\frac{1}{u_{312}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{123}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{231}}\right)-\frac{1}{2}\mathcal{H}\left(1,0,0,1;\frac{1}{u_{312}}\right)+\\
&\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{123}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{132}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{213}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{231}}\right)+\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{312}}\right)+\\
&\frac{1}{4}\zeta_3\mathcal{H}\left(1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{312}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(0,1,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{123}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{213}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{312}}\right)+\frac{1}{4}\mathcal{H}\left(1,0,1,1;\frac{1}{v_{321}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{123}}\right)+\\
&\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{132}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{213}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{231}}\right)+\frac{1}{4}\mathcal{H}\left(1,1,0,1;\frac{1}{v_{312}}\right)+
\end{aligned}
$$

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There should be a better presentation!

Otherwise we should abandon N=4 Yang Mills!

`FullSimplify[]` is not enough, we need a serious weapon to battle this 17-page

How do we proceed?

Classical Polylogs

$$Li_k(z) = \int_0^z Li_{k-1}(t) d\log t \qquad Li_1(z) = -\log(1-z)$$

Satisfy various identities

$$-Li_2(1-\frac{1}{x}) = Li_2(1-x) + \frac{1}{2}\log(x)^2$$

Goncharov polylogs satisfy huge number of identities

$$G(a_k, a_{k-1}, \ldots; z) = \int_0^z G(a_{k-1}, \ldots; t) \frac{dt}{t-a_k}, \qquad G(z) \equiv 1$$

How can we bring the identities under control?

We will use motivic high tech!!!

Symbol of Transcendental Function

Goncharov

$$T_k \to S(T_k) = R_1 \otimes \cdots \otimes R_k$$

Symbol is an element of the k-fold tensor product of the multiplicative group of rational functions, it can be defined recursively

$$dT_k = \sum_i T^i_{k-1} d\log R_i \to S(T_k) = \sum_i S(T^i_{k-1}) \otimes R_i$$

Properties

$$R_1 \otimes (R_2 R_3) = R_1 \otimes R_2 + R_1 \otimes R_3$$

$$R_1 \otimes (cR_2) = R_1 \otimes R_2 \quad \mathrm{c} = \mathrm{const}$$

Examples

Function	Differential	Symbol
$\log R$	$d\log R$	R
$\log R_1 \log R_2$	$\log R_1 d\log R_2 +$ $\log R_2 d\log R_1$	$R_1 \otimes R_2 + R_2 \otimes R_1$
$Li_2(R)$	$-\log(1-R)d\log R$	$-(1-R)\otimes R$
$Li_k(R)$	$Li_{k-1}(R)d\log R$	$-(1-R)\otimes R\cdots\otimes R$

Why is the Symbol Very Useful?

Symbol converts polylog functional equations into rational function identities

- $$Li_2(x) + Li_2(-x) = \frac{1}{2} Li_2(x^2)$$

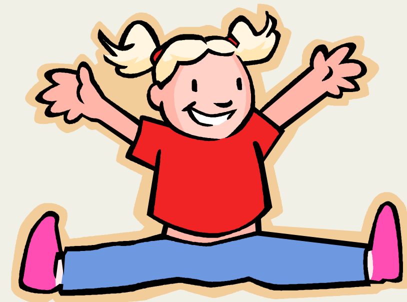
$$-(1-x)\otimes x - (1+x)\otimes(-x) = -(1-x^2)\otimes x = -\frac{1}{2}(1-x^2)\otimes x^2$$

- $$Li_2(z) + Li_2(1-z) + \log(z)\log(1-z) = \frac{\pi^2}{6}$$

$$-z\otimes(1-z) - (1-z)\otimes z + z\otimes(1-z) + (1-z)\otimes z = 0$$

Symbol fixes only the leading functional transcendentality piece

Final Result



Goncharov, Spradlin, Vergu, AV '10

Two-loop six-point MHV remainder function is

$$R_6^{(2)}(u_1,u_2,u_3) = \sum_{i=1}^{3}\left(L_4(x_i^+,x_i^-) - \frac{1}{2}Li_4(1-1/u_i)\right) - \frac{1}{8}\left(\sum_{i=1}^{3} Li_2(1-1/u_i)\right)^2 + \frac{1}{24}J^4 + \frac{\pi^2}{12}J^2 + \frac{\pi^4}{72}.$$

$$L_4(x^+,x^-) = \sum_{m=0}^{3}\frac{(-1)^m}{(2m)!!}\log(x^+x^-)^m(\ell_{4-m}(x^+) + \ell_{4-m}(x^-)) + \frac{1}{8!!}\log(x^+x^-)^4$$

$$\ell_n(x) = \frac{1}{2}\left(Li_n(x) - (-1)^n Li_n(1/x)\right)$$

$$J = \sum_{i=1}^{3}(\ell_1(x_i^+) - \ell_1(x_i^-))$$

$$x_i^\pm = u_i x^\pm$$

$$x^\pm = \frac{u_1+u_2+u_3-1\pm\sqrt{\Delta}}{2u_1u_2u_3}$$

$$\Delta = (u_1+u_2+u_3-1)^2 - 4u_1u_2u_3$$

Bartels, Prygarin, Lipatov '10: analyzed Regge limit
Gaiotto, Maldacena, Sever, Veira '11: OPE approach

What's next?

- Our formula provides hope to the idea that we might be able to really unlock the secrets of multi-loop SYM amplitudes, and connect to strong coupling **Alday, Gaiotto, Maldacena, Sever, Veira**
- We need a technology for writing down symbols without first evaluating integrals at all
- It would be nice to have a derivation from integrals: work on two-loops is in progress, but solved for one-loop higher dimensional polygons

More Motivic Magic

Spradlin, AV 05/11

- Consider one-loop 2m-gon in 2m-dimensions:

  appear in dim regulated one-loop MHV & can also be related to higher-loop four-dimensional integrals

- One-loop hexagon in six dimensions have been evaluated for massless & three-mass case:

  surprisingly the same L_3 function appears.....

Dixon, Drummond, Henn; Del Duca, Duhr, Smirnov 05/11

- It turns out its symbol for any m can be read of from theorem by Goncharov'96 on mixed Tate motives

Recursions for Symbols

- After Feynman parameterization, any one-loop 2m-gon integral in D=2m takes the form

$$F(Q) = \int_{\mathbb{CP}^{2m-1}} \frac{D^{2m-1}W}{(W \cdot Q \cdot W)^m} \qquad Q_{ij} = (p_i + \cdots + p_j)^2$$

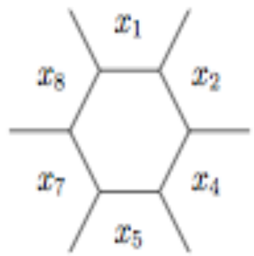
in terms of a quadric Q in $\mathbb{CP}^{2m-1}$

- The symbol of the integral is given recursively by application of a result on mixed Tate motives Goncharov 96

$$S_m(Q) = \sum_{i<j} S_{m-1}(\overline{Q}_{ij}) \otimes \frac{Q_{ij}^{-1} - \sqrt{(Q_{ij}^{-1})^2 - Q_{ii}^{-1} Q_{jj}^{-1}}}{Q_{ij}^{-1} + \sqrt{(Q_{ij}^{-1})^2 - Q_{ii}^{-1} Q_{jj}^{-1}}}$$

$(\overline{Q} = Q \text{ with rows and columns } i, j \text{ deleted})$ Spradlin, AV 05/11

Example: 1-loop 3-mass hexagon



$$Q = \begin{pmatrix} 0 & 0 & x_{14}^2 & x_{15}^2 & x_{17}^2 & x_{18}^2 \\ 0 & 0 & x_{24}^2 & x_{25}^2 & x_{27}^2 & x_{28}^2 \\ x_{14}^2 & x_{24}^2 & 0 & 0 & x_{47}^2 & x_{48}^2 \\ x_{15}^2 & x_{25}^2 & 0 & 0 & x_{57}^2 & x_{58}^2 \\ x_{17}^2 & x_{27}^2 & x_{47}^2 & x_{57}^2 & 0 & 0 \\ x_{18}^2 & x_{28}^2 & x_{48}^2 & x_{58}^2 & 0 & 0 \end{pmatrix}$$

$$\begin{aligned}(1+g+g^2)\Bigg[& S_2(Q_{\overline{12}}) \otimes \frac{\langle 2358\rangle\langle 12\bar{5}\cap\bar{8}\rangle}{\langle 1258\rangle\langle 23\bar{5}\cap\bar{8}\rangle} \\ & + S_2(Q_{\overline{13}}) \otimes \frac{\langle 1238\rangle\langle 2568\rangle\langle 5789\rangle\langle 23\bar{5}\cap\bar{8}\rangle}{\langle 2358\rangle\langle 2789\rangle\langle 4568\rangle\langle 56\bar{2}\cap\bar{8}\rangle} \\ & + S_2(Q_{\overline{14}}) \otimes \frac{\langle 2358\rangle\langle 2789\rangle\langle 4568\rangle\langle 45\bar{2}\cap\bar{8}\rangle}{\langle 1238\rangle\langle 2458\rangle\langle 5789\rangle\langle 23\bar{5}\cap\bar{8}\rangle} \\ & + S_2(Q_{\overline{16}}) \otimes \frac{\langle 2358\rangle\langle 2456\rangle\langle 5789\rangle\langle 78\bar{2}\cap\bar{5}\rangle}{\langle 1235\rangle\langle 2578\rangle\langle 4568\rangle\langle 23\bar{5}\cap\bar{8}\rangle} \\ & + S_2(Q_{\overline{24}}) \otimes \frac{\langle 1238\rangle\langle 2458\rangle\langle 5789\rangle\langle 12\bar{5}\cap\bar{8}\rangle}{\langle 1258\rangle\langle 2789\rangle\langle 4568\rangle\langle 45\bar{2}\cap\bar{8}\rangle}\Bigg]\end{aligned}$$

$$\begin{aligned} S_2(Q) = {} & x_{il}^2 x_{jk}^2 \otimes \frac{-x_{il}^2 x_{jk}^2 + x_{ik}^2 x_{jl}^2 + x_{ij}^2 x_{kl}^2 - \Delta}{-x_{il}^2 x_{jk}^2 + x_{ik}^2 x_{jl}^2 + x_{ij}^2 x_{kl}^2 + \Delta} \\ & + x_{ik}^2 x_{jl}^2 \otimes \frac{+x_{il}^2 x_{jk}^2 - x_{ik}^2 x_{jl}^2 + x_{ij}^2 x_{kl}^2 - \Delta}{+x_{il}^2 x_{jk}^2 - x_{ik}^2 x_{jl}^2 + x_{ij}^2 x_{kl}^2 + \Delta} \\ & + x_{ij}^2 x_{kl}^2 \otimes \frac{+x_{il}^2 x_{jk}^2 + x_{ik}^2 x_{jl}^2 - x_{ij}^2 x_{kl}^2 - \Delta}{+x_{il}^2 x_{ik}^2 + x_{ik}^2 x_{il}^2 - x_{ij}^2 x_{kl}^2 + \Delta} \end{aligned}$$

$$\Delta = \sqrt{+\det Q}$$

$$g : i \to i+3$$

$$\langle ij\bar{k}\cap\bar{l}\rangle = \langle i\; k{-}1\; k\; k{+}1\rangle\langle j\; l{-}1\; l\; l{+}1\rangle - \langle j\; k{-}1\; k\; k{+}1\rangle\langle i\; l{-}1\; l\; l{+}1\rangle$$

Higher Loops and Points Symbols

- Similar story should exist for higher loops: recursions for symbols? Geometrical meaning?
- Higher points: Caron-Huot Wilson loop approach to symbol for all n MHV 2-loops
- Higher loops: Dixon, Drummond, Henn ansatz for a symbol for 6-points 3-loops from OPE and other constraints, assuming the same cross-ratios as at 2-loops
- Non-MHV ratio function: Dixon, Drummond, Henn

Deriving function from symbol requires work.

This is a very nontrivial step.

Duhr, Gangl, Rhodes

Two-loop All-n MHV Symbol

Caron-Huot

The differential of the n-point function is expressed as

$$dR_n = \sum_{i,j} C_{i,j}\, d\log\langle i{-}1\,i\,i{+}1\,j\rangle \tag{A.1}$$

where $C_{2,i}$ is the sum of the four contributions

$$\begin{aligned}
C_{2,i}^{(1)} &= \log u_{2,i-1,i,1} \times \sum_{j=2}^{i-1}\sum_{k=i}^{n+1}\left[\mathrm{Li}_2(1-u_{j,k,k-1,j+1}) + \log\frac{x_{j,k}^2}{x_{j+1,k}^2}\log\frac{x_{j,k}^2}{x_{j,k-1}^2}\right],\\
C_{2,i}^{(2)} &= \sum_{j=4}^{i-2}\Delta(1,2;j{-}1,j;i{-}1,i),\\
C_{2,i}^{(3)} &= \sum_{j=i+2}^{n}\Delta(2,1;j,j{-}1;i,i{-}1),\\
C_{2,i}^{(4)} &= -2\mathrm{Li}_3(1-\frac{1}{u}) - \mathrm{Li}_2(1-\frac{1}{u})\log u - \frac{1}{6}\log^3 u + \frac{\pi^2}{6}\log u,
\end{aligned} \tag{A.2}$$

and other $C_{i,j}$ are obtained by cyclic symmetry. In the first line, $x_{j+1} \equiv x_2$ when $j = i{-}1$, and $x_{k-1} \equiv x_1$ when $k = i$, and in the last line, $u = u_{2,i-1,i,1}$. The symbol of Δ is

$$\begin{aligned}
&\mathcal{S}\Delta(1,2;j{-}1,j;i{-}1,i)\\
&= \left(\ \mathcal{S}[I_5(i;1,2;j{-}1,j)] \otimes \frac{\langle i\,i{+}1(\bar{2})\cap(\bar{j})\rangle\langle 23ij\rangle}{\langle j{-}1\,j\,j{+}1\,i\rangle\langle 123j\rangle\langle 23\,i\,i{+}1\rangle} - ((i\,i{+}1)\to(i{-}1\,i))\right)\\
&+\left(\begin{array}{l}
\frac{1}{2}\mathcal{S}[\mathrm{Li}_2(1-u_{j,2,1,i-1}) - \mathrm{Li}_2(1-u_{j,2,1,i})] \otimes \left(\frac{\langle 123i\rangle\langle j{-}1\,j\,j{+}1\,2\rangle\langle 23ij\rangle}{\langle 123j\rangle\langle j{-}1\,j\,j{+}1\,i\rangle\langle 23\,i\,i{+}1\rangle}\right)^2 \frac{\langle j\,j{+}1(\bar{2})\cap(\bar{i})\rangle\langle i\,i{+}1\,j\,j{+}1\rangle}{\langle 2\,i\,j\,j{+}1\rangle\langle 13(2\,i{-}1\,i)\cap(2\,j\,j{+}1)\rangle}\\
+\frac{1}{2}\mathcal{S}[\mathrm{Li}_2(1-u_{j,i-1,i,2}) - \mathrm{Li}_2(1-u_{j,i-1,i,1})] \otimes \left(\frac{\langle 12\,i{-}1\,i\rangle\langle 23ij\rangle}{\langle 123i\rangle\langle i{-}1\,i\,i{+}1\,j\rangle\langle 23\,i{-}1\,i\rangle}\right)^2 \frac{\langle j\,j{+}1(\bar{2})\cap(\bar{i})\rangle\langle i{-}1\,i{+}1(i23)\cap(i\,j\,j{+}1)\rangle}{\langle 2\,i\,j\,j{+}1\rangle\langle 12\,j\,j{+}1\rangle}\\
+\frac{1}{2}\mathcal{S}[\mathrm{Li}_2(1-u_{2,i-1,i,1})] \otimes \frac{\langle j\,j{+}1(\bar{2})\cap(\bar{i})\rangle\langle i{-}1\,i{+}1(i23)\cap(i\,j\,j{+}1)\rangle}{\langle 2\,i\,j\,j{+}1\rangle\langle 13(2\,i{-}1\,i)\cap(2\,j\,j{+}1)\rangle}\\
+\frac{1}{2}\mathcal{S}[\log u_{j,i-1,i,2}\log u_{j,2,1,i-1}] \otimes \left(\frac{\langle 23ij\rangle}{\langle 123j\rangle}\right)^2 \frac{\langle j\,j{+}1(\bar{2})\cap(\bar{i})\rangle\langle 13(2\,i{-}1\,i)\cap(2\,j\,j{+}1)\rangle}{\langle 2\,i\,j\,j{+}1\rangle\langle 23\,i{-}1\,i\rangle\langle i{-}1\,i{+}1(i23)\cap(i\,j\,j{+}1)\rangle}\\
-((j\,j{+}1)\to(j{-}1\,j))
\end{array}\right)\\
&+\mathcal{S}[I_5(1;i{-}1,i;j{-}1,j)] \otimes \frac{\langle 12ij\rangle\langle 23\,i{-}1\,i\rangle}{\langle 12\,i{-}1\,i\rangle\langle 23ij\rangle}\\
&+\mathcal{S}[\log u_{i,j-1,j,1}\log u_{2,i-1,i,1}] \otimes \frac{\langle j{-}1\,j{+}1(j12)\cap(j\,i\,i{+}1)\rangle\langle 123i\rangle\langle 23\,i{-}1\,i\rangle}{\langle 123j\rangle\langle j{-}1\,j\,j{+}1\,i\rangle\langle 12\,i{-}1\,i\rangle\langle 23\,i\,i{+}1\rangle}.
\end{aligned} \tag{A.3}$$

How can we integrate it?

- Constructing the function which has the same symbol is nontrivial. We need luck/guessing.

$$(1+x+x^2)\otimes x = \frac{1-x^3}{1-x}\otimes x = \frac{1}{3}(1-x^3)\otimes x^3 - (1-x)\otimes x$$

$$\Longrightarrow \quad -\frac{1}{3}Li_2(x^3) + Li_2(x)$$

- In 2-loop 6-point case we had Goncharov's theorem to guide us $A\wedge A = 0 \to \text{classical Li's only}$
- More generally we can have for degree 4:

$$Li_{2,2}(x,y) = \sum_{0<n<m}\frac{x^n}{n^2}\frac{y^m}{m^2} \qquad\qquad Li_4(x) = \sum_{n=1}^{\infty}\frac{x^n}{n^2}$$

Motivic Content

$$S = \sum_i c_i^L \otimes \Delta_i = \sum_i \Delta_i \otimes c_i^R$$

$$c_i^L - c_i^R = \sum_j b_{ij}^L \otimes \Delta_j = \sum_j \Delta_j \otimes b_{ij}^R$$

Motivic content is the antisymmetrization of

$$\sum_j (b_{ij}^L - b_{ij}^R) \otimes \Delta_j$$

with Goncharov, Spradlin, Vergu

This operation kills all terms except for Li_4 and Li_2,2

Examples

$$\Delta = x: \quad Li_4(x) \to -(1-x) \wedge x \otimes x$$

$$\Delta = 1-xy : Li_{2,2}(x,y) \to -2(1-y) \wedge y \otimes y + 2(1-x) \wedge x \otimes x$$

$$\Delta = x; \Delta = y : Li_{2,2}(x,y) \to -(1-R) \wedge R \otimes R$$

- Motivic content will help identify good set of cross-ratios out of 2310 for 7-points
- We also found new 40-term Li_3 identities

$$\sum a_i(1-R_i) \wedge R_i \otimes R_i = 0$$

with Goncharov, Spradlin, Vergu

Regge limit

- While hard to evaluate the amplitude in general, it can be done in the Regge limit. For n=7, this is

$$u_{26} \sim 1 - \mathcal{O}(\epsilon^2); \; u_{25}, u_{36} \sim 1 - \mathcal{O}(\epsilon); \; u_{ij} \sim \mathcal{O}(\epsilon)$$

- The leading log of the imaginary part is $\log \epsilon \; F_2$

$$F_2 = \sum_{i=4}^{5} \log \frac{|\beta_3|^2 |\beta_i - \beta_6|^2}{|\beta_i|^2 |\beta_3 - \beta_6|^2} \; \log \frac{|\beta_i|^2 |\beta_3 - \beta_6|^2}{|\beta_6|^2 |\beta_3 - \beta_i|^2}$$

$$\beta_i = \sum_{k=3}^{i} \vec{p}_k$$

Computed by Prygarin using BFKL theory.

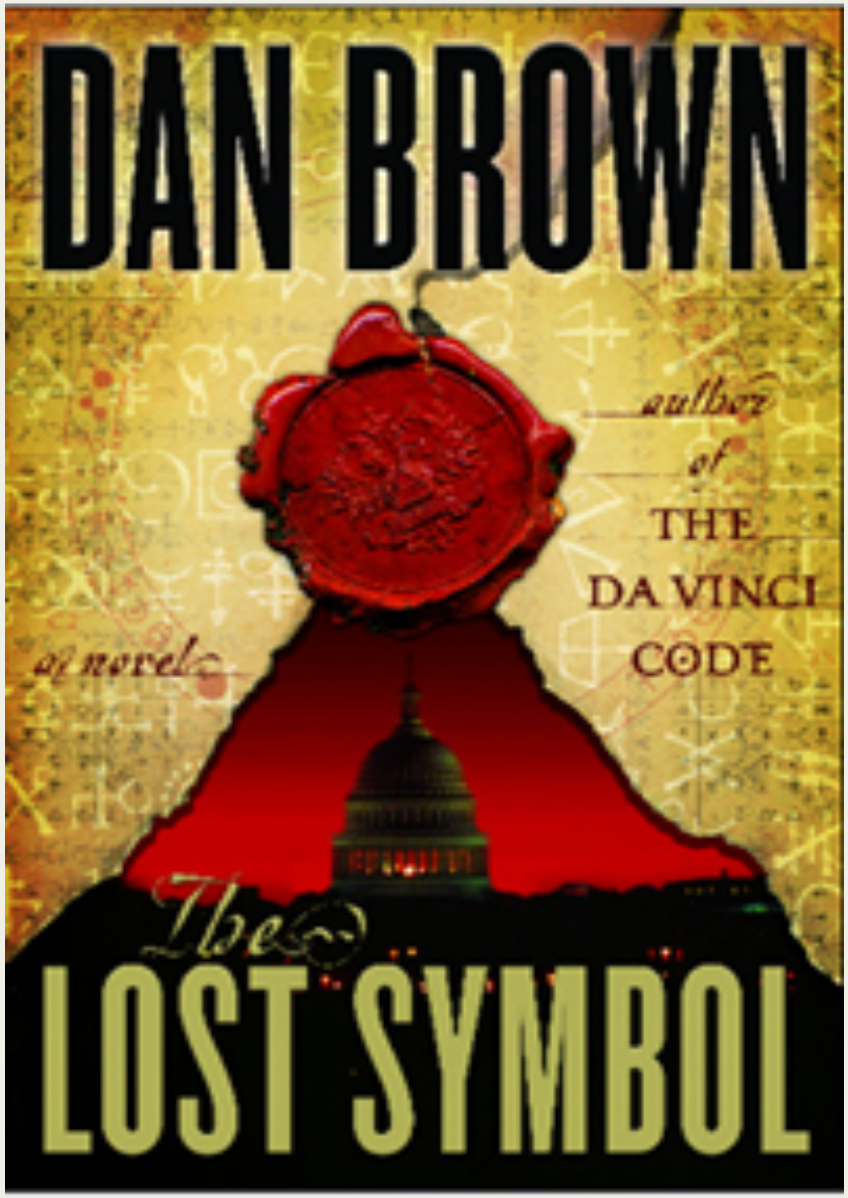
- Regge limit of the discontinuity $u_{26} \to u_{26} \; e^{2\pi i}$ of Simon's symbol reproduces this result exactly.

with Spradlin, Prygarin, Vergu

- More legs and loops in Regge limit ?

Conclusions

- If a problem is too hard, look for something simpler. The symbol is a useful stepping stone half way between an integrand and its integral.
- We need a technology for writing down symbols without first evaluating integrals at all.
- We need better technology to find functions from symbols, motivic content is very helpful in addressing this question.
- Symbols are not the ultimate goal, but serve the simplest incarnation of a deeper motivic structure of amplitudes which we are only beginning to see.


DAN BROWN
author of THE DA VINCI CODE
a novel
The
LOST SYMBOL